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WILLIAM JOHN MACQUORN RANKINE,
CIVIL ENGINEER; LL.D.; F.R.SS. LOND. AND EDIN.; F.R.S.S.A.; ASSOC. I. N. A.;
REGIUS PROFESSOR OF CIVIL ENGINEERING AND MECHANICS IN THE UNIVERSITY OF GLASGOW;
PAST-PRESIDENT OF THE INSTITUTION OF ENGINEERS IN SCOTLAND; PRESIDENT OF
THE PHILOSOPHICAL SOCIETY OF GLASGOW; HONORARY MEMBER OF THE
LITERARY AND PHILOSOPHICAL SOCIETY OF MANCHESTER, OF
THE ROYAL SOCIETY OF TASMANIA, ETC., ETC.

With Numerous Diagrams.

LONDON:
GRIFFIN, BOHN, AND COMPANY.
STATIONERS' HALL COURT.
1862.

[*The Author reserves the right of Translation.*]

3.M

19483.62

BELL AND BAIN, PRINTERS, GLASGOW.

PREFACE.

THIS work is divided into three parts. The first relates to those branches of the operations of engineering which depend on geometrical principles alone, that is to say, SURVEYING, LEVELLING, and the SETTING-OUT of works, comprehended under the general name of ENGINEERING GEODESY, or FIELD-WORK. The second part relates to the properties of the MATERIALS used in engineering works, such as earth, stone, timber, and iron; and the art of forming them into STRUCTURES of different kinds, such as excavations, embankments, bridges, &c. The third part, under the head of COMBINED STRUCTURES, sets forth the principles according to which the structures described in the second part are combined into extensive works of engineering, such as Roads, Railways, River Improvements, Water-Works, Canals, Sea Defences, Harbours, &c.

The first chapter of the second part, entitled a *Summary of the Principles of Stability and Strength*, forms not so much an integral part of the book, as a collection of mechanical principles and formulæ, introduced for the sake of being conveniently referred to in the subsequent chapters, so as to prevent their being encumbered with mathematical investigations to a greater extent than is absolutely necessary.

The third part, so far as the details of the designing and execution of works are concerned, consists, to a great extent, of references to the first and second parts, its special object being to explain those principles which are peculiar to each class of great works of engineering, and which regulate the general plan of such works.

The tables of the strength of materials at the end of the volume give, as regards iron and stone, average and extreme results only. Detailed information as to the strength of different kinds of stone and iron is given in the course of the text, under the proper headings.

I have, throughout the book, adhered to a systematic arrangement as far as was practicable, and have only departed from it in a few instances, when it became necessary to introduce questions that had arisen, or facts that had been ascertained, after the completion of the part of the work to which they properly belonged. In drawing up the table of contents and the alphabetical index care has been taken to show where such detached pieces of information are to be found.

W. J. M. R.

GLASGOW COLLEGE, *6th January*, 1862.

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ADDENDUM

TO ARTICLE 357, p. 509.

Mean results of experiments by W. H. Barlow, Esq., F.R.S. :—

	Tenacity. Lbs. on the Square Inch.	Proof Strength, Transversely Loaded. Lbs. on the Square Inch.	Modulus of Elasticity, under Transverse Load. Lbs. on the Square Inch.
Puddled steel, specimen I., ...	95,233	....	....
„ specimen II.,....	116,336	62,500	22,964,000
„ „ } Cast in ingots, ...	101,753	....	....
Puddled steel, specimen III.,	....	60,000	20,544,000
„ specimen IV.,	....	63,750	24,802,000
„ specimen V.,	....	52,500	22,846,400

Weight of a cubic foot of puddled steel, 485·5 lbs.; of steely iron 483·6 lbs.

Homogeneous metal, ...	100,994	57,500	23,833,600
Steely iron, ...	69,456	52,500	22,846,400

(See the *Engineer* of 3d January, 1862.)

ERRATA AND ADDENDA.

Page 6, in the table, example 10, last column, *insert* "smallest scale prescribed by law for land or contract plans in Ireland."

„ 8, line 12, *for* "the filling up details" *read* "filling up the details."

„ 32, line 10 from bottom, *for* "dwelling-house" *read* "dwelling-houses."

„ 47, at the end of the note, *insert*—"In surveys of newly settled districts, where it is impracticable to obtain a base by direct measurement with sufficient precision, a base may be measured with accuracy sufficient for ordinary purposes in the following manner:—Choose as ends of the base two elevated stations which can be seen from each other, as nearly as possible in the same meridian, and, if possible, 50 or 60 miles asunder; find their latitudes (as explained in Article 86 c, p. 129); also find the true meridian, and the azimuth of the base, from the mean of observations made at each of the stations (as explained in Article 42, p. 71); compute, by equation 1 of this note, the length m of a minute of the meridian corresponding to the *mean* latitude; then *length of base* nearly $= m \times$ difference of latitude in minutes $\times$ secant of azimuth."

„ 64, line 8, *for* "silvered" *read* "unsilvered."

„ 92, equation 2, *for* "60,000" read "56,300," *and for* "1,000" *read* "900."

„ 137, line 18, *for* "49" *read* "94."

„ 153, equation 2, *for* "$\int wsdx$" *read* "$\int xwsdx$."

„ 157, line 1, *for* "DOE" *read* "DOB."

„ 196, line 12, *for* "$\frac{x^2}{m}$" *read* "$\frac{y^2}{m}$."

„ 198, line 11 from bottom, *for* "1824" read "1826."

„ 211, line 13 from bottom, *for* "$y_1 - \frac{y_1^3}{30a}$" *read* "$y_1 + \frac{y_1^3}{30a}$."

„ 256, line 7, *for* "4 A_2" *read* "4 A_3."

„ 273, equation 13 A, *for* "c" *read* "c^2."

„ 278, line 4, *for* "squares" *read* "cubes."

„ 304, line 4, *for* "$\frac{4\,k^2}{3\,l}$" *read* "$\frac{16\,k^2}{3\,l}$."

„ 305, lines 1, 4, and 7 from bottom; p. 308, line 10 from bottom; p. 309, lines 7, 11, and last; p. 310, lines 4 and 6, *for* "$\frac{4\,k^2}{3\,l^2}$" *read* "$\frac{16\,k^2}{3\,l^2}$."

„ 212, line 9, *for* "$\frac{49}{48}$" *read* "$\frac{13}{11}$."

„ 390, line 10, *for* "broached" *read* "stroked."

„ 417, last line, *for* "receive" *read* "conceive."

„ 449, line 2 from bottom, *for* "oak" *read* "ash."

„ 540, line 14, *for* "beam" *read* "rib."

„ 608, line 25, *for* "D" *read* "E."

PART I.

OF ENGINEERING GEODESY; OR, SURVEYING, LEVELLING, AND SETTING-OUT.

CHAPTER I.

GENERAL EXPLANATIONS.

1. **Surveying, Levelling, and Setting-out,** comprehend the principal operations of Engineering Geodesy: the object of surveying and levelling being to make a representation on paper of the ground on which the proposed engineering work is to be executed; and the object of setting-out being, to mark upon the ground the situation of the proposed work preparatory to its execution.

The term "surveying," when used in a comprehensive sense, includes levelling; but in a restricted sense, *surveying* is used to denote the art of ascertaining and representing the form of the ground and the relative positions of objects upon it, as projected on a horizontal surface; and *levelling*, to denote the art of ascertaining and representing the relative elevations of different parts of the ground, and of objects upon it.

2. **Plan and Section.**—The results of surveying, laid down on paper by the operations of "plotting" and drawing, constitute a *plan* or *ground plan;* those of levelling are usually laid down in the form of a *vertical section*, called more briefly a *section* (although there are other ways of representing them, as will afterwards be explained).

A plan is a miniature representation of the ground and the objects upon it, and of the proposed engineering work, as projected on a horizontal surface, that surface being represented by the surface of the paper on which the plan is drawn. A plan differs from a map chiefly in the scale on which it is drawn, the scale of a plan being large enough to serve for the designing of engineering works, while that of a map is so small as to make it serviceable for the purposes of travelling and geography only.

A vertical section shows the figure of a certain line or track on the natural surface of the ground, and of the proposed work to be executed along that line, and sometimes also that of the internal strata, as projected on a vertical surface,—that vertical surface being represented by the surface of the paper on which the section is drawn. A certain straight line on that paper, called the "*datum-line*," represents a fixed horizontal surface at any convenient height above or depth below some fixed and known point, called the "*datum-point*." Lines parallel to the datum-line represent in miniature, distances measured horizontally along the line or track on the earth's surface to which the section relates. Lines perpendicular to the datum-line represent in miniature, heights above or depths below the datum horizontal surface. The natural surface of the ground, and the proposed work, are represented by lines, straight, curved, or angular, which at each point are at the proper vertical distance from the datum-line.

In the same section the scale for horizontal distances and the scale for heights may be different, if convenience requires it, as will afterwards be more fully explained.

3. **A Horizontal Surface** is a surface which is everywhere perpendicular to the direction of the force of gravity; such as the surface of a piece of still water. Its true figure is very nearly that of a spheroid. For a horizontal surface at the mean level of the sea, the dimensions of that spheroid are as follows, according to recent calculations:—*

	Feet.		Statute Miles.
Polar axis, ...	41,707,324	=	7899·114
Equatorial diameter, ...	41,847,192	=	7925·604
Difference, or polar flattening, ...	139,868	=	26·490

The portions of the earth's surface represented by plans for engineering purposes are usually so small compared with the whole earth, that a horizontal surface may, in most cases, be treated as if it were plane, without any error of practical importance. In those cases in which it is necessary to take the earth's curvature into account, the ellipticity or polar flattening may be neglected, and the figure of a horizontal surface may be treated as if it were a sphere of the same *mean diameter* with the spheroid before described; that is to say, very nearly

41,778,000 feet = 13,926,000 yards = 7,912½ statute miles.

It is in certain calculations only that the earth's curvature requires to be taken into account for engineering purposes. In plans, a flat piece of paper, and in vertical sections, a straight line, represent a horizontal surface with as much accuracy as is practicable.

4. **Measures of Length.**—The standard measure of length established by law in Britain is the *yard*, being the distance, at the temperature of 62° of Fahrenheit's thermometer, and under the

---

* Bessel, quoted by Herschel (*Outlines of Astronomy*).

mean atmospheric pressure, between two marks on a certain bar which is kept in the office of the Exchequer, at Westminster.

In addition to the yard, the following units of length are employed for purposes of civil engineering in Britain:—

The **Inch**, one thirty-sixth part of the standard yard; with binary, decimal, or duodecimal subdivisions.

The **Feet**, one-third part of the standard yard; with decimal or duodecimal subdivisions.

The **Fathom** of two yards.

The **Chain** of 66 feet or 22 yards; divided into four *poles* of 5½ yards, and 100 *links* of 7·92 inches.

The **Statute Mile** of 1,760 yards = 5,280 feet = 80 chains, divided into 8 *furlongs*. To these may be added, in cases of harbour engineering—

The **Nautical** or **Sea Mile**, being the length of one minute of a degree of latitude at the mean level of the sea. The length of this mile varies in different latitudes, from about 6,107 feet at the poles to about 6,046 feet at the equator, its mean value being nearly 6,076 feet, or 1·1508 statute mile. A value commonly taken for the nautical mile is that of a minute of longitude at the equator, or 6086·44 feet = 1·1528 statute mile. The nautical mile is sometimes subdivided into 10 *cables*, and 1,000 *fathoms;* the fathom thus obtained being, on an average, about $\frac{1}{80}$th longer than the common fathom.

Amongst obsolete measures of distance the following may be mentioned, as they occasionally occur in old plans:—

The Irish *Perch* of 7 yards, being greater than the imperial perch in the proportion of 14 to 11.

The Irish *Mile* of 320 Irish perches = 2,240 yards = 6,720 feet, bearing to the statute mile the same proportion of 14 to 11.

The Scottish *Ell* of 37·06 imperial inches.

The Scottish *Fall* of 6 ells, or 18·53 imperial feet.

The Scottish *Mile* of 1,920 ells = 5929·6 feet.

Each of these miles is divided, like the statute mile, into 8 furlongs, and 80 chains, so that the Irish, Scottish, and imperial mile, furlong, and chain, bear to each other the proportions—

$$6720 : 5929{\cdot}6 : 5280$$
$$:: 1{\cdot}\dot{2}\dot{7} : 1{\cdot}123 : 1{\cdot}000$$

The French measures of length are all decimal multiples and submultiples of the **METRE**, which is approximately one ten-millionth part of the distance from one of the earth's poles to the equator. The value of the mètre in British measures is

3·2808992 feet, or 39·37079 inches.

The **Kilometre** of 1,000 mètres, or 3280·8992 British feet, is 0·621383 of a statute mile.

For further information of the same kind, see the Comparative Table of French and British Measures at the end of the volume.

5. The **Measures of Area** used in British civil engineering are—

The **Square Inch.**

The **Square Foot** of 144 square inches.

The **Square Yard** of 9 square feet.

The **Acre** of 10 *square chains*, or 100,000 *square links*, or 4,840 *square yards*, subdivided either decimally, or into 4 *roods* of 1,210 square yards, and 160 *perches* of $30\frac{1}{4}$ square yards.

The **Square Mile** of 640 acres, or 3,097,600 square yards, or 27,878,400 square feet.

The Irish *acre*, subdivided into 4 *roods* and 160 *perches*, and the Scottish *acre*, subdivided into 4 *roods* and 160 *falls*, bear to the imperial acre proportions which are the squares of the proportions borne by the Irish and Scottish miles respectively to the statute mile; that is to say,

Irish acre : Imperial acre : : 196 : 121;
Also, Irish acre : Scottish acre : Imperial acre
: : 1·6198 : 1·2612 : 1·0000 nearly.

6. The **Measures of Volume** used in British civil engineering are—

The **Cubic Inch.**

The **Cubic Foot** of 1,728 cubic inches.

The **Cubic Yard** of 27 cubic feet.

In the engineering of water-works, the **Gallon** is used in stating quantities of water. Its statutory value is

277·274 cubic inches, or 0·16046 cubic foot;

but it is convenient in calculation, and in general sufficiently accurate for purposes of water supply, to use the approximate values,

One gallon ... = 0·16 cubic foot, nearly; and
One cubic foot = $6\frac{1}{4}$ gallons, nearly.

Other special measures of volume are employed for certain kinds of materials and work; but these will be explained further on.

7. **Scales for Plans.**—The scale on which a plan is drawn means the proportion which distances, as represented on the plan, bear to the corresponding distances on the ground. Amongst continental European nations it is customary to express that proportion by means of a fraction, such as 1-10,000th. In Britain, it is customary to refer to two units of length, a short unit for the paper, and a

long unit for the ground. For example—"*six inches to one mile*" expresses the scale which, according to the continental system, would be called 1-10,560th. Amongst continental nations, also, the scales most commonly used are those in which the proportion of the dimensions of the plan to those of the ground is some exact decimal fraction, such as 1-10,000th = ·0001, 1-2,500th = ·0004, 1-500th = ·002, &c.; in Britain, the scales most commonly used are those in which a distance of a certain number of miles, chains, or feet on the ground is represented by a distance of a certain number of inches, or aliquot parts of an inch, on the paper.

The magnitude of the scale which is best suited for the plan of a particular survey varies according to the minuteness and complexity of the objects to be represented. Thus, a larger scale is required in plans of towns than in those of the open country; and the smaller and more intricate the buildings and the divisions of property are, the larger should the scale be; and a plan to be used in the final designing and setting-out of works should be on a larger scale than one to be used for the selection of a line of communication, and for preliminary or parliamentary purposes.

The following table enumerates some of the scales for plans most commonly used in Britain, together with a statement of the purposes to which they are best adapted:—

Ordinary Designation of Scale.	Fraction of real Dimensions.	Use.
(1.) 1 inch to a mile,..........	$\frac{1}{63,360}$	Scale of the smaller ordnance maps of Britain. This scale is well adapted for maps to be used in exploring the country.
(2.) 4 inches to a mile,........	$\frac{1}{15,840}$	Smallest scale permitted by the standing orders of parliament for the deposited plans of proposed works.
(3.) 6 inches to a mile,........	$\frac{1}{10,560}$	Scale of the larger ordnance maps of Great Britain and Ireland. This scale, being just large enough to show buildings, roads, and other important objects distinctly in their true forms and proportions, and at the same time small enough to enable the eye of the engineer to embrace the plan of a considerable extent of country at one view, is on the whole the best adapted for the selection of lines for engineering works, and for parliamentary plans and preliminary estimates.
(4.) 6·336 inches to a mile,...	$\frac{1}{10,000}$	Decimal scale possessing the same advantages.

Ordinary Designation of Scale.	Fraction of real Dimensions.	Use.
(5.) 400 feet to an inch,.......	$\frac{1}{4{,}800}$	Smallest scale permitted by the standing orders of parliament for "enlarged plans" of buildings and of land within the curtilage of buildings.
(6.) 6 chains to an inch,......	$\frac{1}{4{,}752}$	Scale answering the same purpose.
(7.) 15·84 inches to a mile,...	$\frac{1}{4{,}000}$	Scales well suited for the working surveys and land plans of great engineering works, and for enlarged parliamentary plans.
(8.) 5 chains to an inch, or 16 inches to a mile,	$\frac{1}{3{,}960}$	(Scale 8 is that prescribed in the standing orders of parliament for "cross sections" of proposed railways, showing alterations of roads.)
(9.) 25·344 inches to a mile,	$\frac{1}{2{,}500}$	Scale of plans of part of the ordnance survey of Britain, from which the maps before mentioned are reduced. Well adapted for land plans of engineering works and plans of estates.
(10.) 200 feet to an inch,......	$\frac{1}{2{,}400}$	Scale suited for similar purposes.
(11.) 3 chains to an inch,......	$\frac{1}{2{,}376}$	Scale of the Tithe Commissioners' plans. Suited for the same purposes as the above.
(12.) 100 feet to an inch,......	$\frac{1}{1{,}200}$	Scale suited for plans of towns, when not very intricate.
(13.) 88 feet to an inch, or 60 inches to a mile,	$\frac{1}{1{,}056}$	Scale of ordnance plans of the less intricately built towns.
(14.) 63·36 inches to a mile,..	$\frac{1}{1{,}000}$	Decimal scale having the same properties.
(15.) 44 feet to an inch, or 120 inches to a mile,	$\frac{1}{528}$	Scale of ordnance plans of the more intricately built towns.
(16.) 126·72 inches to a mile,	$\frac{1}{500}$	Decimal scale having the same properties.
(17.) 30 feet to an inch,........	$\frac{1}{360}$	Scales for special purposes.
(18.) 20 feet to an inch,........	$\frac{1}{240}$	
(19.) 10 feet to an inch,........	$\frac{1}{120}$	
&c.	&c.	

8. **Scales for Sections.**—Except in a few cases of rare occurrence, the *scale for horizontal distances* on a section should be the same with the scale of the plan with which it corresponds. One of the exceptions is that of the parliamentary section of a road upon the level or position of which it is intended to make an alteration for the purpose of carrying a railway across it, whether over or under; in this case, the horizontal scale of the section, as prescribed by the standing orders, is to be *five chains to an inch* (see No. 8 in the table of the last article). The plan *may* be on the same scale, but not necessarily so; in fact, its scale in general is much smaller.

The *vertical scale*, or *scale for heights*, is almost always much greater than the horizontal scale, because the differences of elevation between points on the ground are in general much smaller than their distances apart, and require to be represented on a greater scale on paper, in order that they may be equally conspicuous to the eye; and also, because in the execution of engineering works, accuracy in levels is of more importance than accuracy in horizontal position, and vertical heights should be represented with greater precision than horizontal distances. The proportion in which the vertical scale is greater than the horizontal scale is called the *exaggeration* of the scale. The following table gives some examples:

Ordinary Designation of Vertical Scale.	Fraction of real Height.	Horizontal Scales with which the Vertical Scale is usually combined.	Exaggeration.	Use.
(1.) 100 feet to an inch,	$\frac{1}{1,200}$	$\frac{1}{15,840}$ to $\frac{1}{10,560}$	From 13·2 to 8·8	Smallest scale permitted by the standing orders of parliament for sections of proposed works.
(2.) 40 feet to an inch,	$\frac{1}{480}$	$\frac{1}{4,800}$ to $\frac{1}{3,960}$	10 to 8·25	Smallest scale permitted by the standing orders of parliament for cross sections, showing alterations of roads.
(3.) 30 feet to an inch,	$\frac{1}{360}$	$\frac{1}{3,960}$ to $\frac{1}{2,376}$	11 to 6·6	Scales suitable for working sections.
(4.) 20 feet to an inch,	$\frac{1}{240}$	$\frac{1}{3,960}$ to $\frac{1}{2,376}$	16·5 to 9·9	
&c.	&c.	&c.	&c.	

Vertical sections *without exaggeration*, showing the horizontal and vertical dimensions of the ground in their real proportions to each other, are required at the sites of proposed large works in masonry, timber, and iron, such as viaducts. These sections are in general drawn on a larger scale than the vertical scale of the ordinary working sections.

9. **Methods in Surveying.**—There are two principal methods followed in surveying, each characterized by the elementary mathematical process which it involves: *the method of distances and offsets*, used for filling up the details of a survey, and the *method of triangles*, used chiefly for ascertaining the positions of certain *stations*, but occasionally applied to the filling up details also.

FIRST METHOD—BY DISTANCES AND OFFSETS.

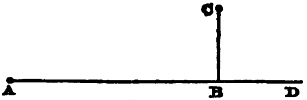

Fig. 1.

In fig. 1, A is the representation on paper of a station, or fixed and marked point on the ground, and A D that of a line extending from A in a known direction. To ascertain and lay down the position of a point C relatively to A, a perpendicular is let fall on the ground from C upon A D, meeting that line in B; the *distance* $\overline{A\,B}$ and *offset* $\overline{B\,C}$ are measured, and these being laid down on the plan to a suitable scale, the point C on the plan which represents C on the ground is marked or *plotted*. In some cases the angle at B may be some measured oblique angle instead of a right angle; but in most cases it is a right angle. This is the method of surveying by distances and offsets, and is that by which the details of a survey are in almost all cases filled in.

The same figure may be taken as representing the elementary operation of levelling, if A D be held as marking the datum horizontal surface, and C B the height above that surface of a point C, whose horizontal distance from A, the commencement of the section, is $\overline{A\,B}$.

SECOND METHOD—BY TRIANGLES.

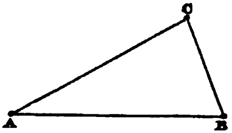

Fig. 2.

A and B, upon the paper, represent two stations or points on the ground, whose relative position—that is, their distance apart, and the direction of the line joining them—has been ascertained. It is required to ascertain and lay down on the paper the position of a third point C relatively to those two. This is to be done by measuring any two out of the following four quantities:—

the distances A C and B C;—
the angles C A B and C B A,—

and plotting or laying down on the paper the representation either of the quantities actually measured, or of others calculated from them. The object of such calculation is in most cases to lay down the distances A C and B C on paper, when the angles at A and B have been measured on the ground; for on the ground, angles are more easily measured with precision than distances; and on paper, distances can be laid down more accurately than angles.

10. **Use of Trigonometry.**—The figure to be measured on the ground and laid down on the paper being in most cases a *triangle*, the branch of mathematics by which the necessary calculations are to be performed is that which relates to the figures and dimensions of triangles: that is, TRIGONOMETRY.

When the triangle formed by the three points is of such extent that the curvature of the earth may be neglected, its sides are sensibly straight lines, and the rules of *Plane Trigonometry* are to be used. When the curvature of the earth has a sensible effect, the sides of the triangle are to be considered as being nearly arcs of circles, of a radius equal to that of the earth, and recourse must be had to *Spherical Trigonometry*. This, however, is of rare occurrence in surveys made expressly for engineering purposes. The principles of spherical trigonometry are also occasionally required, when an angle has been measured on an inclined plane, to compute the corresponding angle as projected on a horizontal plane.

In Chapter III. will be given a summary of those trigonometrical formulæ which are useful in surveying.

11. The **General Order of Operations in Engineering Geodesy** is the following, or nearly so:—

I. The *reconnaissance* or *exploring* of the country by the engineer, with a view to ascertaining in a general way the facilities which it affords for the proposed work, and determining approximately the best site or course for that work. In this process the engineer will pay attention to the geological structure of the ground, and the sources from which useful materials may be obtained: he will be aided by obtaining the best existing maps or plans upon a suitable scale, if any such are to be had, and by the taking of—

II. *Flying levels.*—These are observations for ascertaining the elevations of detached points of primary importance as regards the practicability and cost of the work, and the selection of the line for it; such as passes across ridges and valleys, and points where structures of magnitude may be required.

The engineer having thus determined generally where his proposed work will be situated, proceeds to make a more definite selection of its site, by the aid of—

III. *Preliminary Trial Sections*, made by taking continuous lines of levels in which distances as well as heights are measured. These may, or may not, be accompanied by a *rough survey and plan*,—the necessity for which will depend very much on the character of the existing maps. The engineer is now enabled to determine the site of the work with a degree of precision depending on the care and skill that have been bestowed on the preliminary operations, and to fix accordingly what extent of ground is to be embraced in the—

IV. *Detailed Survey and Plan*, as to the conduct of which further remarks will be made in Article 12. The time, labour, and money expended on this survey will be the less, the greater the precision with which the best line has been found by means of the preliminary operations.

V. *Additional Trial Sections*, both longitudinal and transverse, are now to be made with the aid of the detailed plan, so as to fix exactly the best line for the proposed work that can be found.

VI. *Marking the Line.*—The line so fixed is to be drawn on the plan, and marked on the ground by stakes, or other suitable objects. (See Article 13.)

VII. The *Detailed Section* is now to be prepared by careful and accurate levellings, so as to exhibit a datum horizontal line, a line representing the surface of the ground, and a line, or lines, marking the levels of the proposed work. Certain heights and other information should be marked in figures, as will afterwards be explained. (See Articles 14, 15, and 16.)

VIII. *Trial-Pits and Borings* will be proceeded with, while the levelling for the detailed section is in progress, in order to ascertain the strata of the ground. Borings are the less costly, in time, labour, and damage to the ground; but pits are the more satisfactory to the engineer and the contractor. The results of the trial-pits and borings may be marked on a plan and section for the use of the engineer. (See Article 17.) Further remarks will be made on these matters under the head of earthwork.

IX. *Designs and Estimates.*—The engineer will now design the structures required for the proposed work with sufficient precision to enable him to estimate their probable cost. (See Article 17.)

X. *Parliamentary Proceedings.*—In the event of its being necessary to apply for an act of parliament for the execution of the work, a plan and section and book of reference to the plan will be prepared, and copies of them deposited in certain public offices, in conformity with the standing orders of the House of Lords, and also with those of the House of Commons. No attempt is made in this treatise to give any summary of those standing orders, because, as they are liable to be amended and added to in each session of parliament, the only means of ensuring compliance with them is for the engineer to provide himself with a copy of the standing orders for the session during which the act is to be applied for. Those for a previous session, even for that immediately preceding, are unsafe guides.

XI. *Improving Lines and Levels, under Powers of Deviation.*—In the first preparation of the plan and section of a work requiring the authority of parliament, there is seldom or never time to select the best line and levels with precision. In order to afford an opportunity for afterwards amending the line and levels, powers of deviating from those shown on the parliamentary plan and section are taken, the extent of the power of lateral deviation being indicated on the plan by dotted lines. The usual extent of those powers of deviation is, laterally, 100 yards either way in the country, and 10 yards either way in towns; and vertically, five feet upwards or downwards in the country, and two feet upwards or downwards in towns; but greater or less powers are conferred in special cases. ·After the act of parliament has been obtained, the engineer will avail himself of the power of deviation to make the work more economical, or otherwise to improve it.

The following four operations will then proceed together:—

XII. *Survey for Land Plans.*—If, as is often the case, the previous survey referred to under Operation IV., has been executed too hastily, or plotted on too small a scale, to serve for the plans that are to be used in the purchase of land and execution of the work, a more accurate survey must now be made for that purpose; but this new survey being confined to the ground finally selected for the site of the work, will be of comparatively small extent. (See Article 18.)

XIII. *Ranging and Setting-out the Line*, consists in marking, by stakes or otherwise on the ground, the centre line of the proposed work, as finally fixed.

XIV. *Working Sections* are prepared by taking, with great care and precision, the levels of the ground along the finally selected centre line, and as many lines of transverse sections as may be necessary, plotting the results on a sufficiently large scale (see Article 8, p. 7), and drawing on the sections of the ground so made, lines to represent the intended levels of the work. (See Articles 14, 15, and 16.)

XV. *Setting-out the Breadths of Land* required for the work is performed both on the ground and on the land plans after those breadths have been calculated.

The land required can now be fenced, and the execution of the work proceeded with.

12. **Order of Operations in the Detailed Survey.**—It will now be

stated, in greater detail, what steps are taken in making the survey referred to under Head IV. of Article 11, p. 10.

(*a.*) *Selecting Principal Stations.*—The surveyor, making a general exploration of the ground to be surveyed, will choose a series of stations placed generally on the highest and most open ground; so that each station may command as extensive a view as possible of the ground to be surveyed, and that a pole or other signal placed at each station may be distinctly visible from the neighbouring stations. These stations should also be chosen so that the imaginary lines connecting them with each other, and with a series of conspicuous objects in their neighbourhood, such as towers and spires, may cover the district to be surveyed with a network of large triangles, having no angle less than 30°, or more than 150°; two angles at least of each triangle being accessible stations.

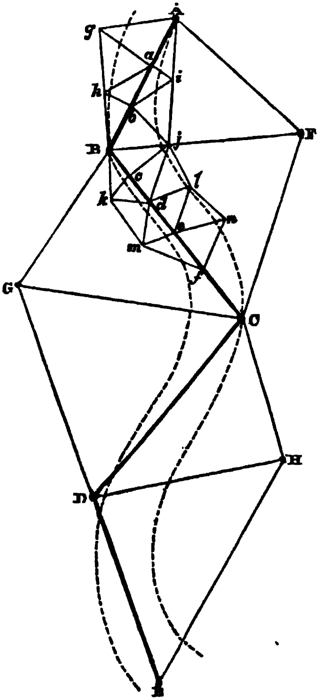

Fig. 3.

With the exception of harbours, most great engineering works are long lines of communication, such as railways, roads, and canals; and the survey required for a work of that sort embraces, in general, a long narrow band of country, usually about a quarter of a mile, and seldom more than half-a-mile wide. Let the two dotted lines in fig. 3 represent part of the band of country to be surveyed; the principal stations, A, B, C, D, E, &c., are to be chosen so as to form the junctions of a series of straight lines running along that band, each line as long as may be practicable consistently with obtaining good points for stations. These are called *base lines,* or *principal station lines.* The network of triangles is to be completed by selecting a series of lateral objects, F, G, H, &c., which may be high buildings, conspicuous trees, &c.

The principal stations are to be marked permanently by stakes, and temporarily, when required, by poles and flags.

(*b.*) *Ranging Principal Station Lines.*—When the lines are of great length, or have uneven ground and other obstacles in their course, it may be necessary to mark intermediate points in them by stakes and poles, as well as the extremities. This is always necessary when there are parts of a station line from which its ends are not visible.

(c) *Main Triangulation. Chaining Base Lines.*—The survey of the network of great triangles might be made by measuring one base line only, and finding the lengths of all the other sides of triangles by calculation from their angles. But for the purposes of the long narrow surveys required for engineering projects, it is more convenient to measure each of the principal station lines AB, BC, CD, &c., by the chain, in order to ascertain the positions of intermediate points suitable for secondary stations, and also of the points where the principal station-lines cross roads, fences, streams, and other objects on the ground. The term "base line" is specially applied to station lines which are thus directly measured. The relative directions of the base lines are determined by measuring the angles $\angle ABC$, $\angle BCD$, $\angle CDE$, &c. The measurements of the angles made by distant objects with the base lines, such as $\angle ABF$, $\angle FBC$, $\angle CBG$, $\angle BCF$, $\angle BCG$, $\angle GCD$, $\angle DCH$, &c., serve, by the aid of trigonometrical calculation, to check the accuracy of the other linear and angular measurements, as will afterwards be shown.

This combination of linear and angular measurement is called *traversing*. It has now been described as practised on a great scale, with principal station lines of several miles in length; but it is also practised on a small scale in surveying objects which are long, narrow, and winding, such as roads and streams.

(d) *Secondary Triangles.*—The surveyor will choose a set of secondary stations, some in the course of the principal station lines; as, *a, b, c, d, e, f*,—others at convenient lateral points; as, *g, h, i, j, k, l, m, n*; the whole so situated that the lines connecting them, which form a network of smaller or *secondary* triangles, may lie sufficiently near to the fences, streams, buildings, and other objects of detail, to enable these to be surveyed from them by the first method of Article 9, p. 8,—that is, by distances and offsets.

(e) *Survey of Details.*—This may be performed wholly by means of distances and offsets; but time and trouble may often be saved by the occasional use of angular measurements.

In order to save time, trouble, and money as far as possible, the five operations which have just been enumerated should be carried on either together or alternately.

13. **Distances, Levels, and other information written on Plan.**—When the plan shows the centre line of a railway, canal, or other line of communication, a scale of distances is to be marked along

the whole of its length, commencing at one of its ends or "termini." According to standing orders which have been in force for many years, that scale of distances on the plan of a proposed railway, is to show each *mile* and *furlong* from the commencement of the centre-line; all radii of curves which *do not exceed one mile* are to be written on the plan in *furlongs* and *chains;* and the lengths of proposed tunnels in *yards.* The information thus written on the plan is useful to the engineer, independently of its being prescribed. It is also useful to the engineer, although not prescribed, to have the levels of important points written on the plan, or shown by the aid of *contour lines* (which will be further explained afterwards), especially when the plan is to be used in selecting a line.

14. **Distances, Datum-point, Heights, and other information written on the Section.**—The horizontal datum-line of the section should have marked on it a scale of distances corresponding with those marked along the centre-line on the plan, in order that corresponding points on the plan and section may be readily found; and great care should be taken that horizontal distances on the plan and section exactly agree.

Alongside the datum-line on the section there should be a written statement of the elevation or depression of the horizontal surface which it represents as compared with what is called the "Datum fixed point;" that is, a well-marked and easily found point on some permanent object, which (as prescribed in the standing orders of parliament) should be "near one of the termini" of the proposed work. The chief requisites of an object for that purpose are, permanence of position and easy identification; so that, on the whole, some portion of the masonry of a building (a public building, if possible), such as the upper side of a window-sill, plinth, or string-course, may be considered as the best. Door-sills are deficient in permanence because of their liability to be worn down by the feet of persons passing in and out; nevertheless, they are frequently used as datum-points, and not objected to. The upper surface of the rails of a railway at some specified point is often referred to as a datum, and considered sufficient, although its elevation is far from being permanent. Amongst objects utterly unsuitable for this purpose may be mentioned, all surfaces whose levels are continually changing, how slight soever the change may be, such as the "top water level" of a canal, and all *ideal* horizontal surfaces.

Amongst other information to be marked in writing on a section are, the heights of the principal parts of the proposed work above the horizontal datum-line, and in particular, in the case of a railway, those of the upper surface of the rails at the points where the inclination varies; the several rates of inclination of proposed railways, and of roads to be altered for the purpose of making them; the greatest depths of cuttings and heights of embankments; the lengths of tunnels and viaducts; the alterations of level and inclination to be made in existing lines of communication; the character of the structures to be used for passing them, whether bridges over or under, or level crossings; and in the case of proposed bridges for existing roads, the width of roadway which they will provide, and if they pass over the roads, the height of headroom. So far as those items of information are required by the standing orders of parliament, reference must be made for details to those standing orders themselves, as has been already stated under Head X. of Article 11.

A working section should state, in writing, the level of the ground, the level of the proposed work, and the height of embankment or depth of cutting, at every point of the ground whose level has been taken; those quantities being found by calculation, not by measurement on the paper. It should also state the positions and levels of all "Bench marks."

15. **Bench Marks** are fixed objects whose levels are known,—in fact, subordinate datum-points,—distributed along the course of the intended work, at distances of from half-a-mile to a mile, and near the sites of all intended structures of importance, such as bridges. If suitable existing objects cannot be found, the heads of large stakes driven for the purpose will answer. They should be placed where they will not be disturbed during the execution of the work.

16. **Checking Levels** consists in taking the levels of points over again, to test the correctness of previous levelling. In preliminary and parliamentary sections, the levels of the more important points only, such as summits of hills and bottoms of valleys, crossings of existing lines of communication, and bench marks, require to be checked; for working sections, every level taken should be checked.

17. **Estimates and Borings marked on Plan and Section.**—It is useful to the engineer to have a copy of the plan and section of a proposed work on which the results of trial-pits and borings are marked, and the estimated cost of each part of the work written opposite to its position on the paper.

18. **Centre Line as a Base for Land-Plan Survey.**—When the centre line of a proposed railway has been carefully ranged and staked out, it may be used, whether straight or curved, as a base for the secondary triangulation of the survey for the land-plans, the great triangulation being dispensed with, and each stake regarded as a station in the survey.

19. **Damage to Property to be avoided.**—All operations of engi-

neering field-work ought to be so conducted as to do as little damage as possible to the property traversed.

20. **Arrangement of the ensuing Chapters.**—The operations of surveying, levelling, and setting-out, having been enumerated and explained in a general way in the present chapter, the remaining chapters of this part will be devoted to the explanation of details relative to certain branches of the subject, in the following order:—

Surveying with the chain.
Surveying by angular measurements.
Levelling.
Setting-out works.
Marine surveying.
Copying, enlarging, and reducing plans.

The explanation of some of the peculiarities of surveys for particular classes of works will be reserved until those works themselves come to be considered.

CHAPTER II.

OF SURVEYING WITH THE CHAIN.

21. **Marks and Signals.**—The marks fixed at stations to enable them to be readily found are usually stakes, of size and strength sufficient to guard against the risk of their being disturbed. In most cases they should be driven to the head, or nearly so. If, for a particular station-mark, greater permanence is desired than can be obtained by means of a stake, a block of stone may be used, having a cross cut on its upper surface.

When a mark fixed at the station itself would be liable to be disturbed, four stakes may be driven so that the intersection of the straight lines joining them diagonally may mark the station; or two or more stakes may be driven, and the distances of the station from them measured and noted down; or the distance of the station from any two or more well-defined permanent objects, such as corners of buildings, may be measured and noted down; or if two permanent objects can be found which lie in one straight line with the station, that fact can be noted, together with the distance of the station from one of the objects. The points where station-lines cross fences are marked by notches upon timber and grooves upon stone.

The signals set up at stations to make them visible from a distance usually consist of poles, with or without flags. Ordinary poles, to be carried about in the field, may be from six to nine feet long, painted in alternate lengths of black and white, and shod with iron. For flags, although white is the colour that is seen farthest, red is more generally employed, as being more easily distinguished from surrounding objects by those who have no defect in the perception of colour. To mark the ends of long station-lines, poles of greater lengths, such as twenty or thirty feet, are often required; these generally need rope stays to keep them upright.

Great care should be taken to set up and keep all poles in a truly vertical position; and tall permanent poles should be adjusted by means of a plumb-line.

For the temporary marking of points in surveying details, bits of paper are used, held in cleft sticks. These are called "whites."

To facilitate the ranging of long station-lines, it is useful to choose them, when opportunities occur, so as to run directly towards some conspicuous existing object, such as a tree, a spire, or a large chimney.

22. **The Surveying Chain.**—For measuring with extraordinary accuracy the bases of national trigonometrical surveys, rods of glass and of metal have been used—a correction for expansion by heat being made either by calculation or by mechanism: also, steel chains, made of flat links connected at the ends by pins, and supported in accurately levelled troughs, the tension being maintained constant by a weight hanging over a pulley, and the correction for expansion made by calculation.

In ordinary surveys for engineering works so great a degree of accuracy is unnecessary; and the instrument generally used for measuring distances is the common surveying chain, which consists of one hundred straight links of iron or steel wire of equal length, having eyes on their ends, and connected together by oval rings. There are usually three of those rings between each pair of straight links. The joints of the rings, and those of the eyes of the links, should be welded: the chain is thus rendered much less liable to stretch than if those joints are open. Each distance of ten links from either end of the chain is marked by a peculiarly shaped piece of brass, so that the mark at ninety links from one end is similar to that at ten links, that at eighty links to that at twenty, and so on, the middle of the chain being marked by a round piece of brass. At each end of the chain is a handle.

The chain should measure its correct length from *outside to outside of the handles.*

As every chain which is in daily use in the field is liable to have its length increased by the continual strain upon it, and diminished by the bending of the links, and by dirt getting into the rings, it ought to have its length tested every day by comparison with a "standard chain," used for the sole purpose of testing other chains, or with two marks on a wall, or on a pair of stakes, whose distance apart has been very accurately adjusted. The length of the working chain, when found to be erroneous, can be corrected by straightening the links and cleansing the rings, and by hammering the latter so as to make them longer or shorter as may be required.

The chains most commonly used in Britain are, "Gunter's Chain" of 66 feet (in which each link is ·66 of a foot or 7·92 inches), and the chain of 100 feet. The advantages of Gunter's chain are, its being an exact decimal fraction of a mile (one-eightieth, or ·0125), and the square described upon it being one-tenth of an acre. The 100-foot chain has the advantage of giving at once dimensions in feet, which are convenient in the calculation of quantities of work.

When a "chain" is spoken of without qualification, Gunter's chain is meant.

The chain is usually accompanied by ten skewers called "arrows," made of iron or steel wire, having a point at one end and a large ring at the other, marked with a piece of red cloth to make it visible from a distance. Some surveyors prefer to use, in chaining long lines, nineteen arrows, nine of iron or steel, and ten of brass.

The chain is carried by two men, called respectively the "leader" and the "follower." In measuring the length of a station-line, the follower, in a crouching attitude, holds one end at the commencement of the line, and the leader, carrying with him all the arrows, fixes his eyes on the object which marks the distant end of the line, and walks straight towards it, dragging the other end of the chain along with him. When the chain is tightened, the leader crouches down at one side of the line, holding near the ground an arrow exactly upright, in the same hand which grasps the handle of the chain. The follower sees that the chain is tight, straight, and unentangled, and directs the leader by words or gestures so as to make him stick the arrow into the ground exactly in the alignment.* The leader and follower then rise, and advance until the follower reaches the arrow that marks the end of the first chain-length, and proceed to lay off a second chain-length and fix a second arrow as before, and so on. The follower picks up the arrows as he advances, so that by counting the arrows in his hand he can tell at any moment how many entire chain-lengths have been measured. On fixing the tenth arrow, the leader cries in a loud voice "ten," or "change;" the surveyor notes in his field-book that ten chains have been measured; the leader stands still until the follower has advanced to him and handed him the nine previously picked-up arrows; the follower holds his end of the chain at the mark made by the tenth arrow, which the leader (if there are ten arrows only) then picks up, and advances with all the ten arrows in his hand to commence the measurement of the next ten chains. If there are nine iron arrows and ten brass ones, the leader, having expended all the iron arrows in marking the first nine chains, marks the end of the tenth chain with a brass arrow; and when the follower comes up to him, takes only the nine iron arrows, leaving the brass arrow to be picked up by the follower when the next chain-length has been measured. In this case the follower, at any moment, can tell the number of entire tens of chains which have been chained by counting the brass arrows in his hand, and the number of chains over and above the entire tens of chains by counting the iron arrows; and thus a check is kept upon the number of entire tens of chains noted in the surveyor's field-book. At the end of each hundred chains the leader receives back all the brass arrows as well as the iron ones.

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* Mr. Haskoll (*Engineering Field-work*) judiciously recommends that words alone be used for this purpose, in order that the leader may fix his eyes on the arrow, and keep it exactly vertical, the follower directing him to move it to one side or the other by saying "to you" and "from you," and to fix it in the ground by the word "mark."

If the leader takes care while advancing to keep his eyes fixed on the signal at the distant end of the line, he will be able to drag the chain forward in the true alignment with very little direction from the follower.

The follower while advancing should allow the chain to slacken, and should take care to keep it clear of the arrow, and of objects which may entangle it.

As the chaining goes on, the surveyor notes the distances from the commencement at which the station-line crosses all fences, boundaries, banks of streams, sides of roads, and other objects to be shown on the plan; also where it crosses other station-lines, and where points occur suitable for intermediate stations in the survey.

23. **Chaining on a Declivity—Reduction to the Level.**—In chaining up or down a slope, the distance actually measured must be reduced on the plan to the projection of that distance on a horizontal plane. The most convenient way of effecting this is by means of a *correction* in links and fractions of a link to be *deducted* from each chain. This correction being known, may be applied mechanically during the chaining, by pulling the chain forward at each chain-length through a distance equal to the required correction.

The following are various formulæ for computing the correction:—

When the angle of inclination has been measured by a "clinometer" or other angular instrument;

$$\text{Correction in links per chain,} = 100 \times \text{versed sine of inclination,} \quad (1.)$$

When the vertical fall in links for each chain of distance on the slope is known;

$$\text{Correction in links per chain} = 100 - \sqrt{10{,}000 - \text{fall}^2}; \quad (2.)$$

and when the slope is gentle, the following approximate formula will answer:—

$$\text{Correction in links per chain} = \frac{\text{fall}^2}{200} \text{ nearly} \ldots\ldots\ldots (3.)$$

To save calculation, most clinometers and theodolites have the correction for declivity marked on the "limb" or graduated arc on which angles in a vertical plane are measured.

Experienced surveyors learn to estimate this correction with considerable accuracy by the eye.

Its use may often be dispensed with by stretching the chain in a horizontal position; the up-hill end touching the ground, and the point on the ground exactly below the down-hill end being found by means of a plumb-line, or a ranging pole held vertically, or by dropping an arrow or a stone. This process is called *stepping*, and may be carried on by half-chains or shorter distances, instead of whole chains, on very steep ground.

24. **Offsets** (to which reference has already been made in Article 9, Division I., p. 8) are ordinates or transverse distances, measured from known points in a station-line to objects whose position is to be ascertained; such as bends and intersections of fences, of the sides of roads, of the banks of streams, and of other boundaries, corners of buildings, and so forth. The surveyor notes in his field-book the distance in links from the commencement of the station-line at which the offset is made (A B, fig. 1, p. 8), and the length of the offset (B C in the same figure); the side of the page on which the latter is noted showing at which side of the station-line the offset lies, as will be further explained in Article 28.

Offsets are almost always at right-angles to the station-line. To ensure accuracy they should seldom exceed about one chain in length (although offsets of two or three chains may be made to boundaries which are nearly parallel to the station-line); and the secondary station-lines from which the details of the ground are surveyed should be laid out accordingly. The position and direction of short offsets may be laid off by the eye; but the longer offsets, especially if they run to important objects, should be laid off by letting fall a perpendicular from the object (at which, if necessary, a pole or a "white" may be placed) upon the station-line, by means of the "cross-staff" or of the "optical square."

The *Cross-Staff* is simply a staff with a spike on the lower end, and two pair of sights at right angles to each other at the upper end.

The *Optical Square*, which has almost superseded the cross-staff, is a brass box, containing two small silvered plate-glass mirrors, whose planes make with each other an angle of 45°; so that every ray of light which falls upon the first mirror, and is thence reflected to the second mirror, is again reflected from the second mirror in a direction at right angles to its original direction. A portion of the second mirror is unsilvered, so that the surveyor can see through it. He places himself on the station-line, and looks through the unsilvered glass towards the signal at one end of it, and then moves backwards and forwards along the station-line until he sees the reflected image of the lateral object apparently coinciding in direction with the signal on the station-line; the directions of those two objects are then at right angles, and the point on the ground directly below the optical square is the commencement of the offset required.

To adjust the optical square, make a rest for it by driving a picket or small post (which may be called A) four and a-half or five feet high, with a flat top. Set up a pole two or three chains off in any convenient direction (which pole may be called B); look towards it through the unsilvered glass; send an assistant to set up a second pole (C) in such a direction that its reflected image apparently coincides in direction with B. Then the lines A B and A C are or ought to be at right angles. In the same way, let the assistant set up a third pole, D, at the same angular distance from C, and a fourth pole, E, at the same angular distance from D. Then on looking directly towards E, if the optical square is correctly adjusted, the reflected image of B will be seen apparently coinciding in direction with E. Should it not be so, correct one quarter of the error by means of the adjusting screw which acts upon one of the mirrors, and repeat the whole operation until the adjustment is exact.

The purpose of an optical square may be answered by a *box-sextant*, the index being set to 90°. This instrument will be described in Chapter III. Lines at right angles to each other may sometimes be marked on the ground by setting out with the tape-line or chain a right-angled triangle of any convenient dimensions; the proportions of the sides being determined by the principle, that the sum of the squares of the sides which enclose the right angle is equal to the square of the hypothenuse, or side opposite the right angle.

Amongst the proportions of whole numbers which fulfil that condition are the following:—

Sides enclosing the right angle.				Hypothenuse.
3	:	4	:	5
5	:	12	:	13
7	:	24	:	25
8	:	15	:	17
20	:	21	:	29

The most useful of these proportions is the first and simplest,

3 : 4 : 5.*

---

* The following is a general method for finding any number of sets of whole numbers which are proportional to the sides of right-angled triangles.

Choose any two numbers whatsoever, m and n, m being the greater; and if they are either both even, or both odd, make

$$x = mn; \quad y = \frac{m^2 - n^2}{2}; \quad z = \frac{m^2 + n^2}{2};$$

but if one is even and the other odd, multiply each of the above expressions by 2. Then,

$$x^2 + y^2 = z^2;$$

and x, y, and z are proportional to the three sides of a right-angled triangle, z corresponding to the hypothenuse.

When two persons are available to measure the lengths of offsets, either a second chain or a *Tape-line* may be used. The surveyor may measure offsets without assistance with the *offset-staff*,—a light and strong wooden pole tipped with brass or iron, ten links long *from end* to end, and divided into links.

25. **Oblique Offsets** may be made, if convenient, with the aid of an angular instrument, such as a box-sextant or a light theodolite, to measure the angles which they make with the station-line. But in surveying by linear measurements alone, oblique offsets are made in pairs from different points in the station-line to the same object, in order to determine its position with more accuracy than is attainable by a single rectangular offset. For example (see fig. 4.), the position of the object D is found by measuring to it a pair of offsets, B D, C D, from two different points, B and C, in the station-line A B C. This process, in fact, belongs to the method of surveying *by triangles*, B D C being a triangle of which the three sides are measured.

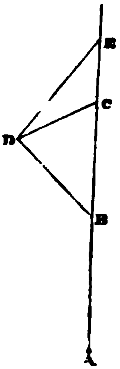

Fig. 4.

The nearer the angle between the two offsets, ∠ B D C, approaches to a right angle, the more accurately is the position of the object determined; and care should therefore be taken to make that angle neither very acute nor very obtuse.

If a check on the accuracy of the operation is desired, a third offset, E D, may be measured to the object from a third point, E, in the station-line.

The principal objects for which the additional accuracy given by oblique offsets is desirable, are corners and intersections of boundaries, angles of buildings, mile-posts, and the like. When the object is a corner of a building, such as D in fig. 5, it is convenient to make each of the offsets, if possible (or at all events one of them), lie in a straight line with a face of the building, and so to determine the direction of such face or faces.

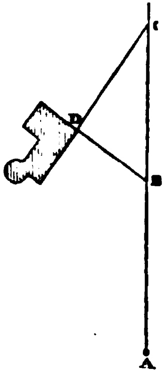

Fig. 5.

No general rules can be laid down for surveying the details of an intricate building, except that in many cases a rectangle may be set out so as to enclose it, and the sides of that rectangle used as station-lines from which to take offsets to the faces and corners of the building. To survey some buildings completely it is necessary to have access to the inside.

26. **Chained Triangles.**—It has already been stated in Articles

9 and 12, that the relative positions of different station-lines, and of the stations which they connect, are determined by so arranging them as to form a complete network of triangles over the district surveyed. In the absence of angular instruments, the figure of each of those triangles must be determined by measuring with the chain the length of each of its sides.

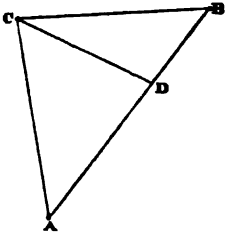

Fig. 6.

In fig. 6, let A B represent a station-line whose length and position are known; C, a third station lying out of the line. Then by measuring the two remaining sides, A C, B C, of the triangle A B C, so that the lengths of all its three sides may be known, the position of C is determined.

Agreeably to the principle already noted in the last article, that determination is the more accurate the less the angle A C B differs from a right angle. Supposing a certain error to have been committed in measuring one of the lines B C or A C, the consequent error in finding the position of C is equal to the original error if A C B is a right angle; but if that angle is either acute or obtuse, the error in the position of C is greater than the original error in the proportion of the *cosecant* of the angle A C B to radius.

Triangles in which the angle at the point to be determined is less than 30°, or more than 150°, are said to be "*ill-conditioned*," and are avoided by skilful surveyors. In an ill-conditioned triangle, the error in the position of C is more than double of the corresponding error in the measurement of a side of the triangle.

The accuracy of the measurements in every important triangle should be checked by measuring a "*tie-line*," from one of its angles to a known point in the opposite side, such as C D in fig. 6. The agreement of the length of that line with the result of the measurements of the sides may be tested on the plan when plotted. It may also be tested by calculation; for if all the measurements are correct, the following equation will be verified,

$$CD^2 = \frac{AC^2 \cdot BD + BC^2 \cdot AD}{AB} - AD \cdot DB \ldots\ldots (1).$$

27. **Gaps in Station-Lines.**—A long station-line, otherwise well adapted for its purpose, may have one or more places in its course through which, owing to the intervention of buildings, woods, precipices, water, swamp, or other obstacles, it may be difficult or impossible to chain along the line with accuracy; and in some cases also it may be impossible to range the line directly across the obstacle. These difficulties are most readily met by the use of

angular instruments; but in the absence of such instruments, the chain alone may be used, according to methods which may be varied to suit the circumstances of each particular case.

Three kinds of cases may be distinguished:—*First*, those in which the obstacle can be seen over from side to side, and chained round, but not chained across. *Secondly*, those in which it can neither be seen over nor chained across, but can be chained round; and *Thirdly*, those in which the obstacle can be seen over, but neither be chained across nor chained round.

In each of the figures that illustrate this article, the inaccessible part of the station-line is marked by dots, and the direction in which the measurement proceeds is indicated by an arrow.

CASE I.—*When the obstacle can be seen over*, the first operation is to plant a ranging pole in the station-line at the further side of the obstacle; and the problem to be solved is, to find the distance to that pole from some point already chained to on the nearer side.

FIRST METHOD (By a parallel line, see fig. 7).—Let A and D be marks at the nearer and further sides of the obstacle respectively. By the optical square or otherwise, range A B, D C, at right angles to the station-line; make these perpendiculars equal to each other, and of any length that may be requisite in order to chain past the obstacle along B C, which will be parallel and equal to A D, the distance required; that is to say,

$$A\,D = B\,C \text{ .... } (1.)$$

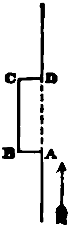

Fig. 7.

SECOND METHOD (By a triangle, see fig. 8).—A and D as before being points in the station-line at the nearer and further sides of the obstacle, set out a triangle A B C of any form and size that will conveniently enclose the obstacle, subject only to the conditions, that B and C are to be ranged in one straight line with D, and that the angles at B and C are neither to be very acute nor very obtuse. Measure with the chain the lengths A B, A C, B D, D C. Then the inaccessible distance A D is given by the formula.

$$AD = \sqrt{\left\{\frac{AB^2 \cdot CD + AC^2 \cdot BD}{BC} - BD \cdot CD\right\}};\ (2.)$$

the computation of which will be much facilitated by the use of a table of squares.

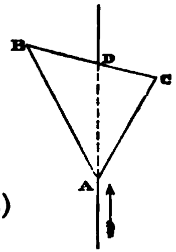

Fig. 8.

That distance may also be found by plotting the triangle and the point D in its base on a sufficiently large scale, and measuring A D on the paper.

The figure of the obstacle may be surveyed by offsets from the sides of the triangle.

THIRD METHOD (By two triangles, see fig. 9).—Let b and c be points in the station-line at the nearer and further side of the obstacle respectively. From a convenient station A, chain the lines A b, A c, being two sides of the triangle A b c; connect those lines by a line B C in any position which will form a well-conditioned triangle A B C, of as large a size as is practicable: measure its three sides. Then the inaccessible distance is given by the formula,

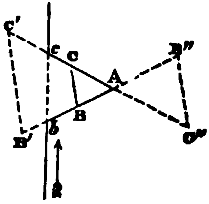

Fig. 9.

$$b\,c=\sqrt{\left\{\mathrm{A}\,b^2+\mathrm{A}\,c^2-\frac{\mathrm{A}\,b\cdot\mathrm{A}\,c}{\mathrm{A\,B}\cdot\mathrm{A\,C}}\cdot(\mathrm{A\,B}^2+\mathrm{A\,C}^2-\mathrm{B\,C}^2)\right\}}\;..\;(3.)$$

The following modification of this formula, though less simple in appearance, is better adapted to computation by the help of a table of squares;

$$b\,c=\sqrt{\left\{\mathrm{A}\,b^2+\mathrm{A}\,c^2-\frac{(\mathrm{A}\,b+\mathrm{A}\,c)^2-(\mathrm{A}\,b-\mathrm{A}\,c)^2}{(\mathrm{A\,B}+\mathrm{A\,C})^2-(\mathrm{A\,B}-\mathrm{A\,C})^2}\cdot(\mathrm{A\,B}^2+\mathrm{A\,C}^2-\mathrm{B\,C}^2).\right\}}\;\ldots\ldots\ldots\ldots\ldots\ldots(3\mathrm{A}.)$$

The points B and C are shown in the first instance as lying between A and the station-line; but if necessary, they may be taken in the prolongations of A c and A b beyond the station-line, as at B′ and C′, or in their prolongations beyond A, as at B″ and C″, and the same formula will still apply.

The formula is much simplified if A B and A C can be laid off so as to be respectively proportional to A b and A c; for then the triangles A B C and A b c become similar, B C is parallel to b c, and the inaccessible distance is simply

$$b\,c=\mathrm{B\,C}\cdot\frac{\mathrm{A}\,b}{\mathrm{A\,B}}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\;(4.)$$

In this method, as well as in the preceding, the inaccessible distance may be found by plotting.

CASE II.—*When the obstacle can be chained round, but not chained across nor seen over.*

FIRST METHOD (By parallel lines, see fig. 10).—From A and

B, two points in the station-line on the nearer side of the obstacle, and at least as far apart as the distance across it is judged to be, lay off, by the optical square or otherwise, the equal perpendiculars A C, B D, of length sufficient to enable a straight line C D E F, parallel to the station-line, to be ranged and chained past the obstacle. Commence the chaining of this parallel line at D, in continuation of that of the station-line at B. As soon as the obstacle is passed, lay off the perpendicular E G equal to A C and B D; then G will be a point in the station-line beyond the obstacle, and the inaccessible distance will be

$$B\,G = D\,E \text{ ....} \quad (5.)$$

By continuing the parallel line and repeating the same process, additional points in the station-line, such as H, may be found.

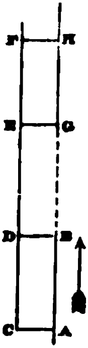

Fig. 10.

Second Method (By similar triangles, see fig. 11).—From a point A, as far back as practicable from the end B of the chained station-line on the nearer side of the obstacle, range two diverging lines A F, A E, past the two sides of the obstacle, in which measure the distances A D, A C, of two points D and C, which lie in one straight line with B. Continue the chaining of A F and A E, and make those distances respectively proportional to A D and A C, so that A D C and A F E may be similar triangles. Measure D C, in which note the position of B. Measure E F, in which take the point G, dividing E F in the same ratio in which B divides C D; then G will be a point in the station-line beyond the obstacle; and points still further on may be found, if necessary, by a similar process.

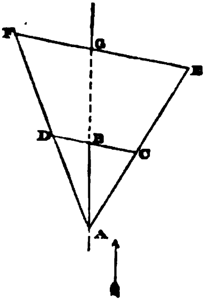

Fig. 11.

The inaccessible distance B G is found by the formula,

$$B\,G = \frac{A\,B \cdot C\,E}{A\,C} \text{ ....} (6.)$$

The boundaries of the obstacle can be surveyed by offsets from the sides of the quadrilateral C D F E.

Third Method (By transversals, see figs. 12, 13).—Let *a* and *b* be two points in the chained station-line at the near side of the obstacle, about as far apart as the inaccessible distance *b c* is judged to be. Mark a station C so as to form a well-conditioned triangle with a and b; prolong the lines b C and a C until two points A and B are reached through which a straight line can be ranged and chained past the further side of the obstacle.

In some cases it may be advisable to begin by choosing the stations A and B, then to choose C, and then to range the lines B C a, and A C b (as in fig. 12), or A b C (as in fig. 13).

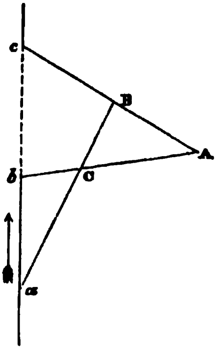

Fig. 12.

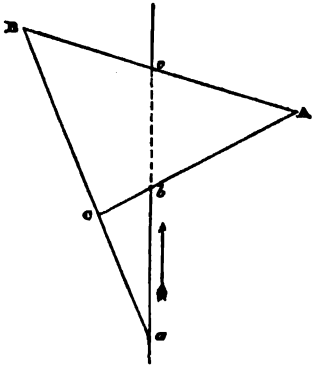

Fig. 13.

All the sides of the two triangles A B C, a b C, are to be measured.

Then, *to find the point* c *at the intersection of the station-line with* A B, compute the distance of that point from B by one or other of the following formulæ :—

If c lies in A B produced, as in fig. 12,

$$\mathrm{B}c = \frac{\mathrm{AB}\cdot a\mathrm{B}\cdot b\mathrm{C}}{\mathrm{C}a\cdot \mathrm{A}b - a\mathrm{B}\cdot b\mathrm{C}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7.)$$

If c lies between A and B, as in fig. 13,

$$\mathrm{B}c = \frac{\mathrm{AB}\cdot a\mathrm{B}\cdot b\mathrm{C}}{\mathrm{C}a\cdot \mathrm{A}b + a\mathrm{B}\cdot b\mathrm{C}}. \quad \ldots\ldots\ldots\ldots\ldots \quad (7\ \mathrm{A}.)$$

Next, *to find the inaccessible distance* b c, use the following formula (which is applicable to both figures) :—

$$bc = \frac{ab\cdot \mathrm{A}b\cdot \mathrm{BC}}{\mathrm{CA}\cdot a\mathrm{B} - \mathrm{A}b\cdot \mathrm{BC}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8.)$$

The same problems may also be solved by plotting the figure a b c A B C a, and producing a b till it cuts A B, as in fig. 13, or A B produced, as in fig. 12. In a purely mathematical point of view, it is unnecessary to measure both A B and a b, as either of those lines might be calculated from the other; but both should nevertheless be chained, as a check on possible errors.*

CASE III.—*When the obstacle can be seen over, but neither chained across nor chained round.* This is the case of a station-line interrupted by a deep ravine, or a deep and rapid river. The first operation, as in Case I, is to range and fix a pole at *c* (fig. 14) in the station-line beyond the obstacle. The next is to find the distance *b c*.

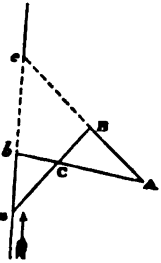

Fig. 14.

FIRST METHOD (By transversals).—On the nearer side of the obstacle, range the stations A and B in a straight line with *c*, making the angle *b c* B greater than 30°, and place them so that the intersecting lines A *b*, B *a*, connecting them with two points *a* and *b* in the station-line, shall form a pair of well-conditioned triangles *a b* C, A B C, as in the last problem. Measure the sides of these triangles, and compute the inaccessible distance *b c* by equation 8, already given.

As a check upon the position thus found for the point *c*, compute also the inaccessible distance B *c* by means of equation 7.

This problem is solved graphically by plotting the figure *a b c* A B C *a*, and producing *a b* and A B till they intersect in *c*.†

SECOND METHOD (By the optical square, when the inaccessible distance does not much exceed three or four chains, see fig. 15). B D being the inaccessible distance, at B, with the optical square, set out B C perpendicular to the station-line, and of a length such as to make B C D a well-conditioned triangle. At C, with the optical square, range C A perpendicular to C D, cutting the station-line in A. Measure A B, B C; then

$$\mathrm{BD} = \frac{\mathrm{BC}^2}{\mathrm{AB}} \quad \text{..................} \quad (9.)$$

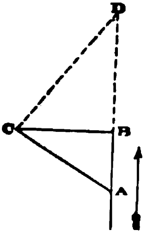

Fig. 15.

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* The following are the formulæ for calculating A B from *a b*:—

In Fig. 12; $\mathrm{AB} = \sqrt{\left\{\mathrm{BC}^2 + \mathrm{CA}^2 - \frac{\mathrm{BC}\cdot\mathrm{CA}}{b\mathrm{C}\cdot\mathrm{C}a}(b\mathrm{C}^2 + \mathrm{C}a^2 - ab^2)\right\}}$.

In Fig. 13; $\mathrm{AB} = \sqrt{\left\{\mathrm{BC}^2 + \mathrm{CA}^2 + \frac{\mathrm{BC}\cdot\mathrm{CA}}{b\mathrm{C}\cdot\mathrm{C}a}\cdot(b\mathrm{C}^2 + \mathrm{C}a^2 - ab^2)\right\}}$.

To compute *a b* from A B, interchange the positions of A and *a*, B and *b*, throughout the above formulæ.

† The solutions of this and the preceding problem are founded on the first theorem in Carnot's celebrated essay *On the Theory of Transversals*; a branch of Geometry at once simple in its principles and useful in its applications, but little known or studied.

The calculation represented by the formula 7, when each of the given distances is expressed by four figures, has been found to occupy about five minutes.

Methods for measuring gaps in station-lines by the aid of angular instruments will be explained in Chapter III.

28. **Field-Book.**—The writing and sketching in field-books is made either with ink or with an indelible pencil. If the book can be protected from rain, ink is to be preferred.

The field-book of a survey should commence with a sketch showing the general arrangement of the stations and station-lines relatively to the more conspicuous objects on the ground to be surveyed, made by the surveyor when he explores the country, as mentioned under head (*a*) of Article 12, page 12. Those stations may be distinguished by letters or by numbers. Principal stations are usually marked thus ◬. The remainder of the book will contain the detailed notes of the distances chained along the several station-lines, and the offsets measured from them.

In order that forward and backward, right and left, on the ground, may be represented by forward and backward, right and left, in the book, the successive notes written on each page begin at the bottom and proceed towards the top; and the pages are numbered from right to left. In the middle of each page is a vertical column broad enough to contain numbers of five or six figures. That column represents the station-line.

The surveyor begins at the bottom of the first page, by writing in the central column a letter, or other mark, to denote the station at which the line about to be chained commences, and beside it, a note stating between what stations the line runs: for example, "from A to B." As the chaining advances, he notes in the central column, proceeding upwards, the distances at which the station-line crosses boundaries, and traverses intermediate stations, and at which offsets are taken. Each distance of an intermediate station from the commencement is distinguished by enclosing it in an oblong or oval, and writing opposite to it the designation of the station, together with a reference to the other pages of the field-book in which the same station is referred to, and a note of its position upon other station-lines which traverse it. To the right and left of the central column are written the offsets measured to the right and left respectively, each opposite the figures denoting its distance from the commencement of the line; and those offsets are accompanied by a sketch-plan of the objects to which they are measured, with explanatory notes when required.

On arriving at the end of a station-line, the relative direction of the next line chained may either be stated in words—as, "turn to the right," "turn to the left"—or indicated by symbols like the following: ↱ ↰. At the commencement of each new station-line will be stated the position of the point from which it starts upon a former station-line.

Oblique offsets, small triangles, measurements of buildings, and the like, are best recorded by sketching a diagram of the lines measured, and writing their lengths along them.

The preceding explanation shows the general principles according to which field-books of chained surveys are kept. The details vary very much in the practice of individual surveyors. It is to be recommended that every surveyor should keep his field-book so distinctly that it may be possible for a draughtsman to plot the survey from the field-book without receiving any explanation from the surveyor.

29. **Plotting a Chained Survey.**—In plotting a survey, great attention should be paid to the absolute flatness of the drawing-board or table on which the paper is to be strained or laid, and to the perfect straightness of the straight-edge by which station-lines are to be ruled.

If the plan is to be mounted on cloth, the paper should be mounted before the plan is plotted; otherwise the mounting will alter its dimensions. On the whole, it is better *not* to "strain" the paper on which a survey is plotted on a drawing-board, in the way practised for architectural and mechanical drawings; because, when the paper is cut away from the board, and so relieved from the strain, it will contract, and perhaps contract unequally in different directions.

Each day's work should be plotted as soon as possible after having been surveyed.

The scale according to which the survey is plotted should at once be drawn on the plan, when it will contract and expand along with the paper.

The plotting is commenced by marking with a needle or pricker a point to represent the first station; drawing a straight line through that point to represent the first station-line, and laying down on that line, with a pair of beam-compasses, the positions of the other stations which it traverses.

The operations which follow consist chiefly in plotting triangles, and plotting distances and offsets.

30. **Plotting Triangles.**—The great triangles, whose sides connect the principal stations, are to be first plotted: then the secondary triangles, until the whole network is completed. The operation of plotting a triangle whose three sides have been measured is as follows:—A and B, fig. 16, represent two stations already plotted; the distances A C, B C, of a third station from those stations are known. With these distances as radii, describe with the beam-compasses a pair of small circular arcs about A and B respectively; the intersection of those arcs

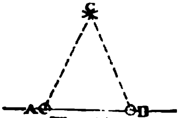

Fig. 16.

marks the required station C on the plan. Then with the straight-edge rule the lines A C, B C, and the triangle is complete.

It is usual, for the satisfaction of the engineer, and for future reference, to draw permanently on the plan, in a faint red colour, the principal station-lines, forming the primary network of triangles. Those lines are sometimes called "lines of construction." In some cases it is useful to draw permanently in the same way a portion of the secondary network of triangles: so far, at least as they can be used in computing areas.

When the plan of a survey extends over several sheets, it is necessary, in order to show the connection between two adjacent sheets, that a portion of at least one station-line, containing at least one principal station, should be plotted on each of the two sheets.

•31. In **Plotting Distances, Offsets, and Details,** a flat ivory or box-wood scale is laid on the paper exactly parallel to the station-line, and loaded to keep it at rest: the divisions marked on its edge represent distances. A shorter flat scale, having broad ends exactly perpendicular to its edges, is laid on the paper with one end against the edge of the scale for distances: it is slid successively to the several distances from the station noted in the field-book, and the offsets are laid down by pricking with a needle opposite the proper graduations on one of its edges. Care should be taken that the offset-scale is exactly rectangular.

Oblique offsets are plotted like the sides of triangles.

In estate plans, on a large scale, different kinds of fences, such as stone-walls, hedges, palings, &c., are distinguished from each other by conventional modes of marking; but in plans for engineering projects, it is sufficient to distinguish between fenced and unfenced lines of division of land, marking the former by plain and the latter by dotted lines. In working plans on a large scale, walls may be shown of their proper thickness, and coloured red. Boundaries of parishes, counties, boroughs, and other legal divisions of the country, are marked with peculiarly shaped and arranged dots. Roads are coloured drab; streams and pieces of water light blue, with a darker shade along their edges. Dwelling-houses are coloured light red, out-buildings dark grey, public buildings light grey. In engraved plans buildings are shaded by diagonal hatching. Railways are marked by parallel lines representing rails; and in some cases these are crossed by short fine lines to indicate sleepers. Canals are distinguished from streams by their greater uniformity of width and regularity of course. Trees are indicated by sketching small figures somewhat resembling them.

There are conventional modes of indicating the nature of the surface of the ground, whether garden ground, arable land, pasture, marsh, heath, and the like; but in plans for engineering projects it is sufficient to refer by numbers written on the plan to corresponding numbers in the book of reference, in which are stated the owner or reputed owner, lessee or reputed lessee, occupier, and description of each portion of property shown on the plan.

32. **Measuring Areas.**—The elementary methods of measuring areas which are useful in surveying are of three kinds:—the method of triangles,—the method of ordinates,—and the method by mechanism.

I. *Method of Triangles.*—Let a, b, c, denote the lengths of the sides of a triangle, and

$$s = \frac{a+b+c}{2},$$

the *half-sum* of those lengths; the area of the triangle is given by the formula—

$$\text{Area} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}; \ldots\ldots\ldots\ldots\ldots (1.)$$

or, using logarithms—

$$\text{log. area} = \frac{1}{2}\left\{\text{log. } s + \text{log. } (s-a) + \text{log. } (s-b) + \text{log. } (s-c)\right\} \quad (2.)$$

Another formula is as follows: let a be any one of the sides of a triangle; p the perpendicular upon that side from the opposite angle; then—

$$\text{Area} = \frac{a\,p}{2}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

Every right-lined figure can have its area calculated by dividing it into triangles, computing their areas by one or other of the preceding formulæ, and adding them together.

The areas of figures with curved outlines can be found approximately by this method, preceded by the process called "equalizing;" which consists in drawing through the curved boundaries a set of straight lines so as to enclose, as nearly as the eye can judge, the same area.

II. *The Method of Ordinates* is applicable to a long piece of ground of varying breadth, such as the stripe of land required for a railway; or the area represented in fig. 17. An axis is drawn along the greatest length of the figure; breadths are measured along ordinates at right angles to that axis, sufficiently close together to make the spaces between them approximate to trapezoids. Then let d be the distance along the axis between two adjacent ordinates, and b, b', the breadths of the figure at those ordinates; the area contained between that pair of ordinates is—

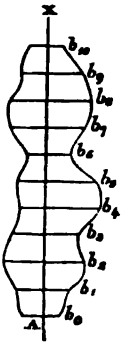

Fig. 17.

$$\frac{b+b'}{2}\cdot d;$$

and the area of the whole figure, being the sum of the areas of the parts into which it is divided by ordinates, is expressed as follows:—

$$\text{Area} = \Sigma \cdot \left(\frac{b+b'}{2}\cdot d\right);\dots\dots\dots\dots\dots\dots\dots(4.)$$

Σ being a symbol of summation.

If the ordinates are at equal distances apart, all the values of d are equal, and the preceding formulæ becomes

$$\text{Area} = \left(\frac{b_o}{2} + b_1 + b_2 + b_3 + \&c\dots + \frac{b_n}{2}\right)\cdot d;\dots\dots\dots\dots(5.)$$

b_o and b_n being the breadths at the two ends of the figure, and b_1, b_2, &c., the intermediate breadths.

A modification of the last formula, founded on the assumption that the lateral boundaries of the figure consist of short parabolic arcs, is as follows, the number of divisions being even:—

$$\text{Area} = \left\{b_o + b_n + 2(b_2 + b_4 + \&c.\dots) + (4\,b_1 + b_3 + \&c.\dots)\right\}\cdot\frac{d}{3}\dots(6.)$$

The most accurate way to find the areas of all the pieces of land included in a survey, is to use the dimensions as given in the field-book alone, calculating the areas of the triangles by formula 1 or 2, and the areas of the stripes of land lying between the station-lines and the fences surveyed from them by formula 4, in which b and b' are to be taken to represent a pair of adjacent offsets, and d the distance between them.

This process, however, is very laborious, and may in many cases be dispensed with, by equalizing boundaries and taking measurements on the plan.

III. *Method by Mechanism.*—Instruments for measuring areas on plans by mechanism are called "Planimeters" and "Platometers;" and several have been contrived by different inventors; amongst others, General Morin and Mr. Sang.

The simplest Planimeter is Amstler's, of which a sketch, showing its general principle, is given in fig. 18. A is a loaded disc which rests on the table, and serves as a fixed support for the instrument. In its centre, at B, is an upright pin, upon which turns the arm B C, to which at C is hinged the arm C D; so that the tracing

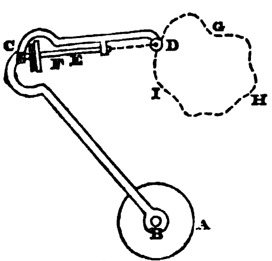

Fig. 18.

point at D can be moved in all directions over the paper. Exactly in the straight line C D is the axis E of the small wheel F, whose edge rests on the paper.

When the tracing point, D, is carried round the outline of any figure, such as G H I, so as to return finally to the point from which it started, it can be proved, that

$$\textit{Distance rolled by the edge of the wheel } \mathrm{F} = \frac{\textit{Area of Figure}}{\mathrm{C\,D}};$$

and consequently that

$$\text{Area of Figure} = \mathrm{C\,D} \times \text{Distance rolled by the wheel F.}$$

C D is a measured constant length. The distance rolled by the wheel is measured by a graduated circle and vernier at one side of the wheel; the number of complete revolutions being recorded by another wheel, driven by an endless screw on the shaft E. This wheel and screw are omitted in the sketch. In Britain, the graduations on the circle usually represent square inches of area on the paper.

CHAPTER III.

OF SURVEYING BY ANGULAR MEASUREMENTS.

33. Summary of Trigonometrical Formulæ used in Surveying.

I. *Relations between Angles and Arcs.*—The *angle* or *difference of direction*, B A C, between two straight lines, A B, A C, which meet at the point A, is expressed as a quantity, as is well known, by stating how many of certain aliquot parts of a right angle it contains; those parts being, the *degree*, or ninetieth part of a right angle, the *minute*, or sixtieth part of a degree, the *second*, or sixtieth part of a minute, and the decimal fractions of a second. This mode of expressing angles is the most convenient for trigonometrical calculation. Another way of representing the same method is to conceive that a circle D E F is described about A in the plane of A B and A C with any radius; that the circumference of that circle is divided into 360 equal arcs called degrees, each degree into 60 minutes, each minute into 60 seconds, and so on: and that the number of such divisions of the circle contained in the arc D E which subtends the angle B A C is ascertained.

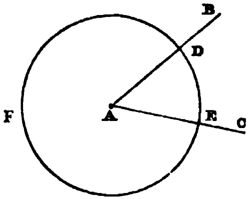

Fig. 19.

A second method of expressing the angle B A C is to take the ratio which the circular arc D E subtending it bears to the radius A D. In this case the angle is said to be expressed in terms of *arc to radius unity*. This method, though less simple than the former, and less commonly employed, is useful in certain cases.

The two methods of expressing the same angle are compared with each other by the aid of our knowledge of the ratio which the circumference of a circle bears to its diameter: which ratio, although it cannot be expressed with absolute exactness by any number of arithmetical figures, can be calculated to any required degree of accuracy by successive approximations. It has been computed to about 250 places of decimals; but seven places of decimals are sufficient for ordinary purposes. The following table gives that ratio to eight places of decimals, with its common logarithm, and several ratios and logarithms deduced from it:—

	Ratios.	Logarithms.
Circumference of a circle to diameter 1; = Length of a semicircle to radius 1; = Area of a circle to radius 1 = π;	3·14159265	0·4971499
Quadrant, or arc subtending a right angle, to radius 1;	$\frac{\pi}{2}$ = 1·57079632	0·1961199
Arc subtending one degree to radius 1;..............................	= $\frac{\pi}{180}$ = 0·01745329	8·2418774
Arc subtending one minute to radius 1;..............................	0·0002908882	6·4637261
Arc subtending one second to radius 1;..............................	0·000004848137	4·6855749
Arc equal to radius, expressed in degrees,	57°·2957795	1·7581226
— — — in minutes,	3437′·747	3·5362739
— — — in seconds,	206264″·8	5·3144254
— — — in degrees, minutes, and seconds,	57° 17′ 44″·8	
— — — in decimal fractions of the circumference,	$\frac{1}{2\pi}$ = 0·1591551	9·2018201
Surface of a hemisphere to radius 1; ...	2π = 6·2831853	0·7981799

The indices of the logarithms of fractions in the above table are affixed according to the system which is employed in trigonometrical calculations in order to avoid negative indices; that is to say, the index in each case is the complement of the proper negative index to 10; or the logarithm is that of the product of the fraction into 10,000,000,000.

The "centesimal" division of the quadrant into 100 degrees or "grades," 10,000 minutes, and 1,000,000 seconds is now nearly obsolete, even in France.

II. *Relations amongst Trigonometrical Functions of One Angle.*—The simplest mode of defining the trigonometrical functions of a given angle, such as the sine, cosine, &c., is to state that they are the ratios to each other of the sides of a right-angled triangle containing the given angle. Another mode, and the more common, is to state that they are represented by lines drawn in particular positions with respect to a circular arc of the radius unity, subtending the given angle.

In fig. 20, and also in fig. 21, A B, A C, are a pair of straight lines making with each other an *acute* angle, B A C.

In fig. 20, C is any point whatsoever in one of those lines, and C B a perpendicular let fall from that point upon the other line, so as to form a right-angled triangle, A B C.

In fig. 21, a circle of the radius unity is described about A, cutting off from each of the two lines a part equal to the radius, viz.:—

$$AD = AC = 1.$$

A F is a third radius, perpendicular to A D.

C B and C H are perpendiculars let fall from C upon A D and A F respectively; D E and F G are straight lines touching the circle at D and F (perpendicular, therefore, to A D and A F), and cutting A C produced in E and G respectively.

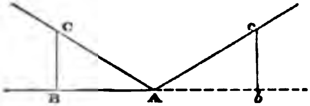

Fig. 20.

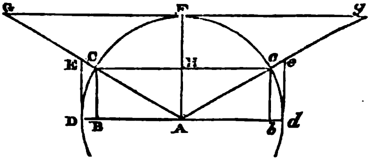

Fig. 21.

Then the definitions of the several trigonometrical functions of the angle B A C, according to the two methods, are as follows:—

	In Fig. 20.	In Fig. 21.
Sine, ...	$\frac{BC}{AC}$	..................$BC = AH$
Cosine, ...	$\frac{AB}{AC}$	..................$AB = CH$
Versed Sine, ...	$\frac{AC - AB}{AC}$	..................B D
Coversed Sine, ...	$\frac{AC - BC}{AC}.$	..................H F
Tangent, ...	$\frac{BC}{AB}$	..................D E
Cotangent, ...	$\frac{AB}{BC}$	..................F G
Secant, ...	$\frac{AC}{AB}$	..................A E
Cosecant, ...	$\frac{AC}{BC}$	..................A G

In fig. 20, the angles B A C and B C A are *complementary* to each other, being together equal to a right angle; so also are the angles B A C and C A F in fig. 21; and when this relation exists between a pair of angles, the sine of each is the cosine of the other, and so of all the other functions by pairs.

Denoting the angle B A C for brevity's sake by A, the following equations give the most important relations amongst its trigonometrical functions:—

$$\sin A = \sqrt{1-\cos^2 A} = \frac{\tan A}{\sec. A} = \frac{1}{\operatorname{cosec} A};\ldots\ldots (1.)$$

$$\cos A = \sqrt{1-\sin^2 A} = \frac{\operatorname{cotan} A}{\operatorname{cosec} A} = \frac{1}{\sec A};\ldots\ldots (2.)$$

$$\operatorname{versin} A = 1 - \cos A;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

$$\operatorname{coversin} A = 1 - \sin A;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{1}{\operatorname{cotan} A} = \sin A \cdot \sec A = \sqrt{\sec^2 A - 1};\ (5.)$$

$$\operatorname{cotan} A = \frac{\cos A}{\sin A} = \frac{1}{\tan A} = \cos A \cdot \operatorname{cosec} A = \sqrt{\operatorname{cosec}^2 A - 1};\ (6.)$$

$$\sec A = \frac{1}{\cos A} = \sqrt{1+\tan^2 A};\ldots\ldots\ldots\ldots\ldots (7.)$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \sqrt{1+\operatorname{cotan}^2 A}.\ldots\ldots\ldots\ldots (8.)$$

The trigonometrical functions of an obtuse angle are defined as follows:—

In fig. 20, and also in fig. 21, let A c be a straight line making with the line B A *produced beyond* A an angle b A c = B A C. Then the obtuse angle B A c is the *supplement* of the acute angle B A C, and is denoted by

$$180^\circ - A.$$

From c in both figures let fall $c\,b$ perpendicular to A b. In fig. 21. draw c H perpendicular to A F, and the tangents $d\,e$, F g, cutting A c produced in e and g.

Then in fig. 20, the right-angled triangle A b c is similar to A B C; and in fig. 21, the combination of lines on the right of A F is similar and equal to the combination of lines on the left; from which it appears, that all the trigonometrical functions of the obtuse angle 180° — A (with one exception to be presently pointed out), are equal in *numerical value* to the corresponding functions of its supplementary acute angle A. The one exception is the versed sine, which in fig. 21 is represented by $D\,b = A\,D + A\,b = 2\,A\,D - D\,B$.

In order the better to distinguish between trigonometrical functions of acute and obtuse angles, the principle is adopted, that inasmuch as A B and A b (in both figures) lie in *opposite directions* from A, they shall be regarded as having opposite signs:—that is, A B being positive, A b is negative; which amounts to laying down the rule, that *cosines of obtuse angles are negative.* The following are the relations between the trigonometrical functions of an obtuse angle 180° — A, and its supplementary acute angle A, which arise from that rule:—

$$\left.\begin{aligned}
&\sin (180^\circ - A) = \sin A; \\
&\cos (180^\circ - A) = -\cos A; \\
&\text{versin} (180^\circ - A) = 1 + \cos A = 2 - \text{versin } A; \\
&\text{coversin} (180^\circ - A) = \text{coversin } A; \\
&\tan (180^\circ - A) = -\tan A; \\
&\text{cotan} (180^\circ - A) = -\text{cotan } A; \\
&\sec (180^\circ - A) = -\sec A; \\
&\text{cosec} (180^\circ - A) = \text{cosec } A.
\end{aligned}\right\} \quad (9.)$$

From these equations it is to be understood, that in applying to obtuse angles trigonometrical formulæ which were originally intended for acute angles, the algebraical signs of all sines and cosecants of such angles are to be kept unchanged, and those of cosines, tangents, cotangents, and secants reversed.

In analytical geometry a further distinction is drawn between the *sines* of angles, whether acute or obtuse, lying to the right and left of a fixed direction, which are regarded as positive and negative respectively. In geodesy it is unnecessary to introduce that distinction, except in one case, to be explained afterwards.

III.—*Trigonometrical Functions of Two Angles.*

$$\sin A = 2 \sin \frac{A}{2} . \cos \frac{A}{2} = \frac{2}{\text{cotan} \frac{A}{2} + \tan \frac{A}{2}} = \sqrt{\frac{1 - \cos 2 A}{2}} . \quad (10.)$$

$$\left.\begin{aligned}
&\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} = 1 - 2 \sin^2 \frac{A}{2} = 2 \cos^2 \frac{A}{2} - 1 = \\
&\qquad \frac{\text{cotan} \frac{A}{2} - \tan \frac{A}{2}}{\text{cotan} \frac{A}{2} + \tan \frac{A}{2}} = \sqrt{\frac{1 + \cos 2 A}{2}} .
\end{aligned}\right\} \quad (11.)$$

$$\left.\begin{array}{l}\tan A = \dfrac{2}{\operatorname{cotan}\frac{A}{2} - \tan\frac{A}{2}} = \sqrt{\dfrac{1-\cos 2A}{1+\cos 2A}} = \\ \qquad\dfrac{\sin 2A}{1+\cos 2A} = \dfrac{1-\cos 2A}{\sin 2A}.\end{array}\right\}(12.)$$

Let A and B be any two angles.

$$\sin(A+B) = \sin A \cos B + \cos A \sin B; \ldots\ldots (13.)$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B; \ldots\ldots (14.)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B; \ldots\ldots (15.)$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B; \ldots\ldots (16.)$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}; \ldots\ldots\ldots\ldots\ldots\ldots (17.)$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}; \ldots\ldots\ldots\ldots (18.)$$

$$\sin A + \sin B = 2 \sin\frac{A+B}{2} \cdot \cos\frac{A-B}{2}; \ldots (19.)$$

$$\sin A - \sin B = 2 \sin\frac{A-B}{2} \cdot \cos\frac{A+B}{2}; \ldots\ldots (20.)$$

$$\cos A + \cos B = 2 \cos\frac{A+B}{2} \cdot \cos\frac{A-B}{2}; \ldots (21.)$$

$$\cos B - \cos A = 2 \sin\frac{A-B}{2} \cdot \sin\frac{A+B}{2}; \ldots (22.)$$

$$\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cdot \cos B}; \ldots\ldots\ldots\ldots (23.)$$

$$\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}; \ldots\ldots\ldots\ldots (24.)$$

$$\operatorname{cotan} A + \operatorname{cotan} B = \frac{\sin(A+B)}{\sin A \cdot \sin B}; \ldots\ldots\ldots (25.)$$

$$\operatorname{cotan} B - \operatorname{cotan} A = \frac{\sin(A-B)}{\sin A \sin B}; \ldots\ldots\ldots (26.)$$

$$\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A - B) \cdot \sin(A + B); \ldots\ldots \quad (27.)$$

$$\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A - B) \cos(A + B); \ldots\ldots \quad (28.)$$

$$\tan^2 A - \tan^2 B = \frac{\sin(A - B)\sin(A + B)}{\cos^2 A \cdot \cos^2 B}; \ldots\ldots \quad (29.)$$

$$\cot^2 B - \cot^2 A = \frac{\sin A - B \cdot \sin A + B}{\sin^2 A \cdot \sin^2 B}; \ldots\ldots \quad (30.)$$

IV. *Formulæ for the Solution of Plane Triangles.*—All these formulæ are deduced from the two following principles :—

The sum of the three angles of a plane triangle is equal to two right angles.

The sides of a plane triangle are proportional to the sines of the opposite angles.

When the computations are to be made without the aid of logarithms, the simplest formulæ are the best; but when logarithms are used, formulæ of greater complexity are often employed, in order, as far as possible, to dispense with additions and subtractions, and make the calculation consist of multiplications and divisions.

Fig. 22 represents a plane triangle, whose three angles are denoted by A, B, C, and the three sides respectively opposite them by *a*, *b*, *c*.

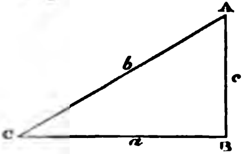

Fig. 22.

The following equations express in various forms the relation between the three angles, and enable this problem to be solved, *given, two of the angles, or trigonometrical functions of them : to find the third angle, or a trigonometrical function of it.*

$$A + B + C = 180^\circ; \ldots\ldots\ldots\ldots \quad (31.)$$

Let A and B be given; then

$$C = 180^\circ - A - B; \ldots\ldots\ldots\ldots \quad (32.)$$

$$\sin C = \sin(A + B); \ \cos C = -\cos(A + B); \ldots \quad (33.)$$

$$\tan C = \frac{\tan A + \tan B}{\tan A \tan B - 1}; \ \tan\frac{C}{2} = \cot\frac{A + B}{2}. \ldots \quad (34.)$$

PROBLEM FIRST.—**When the Angles and one Side are given,** let a be the given side; then the other two sides are

$$b = a \cdot \frac{\sin B}{\sin A}; \quad c = a \cdot \frac{\sin C}{\sin A}; \quad \ldots\ldots\ldots\ldots\ldots \quad (35.)$$

or by logarithms,

$$\left.\begin{array}{l} \log b = \log a + \log \sin B - \log \sin A; \\ \log c = \log a + \log \sin C - \log \sin A. \end{array}\right\} \ldots\ldots \quad (35\ \text{A.})$$

PROBLEM SECOND.—**When Two Sides and the Included Angle are given,** let a, b, be the given sides, C the given included angle; then

1. *To find the third side*, the simplest formula is,

$$c = \sqrt{(a^2 + b^2 - 2\,a\,b \cos C)}; \quad \ldots\ldots\ldots\ldots\ldots \quad (36.)$$

(observing, that if C is obtuse, the third term within the brackets is to be added instead of subtracted).

But this formula being unsuitable for logarithmic calculation, one or other of the following processes is substituted for it.

First Method:—

$$\text{make } \sin D = \frac{2\sqrt{a\,b}}{a+b} \cdot \cos\frac{C}{2}; \text{ then}$$

$$c = (a+b) \cos D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (37.)$$

Second Method:—

$$\text{make } \tan E = \frac{2\sqrt{a\,b}}{a-b} \cdot \sin\frac{C}{2}; \text{ then}$$

$$c = (a-b) \sec E \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (38.)$$

2. *To find the remaining angles*, A and B.

If the third side has been computed,

$$\sin A = \frac{a}{c} \cdot \sin C; \quad \sin B = \frac{b}{c} \cdot \sin C \quad \ldots\ldots\ldots\ldots \quad (39.)$$

If the third side has not been computed,

$$\left.\begin{array}{l} \tan \cdot \dfrac{A+B}{2} = \operatorname{cotan}\dfrac{C}{2}; \ \tan\dfrac{A-B}{2} = \dfrac{a-b}{a+b} \operatorname{cotan}\dfrac{C}{2}; \\ A = \dfrac{A+B}{2} + \dfrac{A-B}{2}; \ B = \dfrac{A+B}{2} - \dfrac{A-B}{2}. \end{array}\right\} \quad (40.)$$

PROBLEM THIRD.—When the Three Sides are given.

To find any one of the angles, such as C, the simplest formula is the following :—

$$\cos C = \frac{a^2 + b^2 - c^2}{2\,a\,b}\,; \;\dots\dots\dots\dots\dots\dots\; (41.)$$

but this formula being unsuited for logarithmic calculation, one or other of the four following formulæ is employed instead when logarithms are used. Let the half-sum of the sides of the triangle be denoted by

$$s = \frac{a + b + c}{2}\,; \text{ then}$$

$$\left.\begin{array}{l} \cos\dfrac{C}{2} = \sqrt{\dfrac{s\,(s - c)}{a\,b}}\,;\ \sin\dfrac{C}{2} = \sqrt{\dfrac{(s - a)\,(s - b)}{a\,b}}. \\ \operatorname{cotan}\dfrac{C}{2} = \sqrt{\dfrac{s \cdot (s - c)}{(s - a)\,(s - b)}}\,;\ \tan\dfrac{C}{2} = \sqrt{\dfrac{(s - a)\,(s - b)}{s\,(s - c)}}. \end{array}\right\}\ (42.)$$

When $\frac{C}{2}$ is a large angle, the expressions for cos $\frac{C}{2}$ and cotan $\frac{C}{2}$ are the most convenient in calculation; when it is a small angle, those for sin $\frac{C}{2}$ and tan $\frac{C}{2}$ are to be preferred. A fifth formula, less used than the preceding, is

$$\sin C = \frac{2\sqrt{s\,(s - a)\,(s - b)\,(s - c)}}{a\,b}\,; \;\dots\dots\dots\dots\; (43)$$

but this is unsuitable if C is nearly a right angle.

PROBLEM FOURTH.—Two Sides given, and the Angle opposite one of them.—In fig. 23, let A be the given angle, and a, c, the given sides, of which a is opposite A. The sine of the angle opposite c is given by the expression,

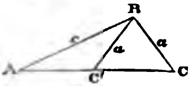

Fig. 23.

$$\frac{c}{a}.\sin A\,; \;\dots\dots\dots\dots\dots\dots\dots\dots\; (44.)$$

but this may apply either to the acute angle C or to its supplement, the obtuse angle C′= 180° – C; C and C′ being the two points in the straight line A C′ C which are at the distance a from B. Unless, therefore, it is known by observation whether the angle opposite the side c is acute or obtuse, the solution of the problem is ambiguous. Should that, however, be known, the angles can be computed, and thence the remaining side, by the method of the first problem. In general, problems that fall under the fourth case ought to be avoided in surveying, especially when the angle opposite c is nearly a right angle.

In all trigonometrical problems, it is to be borne in mind, that small acute angles, and large obtuse angles, are most accurately determined by means of their *sines*, *tangents*, and *cosecants*, and angles approaching a right angle by their *cosines*, *cotangents*, and *secants*.

PROBLEM FIFTH.—**To solve a Right-angled Triangle.**—All the preceding formulæ are applicable to this case; but they become very much simplified owing to the values assumed by the trigonometrical functions of the right angle, viz.:—

Sin $90° = 1$; cos $90° = 0$; tan $90°$ infinite; cotan $90° = 0$; sec $90°$ infinite; cosec $90° = 1$.

Let C denote the right angle; c the hypothenuse; A and B the two oblique angles; a and b the sides respectively opposite them. Then A and B are *complementary angles*, and the sine of each is the cosine of the other, as explained under Head II. of this article. The following cases may be distinguished:—

1. Given, the right angle, another angle B, the hypothenuse c. Then

$$A = 90° - B;\ a = c \cdot \cos B;\ b = c \sin B. \ \dots\dots\dots \ (45.)$$

2. Given, the right angle, another angle B, a side a,

$$A = 90° - B;\ b = a \cdot \tan B;\ c = a \cdot \sec B \ \dots\dots \ (46.)$$

3. Given, the right angle, and the sides a, b,

$$\tan A = \frac{a}{b};\ \tan B = \frac{b}{a};\ c = \sqrt{a^2 + b^2} \ \dots\dots\dots \ (47.)$$

4. Given, the right angle, the hypothenuse c; a side a,

$$\sin A = \cos B = \frac{a}{c};\ b = \sqrt{c^2 - a^2}. \ \dots\dots\dots\dots \ (48.)$$

5. Given, the three sides a, b, c, which fulfilling the equation $c^2 = a^2 + b^2$, the triangle is known to be right-angled at C.

$$\sin A = \frac{a}{c};\ \sin B = \frac{b}{c}. \ \dots\dots\dots\dots\dots\dots \ (49.)$$

Problem Sixth.—To express the area of a plane triangle in terms of its sides and angles.

Case 1. Given, one side, c, and the angles.

$$\text{Area} = \frac{c^2}{2} \cdot \frac{\sin A \sin B}{\sin C} \quad \ldots\ldots\ldots\ldots\ldots \tag{50.}$$

Case 2. Given two sides, b, c, and the included angle A.

$$\text{Area} = \frac{b\,c \cdot \sin A}{2} \quad \ldots\ldots\ldots\ldots\ldots \tag{51.}$$

Case 3. Given, the three sides. See Article 32, page 33.

V. *Formulæ for the Solution of Spherical Triangles.*

These formulæ are all consequences of the two following principles:—

The sum of the three angles of a spherical triangle exceeds two right angles by an angle which bears the same proportion to four right angles that the area of the triangle bears to the surface of the hemisphere.

The sines of the angles of a spherical triangle are proportional to the sines of the angles subtended at the centre of the sphere by the sides to which they are respectively opposite.

Problem First.—To compute approximately the angles subtended by arcs on the earth's surface, and *vice versâ*.

In this calculation it is sufficiently accurate for the purposes of engineering geodesy to treat the earth's surface as a sphere of the diameter stated in Article 3, p. 2, viz.:—

$$41{,}778{,}000 \text{ feet} = 7912\tfrac{1}{2} \text{ statute miles;}$$

so that, referring to the present Article, Division I., p. 37, for the proportions borne to the radius by arcs subtending various units of angle, we find, for the mean lengths of such arcs on great circles of the earth's surface, the following values,—[*]

---

[*] If it is desired to compute the lengths of small arcs on great circles more precisely, the following formulæ may be used:—

Let δ denote the difference between the latitude of the place and 45°, the sign + or — indicating whether that latitude is greater or less than 45°. Then the length in feet of an arc of the meridian which subtends one minute is

$$m = 6076{\cdot}36 \left(1 \pm \frac{\sin 2\,\delta}{200}\right); \quad \ldots\ldots\ldots\ldots\ldots \tag{1.}$$

	Mean length in Feet.	Logarithm.
Arc equal to radius,..................	20,889,000	7·3199176
Arc subtending one degree,.........	364,582	5·5617950
Arc subtending one minute,........	6076·36	3·7836437
Arc subtending one second,.........	101·273	2·0054925

Hence, let a be the length in feet of an arc on the earth's surface; α, the angle subtended by it in seconds; then

$$\alpha = a \div 101{\cdot}273 \text{ nearly. .... (52.)}$$

Problem Second.—To compute approximately the *sines* of the angles subtended by small arcs on the earth's surface, and *vice versâ*.

Let $\frac{a}{r}$ be the ratio of a small arc a to the earth's radius r; α the angle subtended by it. Then it is known that the two following

---

the length in feet of an arc subtending one minute, on a great circle perpendicular to the meridian, is

$$m' = 6076{\cdot}36\left(1 + \frac{1}{300} \pm \frac{\sin 2\lambda}{600}\right); \text{ .... (2.)}$$

and the length in feet of an arc subtending one minute on a great circle which makes an angle θ with the meridian, is

$$m'' = m \cos^2\theta + m' \sin^2\theta. \text{ .... (3.)}$$

The mean length of all the arcs subtending one minute on great circles which can be drawn through a given point is

$$\frac{m + m'}{2} = 6076{\cdot}36\left(1 + \frac{1}{600} \pm \frac{\sin 2\lambda}{300}\right). \text{ .... (4.)}$$

At the parallel of 30° of latitude, which divides the surface of the hemispheroid into two nearly equal parts, the factor of this expression within the brackets is reduced to unity, and the length of the arc to its mean value; and the area of the surface of the spheroid is almost exactly equal to that of a sphere of the radius of 20,889,000 feet, corresponding to that value of the arc. It is for these reasons that 6076·36 feet has been adopted in this work as the true mean length of a nautical mile, rather than the length of a minute of the equator.

The area in square feet of a square, each of whose sides subtends one minute at a given latitude, is

$$m\,m' = (6076{\cdot}36)^2 \cdot \left(1 + \frac{1}{300} \pm \frac{\sin 2\lambda}{150}\right) \text{nearly; .... (5.)}$$

and this quantity also has its mean value at the parallel of 30°; viz. $(6076{\cdot}36)^2$.

The length in feet of a minute of longitude is given by the formula

$$m''' = m' \cdot \cos \cdot \text{latitude} \text{ .... (6.)}$$

In the preceding formulæ, the figure of a level surface is treated as if it were an exact spheroid of revolution with the polar and equatorial diameters in the ratio of 599 to 601, and no account is taken of various irregularities in the form of that surface, whose existence has been proved, but which have not yet been reduced to any general principle.

series give approximations to the value of each of those quantities in terms of the other;

$$\sin \alpha = \frac{a}{r} - \frac{1}{1\cdot 2\cdot 3}\cdot\frac{a^3}{r^3} + \frac{1}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot\frac{a^5}{r^5} - \&c. \ldots\ldots \text{(53.)}$$

$$\frac{a}{r} = \sin \alpha + \frac{1}{1\cdot 2\cdot 3}\cdot \sin^3 \alpha + \frac{1\cdot 3}{2\cdot 4\cdot 5} \sin^5 \alpha + \&c. \ldots \text{(54.)}$$

In most cases which occur in engineering geodesy, the first two terms of each of those series are sufficient, and they may be thus expressed :—

$$\sin \alpha = \frac{a}{r}\left(1 - \frac{a^2}{6r^2}\right) \text{nearly} ; \ldots\ldots\ldots\ldots\ldots \text{(55.)}$$

$$\frac{a}{r} = \sin \alpha \left(1 + \frac{\sin^2 \alpha}{6}\right) \text{nearly}. \ldots\ldots\ldots\ldots \text{(56.)}$$

For logarithmic calculation the following approximate formulæ are convenient :—

$$\log \sin \alpha = \log \frac{a}{r} - \cdot 0723824 \frac{a^2}{r^2}$$

$$= \log a \text{ (in feet)} - 7\cdot 3199176 - \cdot 0723824 \frac{a^2}{r^2} ; \ldots \text{(55 A.)}$$

$$\log a \text{ (in feet)} = 7\cdot 3199176 + \log \sin \alpha + \cdot 0723824 \sin^2 \alpha \text{ (56 A.)}$$

(· 0723824 = modulus of the common logarithms ÷ 6.)

PROBLEM THIRD.—Given, the area of a spherical triangle on the earth's surface; to find the excess of the sum of the three angles above two right angles (or as it is called, the "spherical excess").

Let S be the area of the triangle, r the earth's radius, X the spherical excess; then

$$X = 360^\circ \cdot \frac{S}{2\pi r^2}$$

$$= \text{angle subtended by arc equal to radius} \cdot \frac{S}{r^2} ; \ldots\ldots \text{(57.)}$$

that is to say—

$$X \text{ (in seconds)} = \frac{206264\cdot 8 \text{ S (in square feet)}}{436{,}350{,}321{,}000{,}000}$$

$$= \frac{\text{S (in square feet)}}{2{,}115{,}500{,}000 \text{ nearly}} ; \ldots\ldots\ldots\ldots\ldots \text{(58.)}$$

or by logarithms,

$$\log X \text{ (in seconds)} = \log S \text{ (in square feet)} - 9\cdot 3254101 \ldots \text{(58 A.)}$$

In stating rules for the solution of spherical triangles, the word "side" is used for brevity's sake, when "the angle subtended by a side at the centre of the sphere" is meant. In fig. 24, A, B, C are the three angles of a spherical triangle; a, b, c, the sides respectively opposite them. The angles subtended respectively by these sides, which angles are called "the sides" in stating the rules, will be denoted by α, β, γ.

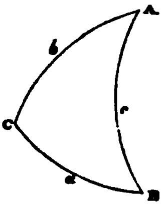

Fig. 24.

Problem Fourth.—Given, two angles of a spherical triangle, and the side between them; to find the remaining sides and angle—

Let A, B be the given angles, and γ the given side. Then to find the remaining sides α and β—

$$\left.\begin{aligned} \tan\frac{\alpha+\beta}{2} &= \tan\frac{\gamma}{2}\cdot\frac{\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}}; \\ \tan\frac{\alpha-\beta}{2} &= \tan\frac{\gamma}{2}\cdot\frac{\sin\frac{A-B}{2}}{\sin\frac{A+B}{2}}; \\ \alpha=\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2};\ \beta &= \frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}. \end{aligned}\right\}\dots\dots\dots(59.)$$

To find the remaining angle C, we have the proportion—

$$\sin\alpha : \sin\beta : \sin\gamma :: \sin A : \sin B : \sin C.\dots\dots\dots(60.)$$

Problem Fifth.—Given, two sides of a spherical triangle and the angle between them; to find the remaining side and angle—

Let α, β be the given sides; C, the given angle.

First Method.—To find the remaining side γ;

$$\cos\gamma = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta\cdot\cos C;\ \dots\dots\dots(61.)$$

but this formula being unsuited to calculation by logarithms, the following has been deduced from it;

$$\text{Make } \sin D = \cos\frac{C}{2}\cdot\sqrt{\sin\alpha\cdot\sin\beta};\ \text{then}$$

$$\sin\frac{\gamma}{2} = \sqrt{\left\{\sin\left(\frac{\alpha+\beta}{2}+D\right)\cdot\sin\left(\frac{\alpha+\beta}{2}-D\right)\right\}};\ (62.)$$

and to find the remaining angles, we have the proportion,

$$\sin\gamma : \sin\alpha : \sin\beta :: \sin C : \sin A : \sin B.\ \dots\dots(63.)$$

Second Method.—To find the remaining angles, A, B.

$$\left.\begin{array}{l}
\tan\dfrac{A+B}{2}=\dfrac{\cos\dfrac{\alpha-\beta}{2}\,.\,\mathrm{cotan}\,\dfrac{C}{2}}{\cos\dfrac{\alpha+\beta}{2}};\\[2ex]
\tan\dfrac{A-B}{2}=\dfrac{\sin\dfrac{\alpha-\beta}{2}\cdot\mathrm{cotan}\,\dfrac{C}{2}}{\sin\dfrac{\alpha+\beta}{2}};\\[2ex]
A=\dfrac{A+B}{2}+\dfrac{A-B}{2};\ B=\dfrac{A+B}{2}-\dfrac{A-B}{2}.
\end{array}\right\}\ldots\ldots(64.)$$

The remaining side γ is found by the proportion (63).

Problem Sixth.—The three sides of a spherical triangle being given; to find the angles—

Let C be the angle sought in the first instance. Then

$$\cos C=\frac{\cos\gamma-\cos\alpha\cdot\cos\beta}{\sin\alpha\cdot\sin\beta};\quad\ldots\ldots\ldots\ldots\ldots(65.)$$

but as this formula is not adapted for logarithmic calculation, one or other of the following, which are deduced from it, is to be employed for that purpose:—

Let $\sigma=\dfrac{\alpha+\beta+\gamma}{2}$ denote the half sum of the sides;

$$\cos\frac{C}{2}=\sqrt{\frac{\sin\sigma\cdot\sin(\sigma-\gamma)}{\sin\alpha\cdot\sin\beta}};\ \sin\frac{C}{2}=\sqrt{\frac{\sin(\sigma-\alpha)(\sin\sigma-\beta)}{\sin\alpha\cdot\sin\beta}}.(66.)$$

$\cos\frac{C}{2}$ is best when $\frac{C}{2}$ approaches a right angle; $\sin\frac{C}{2}$ when $\frac{C}{2}$ is small.

These formulæ will serve alike to compute any angle. If it is desired to express the angle sought by A or by B, the following substitutions are to be made in the formulæ:—

For the following symbols in the formulæ for C, …	α	β	γ
Substitute respectively in the formulæ for A, …	β	γ	α
— — — — for B, …	γ	α	β

Problem Seventh.—In a right-angled spherical triangle, the right angle and any two other parts being given, to find the remaining parts.

Let C be the right angle, and γ the side opposite to it.

Case 1.—Two sides being given, the third is found by the equation—

$$\cos\alpha\cdot\cos\beta=\cos\gamma;\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(67.)$$

and the oblique angles by the equations—

$$\cos A = \operatorname{cotan} \gamma \cdot \tan \beta; \ \cos B = \operatorname{cotan} \gamma \cdot \tan \alpha; \ \ldots (68.)$$

or by the equations—

$$\operatorname{cotan} A = \operatorname{cotan} \alpha \cdot \sin \beta; \ \operatorname{cotan} B = \operatorname{cotan} \beta \cdot \sin \alpha \ldots (69.)$$

Case 2.—Given, a side (α) and the opposite angle (A). Find the side β by the formula—

$$\sin \beta = \tan \alpha \cdot \operatorname{cotan} A; \ \ldots\ldots\ldots\ldots (70.)$$

then find γ by (67) and B by (68) or (69).

Case 3.—Given, a side (α) and the adjacent angle (B). Find the side γ by the formula—

$$\operatorname{cotan} \gamma = \cos A \cdot \operatorname{cotan} \beta; \ \ldots\ldots\ldots\ldots (71.)$$

then find α by (67) and B by (68) or (69).

Case 4.—Given, two angles, A, B—

$$\cos \alpha = \frac{\cos A}{\sin B}; \ \cos \beta = \frac{\cos B}{\sin A}; \ \cos \gamma = \operatorname{cotan} A \cdot \operatorname{cotan} B. \ (72.)$$

VI. *Approximate Solutions of Spherical Triangles, used in Trigonometrical Surveying.*

As the largest triangles formed in trigonometrical surveying do not measure more than 100 miles in the side, and the ordinary triangles much less, the curvature of the arcs forming their sides is very slight, and their areas are very small fractions of that of a hemisphere of the earth; and consequently approximate methods of calculation can be applied to them, by which much of the labour is saved that would be required by a strict adherence to the rules of spherical trigonometry.

PROBLEM FIRST.—Given, in a triangle on the earth's surface the length of one side, c, and the adjacent angles, A, B; to find approximately the third angle, C.

Calculate, by equation 50, p. 46, the *approximate area* of the triangle, as if it were plane. From that area, by equation 58, or 58 A, p. 48, calculate the "spherical excess" X. Then

$$C = 180^\circ + X - A - B. \ \ldots\ldots\ldots\ldots (73.)$$

PROBLEM SECOND.—To find approximately the remaining sides, a, b, of the same triangle. Let α, β, γ be the angles subtended by the sides.

Method First (By spherical trigonometry).—Find the arc γ subtended by the given side c by equation 52, p. 47; or else find $\sin \gamma$ directly from c by equation 55 or 55 A, p. 48. Then

$$\sin \alpha = \frac{\sin A \sin \gamma}{\sin C}; \ \sin \beta = \frac{\sin B \sin \gamma}{\sin C}; \ \ldots\ldots (74.)$$

from which find α and β in seconds; then the lengths of the sides in feet are—

$$a = 101{\cdot}273\,\alpha;\ b = 101{\cdot}273\,\beta; \text{..............}(75.)$$

or a and b may be calculated directly from sin α and sin β by equation 56 or 56 A, p. 48.

Method Second (By plane trigonometry).—From each of the angles subtract one third of the spherical excess, and then treat the triangle as if it were plane. That is to say—

$$a = c \cdot \frac{\sin\left(A - \frac{X}{3}\right)}{\sin\left(C - \frac{X}{3}\right)};\ b = c \cdot \frac{\sin\left(B - \frac{X}{3}\right)}{\sin\left(C - \frac{X}{3}\right)} \text{.........}(76.)$$

PROBLEM THIRD.—Given, in a triangle on the earth's surface, two sides a, b, and the included angle C; to find the remaining side, c, and angles, A, B.

Method First (By spherical trigonometry).—As in the last problem, find the angles α, β, subtended by the sides, by means of equation 52, p. 47, or the sines of those angles by means of equation 56 A, p. 48. Then solve the triangle as a spherical triangle, by means of equations 62 and 63, p. 49, or equation 64, p. 50. Lastly, make

$$c \text{ in feet} = 101{\cdot}273\,\gamma \text{ in seconds. ....}(77.)$$

Method Second (By plane trigonometry).—Compute the *approximate area* by equation 51, p. 46, as if the triangle were plane; thence compute the spherical excess X by equation 58 or 58 A, p. 48, and deduct one-third of it from the given angle. Then consider the triangle as a plane triangle, in which are given the two sides a, b, and the included angle $C' = C - \frac{X}{3}$. Find the third side c by equation 37, or equation 38, p. 43; and the remaining angles A′, B′, of the supposed plane triangle, by the equations 39 or 40, p. 43; and for the remaining angles of the real spherical triangle, take

$$A = A' + \frac{X}{3};\ B = B' + \frac{X}{3}. \text{.................}(78.)$$

PROBLEM FOURTH.—To diminish as far as possible the effects of small errors in angular measurements.

Such small errors are detected by measuring the whole three angles of a spherical triangle, and adding them together. If the measurements are perfectly correct, we shall find

$$A + B + C = 180^\circ + X,$$

(X being the spherical excess, if it is appreciable). But if small errors have been committed in measuring the angles, we shall find

$$A' + B' + C' = 180^\circ + X \pm E,$$

where E is the total error. Then for the most probable values of the corrected angles are to be taken

$$A = A' \mp \frac{E}{3};\ B = B' \mp \frac{E}{3};\ C = C' \mp \frac{E}{3}. \quad \ldots\ldots(79.)$$

The correction of each angle being one-third of the total error, and opposite in sign.

34. The **Theodolite** is an instrument whose chief use is to measure angles in a horizontal plane, or "*azimuths*," and which is occasionally used to measure also vertical angles, or *altitudes and depressions*.

When the word *Azimuth* is used without qualification, it usually means the number of degrees, minutes, and seconds by which the direction of a vertical plane passing through a station and a given object deviates to the *right* of a vertical plane passing through the station and the North Pole. When "Magnetic Azimuth" is specified, the angular deviation is reckoned from the magnetic meridian instead of the true meridian.

But the relative azimuth of any two objects may be measured at a given station; that is to say, the number of degrees, minutes, and seconds by which a vertical plane traversing the station and one of the objects deviates to the right of a vertical plane passing through the station and the other object.

An azimuth exceeding 180° denotes that the direction of the object to which it is measured lies to the *left* of the direction from which azimuths are measured, by an angle equal to the difference between the azimuth and 360°.

For example: in fig. 25, let A denote a station; A B the direction of the pole, or, as the case may be, of the object from which azimuths are measured, and which is held to have the azimuth 0°. C being an object which lies to the right of A B, its azimuth, being the number of degrees, &c., subtended by the arc *b c*, is equal to the angle B A C. On the other hand, D being an object lying to the left of A B, its azimuth, being the number of degrees, &c., subtended by the arc *b′ d*, is equal to the difference between the angle B A D and 360°.

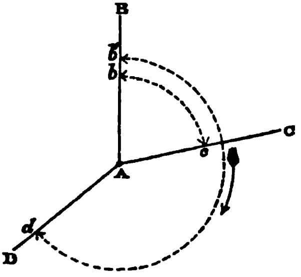

Fig. 25.

The horizontal angle between any two directions is the difference of their azimuths, if that difference is less than 180°; if it is greater than 180°, the excess of 360° above the difference of the azimuths is the angle between the directions.

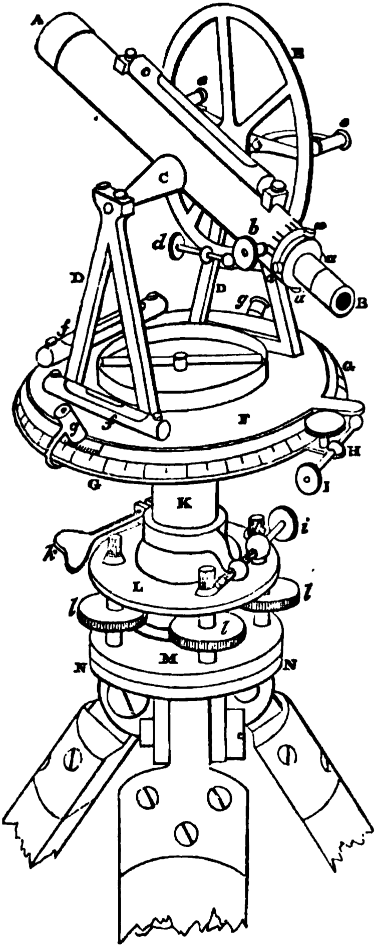

Fig. 26.

Altitudes and depressions are the angles, always acute, which the directions of objects, as seen from a given station, make above and below a horizontal plane. The use of these angles will be further explained under the head of Levelling.

The structure of theodolites varies very much; but there are certain essential parts which are common to all, and which will now be enumerated, commencing at the top, as they are found in the "Transit Theodolite." The usual material is brass, except for the "limb" or graduated ring of each of the circles, which is of silver or palladium.

I. The *Telescope* A B consists of two tubes, one sliding within the other. The outer tube has, at its further end A, the object-glass, which forms at its focus an inverted image of the object looked at. The inner tube has, at its nearer end B, a combination of glasses called the "eye-piece," which magnifies that inverted image. By the use of an additional tube and certain additional glasses, an "erecting eye-piece" may be formed, which makes the object appear erect but this causes loss of light, and possesses no particular advantage. By moving the inner tube inwards and outwards by a rack and pinion, turned by the milled head *b*, the foci of the object-glass and eye-piece are adjusted till they coincide, which is known by the distinct and steady appearance of the image.

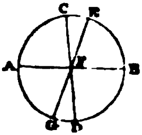

Fig. 27.

At the common focus of the object-glass and eye-piece, where the inverted image is seen, there is a "diaphragm" or partition, with a round hole in the middle crossed by three spider's lines, or equally fine platinum wires (see fig. 27); one horizontal, A B, and the other two, C D, E G, deviating slightly to opposite sides of a vertical plane. The point F where those wires cross each other should be exactly in the axis or "*line of collimation*" of the telescope; and the heads of four screws for adjusting it to that position are shown at *a*, *a*, *a*, *a*, in fig. 26.

II. The *Spirit-Level* *c* is attached to the outer telescope-tube by screws, by means of which it can be set exactly parallel to the line of collimation; so that when the air-bubble is in the centre of the level, the telescope is horizontal. The construction and use of spirit-levels will be further explained under the head of levelling.

III. The *Horizontal Axis* C, when the instrument is in adjustment, is exactly at right angles to the line of collimation, and exactly level; so that the telescope may turn about on the bearings of that axis in a truly vertical plane.

IV. The *Frames* or *Supports* (D, D,) of the horizontal axis are high enough, in the transit theodolite, to admit of the telescope being turned completely over in a vertical plane; a motion which is useful in making certain observations. In Colonel Everest's theodolite the supports are made low, for the sake of compactness; but the telescope may be turned completely over when required, by lifting the horizontal axis out of its bearings. In the common theodolite the telescope is not fixed in the middle of that axis, but is supported in two forked rests called Y's, at the ends of a bar which is fixed at right angles to the horizontal axis; so that the telescope can be turned end for end by lifting it out of the Y's. When not required to be lifted out, the telescope is clasped firmly in its Y's by two semicircular arcs called "clips," which are hinged to the Y's at one side and fastened with pins at the other.

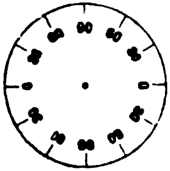

Fig. 28.

V. The *Vertical Circle* or *Altitude Circle* E is fixed upon the horizontal axis. It is divided into four quadrants, the degrees in each of which are numbered from 0° to 90°, as indicated in the sketch, fig. 28. The two 0's are at the ends of the diameter parallel to the line of collimation of the telescope; the two 90's at the ends of the diameter perpendicular to the former. There are two indices with verniers,* at opposite ends of a horizontal bar, read by the microscopes *e*, *e*; when the line of collimation is horizontal, each of those indices reads, or ought to read, 0°.

In directing the telescope to any object, it is turned at first by hand as nearly in the required direction as possible; then the vertical circle is "clamped," by turning a *clamp-screw* which lays hold of its lower edge; and then, by the *tangent-screw d*, a slow motion is given to the circle and telescope until the line of collimation points exactly towards the object.

In Colonel Everest's theodolite, instead of a complete vertical circle, there are two opposite sectors of about 90° each, so as to be capable of measuring altitudes and depressions as far as 45°; and the spirit-level is attached to the index-bar, instead of to the telescope.

In the common theodolite, instead of a vertical circle there is a semicircle only, having but one index and vernier.

VI. The *Vernier-Plate* F (fig. 26), is a circular plate, fixed on the top of, and exactly perpendicular to, the *inner vertical axis* (concealed in the figure). It carries at its sides the supports D, D, of the horizontal axis, in its centre a *magnetic compass* with a glass top, and near its edge a pair of *spirit-levels f*, *f*, at right angles to each other. At two points on its edge, diametrically opposite to each other, are two indices with verniers, read by means of the microscopes *g*, *g*. (In many theodolites there is but one microscope for this purpose, which is shifted round to the one or the other vernier as required.)

In Colonel Everest's theodolite the place of the lower horizontal circle is supplied by three horizontal arms diverging from the top of the inner vertical axis at equal angles of 120°, and having indices and verniers at their ends; and instead of the two spirit-levels *f*, *f*, there is one spirit-level fixed parallel to the horizontal axis.

VII. The *Horizontal Circle* G has its edge or limb bevelled to the figure of the frustum of a cone, and graduated; the degrees being numbered continuously round it towards the right, up to

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* According to the ordinary construction of a vernier, its total length consists of a number of divisions of the primary scale less by one than the number of smaller divisions into which those divisions are to be subdivided. Suppose, for example, that the limb of one of the circles of a theodolite is divided to third parts of a degree, or 20′, and that it is to be subdivided by a vernier to third parts of a minute, or 20″, each subdivision being *one-sixtieth* part of a primary division: the length of the vernier will be 60 — 1 = 59 divisions of the primary scale, and it will be divided into 60 equal parts, each equal to 59-60ths of a division of the primary scale.

360°, as indicated by the sketch, fig. 29. The faces of the verniers are portions of the same conical surface. An arm projecting from the vernier-plate (or in Colonel Everest's theodolite, from the inner vertical axis) carries the *clamp* H for laying hold of the circle after the telescope has been turned approximately towards an object by hand, and the *tangent-screw* I for giving the vernier-plate a slow motion until the line of collimation points exactly towards the object.

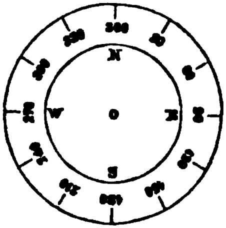

Fig. 29.

The size of the circles of a theodolite, both horizontal and vertical, and the minuteness of their graduations, depends on the extent and accuracy of the operations for which they are intended. Four inches and eight inches in diameter are about the extreme limits of diameter for horizontal circles in those made for any ordinary purpose, though a few have been made of larger sizes. Those most commonly used in surveying have circles of five inches in diameter, divided into half-degrees, and subdivided by the verniers to single minutes, and by estimation with the eye to half or quarter minutes. For such purposes as the principal triangulation of the survey for a line of railway, and for ranging curves, a larger theodolite is requisite: it is generally sufficient to use one with circles of six inches in diameter, divided to twenty minutes, and subdivided by the verniers to twenty seconds, and by estimation with the eye to ten seconds.

VIII. The *Outer Vertical Axis* K is fixed to the horizontal circle, and is a tube, containing within it and accurately fitting the inner vertical axis. It turns round on a ball-and-socket joint at its lower end; and is clamped in any required position by means of a collar with a tightening-screw *k*. From the collar projects an arm, acted upon by means of the tangent-screw *i*, so as to give a slow motion in azimuth to the horizontal circle when the outer vertical axis is clamped. The fixed nut of this screw is attached to—

IX. The *Upper Parallel Plate* L, through a cylindrical socket in which the outer vertical axis passes, so as to be always at right angles to it. The four *plate-screws* *l*, *l*, *l* (and a fourth concealed), serve to place the vertical axes truly vertical, by adjusting the position of the plate L relatively to—

X. The *Lower Parallel Plate* M, to the centre of which the outer vertical axis is attached by means of the ball-and-socket joint before mentioned. This plate is screwed upon—

XI. The *Staff-Head* N, which is supported by three strong wooden legs. In the middle of the lower side of the staff-head, directly under the vertical axis, is screwed a hook (concealed in the figure), from which a plummet is hung, in order to ascertain whether the centre of the theodolite is exactly over the station on the ground.

Instead of the upper parallel plate, Colonel Everest's theodolite has three diverging arms (fig. 30), as in an astronomical circle, with a vertical foot-screw supporting the end of each. The lower end of each screw has a shoulder, by means of which it is held down to the plate which forms the top of the staff-head; and those shoulders form the only attachment between the staff-head and the instrument. The chief advantage of this construction is, that the three foot-screws can be adjusted with one hand; whereas the adjustment of the four plate-screws in the ordinary construction requires both hands.

In some theodolites a second telescope is attached below the horizontal circle, in order, by directing that telescope on an object, to test whether the circle has been disturbed during the interval between two observations.

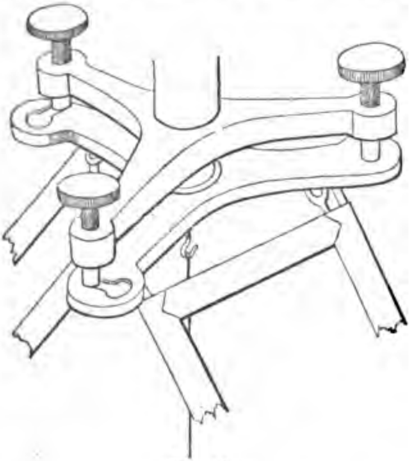

Fig 30.

35. **Adjustments of the Theodolite.**—The adjustments of the theodolite, as well as those of every other surveying instrument, may be distinguished into temporary adjustments, which are made by the user of the instrument each time that it is set up, and permanent adjustments, which are made by the manufacturer, and only tested and corrected occasionally by the user.

I. The *Temporary Adjustments* will now be described, on the supposition that the permanent adjustments are correct.

(1.) Place the theodolite at the station by the aid of the plumb-line mentioned in Division XI. of the last Article.

(2.) To "level the instrument"—that is, to place the vertical axis truly vertical—the easiest process is to make the vernier-plate truly horizontal by means of the spirit-levels *f*, *f*. For that purpose it is to be turned into such a position that the two spirit-levels *l*, *l* shall be parallel respectively to the two diagonals of the square formed by the plate-screws. Then the bubble is to be brought to the centre of each level by turning the pair of plate-screws to whose plane the level lies parallel.

A more exact adjustment, however, can be made by means of the level *c* attached to the telescope, because it is larger and more delicate than those attached to the vernier-plate. To effect this adjustment, turn the vernier-plate till the telescope is over one pair of plate-screws: by the aid of the tangent-screw *d*, adjust the vertical circle carefully to 0°; turn the pair of plate-screws under the telescope until the bubble is brought to the centre of the spirit-level: turn the vernier-plate round through 180°; if the bubble now deviates from the centre of the spirit-level, correct one-half of the deviation by the tangent-screw *d*, and the other half by the plate-screws: turn the vernier-plate through 90°, so as to bring the telescope over the other pair of plate-screws, by means of which bring the bubble to the centre of the level again: the vertical axis is now truly vertical.

If the bubbles are not now at the centres of the vernier-plate levels *f*, *f*, those levels are not truly perpendicular to the vertical axis; but the correction of this error belongs to the permanent adjustments.

In Colonel Everest's theodolite the vertical axis is adjusted by means of the level which is parallel to the horizontal axis, by first placing that level parallel to a line joining any two of the three foot-screws, and bringing the bubble to the centre by turning one or both of them, and then turning the upper part of the instrument through 90°, and bringing the bubble to the centre of the level in its new position by means of the third foot-screw.

(3.) To adjust the telescope for the prevention of "parallax"—that is, to bring the foci of the glasses to the cross-wires,—look through the telescope, and shift the eye-piece in and out until the cross-wires are seen with perfect distinctness. Then direct the telescope to some well-defined distant object, and by means of the milled head *b*, shift the inner tube in and out until the image of the object is seen sharp and clear, coinciding apparently with the cross-wires.

The latter part of this adjustment has to be made anew for each new object at a different distance from the preceding one. The nearer the object, the further must the inner tube be drawn out.

A good test of the adjustment for parallax is to move the head from side to side while looking through the telescope. If the adjustment is perfect, the image of the object will seem steadily to coincide with the cross-wires: if imperfect, the image will seem to waver as the head is moved. If the image seems to shift in the *opposite* direction to the head, the inner tube must be drawn out further; if in the *same* direction, it must be drawn inwards.

II. The *Permanent Adjustments* should be tested from time to time; but in a well-made theodolite they will seldom require correction. Before testing those adjustments, the temporary adjustments should be made with care.

(1.) The *Adjustment of the Line of Collimation*, in a transit theodolite, and also in Colonel Everest's, consists in placing that line precisely at right angles to the horizontal axis. To effect this, direct the line of collimation towards some very distinct distant object, bringing, by means of the tangent-screw of the horizontal circle, the cross-wires to coincide in azimuth with the image of a well-defined point in that object. The vertical circle should be unclamped. Now lift the horizontal axis out of its bearings, and replace it with the ends reversed, so that the telescope is upside down; if the cross-wires now coincide in azimuth with the same object, the line of collimation is perpendicular to the horizontal axis; if not, one-half of the deviation is to be corrected by shifting the cross-wires by means of the horizontal adjusting-screws of the diaphragm, and the other half by the tangent-screw of the horizontal circle. Reverse the horizontal axis again, and repeat the operation till the adjustment is perfect.

In the transit theodolite there is another mode of reversing the telescope to perform this adjustment, which consists in turning the telescope over on its horizontal axis, and then turning it round through exactly 180° in azimuth.

In the common theodolite the line of collimation is adjusted by turning the telescope half round in its Y's about its own axis, and observing whether the cross-wires continue to coincide with the same object. Should they deviate, half the deviation is to be corrected by the diaphragm-screws, and the other half by the tangent-screw of the horizontal circle. This adjustment places the line of collimation in coincidence with the axis of the Y's. The adjustment of the latter line perpendicular to the horizontal axis is left to the instrument maker.

(2.) The *Adjustment of the Level attached to the Telescope* can only be effected, in the transit theodolite, by methods which will be explained in treating of the adjustment of levelling instruments. The same may be said of the adjustment in a vertical direction of the line of collimation. (See Art. 50.)

In the common theodolite, having levelled the level attached to the telescope by the tangent-screw of the vertical circle, lift the telescope out of the Y's and set it down again turned end for end. If the bubble deviates from the centre of the level, correct half the error by the adjusting-screws which connect the level with the telescope, and the other half by the tangent-screw of the vertical circle.

(3.) To ascertain the *Index-error of the Vertical Circle*, set the vertical axis truly vertical with great care, as described under the head of temporary adjustments; set the spirit-level on the telescope exactly level; observe the reading on the vertical circle; if it is 0°, there is no error; if it differs from 0°, the difference is an error in the position of the index of the vertical circle, to be allowed for in each angle measured.

(4.) The *Adjustment of the Horizontal Axis exactly perpendicular to the Vertical Axis* is generally left to the instrument maker; but in some theodolites there are adjusting-screws for the supports of the horizontal axis. In this case the perpendicularity of the horizontal to the vertical axis may be tested by directing the telescope on an object whose altitude is considerable; then turning it round through exactly 180° in azimuth, and turning it over in a vertical plane so as to look at the same object. If the cross-wires can again be brought to coincide with the object, the adjustment is correct; if not, half the deviation is to be corrected by the tangent-screw of the horizontal circle, and the remainder by the adjusting-screws of the supports; and the operation is to be repeated till the adjustment is found to be correct.

This adjustment may also be tested by observing whether, when the instrument is clamped in azimuth, the cross-wires traverse an object and its image as reflected from a level surface of fluid.

36. **Measuring Horizontal Angles with the Theodolite.**—To measure the horizontal projection of the angle subtended at a given station A, by the direction of two objects B and C,—in other words, the difference of azimuth of the two objects,—set up the theodolite at the station, and make the temporary adjustments as described in the preceding Article. The outer vertical axis being clamped, and the vernier-plate and vertical circle A unclamped, direct the telescope towards one of the objects (as B), as accurately as possible by the hand; clamp the vernier-plate, and by its tangent-screw bring the cross-wires to cover the object exactly. Read the degrees, minutes, and seconds indicated by one vernier, and the minutes and seconds indicated by the other, and note them down. Find the mean arc indicated, by setting down the entire degrees as read on the first vernier, and the mean between the additional arcs in minutes and seconds as read by the two verniers.

Unclamp the vernier-plate, direct the telescope towards the other object (C), and proceed as before, taking care to read the entire degrees on the same vernier.

The difference between the mean arcs read off when the line of collimation is directed towards B and C respectively, is the required difference of azimuth, or the horizontal angle B A C.

The object of reading the minutes and seconds on both verniers, and taking the mean, is to correct the effect of any errors which might arise from the vertical axis not being exactly concentric with the graduated limb of the horizontal circle. In fig. 31, let E C and D B be two straight lines cutting each other in A, a point not in the centre of the circle B C D E. The eccentricity of that point produces equal and opposite deviations in the arcs B C and D E from the arc which would subtend an angle equal to B A C at the centre of the circle; so that the mean of those arcs is exactly equal to the arc which correctly measures the angle B A C, how great soever the eccentricity may be.

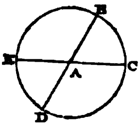

Fig. 31.

The same object is attained in Colonel Everest's theodolite by taking the mean of the arcs read off by the three equidistant verniers, which are used in order to give better security against errors in graduation than two verniers give.

In the transit theodolite, errors arising from the horizontal axis not being exactly perpendicular to the vertical axis may be eliminated by turning the telescope over about the horizontal axis, and half round about the vertical axis, repeating the measurement of the angle in this new position, and taking the mean of the results.

When a series of horizontal angles has been measured at a station between a series of objects, returning at last to the object with which the observations commenced, the accuracy of the observations may be tested by adding the angles together; when their sum ought to be exactly 360°. Should it differ by a small arc from 360°, the most probable values of the several angles will be found by dividing the total error by the number of errors to find the correction, which is to be added to or subtracted from each of them according as their sum is too small or too large.

When very great accuracy is required in measuring a horizontal angle, the effect of errors of graduation may be diminished to any required extent by the process called REPETITION, which is as follows:—

Clamp the *vernier-plate*, and read the verniers.

Unclamp the *vertical axis;* direct the telescope towards B; clamp the vertical axis, and direct the line of collimation exactly towards B by the tangent-screw of the *vertical axis.*

Unclamp the *vernier-plate;* direct the telescope towards C; clamp the vernier-plate, and direct the line of collimation exactly towards C by the tangent-screw of the *vernier-plate.*

Unclamp the *vertical axis, &c.* (as before).

Repeat the whole operation as many times as it is required to reduce the errors of graduation, observing always to direct the line of collimation towards B by turning the vertical axis, and *towards* C by turning the vernier-plate. Finally, the line of collimation being pointed towards C, read the verniers, remembering to reckon 360° for each complete revolution of the vernier-plate upon the horizontal circle.

The difference between the arcs read at the beginning and at the *end* of the process will be equal to the arc subtended by the angle B A C multiplied by the number of *repetitions;* and being divided by that number will give the required angle. The multiplied arc will be affected by only one error of graduation, which will be divided in finding the required arc; so that the error in the final result will be equal to the original error divided by the number of repetitions.

This process diminishes the effect of errors of observation somewhat, but not in the same proportion with errors of graduation; because an observer tends in general to make errors in the same direction at each observation; and such errors accumulate by repetition.

37. **Reflecting Instruments** are used chiefly in navigation and marine surveying; but as they are occasionally used in land surveying also, a general description of their construction and action will be given here.

The principle upon which reflecting instruments act is this:—that if there are two plane mirrors whose reflecting surfaces make a given angle with each other, and a ray of light, in a plane perpendicular to the planes of both mirrors, is reflected from both successively, its direction after the second reflection makes with its original direction an angle which is double of the angle made by the mirrors with each other.

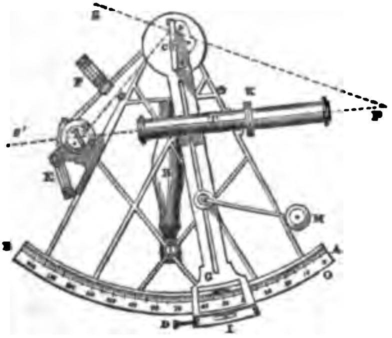

Fig. 32.

One application of this principle—the optical square—has already been described in Article 24, page 21.

The **Sextant** (fig. 32) is of the form of a sector of a circle, of 60°, and sometimes rather more, in angular extent. A B is the graduated limb, on which the degrees are of one-half of the extent of those on a non-reflecting instrument; so that for example, an exact sextant is divided into 120 degrees instead of 60°. C G is the index, having a vernier, and a microscope M for reading the divisions. At the back of the instrument is a clamp-screw, not shown, for holding the index in any required position, and at D is a tangent-screw for giving it a slow motion to complete its adjustment. The two mirrors have their planes at right angles to the plane of the instrument; one of them, called the "*index-glass*," C, is carried by the index at its centre of motion; the other, called the "*horizon-glass*," N, is carried by the frame of the sector; half of it is silvered and the remainder unsilvered. The silvered half is the further from the face of the instrument. Both mirrors should be made of strong plate glass, with its surfaces exactly plane and parallel.

T is a telescope directed towards the horizon glass. It is carried by a ring K, and capable of adjustment to a greater or less distance from the plane of the instrument; and the object of that adjustment is, to vary the proportions of the light received from the silvered part and through the unsilvered part of the horizon-glass, so as to render the images of two luminous objects seen directly and by reflection equally bright, although the objects themselves may be unequally bright. That equalization of brightness is favourable to accuracy in observing angles.

E and F are sets of darkening glasses, of various colours and shades, which are used when required, to moderate the light from very bright objects, such as the sun.

H is the handle by which the instrument is held.

Sextants for nautical purposes usually have the graduated limb of from six to eight inches radius, the graduated limb being subdivided by the vernier to 20" in the smaller sizes, and 10" in the larger. The observer can in each case read to one-half of these arcs by estimation.

The nautical sextant is seldom used for land surveying. For that purpose the *box-sextant* is employed, and for triangles of small extent only, not exceeding about a mile in length of side. The box-sextant is a sextant so small as to be entirely contained within a cylindrical brass box of about three inches in diameter and two inches in depth. It is graduated to half-degrees, and subdivided by the vernier to minutes, and by estimation to half-minutes. It is usually furnished with a small telescope, which, however, it is seldom necessary to employ, a plain sight-hole being used instead. The index is moved by a pinion and toothed sector.

The box-sextant has sometimes a contrivance added for enabling it to measure angles greater than 120°. That contrivance depends on the principle, *that if two reflected rays of light proceed in the same direction from two mirrors which make an angle with each other, the directions of the rays before reflection make double that angle with each other;* and it consists of a small mirror below the index-glass, fixed in such a position that when the index is at the mark numbered 180° upon what is called the "supplementary arc," those two mirrors are at right angles to each other; and the objects whose images as seen in them appear to coincide in direction, lie in fact in diametrically opposite directions.

Troughton's **Reflecting Circle** is an instrument having the mirrors and telescope of a sextant, together with a completely circular limb, and three indices radiating from its centre at angles of 120°. By observing each angle with the instrument in two positions, reading each angle observed upon the three verniers, and taking the mean of the six results, some of the errors of a sextant are avoided, and others diminished.

The **Universal Instrument**, as improved by Professor Piazzi Smyth, is a sort of reflecting circle, in which a spirit-level with a very small bubble is so placed that by means of a lens and a totally reflecting prism an image of the bubble is formed at the focus of the telescope, and the coincidence of the centre of that image with the cross-wires shows when the line of collimation is truly horizontal.

The **Adjustments of the Sextant are as follows**:—

(1.) To place the index-glass exactly perpendicular to the plane of the instrument. This adjustment is made by the maker; but the observer may test it by setting the index to about 60°, and looking at the image of the limb of the instrument as reflected in the index-glass; when the real limb and the image ought to seem to form one continuous arc.

(2.) To place the horizon-glass exactly perpendicular to the plane of the instrument. This adjustment is tested by clamping the index near to 0°; looking at some well-defined far distant object, and turning the tangent-screw of the index till the object as seen directly and its reflected image are made to seem to coincide, if possible. If the horizon-glass is correctly adjusted, it will be possible to make the apparent coincidence exactly; if not, the glass must be corrected by means of adjusting screws with which it is fitted.

(3.) To ascertain the "*index-error*," the angle marked by the index is to be read off when the above-mentioned coincidence has been made. If there is no index-error, the index will mark exactly 0°: any deviation from this is to be noted down as the index-error of the instrument, and allowed for in all future angular measurements. For the purpose of measuring the index-error when it is negative (that is—when the correction for it is to be added), the graduations of the limb are carried a short distance back from 0°. In reading this part of the limb (called the "arc of excess"), the divisions of the vernier are to be reckoned the reverse way.

(4.) The parallelism of the line of collimation of the telescope to the plane of the instrument is tested by placing the index so as to produce the apparent coincidence of two distinct objects whose directions make an angle of 90°, or thereabouts, and observing whether a slight motion of the plane of the sextant about an axis traversing the object seen by reflection disturbs the apparent coincidence, which it should not do if the adjustment is correct.

38. **Use of the Sextant in Surveying.**—To measure a horizontal or nearly horizontal angle with the sextant, hold the instrument so that the plane of its face shall pass through the two objects subtending the angle: look through the telescope or sight-hole at the object which is farthest to the left, so as to see it through the unsilvered part of the horizon-glass; move the index by hand until the reflected image of the right-hand object is seen in the silvered part of the horizon-glass; clamp the index, and move it slowly by the tangent-screw till that image apparently coincides with the left-hand object. (In the box-sextant, the entire motion of the index is produced by turning the pinion.) Then read the angle by means of the index and vernier, and add or subtract the index-error according as it lies behind or in advance of 0°.

In fig. 32, P S′ represents the direction of the left-hand object; P S that of the right-hand object. When the image of the latter appears to coincide with the former, the rays of light coming from the right-hand object are reflected from the mirror C to the mirror N, and thence to the eye in the same direction with those which come directly from the left-hand object; and according to the principle stated at the beginning of the last article, the angle made by the directions of the objects S, P, S′, is double of that made by the planes of the mirrors. When the mirrors are parallel to each other, the index points to 0° (or deviates from that point by the index-error only); and the divisions marked as degrees on the limb are of half the length of actual degrees; so that the angle read off on the limb (index-error being allowed for), is the angle between the directions of the objects. If there is much difference in the distinctness of the objects, the less distinct object should be looked at directly; and should it lie to the right of the other, the face of the sextant must be turned downwards.

In order that the angle measured may be a horizontal angle, the two objects and the observer's eye must be at the same level. When this is not the case, three methods may be followed. The least accurate is, to choose by the eye two objects in the same vertical planes with the objects whose relative azimuth is to be found, and as nearly as possible on a level with the observer's eye, and to measure the angle between these. To attain greater accuracy, two vertical poles are to be ranged and adjusted by the plumb-line, in the directions of the two objects, and the angle between them measured with the plane of the sextant horizontal. In using the box-sextant for the details of a survey, one or other of these methods is in general sufficiently accurate, if the ground is not very hilly.

The most accurate method is to measure the angle between the objects themselves, and to take also the angle of altitude or depression of each. (The taking of such angles will be further considered under the head of levelling.) The *zenith distance* of each object is found by subtracting its altitude from, or adding its depression to, 90°.

In fig. 33, let O represent the observer's station; O B, O C, the directions of the objects; B O C the angle between them; O D E a horizontal plane; D O B and E O C the altitudes of the objects; O A a vertical line, and A D E a spherical surface.

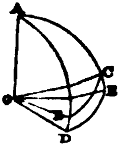

Fig. 33.

Then, in the spherical triangle A B C, the three sides are given, viz., A B and B C, the zenith distances, and B C, the angle between the objects; and the horizontal projection of that angle, being equal to the angle A, may be computed by the proper formula. (See Article 33, Division V., equation 66, p. 50.)

39. **Use of the Compass in Surveying.**—It has already been mentioned that the theodolite is usually provided with a compass, carried on the centre of the vernier-plate. This compass consists of a magnetic needle, hung by an agate cap on a point in the centre of the instrument, and of a flat silver ring fixed round the inside of the compass-box, and divided into degrees and half-degrees, the numbering of which usually commences at a point exactly under the telescope, and proceeding *towards the left*, goes completely round the circle, ending at the point where it started, which is marked 360°. There is a small catch, by pressing which the magnetic needle is lifted off its bearing when not in use, to avoid unnecessary wear; and by which also its vibrations are gradually checked when an observation is made. To find the magnetic bearing of any object from a given station, the line of collimation of the telescope is directed towards it; and the surveyor, when the vibrations of the needle have ceased, reads the angle to which the north end of the needle points, and which denotes *so many degrees to the east of north.* When the angle to the east of north exceeds 90°, it is to be observed that 90° east of north means *east,* 180° east of north, *south,* and 270° east of north, *west.* In some cases, however, the ring is divided into four quadrants, the points in a line directly under the telescope being both marked 0°, and the points in a line perpendicular to the telescope, 90°, as in fig. 28, p. 55; and then the bearing is read so many degrees to the east of north, west of north, east of south, or west of south, as the case may be.

The compass most frequently used in surveying is the *Prismatic*

Compass, consisting of a glass-covered box three or four inches in diameter, in which is hung a magnetic needle: the needle carries a light graduated silver ring fixed upon it, and the box has sights fixed to its rim. The farther sight, when in use, stands upright to a height equal to the diameter of the box, and contains a vertical slit with a vertical wire in the middle. The near sight has a very small slit to look at the object through, below which is a totally reflecting magnifying prism, so placed as to show to the eye of the observer a reflected and magnified image of that part of the edge of the graduated ring which is directly below the line of sight. He directs the sight towards an object, and at the same time, and with the same eye, reads its bearing on the ring. In order to show bearings in degrees to the east of north, the numbering of the degrees on the ring begins at the south end of the needle, proceeds *towards the right*, and goes completely round to 360°.

The "*Circumferenter*" is a compass with sights mounted on a stand, chiefly used in surveys of mines.

The horizontal angle subtended by two objects may be found to a rough approximation by taking the difference of their magnetic bearings.

The compass cannot be read in surveying to less than a quarter of a degree; and considering the continual changes which go on in the earth's magnetism, and the effects of local attraction, it is thought doubtful by the best authorities whether magnetic bearings can be relied upon even to half a degree. Hence, although it is a convenient instrument for filling up small details, and making rough surveys, it is not to be used where accuracy is required.

It is usual to mark the magnetic north upon a plan, and this can easily be done by taking the magnetic bearing of one of the principal station-lines. The true north ought to be shown also, and the means of finding its direction will be explained in Article 42.

40. **Great Trigonometrical Survey.**—The general nature of a survey of this class has already been stated in Article 12, Division (*c*), p. 13, viz.:—measuring one base-line with extreme accuracy, and finding the lengths of all the other sides of triangles by calculation from their angles. A few sides of triangles may be measured in parts of the survey far distant from the original base, in order to test the accuracy of the whole work: these are called *bases of verification.*

The trigonometrical calculations required in a survey of this class consist almost entirely in computing the remaining sides of a triangle when one side and two of its angles are given: as to which computation, if the triangle is sensibly plane, see Article 33, Division IV., equation 35, p. 43, and if it is sensibly spherical, see Article 33, Division VI., pp. 51 to 53.

The following points require some further explanation:—

I. *Ill-conditioned Triangles*, that is, triangles with any angle of less than 30° or more than 150°, are to be avoided in surveying by angles as well as in surveying by the chain, and for the same reason. (See Article 26, p. 24.)

II. *Checking Angles.*—The whole three angles of each great triangle should be measured, in order that the accuracy of the observations may be checked by adding them together, when they ought to amount to 180° (+ the spherical excess, if sensible; see Article 33, Division V., equation 58, p. 48). The treatment of unavoidable errors has been explained in Article 33, Division VI., Problem 4, p. 53.

The accuracy of the measurement of the internal angles of any polygon on the earth's surface may be checked by adding them together; when, if n denotes the number of the sides of the polygon, the angles ought to amount to

$(n-2)\ 180°$ + the spherical excess, calculated from the area of the figure as for a triangle.

III. *Checking Sides.*—In a complete network of triangles, it will always be found that many of the sides are so placed that their lengths can be calculated independently from different sets of data, which gives the means of checking the accuracy of the measurements and calculations.

IV. *Prolonging the Base.*—As it is necessary that the base should be measured on a level piece of ground, it is in general of limited extent, and much shorter than the sides of the great triangles; and its ends, also, are seldom in commanding positions suited for stations. Such a base line is "*prolonged,*" by ranging straight lines in continuation of it, at one or both ends, until a sufficient length has been obtained and suitable stations reached, the length of such additional lines being computed from angular measurements, as follows:—In fig. 34, let A B be the measured base, and B E a line ranged in continuation of it. Choose a lateral station C, so that A B C and B C E shall be well-conditioned triangles; measure the three angles of each of these triangles; from the angles A C B, C A B, and the base A B, compute the side B C; and from that side, and the angles C E B, B C E, compute the additional length B E. Take another lateral station D, at the opposite side of the base, and by solving, in the same manner, the triangles A B D, D B E,

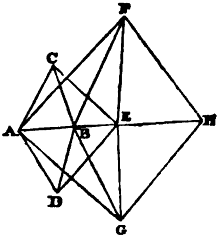

Fig. 34.

compute B E from independent data, so as to check the previous determination of its length.

E H represents a farther prolongation of the base line, and F and G the lateral stations which form the triangles by means of which its length is computed. At each of those stations angles are measured between all the previously determined points, A, B, E, in order that there may be as many ways of verifying the calculations as possible. In the same manner the base may be prolonged either way as far as may be deemed necessary.

V. *Enlarging Triangles.*—A mode of connecting a comparatively short base with the sides of large triangles, without prolonging it, or introducing ill-conditioned triangles, is as follows:—In fig. 35, let A B represent the base. Choose two stations C and D, at opposite sides of the base, and as far from each other as is consistent with making A C B and A D B well-conditioned triangles. From each of those four points measure the angles subtended by the other three. Then calculate the sides A C, C B, B D, D A; when there will be data for computing the length of C D in a variety of different ways, which will check each other. Taking C D as a new base, choose a pair of stations E and F still farther asunder, and proceed as before to determine the distance E F, and so on until a distance has been determined sufficiently long to serve as the side of a pair of triangles in the great triangulation.

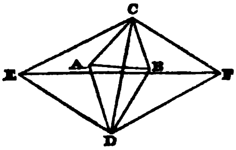

Fig. 35.

41. **Great Traversing Survey.**—The general nature of a survey of this class, as usually required for a long line of communication, has been explained in Article 12, pp. 12, 13, and illustrated by figure 3, p. 12. Some further explanation will now be given on the following points:—

I. *Checking Distances and Angles.*—The lateral objects, such as F, G, H, &c., in fig. 3, are generally inaccessible or unavailable as stations for the theodolite; so that the only angles measured for the main triangulation are those at the stations A, B, C, &c. If errors were impossible, the measurement of the base lines A B, B C, C D, &c., and of the angles between them, A B C, B C D, &c., would be sufficient to determine their lengths and directions. The use of the lateral objects is to check the results of those measurements, in the following manner:—

In the triangle A B F, the side A B, and the angles at A and B having been measured, calculate the side B F. In the triangle B F C, the side B F having been calculated, and the angles at B and C having been measured, calculate the side B C; the result being compared with the length of the same line as measured on the ground, will check the accuracy of the work so far. The process of comparison is precisely similar for each successive main station-line of the survey.

II. *Gaps in the Main Station-lines*, such as have already been referred to in Article 27, are in most cases to be measured by the process already described in Article 40, Division IV, p. 69, and illustrated by fig. 34, for prolonging a base-line. In that figure A B may be held to represent a measured portion of the station-line, and B E, or B H, the gap or inaccessible distance. The sides of the lateral triangles formed in order to determine *that* distance may also be used as station-lines for the details of the survey.

Fig. 36 shows how a distance C D between two objects is to be measured, when both ends of it are inaccessible to chaining. Measure a base A B, having its ends so situated that the six lines connecting them and the objects C and D with each other may form well-conditioned triangles, and at the stations A and B measure the angles C A D, D A B, A B C, C B D. In the triangle C A B, compute the sides A C, B C; in the triangle D A B, compute the sides A D, B D. Then, in the triangle C A D, in which the sides A C and A D, and the included angle at A are given, compute the third side C D, as shown in Article 33, Division IV., Problem 2, equations 37, 38; also compute C D by the same process as the third side of the triangle C B D; the two results will check each other.

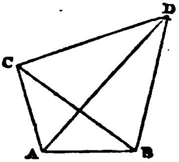

Fig 36.

42. **Finding the Meridian.**—For the purpose of laying down the direction of the true north on the plan of an engineering survey, the angle which one of the principal station-lines makes with the meridian must be determined, though not with the same accuracy that is required for astronomical and geographical purposes. The following are some of the methods:—

I. *By the Two greatest Elongations of a Circumpolar Star.*—This, the most accurate method, consists in observing the greatest and least horizontal angles made by a star near the pole with a station-line of the survey, when the star is at its greatest distances east and west of the pole, and taking the mean of those angles, which is the true azimuth of the station-line, or horizontal angle which it makes with the meridian. In the northern hemisphere the Pole-star, α Ursæ Minoris, is the best for this purpose.

This method, however, is seldom practicable with an ordinary theodolite, as in general, one of the observations must be made by daylight.

II. *By equal altitudes of a Star.*—The theodolite being at a station in the station-line chosen, measure the horizontal angle from the station-line to any star which is not near the highest or lowest point of its apparent daily course, and take also the altitude of that star. Leave the vertical circle clamped, and let the instrument remain perfectly undisturbed until the star is approaching the same altitude at the other side of its apparent circular course. Then, without moving the vertical circle, direct the telescope towards the star, clamp the vernier-plate, and by the aid of its tangent-screw, follow the star in azimuth with the cross wires until it arrives exactly at its former altitude, as is shown by its image coinciding with the cross wires; then measure the horizontal angle between the new direction of the star and the station-line: the mean between the two horizontal angles will be the true azimuth of the station-line.*

In both the preceding processes it is to be understood that the *mean of two horizontal angles* means their *half-sum* when they are at the same side of the station-line, but their *half-difference* when they are at opposite sides.

The second method may be applied to the sun, observing the sun's west limb in the forenoon and east limb in the afternoon, or *vice versâ;* but in that case a correction is required, owing to the sun's change of declination. When the sun's declination is changing towards the $\left\{\begin{matrix}\text{north}\\ \text{south}\end{matrix}\right\}$, the approximate direction of the meridian, as found by the method just described, is too far to the $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$. The correction required is given by the formula,†

$$\frac{\text{change of sun's declination}}{2} \times \sec\cdot\text{latitude} \times \text{cosec}\,\frac{1}{2}\,\text{angular motion of sun between the observations} \quad \ldots\ldots\ldots\ldots \text{(1.)}$$

III. *By One greatest Elongation of a Circumpolar Star.*—To use this method, the polar distance of the star, and the latitude of the place, should be known. Then

$$\sin\cdot\text{azimuth of star at greatest elongation} = \sin\cdot\text{polar distance} \div \cos\cdot\text{latitude}; \quad \ldots\ldots\ldots\ldots \text{(2.)}$$

and this azimuth, being added to or subtracted from the horizontal angle between the station-line and the star, when at its greatest elongation (according as the station-line lies to the same side of the meridian with the star, or to the opposite side) gives the azimuth of the station-line.*

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* In observing at night with the theodolite, it is necessary to throw, by means of a lamp and a small mirror, enough of light into the tube to make the cross-wires visible.

† At the equinoxes, the rate of change of the sun's declination is about 59″ per hour; and it varies nearly as the cosine of the sun's right ascension.

IV. *By observing the Altitude of a Star, and the Horizontal Angle between it and the Station-line.*—The altitude being corrected for refraction, the azimuth of the star is computed by taking the zenith-distance, or complement of that altitude, the polar distance of the star, and the co-latitude of the place, as the three sides of

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* The following is a table of the polar distances of a few of the more conspicuous stars for the 1st of January, 1861, together with the annual rate at which those distances are changing, + denoting increase, and — diminution:—

NORTHERN HEMISPHERE.

Star.	North Polar Distance.	Rate of Annual Variation.
α Andromedæ,	61° 40′ 37″	— 19″·9
α Ursæ Minoris (Pole-Star),	1 25 53	— 19 ·2
α Arietis,	67 11 48	— 17 ·25
α Ceti,	86 27 29	— 14 ·4
α Persei,	40 38 14	— 13 ·2
α Tauri (Aldebaran),	73 46 24	— 7 ·7
α Aurigæ (Capella),	44 8 53	— 4 ·2
α Orionis (Betelgeuze),	82 37 21	— 1 ·1
α Geminorum (Castor),	57 48 38	+ 7 ·4
α Canis Minoris (Procyon),	84 25 18	+ 8 ·9
β Geminorum (Pollux),	61 38 30	+ 8 ·3
α Leonis (Regulus),	77 21 18	+ 17 ·4
α Ursæ Majoris,	27 29 59	+ 19 ·3
η Ursæ Majoris,	39 59 31	+ 18 ·1
α Bootis (Arcturus),	70 5 32	+ 18 ·9
α Ophiuchi,	77 20 9	+ 3 ·0
α Lyræ (Vega),	51 20 37	— 3 ·1
α Aquilæ (Altair),	81 29 46	— 9 ·2
α Cygni,	45 12 53	— 12 ·7
α Pegasi (Markab),	75 32 31	— 19 ·3

SOUTHERN HEMISPHERE.

Star.	South Polar Distance.	Rate of Annual Variation.
β Orionis (Rigel),	81° 38′ 5″	+ 4″·5
α Columbæ,	55 51 0	+ 2 ·2
α Argûs (Canopus),	37 22 44	— 1 ·8
α Canis Majoris (Sirius),	73 28 17	— 4 ·6
α Hydræ,	81 56 31	— 15 ·4
η Argûs,	31 2 46	— 18 ·7
α Crucis,	27 40 21	— 19 ·9
α Virginis (Spica),	79 33 55	— 18 ·9
α Centauri,	29 44 36	— 15 ·1
α Scorpii (Antares),	63 52 48	— 8 ·4
α Trianguli Australis,	21 14 3	— 7 ·4
α Pavonis,	32 49 26	+ 11 ·1
α Gruis,	42 22 5	+ 17 ·2
α Piscis Australis (Fomalhaut),	59 38 31	+ 19 ·0

a spherical triangle; when the azimuth of the star will be the angle opposite the side representing the polar distance. (See Article 33, Division V., Problem 6, p. 50.) The azimuth of the station-line is then to be found as in Method III.

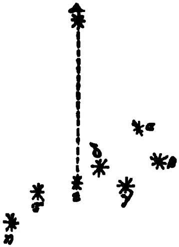

Fig. 37.

V. *Approximate Method by observing certain Stars.*—It is remarked by Mr. Butler Williams, that a great circle traversing the Pole-star (α Ursæ Minoris), and the star *Alioth* in the Great Bear (ε Ursæ Majoris), passes very near the pole. Hence, in the northern hemisphere, a meridian-line may be fixed approximately by observing, with the aid of a plumb-line, the instant when those two stars appear in the same vertical plane, as shown in fig. 37. The Pole-star is marked A.

When two points on the earth's surface have the same latitude, but different longitudes, the horizontal angle made by their meridians with each other is found by the following equation;

$$\sin \frac{1}{2} \text{horizontal angle} = \sin \frac{1}{2} \text{difference of long.} \times \sin \cdot \text{lat.} \quad (3.)$$

43. **Plotting and Protracting.**—The most accurate method of laying down the angles of great triangles on paper is to calculate the lengths of the sides of the triangles, and plot them with beam-compasses like chained triangles (Article 30, p. 31).

To plot, according to this principle, a solitary angle, like that between a station-line and the meridian, a circle is to be drawn, with as large a radius as is practicable, round the station where the angle is to be laid down. Then the distance between the points where the two lines enclosing the angle cut that circle is found by multiplying the radius by the *chord* of the angle—that is, twice the sine of half the angle.

But to save time where less accuracy is required, especially in laying down secondary triangles and details, angles are laid down at once, or "protracted," by the aid of instruments called "protractors;" being flat graduated circles or parts of circles, which are laid on the paper. They are of various constructions and various degrees of accuracy.

The most accurate *Circular Protractor* has a round piece of plate-glass in its centre, through which the paper can be seen. The under side of the glass touches the paper, and has the centre of the graduated circle marked on it by a fine cross. The circle is divided to half-degrees, and subdivided to minutes by the vernier on its index. The index has two diametrically opposite arms, each of which has hinged on its end a branch carrying a pricker, which is held up clear of the paper by a spring. When the index has been turned to any required degree and minute on the circle, the two branches are pressed down, and their prickers mark two points on the paper which are in the required direction, and which *are or* ought to be in one straight line traversing the centre of the circle. It is often convenient to draw, by the aid of those prickers, a graduated circle on the paper, through the centre of which lines making any required angle can be drawn, and their directions transferred, so as to pass through any required station on the paper, by the aid of a large and accurate parallel ruler.

The *Semicircular Protractor* has a straight side, which can be slid along a straight-edge fixed to the table or drawing-board into any required position. Its index has a long arm projecting beyond the circle, with a straight fiducial edge, which is used to rule lines in any required direction through any station on the plan.

44. **Traversing on a Small Scale** has been referred to in Article 12, Division (*c*), p. 13, as a means of surveying long, narrow, and winding objects in detail. The most accurate way of performing it is to form a series of triangles by means of lateral objects, as already described in that article, and in Article 41, the checking of the accuracy of the work being tested by plotting, without calculation. Each lateral object is traversed by at least three lines from different stations in the survey; and those three or more lines will intersect each other in one point on the paper, if the station-lines between the stations and the angles at the stations have been correctly measured and plotted.

In almost all mining surveys, and in some above ground, it is impossible to take suitable angles to lateral objects, the only angles capable of being measured being those which the station-lines make with each other. In such cases the station-lines should be laid out so as to return to the starting-point, and form a "closed polygon." The accuracy of measurement of the angles may then be tested by taking the sum of all the "salient" angles of that polygon—that is, of those which project outwards—and subtracting from it the sum of the "re-entering angles"—that is, of those which project inwards. The result (which is the *algebraical sum* of the angles of the polygon) ought to be

$$180^\circ \times \Big\{\text{number of salient angles} - 2 - \text{number of re-entering angles}\Big\} \ldots (1.)$$

Before plotting such a survey, the angle made by each station-line with one fixed direction ought to be computed (by successive additions or subtractions of the angles which those lines make with each other) and protracted on the paper by drawing a line to represent that fixed direction, placing the zero-points of the protractor on that line, and laying off the directions of the several station-lines as described in Article 43. The accuracy of the measurement of distances, and of the plotting of distances and angles, is tested by the exactness with which the end of the last station-line on the paper coincides with the starting-point of the first.

In surveying by *traversing with the compass and chain,* the angles observed at each station are the directions which the station-lines that meet at it make with a fixed or nearly fixed direction, viz., that of the magnetic meridian. The zero-line on the paper, therefore, represents that meridian; and the angles protracted from it are simply the several magnetic bearings of the station-lines. Traversing with the compass and chain is accordingly an easy and rapid method of surveying; but as explained in Article 39, p. 67, its want of accuracy makes it suitable only for small or rough surveys.

45. **Plotting by Rectangular Co-ordinates,** or by **Northings, Southings, Eastings,** and **Westings,** is the most accurate way of plotting a traverse, because the position of each station is plotted independently, and not affected by the errors committed in plotting previous stations. It consists in assuming two fixed lines or *axes,* as O X and O Y, fig. 38, crossing each other at right angles at a fixed point O, computing the perpendicular distances or *co-ordinates* of each station from those two axes, and plotting the position of each station by the aid of a straight-edged scale fixed parallel to one of the axes, and a T-square sliding along it, so as to rule lines parallel to the other axis, and at any given distance from it, and of any given length. When the direction of the true meridian has been ascertained, it is best to make one of the axes represent it; and in that case the co-ordinates parallel to one axis will be the distances of the stations to the north or south of the fixed point or "*origin*" O, and those parallel to the other axis will be their distances to the east or west of the same point; whence the phrase, "Northings, Southings, Eastings, and Westings." If the true meridian is unknown, any fixed direction will answer the purpose, and may be called "the Meridian" for the occasion, and one of its ends "the North." The calculations to be performed are the following:—In the figure, let O Y represent "the Meridian," Y being towards "the North." One of the stations in the survey is to be taken as the origin O. Let A be the next station, and O A its distance from O. If Y O A, as in the figure, is an acute angle, A is to the northward of O; if an obtuse angle, to the southward; if Y O A, as in the figure, lies to the right of

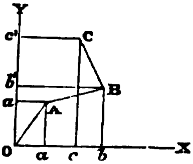

Fig. 38.

the meridian, A is to the eastward of O; if to the left, to the westward; and the co-ordinates of A are as follows (θ denoting the angle Y O A):—

$$\left.\begin{array}{l}\text{Northing O } a' = a \text{ A (or if negative, Southing).} = \text{O A} \cdot \cos\theta; \\ \text{Easting, O } a = a' \text{ A (or if negative, Westing)} .. = \text{O A} \cdot \sin\theta.\end{array}\right\}\ (1.)$$

In the same manner are to be computed the co-ordinates of the third station B *relatively to* A; viz.:

$$a'\,b' = \text{A B} \cdot \cos\theta'; \ a\,b = \text{A B} \cdot \sin\theta'; \ \ldots\ldots\ldots\ldots \ (2.)$$

(where θ' denotes the angle made by A B with the meridian); also the co-ordinates of C *relatively to* B, and so for each successive station. In the figure, it will be observed that the direction of B C deviates to the westward of north, so that $b\,c$ is a "Westing," and is to be considered as negative. The results of these calculations are to be entered in a book, in four columns—for northings, southings, eastings, and westings respectively. Then in four other columns are to be entered the total northing or southing, and easting or westing, of each station from the origin or first station, computed by adding all the successive northings and subtracting the southings, made in traversing to the station, the result being a northing if positive, a southing if negative; and by treating the eastings and westings in the same manner.

These calculations are expressed by symbols as follows:—Let $\pm y$ denote the total $\left\{\begin{matrix}\text{northing}\\ \text{southing}\end{matrix}\right\}$ of a station, and $\pm x$ its total $\left\{\begin{matrix}\text{easting}\\ \text{westing}\end{matrix}\right\}$; L the length of any given station-line, and θ the angle which it makes with the meridian from the north; observing that both θ and $\sin\theta$ are $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right\}$ according as that angle lies to the $\left\{\begin{matrix}\text{east}\\ \text{west}\end{matrix}\right\}$ of the meridian, and that cosines of obtuse angles are negative. Then

$$\left.\begin{array}{l} y = \Sigma \cdot \text{L} \cos\theta; \\ x = \Sigma \cdot \text{L} \sin\theta. \end{array}\right\} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3.)$$

This method is chiefly useful in surveying mines, but may also be applied with advantage to some surveys above ground, such as those of towns. The book forms a record of the position of each station, independently of the plan; and it may be made more complete by the addition of a column containing the elevations of the stations above a datum horizontal surface. This will be further considered under the head of levelling.

46. The **Plane-Table** is a drawing-board, having a sheet of paper

strained on it, mounted on a portable three-legged stand, and capable of turning about a vertical axis, and of being adjusted by screws (like the azimuth circle of a theodolite) to a horizontal position, as shown by a spirit-level laid on its surface.

The vertical axis has a clamp and a tangent-screw to adjust the table to any required position.

The *index* is a flat straight-edged ruler, having upright sights at its ends.

The use of the plane-table resembles trigonometrical surveying on a small scale, except that the angles, instead of being read off on a horizontal circle and afterwards plotted, are at once laid down on paper in the field.*

Fig. 39 illustrates the principle of surveying with the plane-table. The first operation is to measure carefully a base on the ground, A B, and to lay down on the paper a straight line *a b*, to represent that base on a suitable scale. The instrument is then to be placed and levelled at the station A, the point *a* on the paper being directly above the point A on the ground; a needle is to be fixed upright at *a*; and the index being laid on the table, so that its fiducial edge shall lie exactly along the line *a b*, the table is to be turned until the sights of the index are in a line with the farther station B, and adjusted exactly to that position by the tangent-screw. The table remaining steady, the index is to be turned so that while its fiducial edge still touches the needle at *a*, its line of sight shall be successively directed towards all the important objects whose positions relatively to the base A B are to be found, such as C and D; and with a fine hard pencil lines are to be drawn along the edge of the index pointing towards those objects from *a*. The table is now to be shifted from A to B, and the needle from *a* to *b*, the point *b* on the paper being placed exactly over B on the ground; the index being laid along *b a*, the table is to be adjusted till the sights are in a line with A. The index is then to be turned so that while its fiducial edge still touches the needle at *b*, its line of sight shall be successively directed towards the same objects as before, and short lines pointing from *b* towards those objects are to be drawn along its edge, intersecting the lines

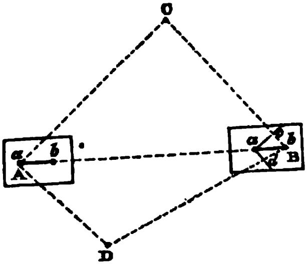

Fig. 39.

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* To protect the paper against the effects of the alternate moisture and dryness of the air, Captain Siborn recommends that its lower side should have spread over it the beat-up white of an egg before it is laid on the board.

previously drawn; the points of intersection, such as *c* and *d*, mark the objects on the paper. The details are filled in by sketching.

The objects thus laid down include poles at points suited for additional stations. On removing the table to one of those new stations, such as C, the needle is to be fixed at the point *c* representing that station on the paper, and the index is to be placed with its edge touching that needle, and traversing also a point representing one of the former stations, such as *a*. The table is then to be turned so that the sights shall be directed towards a pole fixed at that former station; and then all the lines on the paper will be parallel to the corresponding lines on the ground; and the survey of additional objects from the new station may be proceeded with as before.

The plane-table is well suited for surveying where minute accuracy in details is not required, the end in view being to show the relative positions of the more important objects on the ground. It is therefore more useful for topographical and military purposes than for those of engineering. For full information as to its use, see Siborn *On Topographical Surveying and Drawing*.

CHAPTER IV.

OF LEVELLING.

47. **Setting-out a Line of Section.**—Preparatory to taking the levels of the ground along the line of a proposed vertical section, that line is to be "ranged," by marking on the ground with whites, poles, and permanent marks where required, the points where the line of section crosses all streams, lines of communication, boundaries, &c., and a sufficient number of other points to enable it to be exactly followed. For that purpose, a tracing is to be made of so much as may be necessary of the plan on which the intended line of section is drawn, and the distances of that line from corners of fences and other definite objects are to be carefully measured on the original plan, and marked in figures on the tracing. An assistant goes over the ground with this tracing, and marks the points in accordance with it. Should the leveller see fit to alter the line in any respect as he goes along it, or should it be left entirely to his own judgment to choose it, as is often the case with trial sections, the distances of a sufficient number of points in it from objects on the ground can be measured on the spot and noted on the tracing, so as to enable the line of section chosen to be laid down on the plan.

48. The **Spirit-Level** strictly speaking is a glass tube B C, fig. 40, hermetically sealed at both ends, containing some very limpid liquid, such as alcohol, chloroform, or sulphuret of carbon, and a bubble of air A, and having a slight curvature, convex upwards. That curvature is much exaggerated in fig. 40, being in reality so slight as to be imperceptible, or nearly so, to the eye. The air-bubble places itself at the highest point in the tube; and a tangent to the upper internal surface of the tube at that point is horizontal. The glass tube is usually fixed in and protected by a brass case. When the instrument to which the spirit-level belongs is in adjustment, the centre of the bubble is in the middle of the tube. When the bubble deviates from that position, it indicates that a tangent to the middle of the tube deviates from a horizontal position through an angle whose value in seconds is—

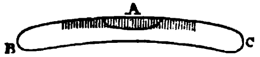

Fig 40.

$$206264''\cdot 8 \times \frac{\text{deviation of bubble}}{\text{radius of curvature of tube}}; \ldots\ldots\ldots (1.)$$

so that the longer that radius, the more delicate is the spirit-level. A scale of equal parts is marked on or attached to the top of the tube, to measure the deviation of the bubble; and the value of those parts in seconds can be found by trial.

Fig. 41 shows a form of spirit-level introduced by Professor Piazzi Smyth, in which the air-bubble A is very small. Another and a larger portion of air is contained in the upper part of the end C of the tube, which is separated from the rest by the partition D, with a nozzle-shaped orifice in its centre, through which air can be transferred so as to enlarge or diminish the bubble at will, by a mode of handling described in the *Transactions of the Royal Scottish Society of Arts* for 1856.

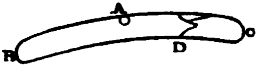

Fig. 41.

The term *spirit-level* or *level*, is also applied to a *levelling instrument*, of which the spirit-level proper is the essential part. Various forms of level are used for engineering purposes; that which is represented in fig. 42 is Mr. Gravatt's, called the "Dumpy Level." A is the spirit-level, attached by screws at *a*, *a* to the telescope B C; by means of those screws it can be adjusted, in order to place a tangent to its middle point parallel to the line of collimation of the telescope. A small circle near the object-end B of the telescope, indicates a small transverse level, used to show whether the horizontal cross-wire is truly horizontal.

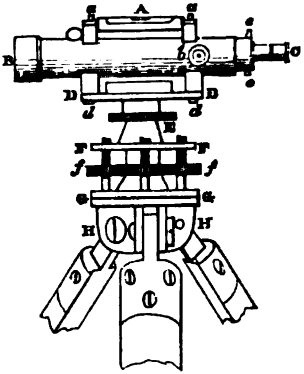

Fig. 42.

The telescope is similar to that of a theodolite (see Article 34, p. 53), except that the diaphragm at the common focus of the object-glass and eye-piece contains one horizontal and two parallel vertical cross-wires, as shown in fig. 43. B is the object-end of the telescope, C the eye-piece; *b* the milled head of the pinion by which the inner tube is drawn in and out; *c*, *c* screws for adjusting the diaphragm so as to bring the horizontal cross-wire exactly to the line of collimation or axis of the telescope. D D is an oblong plate or flat bar fixed on the top of the vertical axis E; to this plate the telescope is

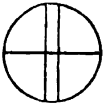

Fig. 43.

connected by adjusting screws at *d*, *d*, by means of which the line of collimation is placed perpendicular to the vertical axis. The vertical axis is hollow, and turns upon a spindle fixed to the upper parallel plate F; that spindle is continued downwards and attached to the lower parallel plate G by a ball-and-socket joint. *f*, *f*, *f* are three of the four plate-screws by which the vertical axis is set truly vertical. The lower plate G is screwed on the staff-head H, which has three wooden legs like those of a theodolite.

In most levels a compass is carried on the top of the plate D D, for the purpose of taking the magnetic bearings of lines of trial sections.

The following are some of the principal variations from the construction above described:—

In Troughton's level, the brass case of the spirit-level is imbedded in the top of the outer telescope-tube, and has no adjusting screws; the adjustment of the spirit-level to parallelism with the axis of the telescope being left to the instrument maker.

In the Y-level, the telescope is carried by two forked supports called Y's. It can be rotated in these about its own axis, and can be lifted out and turned end for end. The spirit-level hangs below the telescope, instead of being supported above it. One of the Y's is supported by a vertical screw with a milled head, by means of which the telescope is adjusted so as to be at right angles to the vertical axis, and which answers the purpose of the screws at *d*, *d*, in the Dumpy level.

The Y-level is more easily adjusted than the others; but it is also considered to be more easily deranged.

Some levels are provided with a small mirror, which being placed in a sloping position above the spirit-level, enables the observer to see the reflected image of the bubble at the same time that he looks through the telescope. Reference has already been made to the contrivance of Professor Piazzi Smyth, by which an image of a small bubble is formed at the cross-wires when the line of collimation is horizontal. (Article 37, p. 65; see also a paper by Mr. Bow, in the *Transactions of the Royal Scottish Society of Arts* for 1858-9.)

49. The **Levelling-Staff** is a rectangular wooden rod, having a face about two inches or two inches and a-half broad, on which is painted in a bold conspicuous manner, a scale of feet, divided into tenths and hundredths, commencing at the lower end of the staff. Its extreme length is usually from fifteen to seventeen feet, and it is made in three pieces, which in some staves can be put together or taken asunder, according as a greater or less length of staff is required, and in others, are made to draw out like telescope tubes. The staff, when in use, is held exactly vertical; for which purpose it sometimes has a plummet enclosed in a groove at one side of it, and

visible through a small piece of glass; it rests on its lower end, which is shod with brass; and in soft ground it is useful to have a small metal plate to place on the ground below the staff, and prevent it from sinking.

When the telescope of the level is directed towards the staff, and the line of collimation is truly horizontal, the number of feet and decimals of a foot at which the horizontal cross-wire crosses the inverted image of the scale on the face of the staff (subject to corrections to be afterwards explained), shows the vertical depth of the point on which the lower end of the staff stands below the line of collimation. If two such observations are made with the staff at different points, and the level at the same station, the difference between the two readings shows, in feet and decimals of a foot, how much the point at which the *less* reading is taken is *higher* than the point where the *greater* reading is taken.

In an old form of levelling-staff, now seldom used, a "sliding-vane" was slid up and down by the staffman, in accordance with signals made by the leveller, until its centre was in the line of collimation prolonged; the staffman then read the height of the vane above the ground. The making the divisions on the staff so distinct that the leveller can read them himself is an invention of Mr. Gravatt.

50. The **Adjustments of the Level** may be distinguished, like those of the theodolite, into *temporary adjustments*, which have to be made anew every time the level is set up, and *permanent adjustments*, which seldom become deranged in a well-made level, but still ought to be tested on each day that the instrument is used.

I. The TEMPORARY ADJUSTMENTS are as follows:—

(1.) *To make the foci of the object-glass and eye-piece coincide with the cross-wires.*—The same as in the theodolite (see p. 59).

(2.) *To place the vertical axis truly vertical.*—The same as in the theodolite (see p. 58). In order to avoid straining the plate-screws, this adjustment ought first to be made as nearly as possible by shifting one of the legs of the stand, and then corrected by the plate-screws.

II. The PERMANENT ADJUSTMENTS are as follows:—

(1.) *To place the cross-wires in the axis of the telescope-tube.*—In the Y-level, the same as in the Y-theodolite (see p. 60). In Troughton's level this adjustment is not made, except indirectly, as will be afterwards explained.

In the Dumpy level,* make three bench marks, by driving stakes or otherwise, in one straight line, at equal distances of four or five chains apart; call those bench marks a, b, and c. Place the level exactly midway between a and b, and make its temporary adjustments: read a levelling-staff placed on each of those bench marks; call the readings a and b; assume a datum-level at any convenient depth A below a; then, because the level is midway between the staves, the errors of adjustment affect the readings a and b alike; and how great soever these errors may be, the correct elevation of b above the datum is

---

* This is an Abstract of Mr. Gravatt's Directions, slightly modified.

$$B = A + a - b. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

Next place the level exactly midway between b and c, and make its temporary adjustments; place a staff on each of those bench marks and read it; call the readings b' and c'; then, as before, the correct elevation of c above the datum is

$$C = B + b' - c' = A + a + b' - b - c. \quad \ldots\ldots (2.)$$

Now place the level at the shortest distance *beyond* a at which it is practicable to read the staff, make the temporary adjustments, and read a staff on each of the three bench marks; call these readings a'', b'', c'', and compute *apparent* elevations of b and c above the datum, as follows:—

$$B' = A + a'' - b''; \; C' = A + a'' - c''; \quad \ldots\ldots (3.)$$

compute the *errors* of those apparent elevations, if any, viz.,

$$B' - B; \; C' - C;$$

and if the cross-wires are in the axis of the telescope, we shall find the latter error to be exactly double of the former; that is

$$C' - C = 2\,(B' - B); \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

but should this not be so, the screws at *c*, *c*, fig. 42, are to be used to shift the diaphragm until it is so.

The following table shows the direction in which the cross-wires require to be shifted for different relations amongst the errors:—

Direction of error C′ — C.	C′ — C more than 2 (B′ — B)	C′ — C less than 2 (B′ — B)
upward	upward	downward
downward	downward	upward

Direction in which the cross-wires are to be moved.

(2.) *To make the line of collimation and the spirit-level parallel to each other.*—In the Y-level, bring the bubble to the middle of the spirit-level by means of the plate-screws; lift the telescope out of the Y's, and set it down with the ends reversed. If the bubble remains in the middle of the spirit-level, the adjustment is correct; if it deviates, correct one-half of the deviation by the plate-screws, and the remainder by the adjusting screws which connect the spirit-level with the telescope.

In Troughton's level, make two bench marks about ten chains apart; set up the level exactly midway between them, and read staves set upon them, so as to find, by the difference of those readings, the true difference of level of the bench marks. Now set up the level beyond one of the bench marks and read both staves; if the difference of the readings deviates from the true difference of level, alter the position of the diaphragm, by means of its adjusting screws, until the readings of the staves give the true difference of level. The cross-wires are thus placed in a line passing through the centre of the object-glass parallel to the spirit-level; and the maker is relied on to make that line the true axis of the telescope. The same adjustment may be made by the aid of a sheet of water on a calm day; because two stakes can be driven at its margin so that their heads, being flush with the water, shall be exactly at the same level.

In the Dumpy level, having ascertained the true difference of level of two bench marks, as already described, and shifted the level to a position beyond one of them, alter, if necessary, the inclination of the telescope by means of the *plate-screws*, until the readings of the staves give the true difference of level, and bring the bubble to the middle of the spirit-level by means of the adjusting screws which connect the spirit-level with the telescope (*a*, *a*, in fig. 42).

(3.) *To place the telescope and spirit-level perpendicular to the vertical axis* (or, as it is called, to make the instrument "traverse") place the telescope over a pair of plate-screws, and by turning them, bring the bubble to the centre of the spirit-level; reverse the direction of the telescope exactly, by turning it through 180° about the vertical axis; if the bubble is still in the middle of the spirit-level, the adjustment is correct: if not, correct half the deviation by the plate-screws, and the other half by means of the screws which connect the telescope with the bar on the top of the vertical axis (*d*, *d*, fig. 42).

51. The **Use of the Level** in finding the difference of elevation between two points has been described in the two preceding articles.

The observations, or readings of the staff, taken by means of the level, are called "*sights*."

When two sights only are taken from one station, one with the staff upon a point whose level has been ascertained, and the other with the staff upon a point whose level is to be ascertained, the former is called the *back-sight*, and the latter, the *fore-sight*.

If the back-sight is the greater, the ground rises, and if the fore-sight is the greater, it falls, from the former point to the latter.

When the levels of a series of points are taken with the level at one station, in order to make a continuous section, the first and the last observations are the principal back and fore-sights respectively; the first back-sight being taken with the staff on a bench mark or other point, whose level has been ascertained by means of a sight from a former station; and the last fore-sight being taken with the staff upon a mark which is to be the object of the first back-sight when the level is shifted to a new station. Of the intermediate sights, taken with the staff upon places where the inclination of the ground changes, on roads, at the bottoms of streams, &c., each is a fore-sight relatively to the preceding sight, and a back-sight relatively to the following one.

For example, in fig. 44, A is a station where the level is set up, and the horizontal line *b* A *c* is the *line of sight*, or straight line in prolongation of the line of collimation. The *first back-sight* is taken with the staff on B, a point whose height above the datum-surface has been ascertained by previous observations; and it gives, as the reading on the staff, B *b*. The *last fore-sight* is taken with the staff on C, a point well suited as a position for the staff when the *first back-sight* with the level at the next station D is taken. It gives, as the reading of the staff, C *c*. The first intermediate sight, at the point marked 1, is a fore-sight relatively to that at B, and a back-sight relatively to that at the point 2, and so on.

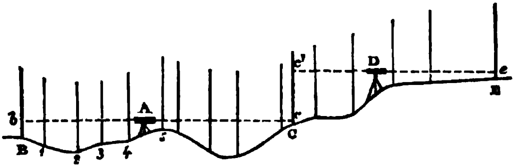

Fig. 44.

The first back-sight and last fore-sight are the most important in point of accuracy; for any error committed in them is carried on through the whole of the remainder of the section; whereas any error committed in taking an intermediate sight affects that sight only.

The first back-sight and last fore-sight taken from each station ought to be at points as nearly as possible at equal distances from the level, in order to neutralize the effects of errors of adjustment, and also those of the curvature of the earth and of refraction, which will be explained in the next article; and those points should be on firm ground, and, if possible, so placed that the readings shall not exceed ten or eleven feet.

When the leveller thinks it desirable to carry his level on to a new station, such as D, the staffman holds his staff steadily at C, only making it face about; the leveller advances to D, sets up and adjusts his level, takes the first back-sight C *c*′, and proceeds as before. E *e* represents the position of the staff when the last fore-sight is taken from D; the staff is held there until the leveller has moved on and planted his level at a third station, and so on. These operations can be performed with one staff; but much time is saved by using two, carried by two staff-holders.

While the levels are thus being taken, two chainmen measure the line of section with the chain, in the manner described in Article 22, pp. 19, 20; except that instead of always chaining in straight lines, they follow the line of section as set out. The leveller notes the distances of all the points at which the staff is set up, as well as those where boundaries are crossed, whether levels are taken there or not: in this he may get useful help from the staff-holders.

In crossing a stream or a sheet of water, the leveller, besides taking enough of levels to give a section of its banks and bed, should take the existing level of the surface of the water, and also the highest and lowest levels of the water, so far as he can ascertain them. Levels of the bottom may be taken by sounding.

When a sight is to be taken to determine the level of a point which is below the line of collimation by more than the entire length of the staff, the staff may be raised up vertically until the leveller can read some division near its upper end, and the height of the lower end of the staff above the ground may, at the same time, be measured with a tape-line or with another staff, and added to the height read. This, however, should only be practised at intermediate sights.

On the subject of **Checking Levels,** see Article 16, page 15. In good ordinary levelling the discrepancy between two sets of levels over the same section may be about a foot in forty miles of distance.

52. **Corrections for Curvature and Refraction.**—Inasmuch as a truly horizontal surface is not plane, but spheroidal (Article 3, p. 2), the line of sight of the level, when truly adjusted, does not exactly coincide with such a surface, but is a tangent to it. The

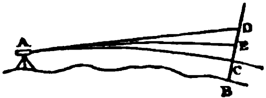

Fig. 45.

height read upon a levelling-staff, therefore, is always greater than it would be if a horizontal surface were plane; and the quantity to be deducted from the height on the staff of the point which is in the prolongation of the line of collimation, in order to reduce it to the height which would have been read had a horizontal surface been plane, is called the *correction for curvature.*

On the other hand, the line of sight, being the line along which light proceeds from the object looked at to the telescope, is not perfectly straight, being made slightly concave downwards by the refracting action of the air. Hence the point seen on the staff apparently in the line of collimation produced, is not exactly in that line, but is below it by an amount called the *error from refraction,* and thus the error arising from curvature is partly neutralized; and the correction to be subtracted for curvature and refraction usually is somewhat less than the correction for curvature alone.

In fig. 45, A represents the level; B, a point on the ground; B C E D, the staff standing on it; A C, a level surface touching the line of collimation, with the curvature very much exaggerated; A D, a straight line in prolongation of the line of collimation; E A, the real line of sight, a curved line in which light proceeds, owing to atmospheric refraction. Then the *correction for curvature* is – C D; the *correction for refraction* + D E; and the joint-correction,

$$-\mathrm{E\,C} = -\mathrm{C\,D} + \mathrm{D\,E}, \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.)$$

The correction for curvature is a third proportional to the earth's diameter and the distance between the level and the staff—that is to say, its value *in feet* is

$$\frac{\text{distance}^2}{41{,}778{,}000} = \frac{2}{3}\,(\text{distance in statute miles})^2 \;\ldots\ldots\; (2.)$$

The error produced by refraction varies very much with the state of the atmosphere, having been found to range from one-half to one-tenth of the correction for curvature, and in some cases to vary even more. Its value cannot be expressed with certainty by any known formula; but when it becomes necessary to allow for it, it may be assumed to be on an average about one-sixth of the correction of a curvature; so that the *joint correction for curvature and refraction,* to be subtracted from the reading of the staff, is on an average,

$$\frac{5}{6} \times \frac{(\text{distance in feet})^2}{41{,}778{,}000} = \cdot 56\,(\text{distance in statute miles})^2 \ldots (3.)$$

The errors produced by curvature and refraction are neutralized when back and fore-sights are taken to staves at equal or nearly equal distances from the level. At distances not exceeding ten chains, they are so small that they may be neglected.

The uncertainty of the correction for refraction makes it advisable to avoid, in exact levelling, all sights at distances exceeding about a quarter of a mile.

53. The **Level Field-Book** is kept in various forms, according to the practice of different engineers. In one of the most usual and convenient, each page is divided into seven columns, headed as follows:—

Rise.	Back-sight.	Fore-sight.	Fall.	Reduced Level.	Distance.	Description of Object.

The first entry made is in the column of reduced levels, being the elevation, in feet and decimals, of the bench mark on which the first back-sight is taken above a datum horizontal surface; and opposite this, in the column of description of objects, is the designation of that bench mark. In the second and all the following lines the only entries usually made in the field are the back-sights and fore-sights, the distances, and the description of objects; so that, beginning at the right side of the page, we have in each line the description of a point or object (if any description is necessary), its distance from the commencement of the line of section, the fore-sight read upon the staff when held at that point, and the back-sight read upon the staff when held at the point immediately preceding. On each occasion when an intermediate sight is taken without shifting the level, it will be entered as a fore-sight opposite the point to which it is taken, and also as a back-sight in the following line. In *reducing* the levels, which ought to be done each evening for the levels taken during the day, the first process is to take the difference between the back-sight and fore-sight in each line, and enter it as a $\left\{\begin{matrix}\text{rise}\\ \text{fall}\end{matrix}\right\}$ according as the $\left\{\begin{matrix}\text{back-sight}\\ \text{fore-sight}\end{matrix}\right\}$ is the greater. The reduced levels are then computed in succession from the level of the first bench mark by the successive addition of the rises and subtraction of the falls.

The calculations in each page are checked by adding up the first four columns; when the difference between the total rise and total fall ought to be equal to the difference between the sum of the back-sights and the sum of the fore-sights, and also to the difference between the first and last reduced levels in the page, the first reduced level in the first page being that of the first bench mark; and the last reduced level in each page being also entered as the first in the following page.

It is sometimes useful to enter in the column of descriptions the magnetic bearings of the lines levelled, and to illustrate it occasionally by sketch sections of the more intricate parts of the ground.

It is often necessary to reduce the levels in the field, especially in taking trial levels. In such cases the calculations should be carefully checked afterwards.

54. **Plotting a Section** is commenced by drawing with a very accurate straight-edge a straight "*datum-line*," to represent the datum horizontal surface from which heights are reckoned, and marking on that datum-line a scale of distances. The vertical scale should be drawn on the paper *at right angles to the datum-line*, in order that it may be parallel to the lines representing heights, and expand and contract along with them. This is of great importance in engraved and lithographed sections, in which the paper often expands or contracts differently in different directions. The plotting of the distances and heights entered in the field-book is performed like that of the distances and offsets in a chained survey. (Article 31, p. 32.) As to scales, see p. 7.

Explanations are usually written above the objects to which they relate, such as roads, railways, canals, rivers, &c.

The nature of the principal information which is required in writing on sections for engineering purposes has been stated in Article 14, pp. 14, 15.

55. **Levelling by the Theodolite** may be performed in three different ways.

I. *By placing the line of collimation horizontal, and using the theodolite like a level.*—This may be done when a proper levelling instrument is not at hand.

II. *By setting the line of collimation at a known angle of inclination, and taking sights in other respects as if with a level.*—This process may save time in taking the levels of steeply sloping ground. In fig. 46, A represents the theodolite, *b* A *c* the sloping line of sight, B *b*, C *c*, and the other vertical lines, heights read off on the staff. The most convenient way to reduce levels taken by this method is first to reduce them as if the line of sight were horizontal, and then, according as its inclination is upward or downward, add to or subtract from each reduced height a *correction for declivity*, found by multiplying the distance of the point from the commencement of the sloping line of sight by the *sine* of the angle of inclination, if distances have been measured on the slope, or by its *tangent*, if they have been reduced to horizontal distances. (Article 23, p. 20).

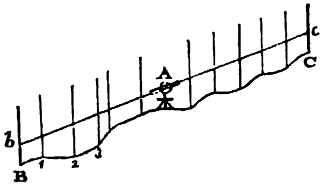

Fig. 46.

III. *By taking angles of altitude and depression.*—The height of an object above, or its depth below, the telescope of the theodolite, is nearly equal to its horizontal distance from the station of observation multiplied by the tangent of its altitude or depression, as the case may be.

The correction for curvature is *one-half of the angle subtended by the distance at the centre of curvature of the earth's surface*, or "contained arc," as it is called; and that correction is to be added to altitudes and subtracted from depressions. (As to the computation of that angle, see Article 33, Division V., pp. 46, 47.)

The correction for refraction is very variable, as has been already explained. On an average, it may be approximated to by diminishing the correction for curvature by one-sixth.

The effects of curvature and refraction may be nearly neutralized by taking *reciprocal angles*, as they are called; that is to say, if A and B be two stations, B being the higher; take the altitude of B as seen from A, and the depression of A as seen from B; half the difference of those angles will be the combined correction; and the *tangent of half their sum*, being multiplied by the distance, will give the difference of level nearly. The reciprocal angles should be taken as nearly as possible at the same instant, lest the refracting power of the air should change in the interval.

Levelling by angles is not to be relied upon for engineering purposes, except occasionally in taking flying levels.

The altitude of an object on land is taken with the sextant, by observing the "*double altitude*"—that is, the angle between the object and its image as reflected in a trough of mercury, called an "*artificial horizon*,"—and taking one-half of that double altitude.

56. **Levelling by the Plane-Table** is performed by adjusting the table with particular care to a horizontal position, measuring the tangent of the altitude or depression, and multiplying it by the distance. To enable such tangents to be measured, the index is constructed as follows:—In fig. 47, E F is the flat bar of the index, F *b* its forward and E *a* its backward sight. Near the bottom of the backward sight is a sight-hole A for observing altitudes; near the top, a sight-hole *a* for observing depressions. A scale of equal parts is marked on the forward sight, and numbered upwards from B opposite A, and downwards from *b* opposite *a*. A slider D is slid up or down till a cross-wire contained in it appears in a line with the object, and the tangent is read by an index and vernier.

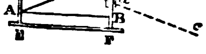

Fig. 47.

This process also is only suited for flying levels.

57. **Levelling by the Barometer and Thermometer** may occasion-

ally be used for engineering purposes to take flying levels in exploring the country. The following formula is sufficiently correct for that object:—

Let the quantities observed be denoted as follows:—

	At the lower station.	At the higher station.
Height of the mercurial column in the barometer,	H	h
Temperature of the mercury in degrees of Fahrenheit, as shown by the "attached" thermometer,	T	t
Temperature of the air in degrees of Fahrenheit, as shown by the "detached" thermometer,	T'	t'

Then the height of the higher station above the lower, in feet

$$\left.\begin{array}{l} = 60360 \left\{\log H - \log h - \cdot 000044\,(T - t)\right\} . \\ \qquad \left(1 + \dfrac{T' + t' - 64}{986}\right). \end{array}\right\} \ldots (1.)$$

For rapid calculation, the following, though less exact, is convenient:—

$$\text{Height in feet} = 60000\,(\log H - \log h).\left(1 + \frac{T' + t'}{1000}\right) \text{nearly. } (2.)$$

In the absence of logarithms, the following formula may be used for heights not exceeding about 3,000 feet. Correct the barometric reading at the higher station as follows:—

$$h' = h\left(1 + \frac{T - t}{10000}\right); \text{ then}$$

$$\text{Height in feet} = 52428\,\frac{H - h'}{H + h'}.\left(1 + \frac{T' + t' - 64}{986}\right) \text{nearly. } (3.)$$

The preceding formulæ are applicable to the mercurial barometer. They are also applicable to the "Aneroid" barometer, with the exception of the correction depending on the temperature by the attached thermometer. The aneroid barometer, if very skilfully constructed, may be made to require no appreciable correction for the effect of its own temperature on its indications. Should it need such correction, the amount can only be determined by an experimental comparison between the individual aneroid barometer and a mercurial barometer.

Another method of taking flying levels, depending, like the barometric method, upon the pressure of the air, is that of determining the boiling-point of pure water by a very sensitive thermometer—a method invented by Dr. Wollaston, and improved by Principal Forbes. (See *Transactions of the Royal Society of Edinburgh*, vols. xv. and xxi.)

That boiling-point falls very nearly at the rate of *one degree of Fahrenheit for every* 543 *feet of ascent*; and still more nearly according to the following formula:—

$$z \text{ in feet} = 517\,(212^\circ - T) + (212^\circ - T)^2; \ldots\ldots (4.)$$

T being the boiling-point on Fahrenheit's scale, and z the height of the station where the experiment is made above a station where the boiling-point is 212°. To compare the levels of two stations, the boiling-point of pure water is to be observed at each, and the quantity z is to be calculated by formula 4 for each of the boiling-points; when the difference between those quantities z will be the approximate height of the station where the boiling-point is lowest above the other station.

58. **Detached Levels—Features of the Country.**—The use to the engineer of "Flying Levels," or observations of the heights of detached points, has already been mentioned in Article 10, p. 9. Such heights cannot be easily shown by means of vertical sections; and the most convenient method of recording them is to write them on a plan of the country.

Detached levels may be taken for the purpose of determining the elevations of important points on existing works, such as bridges, roads, railways, canals, &c., or of objects suitable for bench marks; or they may be taken in order to give the engineer a general knowledge of the form of the surface of the country. In the present Article it will be shown what positions are the best suited for detached levels taken with the last-mentioned purpose.

On the surface of the earth, and, indeed, on any irregularly curved surface, two classes of lines may be distinguished, whose positions and figures are of primary importance in determining the shape of that surface—RIDGE-LINES and VALLEY-LINES.

I. A **Ridge-Line** is distinguished by the property, that along the whole of its course it is higher than the ground immediately adjacent to it on each side;—in other words, the ground slopes downwards from it at both sides. The rain-water which falls on the ground consequently runs away from both sides of a ridge-line; and hence it is also called a "WATER-SHED LINE." Ridge-lines are also sometimes called "the features of the country." The earth's surface is traversed by a number of main ridge-lines, which are the central lines of the great mountain-chains; from these there diverge branch ridge-lines, and from these secondary branch ridge-lines, and so on; until in most cases the final ridge-lines end at promontories, where they sink down into the plains or the valleys. A ridge-line may return into itself, so as to contain within it an enclosed hollow or basin; but this is of comparatively rare occurrence.

A ridge-line is seldom either straight or level throughout any considerable part of its length, being almost always more or less wavy or serrated both vertically and horizontally.

The highest points of ridge-lines form the summits of the hills. The summit of a conical or rounded hill may in some cases be an isolated point, not traversed by a ridge-line; but the summit of a hill is in general traversed by at least one ridge-line, and is very often a point of divergence of several ridge-lines. A summit may sometimes be a flat expanse called a "table-land," with ridge-lines diverging from its edges.

II. A **Valley-Line** is distinguished by the property, that along the whole of its course it is lower than the ground immediately adjacent to it on each side;—in other words, the ground slopes upwards from it at both sides. The water on the surface of the ground consequently runs towards a valley-line from both sides, and except in certain cases, runs along the valley-line in a stream; whence valley-lines may be called "WATER-COURSE LINES." The exceptions are, when the valley-line is in an enclosed basin, so that a lake is formed; and when the surface water disappears by evaporation or absorption. Between each adjacent pair of final ridge-lines there is a valley-line; these valley-lines converge and unite into greater valley-lines, and so on until the final valley-lines end in the sea, or at the bottom of some enclosed basin, or at the edge of a plain. A valley-line, like a ridge-line, is seldom either straight or level throughout any considerable part of its length.

The end of a ridge-line lies in general either in a plain, or between two converging valley-lines, or in the bend of a valley-line. The commencement of a valley-line lies in general between two diverging ridge-lines, or in the bend of a ridge-line, or at a "Pass."

A **Pass** is a place on a ridge-line lower than any neighbouring point on the same ridge-line, and might be described as a point where a ridge-line and a valley-line cross each other at right angles; but it is more in accordance with the ordinary use of the word "valley" to describe the line of lowest elevation at a pass, which crosses the ridge-line at right angles, as consisting of two valley-lines which run downwards from the pass in opposite directions.

From these descriptions of ridge-lines and valley-lines, and of points in and connected with them, it is obvious that the places whose elevations are of most importance towards a knowledge of the figure of the surface of a district are the following:—

The summits of hills, being peaks, table-lands, or highest points of ridge-lines.

The points where the inclinations of ridge-lines change.

The points from which ridge-lines diverge.

The passes, or lowest points of ridge-lines and highest points of pairs of valley-lines.

The lowest points of valley-lines.

The points where the inclinations of valley-lines change.

The points where converging valley-lines meet.

Of all those places, those which are of most importance in the engineering of lines of communication are the *passes;* because they are in general the points at which ridges are to be crossed.

To the levels of the places already enumerated may be added, those of the surfaces of seas, lakes, rivers, and other bodies of water, in their various conditions.

The valley-lines of a district are usually marked with sufficient distinctness on a plan by the water-courses. The position of the ridge-lines is in general indicated by shading the slopes which fall from them in each direction, the best system of shading being that according to which the depth of the shadow varies with the steepness of the slope, being made as nearly as possible proportional to the tangent of the angle of declivity. The bottoms of valleys are, in addition, very slightly shaded, in order to distinguish them from the tops of hills.

59. **Contour-lines** are used as means of enabling a plan to give more complete information as to the figure of the surface of the ground than is possible by means of levels written in figures alone.

A contour-line on a plan represents a contour-line on the earth's surface, which is a line traversing all the points on the ground that are at a given constant height above the datum-level. A contour-line on the ground may be otherwise described as a horizontal section of the earth's surface, or the line where the earth's surface is cut by a given horizontal surface, or the outline of an imaginary sheet of water, covering the ground up to a certain given elevation.

All contour-lines cross the lines of steepest declivity on the surface of the ground at right angles; they also cross at right angles all ridge-lines and valley-lines at which the surface of the ground is sensibly curved, and does not form an absolutely sharp ridge or furrow.

The vertical distance between successive contour-lines on a plan depends on the scale of the plan, the figure of the ground, and the purpose for which the plan is intended; being greater in plans on a small scale than in those on a large scale; greater where the slopes are steep and the hills high than where the slopes are gentle and the hills low; and greater or less, also, according to the precision with which levels have been taken for finding the position of the contour-lines, and the use that is to be made of them in designing works. For example, in the Ordnance Maps of Britain, on the scale of six inches to a mile, contour-lines are drawn at each twenty-five feet of height, and certain of these, called "principal contour-lines," are determined with greater precision than the others; and those principal contour-lines are at every fifty feet of elevation in the flatter parts of the country, and at every hundred feet in the more hilly parts. The closest contour-lines are those which have been laid down in some plans of town districts for purposes of drainage and other improvements: these occur at vertical intervals of from eight feet to two feet.

Different methods of determining the positions of contour-lines may be followed according to the degree of precision required. To lay down principal contour-lines, a series of bench marks should be made at such points in ridge and valley-lines as have been already specified in the preceding Article; the positions of those bench marks should be ascertained in the course of the survey, and laid down on the plan, and their elevations found by levelling. Then by levelling from those bench marks, points are to be marked by pegs, or otherwise, on the ridge and valley-lines, and at as many intermediate places as may appear necessary, at certain definite elevations above the datum-level, such as 50 feet, 100 feet, 150 feet, and so on. The positions of the points so marked being surveyed with the chain and plotted, give a series of points in the contour-lines; and the course of those lines between the points so found by surveying is to be sketched upon a tracing of the plan taken to the ground for the purpose. Bench marks, whose levels ought to be checked, should be made at the places where principal contour-lines cross important ridge-lines and valley-lines.

Intermediate contour-lines can be interpolated between the principal contour-lines by sketching on the ground, aided by the known levels of the points where the rates of inclination of the ridge and valley-lines vary.

The horizontal distance between two adjoining contour-lines being inversely as the tangent of the angle of inclination of the ground, is also inversely as the depth of shadow to be used to express the steepness of the slope. "Hill-sketching," as it is called, consists in shading the slopes of hills upon the ground according to this principle, with the pencil, by drawing horizontal lines parallel to the contour-lines, and with a degree of closeness proportional to that of the contour-lines themselves. Those pencil hatchings are in fact intermediate contour-lines sketched by hand.*

In engraved plans the shading of hills is effected by means of hatched lines at right angles to the contour-lines, and following, therefore, the lines of steepest declivity.

In order that an engineer may know how far he can depend upon the contour-lines on a plan as a means of enabling him to select the best line for a proposed work, it is necessary that he should know by what method, and with what degree of precision, their positions have been determined, and that he should see upon the plan the positions and written levels of the bench marks and other detached points which have been used during that process.

60. **Cross Sections** may cross the centre line, or line of the longitudinal section, of a proposed work either at right angles or obliquely.

The term is applied to longitudinal sections of existing lines of communication which the proposed work has to cross. Such sections have already been referred to in Article 7, p. 6, and Article 8, p. 7.

Cross sections to assist the engineer in choosing the best line (referred to in Article 11, Division V., p. 10) should in general run along the ridge-lines and valley-lines which are to be crossed by the proposed work. They should also be made where the ground has a steep slope in a direction transverse or oblique to that of the centre line of the proposed work.

Cross sections to accompany the working section, for the purpose of enabling quantities of excavation or other work to be measured and calculated exactly (referred to in Article 11, Division XIV.,

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* In the late Mr. Butler Williams's *Practical Geodesy*, p. 190, he describes in the following terms an approximate method of drawing contour-lines by the aid of pencil hill-shadings:—

"Horizontal contours can be traced by the eye with considerable accuracy, especially when the surveyor is assisted by the altitudes obtained in the trigonometrical operations serving for the construction of the outline map. The process . . . which I now proceed to describe, is rapid in execution, and tolerably correct for a small scale (say one inch or two inches to a mile), where experience has trained the eye to accuracy. It is well adapted for reconnoissances of a country, and is much used by military engineers. The civil engineer would, however, frequently find the same advantage in using it in his preliminary examinations of countries for the purpose of selecting general lines of communication.

"In the field, when the eye is alone depended upon, the horizontal lines are traced in pencil, by close parallel hatchings; and when the whole drawing is finished, the normal contours are traced at the required vertical distances apart, by following the general direction of the pencil lines, and checking their truth by means of the trigonometrical elevations or other heights marked on the map. The contours, when a complete circuit is made, must return to the point of departure; and if it were attempted by the eye alone to trace normal contours which are isolated from each other, no degree of previous experience would suffice for the attainment of the object.

p. 11), are in general at right angles to the line of the longitudinal section.

61. The **Water-Level** is an instrument used instead of the spirit-level where long range and great accuracy are unnecessary. It consists of an inverted siphon tube, fixed on the top of a stand, and nearly filled with water, which may be slightly tinged to make it the more easily visible. The horizontal part of the tube (about eighteen inches or two feet long) may be of metal: the two vertical branches (which are only two or three inches high) are of glass. The surfaces of the water stand at the same level in those two branches; and the leveller obtains a horizontal line of sight by looking along a line joining those two surfaces, which may be considered as the "line of collimation" of the instrument. When the distance of the staff is so great that the observer cannot read the divisions, a staff of the old kind, with a sliding vane, may be used. (See Article 49, p. 83.) The water-level is useful for setting-out the points of contour-lines intermediate between the bench marks, being sufficiently accurate for that purpose, and more expeditious than the telescopic levelling instrument.

CHAPTER V.

OF SETTING-OUT.

62. **Ranging Straight Lines.**—It has already been stated in Article 11, Division XIII., p. 11, that the process of *ranging and setting-out the line* consists in marking on the ground the centre line of the proposed work.

That marking consists of two operations: temporary marking, or ranging, by means of poles; and permanent marking, or setting-out, properly so called, in which the principal marks are in general stakes.

The distance apart of the stakes used in setting-out the centre line of a proposed work varies considerably in the practice of different engineers. In some cases, a stake is driven at every chain of 66 feet; in others, at every 100 feet; while on some works the distance from stake to stake has been as great as 300 feet. Time and money are saved by adopting a long interval between the stakes, but at the expense of precision.

For ranging straight lines of moderate length, the most convenient instrument is a large-sized transit theodolite—that is to say, one with circles of six inches in diameter or more (Article 34, pp. 54, 55)—because the telescope is capable of being turned completely over about its horizontal axis, so as to range one continuous straight line in two opposite directions from the station. In order that this operation may be correctly performed, great care must be bestowed on the adjustment of the line of collimation perpendicular to the horizontal axis (Article 35, p. 60), of the horizontal axis perpendicular to the vertical axis (Article 35, p. 61), and of the vertical axis truly vertical (Article 35, p. 58). With a good six-inch theodolite the error in ranging a pole in a straight line should not exceed 10″ in angular direction; that is to say, about three inches at a distance of a mile off.

For very long straight lines, however, the theodolite is not sufficiently exact; and then it becomes advisable to use a small Transit Instrument, consisting simply of a telescope with a horizontal axis, resting on a suitable stand, so as to be capable of being turned over in a vertical plane.

The telescope of a transit instrument for engineering purposes may be from twenty to thirty inches in the focal length of the object-glass. At the middle of the length of the telescope tube is a hollow sphere, to which are joined two hollow cones, forming the arms of the horizontal axis. Those arms taper towards the ends, where they terminate in two hollow cylindrical pivots, which rest in angular bearings called Y's, each supported on the top of one of the standards of the frame. One of these Y's has a vertical adjusting screw, for raising or lowering it till the horizontal axis is truly horizontal; the other has horizontal adjusting screws, for shifting it back or forward until the horizontal axis is truly perpendicular to the vertical plane in which the line of collimation is intended to move. There is a moveable spirit-level for placing the axis horizontal, whose use will presently be described.

At the common focus of the object-glass and eye-piece are a set of cross-wires carried by a diaphragm, which has adjusting screws to move it so as to place the line of collimation (marked by the intersection of the central pair of cross-wires) exactly perpendicular to the horizontal axis. At night the cross-wires are rendered visible by light which enters from a lantern through one of the hollow pivots of the horizontal axis, and is reflected towards the cross-wires by a small oblique mirror. The strong cast-iron stand of the instrument rests on and is screwed to a smooth level stone slab, forming the top of a massive stone or brick pedestal, built on a firm foundation. The building which shelters the instrument should be entirely disconnected from the pedestal; otherwise the vibrations produced in it by the wind will be communicated to the instrument.

To facilitate the placing of the instrument exactly in a given alignment, the frame sometimes rests on a lower frame, like the slide-rest of a lathe, along which it can be slid sideways into the required position by the action of a screw.

The adjustments of the transit instrument are as follows:—

(1.) *To place the line of collimation exactly perpendicular to the horizontal axis.*—Direct the cross-wires towards a well-defined point in a distinct object; lift the telescope with its axis out of the Y's, turn it over, so as to reverse the position of the axis end for end, and set it down again: if the cross-wires cover exactly the same object, the adjustment is correct; if not, correct one-half of the error by the horizontal adjusting screws of one of the Y's, and the other half by the adjusting screws of the diaphragm. Repeat the process till the adjustment is perfect.

(2.) *To place the horizontal axis truly horizontal.*—The spirit-level has two feet, which are to be placed striding across the telescope so as to rest on the two pivots of the horizontal axis respectively. Bring the bubble to the middle of the level by turning the vertical adjusting screw of one of the Y's; reverse the position of the level end for end: if the bubble remains at the centre of the level, the adjustment is correct; if not, correct one-half of the deviation by the vertical adjusting screw of the axis, and the other half by the adjusting screw which regulates the height of one of the feet of the spirit-level. Repeat the operation till the adjustment is perfect; and be careful to remove the spirit-level before moving the telescope.

(3.) *To place the plane of motion of the line of collimation exactly in the vertical plane traversing two distant stations at opposite sides of the instrument.*

The pedestal is to be built as nearly in the true alignment as is practicable by ordinary methods of ranging, and the upper surface of the flat stone which forms the top is to be carefully levelled.

The transit instrument having been set on the pedestal, and its line of collimation adjusted perpendicular to the horizontal axis, is to be moved by hand until the telescope, being turned alternately in opposite directions, points nearly towards the signals marking the distant ends of the line; observing, that when the line of collimation deviates to the *same* side of both signals (for example, in a line running north and south, to the east of both, or to the west of both) such deviation is to be corrected by shifting the stand sideways; and that when the line of collimation deviates to *opposite* sides of the signals (for example, to the east of one, and to the west of the other) such deviation is to be corrected by turning the stand as if about a vertical axis. This is the first approximation to adjustment. A second approximation is made in the same manner, after having levelled the horizontal axis; and then the places are marked for the screw sockets. The instrument having been removed, the holes for those sockets are to be cut, and the sockets fixed in them with lead.

The instrument is then replaced, and approximately adjusted as before, and the screws for fixing it to the pedestal are inserted, but not tightened. The instrument is finally adjusted by the aid of the horizontal adjusting screws of the horizontal axis, and the fixing screws are tightened.*

63. **Ranging and Setting-out Curves.**—The curved parts of railways require to be set out with great precision. The form almost universally adopted for them is that of circular arcs, though in a few instances other forms, such as that of the parabola, have been used. There are reasons for thinking that the best form, in a mechanical point of view, is that called the "elastic curve," which a spring of uniform transverse section takes when bent; but the

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* Ample details on this subject are given in Mr. Simms's work *On Practical Tunnelling.*

great difficulty of setting it out accurately has prevented its being used. The only methods which will be described here are three of those of setting out circular curves—the method by angles—the method by offsets—and the method by bisections of arcs.

METHOD I.—*Setting-out Circular Curves by Angles at the Circumference.*—This is the only method by which circular curves can be set out at once as quickly and as accurately as straight lines.[*] It depends on the well-known principle, that the angle subtended by any arc of a circle at any point in the circumference of the same circle, is one-half of the angle subtended by the same arc at the centre of the circle. For example, in fig. 48, A B is an arc of a circle, C a point in the circumference of the same circle lying beyond the arc. The angle A C B is one-half of the angle subtended by A B at the centre of the circle. When the point at which the angle is measured lies upon the arc, as at E, it is the angle B E F = A E G, between the line drawn from one end of the arc and the prolongation of the line from the other end, that is equal to half the angle at the centre of the circle. When the point at which the angle is measured is one of the ends of the arc, as A, it is the angle D A B, between the tangent of the arc and its chord, that has the same property.

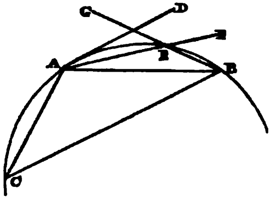

Fig 48.

To express this by a formula; let a denote the length of the arc, r the radius of the circle; then—

Angle at the circumference in minutes

$$= \mathrm{A\,C\,B} = \mathrm{F\,E\,B} = \mathrm{D\,A\,B} = \frac{\text{Angle at the centre}}{2}$$

$$= 1718'{\cdot}873\,\frac{a}{r}.\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

(The co-efficient is the value in minutes of one half of the arc equal to radius; see p. 37.)[†]

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[*] This method of setting-out curves by angles was published for the first time in a paper read to the Institution of Civil Engineers on the 14th of March, 1848, by the author of this work, who had first practised it in 1841. Methods of setting-out curves by the theodolite had previously been employed by Captain Vetch and Mr. Gravatt, but they have never, so far as the author knows, been published.

[†] American engineers describe the sharpness of curves by stating the number of degrees in the angle subtended at the centre by an arc of 100 feet in length, which angle they call the "angle of deflection." Its value is

$$\text{Angle of deflection in degrees} = \frac{5729{\cdot}6}{\text{radius in feet}};$$

In applying that principle to practice, the best instrument is a six-inch transit theodolite, which will range the positions of poles at the distance of half a mile to the accuracy of an inch and a-half. With a smaller instrument, the distances must be shorter, or the precision less.

Problem First.—To set out a circular curve touching two given straight lines, when the point of intersection of those straight lines is accessible.

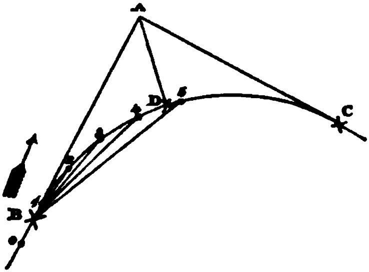

Fig. 49.

In fig. 49, let B A, C A, be the two straight lines, intersecting in A. Set the theodolite at A, and measure the angle there, which denote by A; then lay off the two equal tangents, A B, A C, as calculated by the following formula (in which r is the intended radius of the curve):—

$$AB = AC = r \cdot \operatorname{cotan} \frac{A}{2}; \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

and B and C will be the ends of the curve, where it touches the straight lines.

It is convenient (though not always necessary) to find the middle point of the curve. For that purpose, range, by means of the theodolite, the line A D bisecting the angle at A; and lay off the distance,—

$$AD = r \cdot \left(\operatorname{cosec} \frac{A}{2} - 1\right); \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

then will D be the middle point of the curve.

The points B and C (and also D, if marked) should be marked by stakes distinguished in some way from the ordinary stakes which are driven all along the centre line at equal distances of one chain, or 100 feet, or some other distance.

and conversely—

$$\text{Radius in feet} = \frac{5729{\cdot}6}{\text{Angle of deflection in degrees}}.$$

A curve is called a "one-degree curve," a "two-degree curve," and so on, according to its angle of deflection. Hence,

A "one-degree curve" means a curve of 5729·6 feet radius;
a "two-degree curve" — — 2864·8 — —
a "three-degree curve" — — 1909·9 — —

and so on.

The total length of the curve is found by the formula,

$$\text{Arc B C} = \cdot 0002909\, r \times \text{supplement of A in minutes} \ldots (4.)$$

Any one of the points, B, C, or D, will answer as a station for the theodolite in ranging the curve. The commencement of the curve, B, is the station that involves the simplest operations; but when the length of the curve exceeds about half a mile, the middle point, D, is the best station as regards accuracy and convenience.

The following is the process of ranging the curve with the theodolite planted at its commencement, B:—

For brevity's sake, the distance between the stakes which mark the centre line of the proposed railway will be called "a Chain," whether it is 66 feet, 100 feet, or a greater distance.

Let *o*, in fig 49, represent the last stake in the portion of the straight line immediately preceding the curve; the distance B 1 from the commencement of the curve to the first stake in it will be the difference between one chain and *o* B. The angle at the circumference subtended by the arc B 1 having been calculated by equation 1, is to be laid off by the theodolite from the tangent B A, the zero-point of the azimuth circle being directed towards A. The line of collimation will then point in the proper direction for the first stake in the curve, 1; and its proper distance from B being laid off by means of the chain, its position will be determined at once.

The angles at the circumference subtended by B 1 + 1 chain, B 1 + 2 chains, B 1 + 3 chains, &c., being also calculated, and laid off from the tangent B A in succession, will respectively give the proper directions for the ensuing stakes, 2, 3, 4, &c., which are at the same time to be placed successively at uniform distances of one chain by means of the chain.

The difference between an arc of one chain and its chord, on any curve which usually occurs on railways, is in general too small to cause any perceptible error in practice, even in a very long distance; but should curves occur of unusually short radii, it is easy to calculate the proper chord, and set it off from each stake to the next, instead of one chain, the length of the arc. For this purpose, the following approximate formula is useful. Let *r* be the radius, *a* the arc, and *c* the chord; then—

$$c = a\left(1 - \frac{a^2}{24\, r^2}\right) \text{ nearly. } \ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

When the curve is ranged with the theodolite at D, or at any other intermediate point in the curve, or at its termination C, the process is precisely the same, except that the zero-point of the azimuth circle is to be turned towards B instead of A; and that when the chain passes the theodolite station (for example, in going from stake 4 to stake 5 in fig. 49, with the theodolite at D), the telescope is to be turned completely over.

When the inequalities of the ground make it impossible to range the entire curve from the stations B, D, and C, any stake which has already been placed in a commanding position will answer as a station for the theodolite.

The stakes or poles, after having been ranged by the theodolite, should have their positions finally checked and adjusted by a modification of the method of offsets, which will afterwards be explained.

PROBLEM SECOND.—To set out a circular curve, touching two given straight lines, when the point of intersection of those lines is inaccessible.

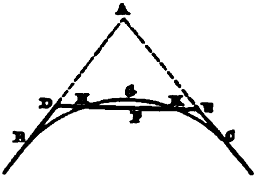

Fig. 50.

In fig. 50, the lines to be chained on the ground are represented by full lines; those whose lengths are to be calculated only are dotted.

Let B A, C A, be the two straight lines, meeting at the inaccessible point A. Chain a straight line D E upon accessible ground, so as to connect those two tangents. The position of the *transversal* D E is arbitrary; but it is convenient so to place it that it will cut the proposed curve in two points, which may be determined, and used as theodolite stations.

Measure the angles B D E, D E C, which may be denoted by D and E. Then the angle at A is

$$\mathrm{A} = \mathrm{D} + \mathrm{E} - 180^\circ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

$$\mathrm{A\,D} = \mathrm{D\,E} \cdot \frac{\sin \mathrm{E}}{\sin \mathrm{A}}; \ \mathrm{A\,E} = \mathrm{D\,E} \cdot \frac{\sin \mathrm{D}}{\sin \mathrm{A}}; \ldots\ldots\ldots (7.)$$

$$\mathrm{D\,B} = r \cdot \mathrm{cotan}\, \frac{\mathrm{A}}{2} - \mathrm{A\,D}; \ \mathrm{E\,C} = r \cdot \mathrm{cotan}\, \frac{\mathrm{A}}{2} - \mathrm{A\,E}; \ldots (8.)$$

and by laying off the distances D B and E C as thus calculated, the ends of the curve B and C are marked, and it can be ranged from either of those stations as in Problem First.

But it is often convenient to have intermediate points in the curve for theodolite stations; and of those the points of intersection with the transversal, H and K, and the point G, midway between these, can easily be found by the following calculations, in making which a table of squares is useful.

Let F be the point on the transversal midway between H and K.

If B D = C E, the point F is at the middle of D E. If B D and C E are unequal, let B D be the greater; then the position of F is given by either of the two following formulæ:—

$$\mathrm{D\,F} = \frac{\mathrm{D\,E}}{2} + \frac{\mathrm{B\,D}^2 - \mathrm{C\,E}^2}{2\,\mathrm{D\,E}}; \quad \mathrm{E\,F} = \frac{\mathrm{D\,E}}{2} - \frac{\mathrm{B\,D}^2 - \mathrm{C\,E}^2}{2\,\mathrm{D\,E}}. \;.. \quad (9.)$$

The points H and K are at equal distances on each side of F, given by either of the following expressions:—

$$\mathrm{F\,H} = \mathrm{F\,K} = \sqrt{\left\{\frac{\mathrm{D\,E}^2}{4} + \frac{(\mathrm{B\,D}^2 - \mathrm{C\,E}^2)^2}{4\,\mathrm{D\,E}^2} - \frac{\mathrm{B\,D}^2 + \mathrm{C\,E}^2}{2}\right\}}$$

$$= \frac{\sqrt{(\mathrm{D\,F}^2 + \mathrm{E\,F}^2 - \mathrm{B\,D}^2 - \mathrm{C\,E}^2)}}{\sqrt{2}}. \;\dots\dots\dots\dots \quad (10.)$$

The equations 9 and 10 are deduced from the two following, which may be used in order to check the calculations, and are given in a form suitable for the use of a table of squares:—

$$\left.\begin{aligned} \mathrm{B\,D}^2 &= \mathrm{D\,H} \cdot \mathrm{D\,K} = \frac{(\mathrm{D\,H} + \mathrm{D\,K})^2 - (\mathrm{D\,K} - \mathrm{D\,H})^2}{4}; \\ \mathrm{E\,C}^2 &= \mathrm{E\,H} \cdot \mathrm{E\,K} = \frac{(\mathrm{E\,H} + \mathrm{E\,K})^2 - (\mathrm{E\,H} - \mathrm{E\,K})^2}{4}. \end{aligned}\right\} \quad (11.)$$

The point G in the curve is found by setting off the ordinate F G perpendicular to D E, of the following length:—

$$\mathrm{F\,G} = r - \sqrt{r^2 - \mathrm{F\,H}^2}. \;\dots\dots\dots\dots\dots\dots \quad (12.)$$

The angles subtended at the *centre* of the curve by the several arcs between the commencement B and the points H, G, K, C, are as follows:—

$$\left.\begin{aligned} \text{Angle subtended at the centre by BH} &= 180^\circ - \mathrm{D} - \operatorname{arc}\cdot\sin\frac{\mathrm{FH}}{r}; \\ \text{— — — — } \quad \mathrm{BG} &= 180^\circ - \mathrm{D}; \\ \text{— — — — } \quad \mathrm{BK} &= 180^\circ - \mathrm{D} + \operatorname{arc}\cdot\sin\frac{\mathrm{FH}}{r}; \\ \text{— — — — } \quad \mathrm{BC} &= 180^\circ - \mathrm{A} = 360^\circ - \mathrm{D} - \mathrm{E}; \end{aligned}\right\} \quad (13.)$$

and the length of any one of those arcs may be computed by means of the formula,

$$\text{Arc} = \cdot 0002909\, r \times \text{angle at centre in minutes.} \;\dots \quad (14.)$$

The use of such computations will appear in the next problem.

Cases may occur in which obstacles upon the ground render it necessary to make one or both of the ends of the transversal D E meet the straight tangents *beyond* the ends of the curve. The whole of the formulæ already given continue to be applicable, with only the following modifications.

When D lies further from A than B does, D B is negative in the first of the equations 8—that is, A D is greater than $r \cdot \text{cotan} \frac{A}{2}$; and the point H, as found by means of equation 10, lies, not on the arc to be ranged, but on the continuation of the same circle beyond B.

When E lies further from A than C does, E C is negative in the second of the equations 8—that is, A E is greater than $r \cdot \text{cotan} \frac{A}{2}$; and the point K, as found by means of equation 10, lies, not on the arc to be ranged, but on the continuation of the same circle beyond C.

The point G always lies on the arc to be ranged. The longer the ordinate F G is, the more carefully must it be set off at right angles to the transversal.

Problem Third.—To set out a circular curve touching two given straight lines, when part of the curve is inaccessible to the chain.

If the point of intersection of the tangents is accessible, the two ends of the curve are to be determined and marked as in Problem First, and also the middle point of the curve, unless it lies on the inaccessible ground; and the length of the curve is to be computed by equation 4.

If the point of intersection of the tangents is inaccessible, the two ends of the curve, and at least one intermediate point, are to be determined and marked by the aid of a transversal, as in Problem Second, and the lengths of the arcs bounded by those points are to be computed by the formulæ 13 and 14.

A transversal may be useful even when the point of intersection of the tangents is accessible.

Each of the points thus marked will serve either as a theodolite station, or as a station to chain from, or for both purposes; and the stakes lying between the obstacle and the next station beyond it are to be planted by chaining backwards from that station.

Suppose, for example, that the commencement of the curve (B), lies at 243 chains 60 links from the commencement of the line, or "peg 0." The first stake in the curve will be 40 links from B, and will be "peg 244." Now, suppose that pegs 245 and 246 can be planted by chaining forwards, but that an obstacle occurs in the course of the next chain. Let G denote a station in the curve beyond the obstacle, found by means of a transversal or otherwise, and let the arc B G, computed by the proper formula, be 6 chains 20 links. Then G is 243 · 60 + 6 · 20 = 249 chains 80 links from "peg 0," and lies between peg 249 and peg 250. Peg 249 is planted by chaining backwards 80 links from G, and pegs 248 and 247 by continuing to chain backwards. Peg 250 is planted by chaining forwards 20 links from G, and pegs 251, &c., by continuing to chain forwards. The ranging of the angular directions of the stakes from a theodolite station presents no peculiarity.

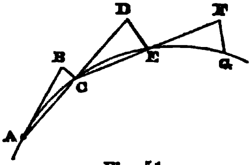

Fig. 51.

Method II.—*Setting-out Circular Curves by Offsets.*

In fig. 51, let A C, C E, E G, be a series of equal or unequal chords inscribed in a circle. Produce A C, to D, making C D = C E; join D E. The distance D E is called the "*offset;*" and its value is almost exactly

$$D\,E = \frac{C\,E \cdot A\,D}{2r}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15.)$$

Let C E and E G be two *equal* chords; then the offset is

$$F\,G = \frac{C\,E^2}{r}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16.)$$

If A B is a tangent to the curve at A, and C B a perpendicular let fall upon it from C, that perpendicular, being the *offset from the tangent*, is

$$B\,C = \frac{A\,C^2}{2r}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17.)$$

Problem Fourth.—To set out a circular curve by offsets, commencing at a given point on a straight line (fig. 51).

Let A be the commencement of the curve, found as in Problem First, and marked with a pole; A B the prolongation of the straight line (being a tangent to the curve), and B the end of the chain when laid along that prolongation from the last stake in the straight line. Plant a small pole at B, calculate the offset B C by equation 17, shift the end of the chain, and the pole along with it, sideways from B to C, keeping the chain tight, and leave the pole at C.

Drag the chain onward in the prolongation of A C; range a pole at D in a straight line with A and C, and at one chain's distance from C; shift the pole and the end of the chain through the offset D E, calculated by equation 15.

Drag the chain onward; range a pole at F in a straight line with C and E, and at one chain's distance from E; shift the pole and the end of the chain through the offset F G, calculated by equation 16; leave the pole at G, and so on.

If this process could be performed with absolute precision, the curve would terminate by exactly touching the further tangent at the point of contact found as in Problem First. But this never takes place at the first trial, except by accident; for any small inaccuracy in laying off the offset produces an error in the position of each stake, increasing nearly as the square of the distance from the commencement of the curve. If the final error is considerable, the curve must be ranged over again, until by successive trials the final error has been reduced to one not exceeding about ten links; then the positions of the stakes are to be finally adjusted by chaining round the curve once more, and shifting each stake sideways through a distance proportional to the square of its distance from the commencement of the curve.

Although this method is clumsy and tedious as a means of ranging curves, it is very useful for testing the uniformity of curvature of curves already ranged, and for rectifying the positions of individual stakes to the extent of an inch or two.

METHOD III.—PROBLEM FIFTH.—*To set out a circular curve by successive bisections of arcs.*

This is a method to be used only in the absence of angular instruments. It depends on the following relation between the versed sine of an angle B and that of its half;

$$\text{versin}\,\frac{B}{2} = 1 - \sqrt{1 - \frac{\text{versin}\,B}{2}}. \qquad \ldots\ldots\ldots (18.)$$

To apply this principle, let B A, C A, in fig. 52, be the two tangents, and B and C the ends of the curve, so placed that A B and A C shall be equal, but leaving the radius to be found by calculation. Measure the chord B C.

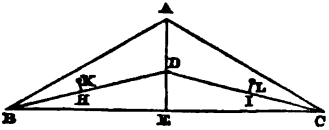

Fig. 52.

Then the simplest process for finding the radius is to use the following formula;—

$$r = \frac{A\,B \cdot B\,C}{2\sqrt{A\,B^2 - \frac{B\,C^2}{4}}}; \qquad \ldots\ldots\ldots\ldots\ldots\ldots (19.)$$

but as the triangle A B C is in general "ill-conditioned," it is more accurate, though more laborious, to bisect B C in E, measure A E, and make

$$r = \frac{\mathrm{A\,B} \cdot \mathrm{B\,E}}{\mathrm{A\,E}}. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(19 A.)}$$

Calculate the versed sine of the angle A B E = B, which is that subtended at the centre by one-half of the curve, as follows:—

$$\operatorname{versin} \mathrm{B} = \frac{\mathrm{A\,B} - \mathrm{B\,E}}{\mathrm{A\,B}}; \quad\ldots\ldots\ldots\ldots\ldots\ldots \text{(20.)}$$

and by means of equation 18 (using a table of squares, if one is at hand) calculate the versed sines of $\frac{\mathrm{B}}{2}, \frac{\mathrm{B}}{4}, \frac{\mathrm{B}}{8}$, &c., in succession, observing that versin B enables one intermediate point in the curve to be found, versin $\frac{\mathrm{B}}{2}$, three points, versin $\frac{\mathrm{B}}{4}$, seven points; and generally, that versin $\frac{\mathrm{B}}{2^n}$ enables $2^{n+1} - 1$ intermediate points in the curve to be found.

From the middle E of the chord B C and perpendicular to it, lay off the offset E D = r versin B; D will be the middle point of the curve.

Chain and bisect the chords B D, D C, and from their middle points and perpendicular to them, lay off the offsets

$$\mathrm{H\,K} = \mathrm{I\,L} = r \operatorname{versin} \frac{\mathrm{B}}{2}; \quad\ldots\ldots\ldots\ldots\ldots\ldots \text{(21.)}$$

K and L will be points in the curve, midway respectively between B and D, and between D and C; and so on until a sufficient number of points have been marked by poles.

Then chain round the curve as ranged by the poles, and drive stakes at equal distances apart.

The uniformity of the curvature may be finally checked by Method II.

64. **Nicking-out** the centre line of a proposed work consists in cutting a small trench about six inches wide, to mark the centre line in the intervals between the stakes. The surface of the ground ought to be left undisturbed for a short distance on each side of each stake.

Where the centre line crosses fences and buildings, it should be distinctly marked by notches or grooves.

65. **Permanent Marks of the Line and Levels** are usually stakes

of larger dimensions than those which mark the centre line, and placed so far to either side of it that there is no risk of their being disturbed during the progress of the work, by which the marks on the centre line itself are obliterated.

The places where permanent marks of the course of the line are chiefly required are on the tangents of curves,—their distances from the ends of the curves being noted, so as to enable both the curves and the straight lines which connect them to be ranged over again at any time. Any important point on a curved or straight part of the centre line may be permanently marked by driving two stakes in a straight line passing through it, one at each side of the site of the work, and noting its distances from them, or by any of the means described in the second paragraph of Article 21, p. 17.

Stakes to be used as permanent bench marks for the levels of the work are about three or four feet long, and four inches square, hooped round the top with iron to prevent the head from being crushed. One of the best ways to form a firm surface for the staff to rest on is to drive into the head of the stake a long iron spike with a large convex head; the uppermost point of the convex surface of that head is the bench mark. Such marks are to be placed near the sites of all proposed pieces of masonry, and other structures of importance, and near the ends of cuttings and embankments; and opposite points where the rate of inclination or *gradient* of a proposed railway is to change.

As soon as any piece of masonry has been built high enough, one or more bench marks should be made on the masonry itself, to regulate the levels to which the remainder of the structure is to be built.

66. **Working Section and Level-Book.**—The nature of a working section has already been explained generally in Article 11, Division XIV., p. 11, and in Article 16, p. 15. The levels taken, in order to prepare it, consist for the most part of those of the stakes planted to mark the centre line, which are driven until their heads are flush with the ground. Should any inequality of the ground occur between two stakes, enough of additional levels and distances must be taken to enable an exact vertical section of it to be plotted; and the levels of every line of communication and other important object must be taken where it is crossed by the centre line. The level of every stake, and of every line of communication crossed, is to be taken twice over.

As to scales for working sections, see p. 7.

Cross Sections have already been referred to in Article 60, p. 97. When the ground is uneven sideways, they may be required at each stake. In general, they should be ranged accurately at right angles to the centre line, and should be plotted without exaggeration; their vertical and horizontal scales being the same with the vertical scale of the longitudinal section. All cross sections should be plotted as seen by looking *forwards* towards them along the centre line.

The **Level-Book** of the working section of a line of communication is a book containing a complete statement of the levels of the ground and of the intended work, and of other information which will presently be specified. Each folio of the book is divided into several columns, whose number, arrangement, and contents differ in the practice of different engineers. The following statement of the contents of the several columns of a level-book may be taken as an example. It is specially adapted to a railway, but may be made, by slight modifications, to suit other kinds of works:—

Column 1. Numbers of the stakes planted at equal intervals of 66 feet, 100 feet, 300 feet, or some other distance along the centre line. The stake at which the line commences is numbered 0.

Column 2. Distances from the commencement of the section, in links or feet, as the case may be.

Column 3. Descriptions of objects between the equidistant stakes, such as fences, streams, roads, canals, railways, intermediate stakes at ends of curves, &c.

Column 4. Levels of the ground at the stakes, and between them when necessary, and of bench marks.

Column 5. Intended level of the upper surface of the railway (or other proposed work.)

Column 6. *Formation level* (that is, level of the ground when prepared by excavation or embankment for the completion of the work).

Column 7. Depths of cutting, } as calculated by taking the differ-
Column 8. Heights of embankment, } ences between the numbers in column 4 and column 6.

Column 9. }
Column 10. } Rate of lateral slope of the ground, if any, to the $\left\{\begin{matrix}\text{left}\\ \text{right}\end{matrix}\right\}$ of the centre line, specifying whether it rises or falls from the centre line. If the slope of the ground is irregular, reference may be made to a cross section.

Column 11. }
Column 12. } Breadths of land required for works only (exclusive of fences) to the $\left\{\begin{matrix}\text{left}\\ \text{right}\end{matrix}\right\}$ of the centre line These are called "*half-breadths.*" The method of calculating them will be described under the head of EARTHWORK, in the sequel.

$\left.\begin{matrix}\text{Column 13.}\\\text{Column 14.}\end{matrix}\right\}$ *Total half-breadths* to the $\left\{\begin{matrix}\text{left}\\\text{right}\end{matrix}\right\}$ of the centre line, found by adding the intended breadth of the fencing to the half-breadths in columns 11 and 12.

Column 15. Angles at which streams, roads, canals, railways, &c., cross the centre line, stating (if the angles are oblique) whether the acute angle lies to the *left* or *right* of the centre line looking forwards.

Column 16. Remarks:—Comprising positions of permanent marks, rates of inclination or gradients, radii of curves, spans and head-room of bridges, tunnels, and arches of viaducts, alterations of level of existing lines of communication, &c.,—the whole accompanied by a sketch of the working section.

When an existing line of communication is to be altered in position or level for the purposes of the proposed work, a working section of the works required for such altered line should be prepared in the same manner with that of the principal work, and its description inserted in the level-book.

67. **Setting-out Slopes and Breadths of Land** (already referred to in Article 11, p. 11) is performed by laying off the *half-breadths* of the work and the *total half-breadths*, as calculated, exactly at right angles to the centre line, marking their ends with stakes, and sometimes also nicking out lines so as to connect those stakes, and show the boundaries of the earthwork and the boundaries of the land to be occupied respectively. A temporary fence is made, as soon as possible, along the outer of those boundaries.

The **Land-Plans** (referred to in the same page) are prepared by plotting the total half-breadths on the plan of the working survey, drawing the boundaries of the pieces of land required for the work, and making separate copies or tracings of them, to be used in dealing with the owners and occupiers.

68. **Permanent Marks of Sites of Works** are stakes planted on nearly the same principle with those already described in Article 65 for marking points on the centre line. For example, suppose that the work to be set-out is a bridge, consisting principally of two abutments which support an arch or a platform. The principal points, upon which the positions of all other points in the bridge depend, are the four corners of its abutments. To enable the positions of those corners to be found at any time, plant four stakes in the prolongations of the faces of the two abutments, at known distances from the four corners, and sufficiently far from them to be clear of the work.

69. In **Setting-out Levels of Excavations** the engineer causes stakes to be driven, whose heads are at the intended formation-level.

To plant a stake at a given level, the staff is to be held upon the nearest bench mark, and read; the difference between the level of that bench mark and that of the new stake to be driven, is to be added to the reading of the staff, if that stake is to be lower than the bench mark,—subtracted, if it is to be higher. This gives the height which will be read upon the staff at the new stake, when that stake has been driven to the proper depth.

Two such stakes, being driven at fifty feet apart or thereabouts, in the centre line, near the commencement of a proposed cutting, enable the excavators to carry on the cutting at the proper level and rate of inclination for some distance, by the operation called "*boning*," which consists in ranging a line of uniform inclination from two given points in it, with T-shaped instruments called "boning-rods." Each of these consists of an upright staff, having a cross-bar at right angles to it at the top: all the boning-rods belonging to one set ought to be exactly of the same height. To range or "bone" the bottom of a cutting with them from two given stakes, two of the rods are to be held upright on the heads of the two stakes, and a third held upright at any point in the cutting which is in the same straight line with the stakes; when, if the bottom of the cutting is at the true formation level, the tops of the three rods will be in one straight line. In this manner the cutting is carried forward at an uniform rate of inclination, until the engineer thinks it advisable to plant a new pair of stakes by the level and staff near its inner end, from which the boning goes on as before.

70. **Ranging and Setting-out Tunnels.**—The centre line of a tunnel having been at first ranged on the surface of the ground, in the manner already described, a row of shafts are sunk in convenient positions along that line.

In order to range the line below ground, it is necessary to have two marks in the centre line at the bottom of each shaft, as far asunder as possible, to enable that line to be prolonged from the bottom of the shaft in both directions. Those marks consist of nails or spikes driven into the cross-timbers.

The former practice was to determine the positions of those marks below ground, by erecting over the shaft a timber frame, from which two plumb-lines were suspended, hanging nearly to the bottom of the shaft, and to range those plumb-lines by the transit instrument; but as that process is difficult or impossible in windy weather, Mr. Simms introduced the following improved method :—[*] The engineer ranges, by the transit instrument, two strong stakes in the centre line above ground, each about sixteen feet from the centre of the shaft, so as to be safe from disturbance while the work is in progress. To mark the exact position of the centre line, each stake has driven into its head a spike, with an eye through its top. The eye of each spike is very carefully ranged in the exact centre line, being made visible to the observer at the instrument by holding a piece of white paper behind it. A cord is stretched through the holes in the spikes, so as to mark the course of the centre line across the mouth of the shaft. At each side of the shaft a plank is laid, at right angles to the string, and with its edge overhanging the edge of the shaft two or three inches, so that a plumb-line may hang from it clear of the side of the shaft. Two plumb-lines are then hung from the planks, directly under the cord that marks the centre line; and the lower ends of those plumb-lines show two points in the centre line at the bottom of the shaft.

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[*] Simms *On Practical Tunnelling.*

The approximate ranging of the "heading" or "drift," or small horizontal mine that connects the lower ends of the shafts, is performed by means of candles, each hung from the timber framing in a sort of stirrup.

The accurate ranging of the centre line, after the heading has been made, is performed by stretching a cord between the marks already ranged at the bottom of the shaft, and fixing, at intervals of thirty or forty feet, either small perforated blocks of wood carried by cross-bars, or stakes with eyed spikes driven into their heads, so that the holes in the blocks or spikes shall be ranged by the cord exactly in the centre line. The centre line of any part of the tunnel can then be marked at any time when required, by stretching a cord through two of those holes. The cross-bars are fixed in a temporary way to the timber framework of the heading, so that they can be removed, to leave a free passage for men and wagons; but their places are so marked that they can be re-fixed exactly in their proper positions at any time when it is required to range part of the line.

Curves can be set-out below ground by means of a theodolite on a short-legged stand, and candles or lamps instead of ranging-poles. In this case, the two marks at the bottom of a shaft indicate the direction of a tangent to the curve at its centre.

When the line of shafts does not follow the centre line of the tunnel, but a line parallel to it, a corresponding line is to be set-out through the heading at the bottom of the shafts; and from that line the centre line, or any given part of the tunnel, can be set-out by laying down offsets in transverse headings.

In order to *set-out the levels of a tunnel*, there should be a bench mark above ground, as described in Article 65, p. 110, near the mouth of each shaft. When the shaft has been sunk, and lined with timber or brickwork, a second bench mark is to be made within the shaft, and near its top, by driving into the timber or brickwork a horseshoe-shaped staple in a horizontal position, the levelling-staff being held on its upper surface in taking its level.

Some part of the masonry or brickwork of the intended tunnel is taken as a standard point by means of which the levels of other points are regulated: for example, the "invert-skew-back," or joint where the inverted arch forming the bottom of the tunnel meets the sides. That joint being at a fixed height above or below the rails (generally below), its depth below the staple is to be calculated. That depth is then to be set-off by hanging through the staple a chain of rods of the proper length. The rods used by Mr. Simms are connected together at the ends by eyes and spring-hooks: the length of each rod, from the inside of the eye at one end to the inside of the hook at the other, is ten feet. To set-off a given depth below the staple, the number of rods to be linked together is one more than the number of entire tens of feet in the depth; the odd feet and decimals of feet are set-off on the uppermost rod by screwing a gland upon it at the proper point. The chain of rods is then dropped through the staple until the gland, resting on the staple, prevents them from passing further, and supports the whole chain; a bench mark, consisting of a flat-sided spike driven horizontally into the timbering, or of a stake with a round-topped spike in its head, driven vertically into the ground, is then adjusted at the bottom of the shaft, so that its upper surface is exactly on a level with the bottom of the lowest rod.

The staple forms a permanent bench mark, through which the rods can be lowered again, whenever it is necessary to make a new bench mark under ground, owing to disturbance of the former bench mark. This is always done after the brickwork has been partly built, in order to make a permanent bench mark, by driving a flat spike into the side of the tunnel.

Further remarks on setting-out will be made under the head of each kind of work for which peculiar methods are required.

CHAPTER VI.

OF MARINE SURVEYING FOR ENGINEERING PURPOSES.

71. **Limitation of the Subject—Landmarks—Buoys.**—Marine surveys are undertaken for purposes of geography and navigation as well as for those of engineering; but the present chapter has reference to the last of those purposes only; and it therefore describes the operations of marine surveying so far only as they are required in preparing plans for engineering works in navigable waters.

The principal objects of such surveying are to determine and represent on a plan the figure of the bottom of the sea, or other piece of water, on a scale suited for designing engineering works, and to ascertain the materials of which the bottom consists, the level, rise and fall of the surface of the water, and the direction and speed of its currents.

The marine survey must be based upon a survey on the adjoining land, by means of which the figure of the coast and the positions of a sufficient number of conspicuous and well-defined objects near the coast have been ascertained. These objects are the *landmarks*, by observations of which the positions of points on the surface of the water are determined.

Stations afloat can be marked by means of buoys, carrying poles and vanes.

To prevent a buoy from deviating to any considerable distance from a position directly above its anchor, the mooring cable, which is fixed at one end to the anchor, passes through a ring called a "thimble," attached to the buoy, and has a weight hung to the other end.

72. **Datum and Bench Marks for Levels.**—There should also be a *datum-point*, or principal bench mark, on land, to which the levels are referred, and a sufficient number of other bench marks, whose elevations relatively to the principal bench marks are to be found by the ordinary process of levelling.

For nautical purposes the *datum-surface*, relatively to which the levels of the bottom are stated, is the *average low-water-mark of spring tides;* and the same datum-surface, when it is sensibly horizontal, will answer for an engineering survey; but on the sea-coast, when the survey is extensive, and in the channels of rivers, the low-water of spring tides is not a horizontal surface; and in such cases, the levels for engineering purposes must be reckoned from an arbitrary horizontal surface, as in sections on land.

73. **Tide-Gauges.**—The successive levels of the surface of the water must be observed and recorded from time to time, as well for their own importance as because the levels of the bottom are ascertained by sounding, and in order to reduce the latter levels to a common datum the variations of the level of the surface of the water must be known.

The tide-gauges used for this purpose, when of the simplest kind, are posts set exactly upright, and having scales of feet and tenths of feet marked upon them, numbered from the bottom upwards. They must be fixed and stayed in such a manner as to be capable of resisting the waves. Sometimes the whole rise and fall of the tide at a given place may be observed on one post; but in general the slope of the beach makes it necessary to have a row of posts extending from low-water-mark to high-water-mark, and forming, in fact, one tide-gauge, divided into several stages or steps. The lowest mark on the lowest post of the row is the zero of the tide-gauge: its level should be ascertained relatively to the nearest bench mark on land by levelling. It will form the commencement of a scale of feet and tenths, numbered upwards. The lowest mark on the second post must be made at a point adjusted by levelling to the same level with the highest mark on the first post, and marked with the same number, and so on; so that the marks on the entire row of posts may form one continuous scale of heights above the zero-mark.

The number of different tide-gauges required, and the places where they are to be erected, will be fixed by the engineer to the best of his judgment, so as to give the means of determining the figure of the surface of the water at any given instant. They must be more numerous, the more the surface of the water at each instant deviates from a horizontal form. Such deviation always exists in river channels; and in them, and also in estuaries, and on the coast, its existence and extent are indicated by differences in the time of high and low-water, and in the extent of rise and fall of the tide. Even when those deviations are not practically appreciable, it is desirable to have two tide-gauges at points distant from each other, in order that the two series of observations may check each other.

The observers of the tide-gauges should be trustworthy and intelligent persons, provided with watches, which should be compared every day with that of the principal surveyor.

For the purpose of reducing soundings only, it is in general sufficient to observe each tide-gauge at each quarter of an hour. When it is desired also to ascertain the laws of the tide at the locality, it is better to observe the height on the tide-gauge at each ten minutes, for an hour before and an hour after high and low-water, and at each half-hour during the remainder of the day.

For engineering purposes, the tide-gauges already described, consisting of simple posts, are in general sufficient; because when the water is smooth enough to take accurate soundings, it is smooth enough to enable the observer of the tide-gauge to estimate the mean level between the crests and troughs of the waves.

When more exact observations are required, the tide-gauge should consist of an upright tube, communicating with the water outside through a few small holes only, and having in it a float with a graduated upright stem, tall enough to be visible above the top of the tube.

In a self-registering tide-gauge, such a float acts through a chain or cord on a train of mechanism, and moves a pencil up or down, which marks a line on a paper-covered cylinder turned by clock-work. (Airy *On Tides and Waves.*)

The observations at the tide-gauges having been copied from the observers' books into one book, are to be reduced to the datum of the survey by the aid of the known levels of the zero-marks of the tide-gauges relatively to that datum.

The mean of all the reduced observations of the tide-gauges taken during one or more entire "lunations," or revolutions of the moon, gives the *mean level of the sea*, which is a truly horizontal surface.

Further remarks on the tides will be made in the sequel.

74. **Determining Stations afloat.**—In fig. 53, let D represent the position at a given instant of a point in a boat, which is to be determined.

This is done by measuring with the sextant in the boat the angles between three known objects on land, A, B, C.

To diminish or prevent the errors that would arise from the boat's shifting its position while the angles are being measured, the surveyor should have three sextants, if possible, with which he should take the angles A D B, B D C, A D C, in rapid succession, reading them off at leisure afterwards. The angle A D C, which should be the sum of the other two, serves as a check upon their accuracy.

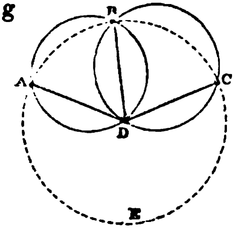

Fig. 53.

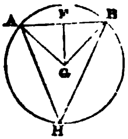

Fig. 54.

Care should be taken that the four points, A, B, C, D, do not lie in or near the circumference of one circle; for in that case the observations would leave the position of D indeterminate, as will presently be explained.

There are different methods of plotting the position of D on the plan.

Method I.—*By Two intersecting Circles.*—To draw through two points, as A and B, fig. 54, a circle which shall *contain* a given angle; that is to say, a circle such that from any point in its circumference, as H, the arc A B shall subtend an angle A H B equal to the given angle, draw through A and B the straight lines A G, B G, making with the straight line A B the angles B A G, A B G, each equal to the complement of the given angle; the intersection of those lines G will be the centre of the circle required.

Let fig. 53 now represent the plan, and A, B, C, the positions of the three landmarks as plotted on it; through A and B draw a circle containing the observed angle A D B; through B and C draw a circle containing the observed angle B D C; those circles will give by their intersection the point D on the plan:—unless they should happen to coincide with each other and with the dotted circle A B C E, when the point D may be anywhere in that dotted circle, and cannot be plotted from the observations taken. Should D lie *near* the dotted circle, the two intersecting circles will cut each other at too acute an angle, like the sides of an ill-conditioned triangle; and the plotted position of D will be liable to inaccuracy.

Method II.—*By the intersection of a Circle and a Straight Line.*—From A draw A E, making the angle C A E = C D B: from C draw C E, making the angle A C E = A D B, and cutting A E in E: through the three points A, C, E, describe a circle: through E and B draw a straight line cutting the circle in D; D will be the required point on the plan.

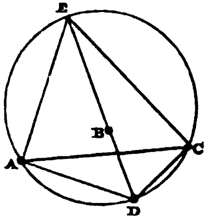

Fig. 55.

The two preceding methods are both too tedious for ordinary use, and the two following are almost always employed instead.

Method III.—*By a Piece of Tracing-paper.*—On a piece of tracing paper draw three straight lines radiating from one point so as to make with each other angles equal to A D B and B D C. Lay it on the plan, and shift it about till the three lines traverse A, B, and C respectively; the point from which they diverge being pricked through on the plan, will give the position of D.

Method IV.—*By the Station-pointer.*—This is an instrument consisting of three long flat arms turning about one centre, and having straight fiducial edges diverging from that centre. Fixed to the middle arm is a graduated circular arc, and fixed to the side arms, two indexes with verniers, by means of which those arms can be set so as to make any required pair of angles with the middle arm. The arms being set so as to form the angles A D B, B D C, the instrument is laid on the plan and shifted about until the three fiducial edges traverse the points representing the three landmarks respectively. The centre of the instrument will then be over the required point D, which is marked by means of a pricker that passes through a hole in the centre of the instrument; or otherwise, three pencil lines may be drawn along the fiducial edges of the arms, and produced after the instrument has been lifted off the paper; when their intersection will give the required point.

Three landmarks are all that are absolutely necessary to determine the position of a station afloat; but when the station is an important one, the surveyor, for the purpose of verification, should measure angles to additional known objects.*

Where a sufficient number of objects on land are not visible, the positions of stations afloat may be determined by taking angles to previously determined stations afloat which are marked by buoys, or at which boats with flags are moored; but this method is wanting in precision, and objects on land are always to be preferred when they can be seen.

75. **Soundings and Levels.**—The instrument generally employed for taking soundings for nautical purposes is the *lead-line*, a tough, hard, and flexible cord, loaded with a conical lead weight, and divided into fathoms. For engineering purposes, where the depth does not exceed about 100 feet, a chain is used. In shallow water, the best instrument is a rod, divided into feet and tenths, and loaded at the lower end.

The sounding-lead is "armed" with a lump of tallow in a hollow at its lower end, by which, when the material of the bottom is loose, specimens of it are brought up. When the material is of a firmer texture, a specimen may be brought up by dropping a heavy iron pike, jagged and barbed at the lower end, called a "plunger," and hauling it up again by a rope; or the nature of the bottom may be ascertained by boring, or by diving—operations which will be again referred to further on.

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* The distances of the station from two of the landmarks might be calculated by the rules of plane trigonometry and plotted; but the process is too tedious for ordinary use. The following are the steps of which it consists (see fig. 55):—

In the triangle A E C, given A C, and the angles E A C (= B D C) and A C E (= A D B), calculate A E and C E.

In the triangle A B E, given A B, A E, and $\angle$ B A E (= B D C — B A C), calculate $\angle$ A E B.

In the triangle B E C, given B C, C E, and $\angle$ B C E (= A D B — B C A), calculate $\angle$ B E C.

In the triangle A D E, given A E and the angles, calculate A D.

In the triangle D E C, given C E and the angles, calculate C D.

Soundings for nautical purposes are noted, and written on the plan, in fathoms of six feet, and half and quarter fathoms; those for engineering purposes, in feet and decimals, or feet and inches.

The levels of the bottom are ascertained by taking several series of soundings along straight lines, in such positions as the engineer judges to be best. In general, the position of those lines is nearly that of the lines of steepest declivity of the bottom, and nearly at right angles to the coast.

As each sounding is taken, the surveyor notes the time, the depth, and the position.

The following are two methods of determining the positions of soundings:—

METHOD I.—*By a Series of Angles.*—In fig. 56, A and B represent two known objects, in a straight line with which a set of soundings are to be taken; C is a third known object lying at a sufficient distance to one side of the line A B. The boat is rowed along the straight line B E, either directly towards or directly from B. The surveyor sees that the boatmen keep B and A exactly in one straight line; and the instant that each sounding is taken, he measures with a sextant the angle which the direction of C makes with the line. For example, if 1, 2, 3, 4, &c., are points where soundings are taken, the angles to be measured at those points are B 1 C, B 2 C, B 3 C, B 4 C, &c. The position of C should be so chosen that the most acute of those angles may be 30° or somewhat greater.

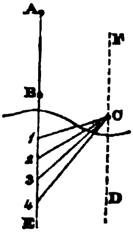

Fig. 56.

To plot the positions found by this method, draw through C on the plan the straight line F C D parallel to A B E, and lay off the angles D C 1 = B 1 C, D C 2 = B 2 C, &c.: the intersections of the lines C 1, C 2, &c., with B E, will give the points required.

METHOD II.—*By two Stations and an uniform speed of Rowing.*—In Fig. 57, A represents a known object on which the line of soundings is to run. The surveyor determines the position of (B or C) the commencement of the line by three angles taken between known objects; the rowers then row as steadily as possible at an uniform speed in a straight line directly from or directly towards A. Soundings are taken at equal intervals of time; and when the line has been carried far enough, the surveyor determines the position of its termination (C or B) by three angles taken between known objects.

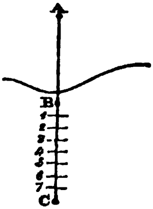

Fig. 57.

In plotting a line of soundings so taken, its two ends B and C are laid down by means of the station-pointer: the straight line B C is drawn, and divided into as many equal parts as there were equal intervals of time between the soundings from the beginning to the end of the line; and thus the intermediate points, 1, 2, 3, &c., are found.

A line of soundings may often be conveniently prolonged on the higher parts of the beach by ordinary levelling. In fact, levelling should be used wherever it is practicable, being the more accurate operation.

76. The **Reduction of Soundings** to the datum of the survey is made by taking the difference between each sounding and the height of the water above that datum at the instant when the sounding was taken, as found by examination of or interpolation in the register of the tides. If the sounding is the greater, that difference is a depth below the datum,—if the less, a height above the datum. (As to what that datum is, see Article 71). When the datum is the mean low-water-level of spring tides, the latter class of reduced soundings are said to be *dry*, and are distinguished in the register and on the plan by a score beneath the figures.

To reduce soundings by calculation, in the absence of direct observations of the tide, it is necessary to know the rise of the tide above the mean water-level, and the time of high-water, for the tide during which the soundings were made, and the *duration* of that tide, or interval of time between high-water and low-water (which on an average is about six hours twelve minutes, but varies considerably at different times and places).

Let H be the height of the mean water-level above the datum:—

r, the rise of the tide above the mean water-level;

D, the duration of the tide;

t, the time before or after high-water at which a given sounding is taken;

h, the height of the surface of the water above the datum at that instant, being the quantity to be subtracted from the sounding.

Then
$$h = \mathrm{H} + r \cdot \cos 180^\circ \frac{t}{\mathrm{D}}; \qquad \text{(1.)}$$

in using which formula it is to be remembered that *cosines of obtuse angles are negative*.

77. **Lines of Equal Depth** are analogous to contour-lines on land (see p. 95), being contour-lines of the bottom of the sea sketched on the plan so as to pass through those points where the reduced soundings are equal. It is customary to mark the *line of one fathom soundings* by single dots, of two fathoms by dots in pairs, of three fathoms by dots in triplets, and so on.

The **High** and **Low-Water-Marks** of average spring tides, which should be drawn on the plan, are also analogous to contour-lines.

78. **Currents—Waves.**—The directions and velocities of tidal currents should be noted by the surveyor, and marked on the plan by arrows; each arrow having figures beside it denoting the speed of the current in nautical miles an hour, and the time after the moon's transit at which it prevails. Flood-currents are denoted by feathered arrows; ebb-currents by unfeathered arrows.

The direction of the current which runs past a moored vessel may be ascertained by dropping some floating body into it, and observing the angle which the direction of motion of that body makes with the direction of some known object. The velocity may be found by means of Massey's Log, an instrument in which the rotations of a fan driven by the current are registered by wheel-work.

The direction and velocity of a current may also be determined by setting a light deal pole, having a weight at the lower end, to float upright in it, and taking simultaneous angles to that object from two known stations. This must be done by two observers, who should take special care to make their angular measurements exactly at the same instants of time.

The usual directions and velocities of waves should be ascertained and noted, and also the greatest height from the crest to the trough of a wave.

79. **Miscellaneous Information on Plan.**—Besides the soundings, levels, currents, and other information already mentioned, the plan of a marine survey for engineering purposes should show at different points the material of the bottom, by such abbreviations as *r.* for rock, *st.* for stones, *s.* for sand, *m.* for mud, &c., and by references to borings and examinations by diving, where such have been made. It should also show all lighthouses, beacons, buoys, fixed moorings, &c.

80. **Taking Altitudes by the Sextant—Dip of the Horizon.**—When the altitude of an object is taken at sea by measuring with a sextant its angular elevation above the visible sea-horizon, a correction must be made by subtracting the *dip* of that horizon—that is, its apparent angular depression below a truly horizontal line traversing the eye of the observer. The amount of that depression is uncertain, owing to the variable refractive power of the atmosphere; but on an average, it is given approximately by the following formula, in which h denotes the height of the observer's eye above the sea, and r the radius of curvature of the surface of the sea.

$$\text{Dip in seconds} = \frac{9}{10} \times 206264''\cdot 8 \sqrt{\frac{2h}{r}}$$

$$= 57''\cdot 4 \sqrt{h \text{ in feet}} \text{ .... (1.)}$$

CHAPTER VII.

OF COPYING, ENLARGING, AND REDUCING PLANS.

81. **Tracing** upon a sheet of thin semi-transparent paper, laid smoothly on the original drawing, is the most accurate method of obtaining a copy of a plan on the same scale with the original. By using a drawing table made of strong plate-glass, called the "copying-glass," with a sloping mirror below, if necessary, to reflect light through it, a tracing may be made on drawing paper of ordinary thickness, provided the original is not mounted on cloth.

When a tracing has been made on thin paper, other copies can be made on thick paper by rubbing the lower side of the tracing with black-lead, or putting a sheet of black-leaded paper below it, laying it on the thick paper, and passing a smooth pointed instrument along all the outlines of the tracing. The new copy has then to be drawn in ink and finished.

Pricking Through is applicable to plans in which the outlines consist chiefly of straight lines, and damage to the original plan is unimportant.

82. **Engraving, Lithographing, and Printing.**—When a plan is to be engraved on copper, a tracing of it is placed on the copper plate, face downwards, and the outlines scratched on the copper with a point which cuts through the tracing. The impressions from copper plates, being printed on damp paper, shrink when they dry, to an extent which varies from 1-400th to 1-200th of the original dimensions. All measurements, therefore, on printed plans should be made by means of the scale engraved along with the plan, and every sheet should have a scale upon it. The shrinking is sometimes slightly different lengthwise and breadthwise. As to the effect of this on sections, see Article 54, p. 90.

Where great accuracy is required in engraved plans (as in those of the ordnance survey), the principal stations are *plotted on the copper*, and the details only laid down on it by tracing.

In lithographing plans, the usual process is to make a copy on "transfer paper" by the aid of a tracing on thin paper, as already described in the preceding Article. The copy so made is drawn and finished with lithographic ink, laid face downwards on a stone, and transferred to the stone by the proper process. The transfer paper being damp during that process, expands to a certain extent, so that the drawing on the stone is somewhat larger than the original; and this expansion is to a certain extent counteracted by the shrinking of the paper on which the impressions are printed; so that the impressions may be slightly larger or smaller than the original in a proportion which it is difficult to assign with precision. As with engraved plans, each sheet should have its own scale, by means of which all measurements upon it should be made.

83. **Reducing Drawings by Hand** is performed, in the case of plans, by forming triangles to connect the stations and other principal points on the original, measuring their sides, and plotting them on a smaller scale on the reduced plan. The details may be reduced by covering the original with a network of squares, and the reduced copy with a network of squares having their sides smaller than those of the original squares in the proportion in which the plan is to be reduced, and sketching the details on the reduced copy in their proper places by the aid of those squares to guide the eye and hand.

In the case of sections, reducing by hand is best performed by plotting the section anew on the smaller scale.

84. **Reducing Drawings by Mechanism** is performed by means of instruments called the "Pantograph" and the "Eidograph." In each of those instruments a tracing-point is made to travel over the outlines of the original drawing; a pencil is so connected with the tracing-point that it is always in a straight line with the tracing-point, and with a fixed centre, and always at a distance from that centre bearing a given constant ratio to the distance of the tracing-point from that centre; and that pencil draws the outlines of a copy of the drawing reduced in the given ratio.

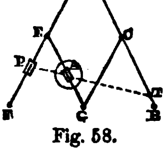

Fig. 58.

Fig. 58 is a skeleton sketch of the PANTOGRAPH. F D, D B, E G, and G C are four flat bars, jointed to each other at E, D, C, and G, so that G E = E D = D C = C G, and the figure G E D C is always an exact rhombus, its opposite sides being parallel, and all of them equal. Those bars are supported by ivory castors, which run on the paper or on the drawing-board. T is the tracing point. A is the fixed centre, having a heavy foot, which rests on the paper or the drawing-board. On its vertical spindle turns a socket through which the bar E G can be slid to any required position, and fixed there by a clamp-screw. P is a square socket, sliding on the bar D F, on which it can be fixed in any required position; the pencil is carried by it. The pencil is loaded on the top with weights, which press its point against the paper; it can be lifted off the paper when required by pulling a string. The dotted line P A T represents the imaginary straight line in which the pencil, the centre, and the tracing point ought to be situated. The bars G E and E F have scales marked on them, showing the proper positions of the sliders for reducing drawings in various proportions. Let $1 : n$ be the proportion in which the plan is to be reduced; so that—

$$n : 1 :: \mathrm{TA} : \mathrm{AP}; \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

then

$$n : 1 :: \mathrm{DE} : \mathrm{EP}; \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

and

$$n + 1 : 1 :: \mathrm{DT} : \mathrm{EA}. \ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

The **Eidograph** is represented by the skeleton sketch, fig. 59. A is its fixed centre, with a heavy leaden foot. On the spindle of this centre turns a square socket, through which slides the bar D E, which can be clamped in any required position. At the ends of that bar are two pulleys, D and E, exactly equal in diameter, and connected by means of a thin steel belt. F and G are screws for *adjusting* the lengths of the two divisions *of that* belt, so as to make the rods B P and T C exactly parallel. These rods slide through square sockets carried by the pulleys, and having clamp-screws. T is the tracing-point, P the pencil, and T A P the imaginary straight line in which the pencil, centre, and tracing-point should always be.

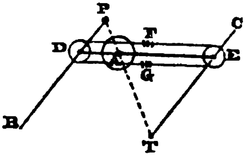

Fig. 59.

Let $1 : n$ as before be the ratio of reduction; then the proper positions of the sockets are given by the formulæ—

$$n : 1 :: \mathrm{A\,E} : \mathrm{A\,D};\ \mathrm{E\,T} = \mathrm{A\,E};\ \mathrm{D\,P} = \mathrm{A\,D}. \ldots (4.)$$

Each bar has a scale of 200 equal parts on it, with 0 marked at the middle of its length, and numbered to 100 each way. These scales are subdivided by the aid of verniers on the sockets. When the instrument is correctly adjusted, each socket is at the same distance from the middle of its bar; and that distance, in divisions of the scale, is found by the following formula:—

$$m = 100 \cdot \frac{n - 1}{n + 1}. \ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

The best test of the accuracy of the adjustments of the Pantograph and Eidograph is to draw the tracing-point T for a certain distance along the edge of a flat straight-edged ruler; when the pencil P ought to draw an exactly straight line, of a length bearing the proper proportion to the length of the original line.

85. **Enlarging Plans** may be performed by hand, in the same manner with reducing; and with the Pantograph or Eidograph, by adjusting either of those instruments so that the pencil shall be further from the centre than the tracing-point is. This, however, is an operation which is not capable of accuracy, except when the ratio of enlargement does not much exceed that of equality.

86. **Reducing Drawings by Photography** is the method employed in reducing the large plans of the ordnance survey, drawn on a scale of $\frac{1}{2500}$, to the scale of six inches to a mile. The details of the plans so reduced are afterwards traced on the copper plates, on which the stations have been previously plotted by the lengths of the sides of the triangles. A process of transferring the reduced outlines to copper, zinc, or stone, without tracing, has lately been introduced. See the *Report on the Progress of the Ordnance Survey*, by Colonel Sir Henry James, R.E.

SUPPLEMENT TO CHAPTER III., ARTICLE 40.

86 A. **Reduction of Angles to the Centre of the Station.**—It sometimes happens that the theodolite cannot be planted exactly at a station in a trigonometrical survey; but has to be placed at a short distance to one side of it. In such cases, the angle actually measured between two objects is reduced to the angle which would have been measured, had the theodolite been exactly at the station, by a correction which is calculated approximately as follows:—

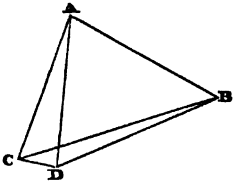

Fig. 59 A.

In fig. 59 A, let C be the station, D the position of the theodolite, A and B two objects; A D B the horizontal angle between them as measured at D; A C B the required horizontal angle at the station C.

Measure C D, and the angle A D C; calculate A C and C B approximately as if A C B were equal to A D B; then

$$ACB = ADB - 206264''.8\ CD \left\{ \frac{\sin ADC}{AC} - \frac{\sin BDC}{BC} \right\} \quad .(1.)$$

The above formula gives the correction in seconds when D lies to the *right* of both C A and C B. When it lies to the left of C B, sin B D C changes its sign; when to the left of C A, sin A D C changes its sign.

SUPPLEMENT TO CHAPTER III., ARTICLE 42, DIVISION IV., PAGE 73.

86 B. **Astronomical Refraction.**—The refracting action of the atmosphere causes the altitudes of the stars to appear greater than they really are. The correction for refraction, therefore, is always to be subtracted from an altitude. Its value may be found in seconds approximately by the following formula:—

$$\text{Refraction} = 58'' \times \text{cotan apparent altitude.}$$

For more exact information on the subject, see a paper by the Rev. Dr. Robinson in the *Transactions of the Royal Irish Academy*, vol. xix. Tables of Refraction are given in treatises on Navigation, such as Raper's.

It is to be borne in mind, that below about 8° or 10° of altitude the changeable condition of the atmosphere makes the correction for refraction very uncertain.

86. C. **To find the Latitude of a Place.**

METHOD I.—*By the Mean Altitude of a Circumpolar Star.*—Take the altitudes of a circumpolar star at its upper and lower culmination (which positions are known by watching for the instants when the altitude is greatest and least). From each of those *apparent* altitudes subtract the correction for refraction; the mean of the *true* altitudes thus found is the latitude of the place.

METHOD II.—*By One Meridian Altitude of a Star.*—Observe the meridian altitude of a star by watching for the instant when its altitude is greatest or least, and subtract the corrections for refraction, and also for dip, if necessary. The complement of the true altitude is the *zenith distance.* Find the declination of the star from the *Nautical Almanac* (which is published four years in advance.)*

Then if the star is between the zenith and the equator,

$$\text{Latitude} = \text{Zenith distance} + \text{Declination}; \ldots\ldots \text{(1.)}$$

If the star is between the equator and the horizon,

$$\text{Latitude} = \text{Zenith distance} - \text{Declination}; \ldots\ldots \text{(2.)}$$

If the star is between the zenith and the elevated pole,

$$\text{Latitude} = \text{Declination} - \text{Zenith distance}; \ldots\ldots \text{(3.)}$$

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[*] The Polar Distances, or complements of the declinations, of a few stars, are given at p. 73.

If the star is between the elevated pole and the horizon,

$$\text{Latitude} = 180° - \text{Declination} - \text{Zenith distance.} \dots (4.)$$

Method III.—*By the Sun's Meridian Altitude.*—In this method the final calculation, from the sun's declination, as found in the *Nautical Almanac*, and the *true* altitude of his centre, is the same as in Method II. But besides the correction for refraction and dip, the altitude requires to be further corrected by subtracting or adding the sun's semidiameter, according as his upper or lower limb has been observed, and by adding the sun's parallax, being the angle subtended at the sun by the distance between the earth's centre and the place of observation.

To find the correction for parallax, find the sun's horizontal parallax on the day of observation, from the *Nautical Almanac*, and multiply it by the cosine of the altitude of the sun's centre.

(The mean value of the sun's horizontal parallax is about 8″·6).

The sun's semidiameter on the day of observation is to be found in the *Nautical Almanac*. It varies from 15′ 46″ to 16′ 18″.

The calculation may be thus set down algebraically—

$$\left\{\begin{array}{l}\text{True altitude} = \text{apparent altitude} - \text{Dip (if the sea-} \\ \quad\text{horizon has been observed)} - \text{Refraction} \pm \text{sun's} \\ \quad\text{semidiameter} + \text{parallax;} \dots\dots\dots\dots \end{array}\right\} (5.)$$

$$\text{Zenith distance} = 90° - \text{true altitude,} \dots\dots\dots (6.)$$

Latitude (see Equations 1, 2, 3, 4).

Equations 1 and 2 are the most frequently applicable to the sun. Equation 3 is occasionally applicable between the tropics; and Equation 4 relates to observations made at midnight, in summer, in the polar regions.

86 D. **List of Authorities on Engineering Geodesy and Subjects connected with it.**—Butler Williams's *Practical Geodesy;* Bruff *On Surveying;* Castle *On Surveying;* Haskoll's *Engineering Field-Work;* Haskoll *On Railway Construction;* Simms *On Mathematical Instruments;* Simms *On Levelling;* Simms's *Practical Tunnelling;* Sir Edward Belcher *On Marine Surveying; Admiralty Manual of Scientific Inquiry,* Article "Hydrography;" Raper's *Navigation;* De Morgan's *Trigonometry;* Airy's *Trigonometry,* edited by Professor Blackburn.

PART II.

OF MATERIALS AND STRUCTURES.

CHAPTER I.

SUMMARY OF PRINCIPLES OF STABILITY AND STRENGTH.

SECTION I.—*Of Structures in General.*

87. A **Structure** consists of portions of solid materials, put together so as to preserve a definite form and arrangement of parts, and to withstand external forces tending to disturb such form and arrangement. As the parts of a structure are intended to remain at rest relatively to each other, the forces which act on the whole structure, and on each of its parts, should be *balanced*, so that the mechanical principles on which the permanence and efficiency of structures depend for the most part belong to STATICS, or the science of balanced forces.

The *materials* of a structure may be more or less stiff, like stone, timber, and metals, or loose, like earth.

The ensuing chapters of this part will be divided according to the materials of which the structures they treat of consist. In the present chapter are given a summary of mechanical principles applicable to all structures. Many passages in it are extracted from a previous Treatise on *Applied Mechanics*, and abridged or amplified as may be required, in order to suit the purpose of the present Treatise. Such passages are indicated by the letters *A. M.*, with a reference to the number of the corresponding Article in that work.

87 A. **Pieces—Joints—Supports—Foundations.** (*A. M.*, 129, 130).—A structure consists of two or more solid bodies, called its *pieces*, which touch each other and are connected at portions of their surfaces called *joints*. This statement may appear to be applicable to structures of stiff materials only; but, nevertheless, it comprehends masses of earth also, if they are considered as consisting of a very great number of very small pieces, touching each other at innumerable joints.

Although the pieces of a structure are fixed relatively to each other, the structure as a whole may be either fixed or moveable relatively to the earth.

A fixed structure is supported on a part of the solid material of the earth, called the *foundation* of the structure; the pressures by which the structure is supported, being the resistances of the various parts of the foundation, may be more or less oblique.

A moveable structure may be supported, as a ship, by floating in water, or as a carriage, by resting on the solid ground through wheels. When such a structure is actually in motion, it partakes to a certain extent of the properties of a machine; and the determination of the forces by which it is supported requires the consideration of dynamical as well as of statical principles; but when it is not in actual motion, though capable of being moved, the pressures which support it are determined by the principles of statics; and it is obvious that they have their resultant equal and directly opposed to the weight of the structure.

88. **The Conditions of Equilibrium of a Structure** are the three following (*A. M.*, 131):—

I. *That the forces exerted on the whole structure by external bodies shall balance each other.*—The forces to be considered under this head are—(1.) the *Attraction of the Earth*—that is, the *weight* of the structure; (2.) the *External Load*, arising from the pressures exerted against the structure by bodies not forming part of it nor of its foundation; (these two kinds of forces constitute the *gross* or *total load*); (3.) the *Supporting Pressures*, or resistance of the foundation. Those three classes of forces will be spoken of together as the *External Forces.*

II. *That the forces exerted on each piece of the structure shall balance each other.*—These consist of—(1.) the *Weight* of the piece, and (2.) the *External Load* on it, making together the *Gross Load;* and (3.) the *Resistances*, or forces exerted at the joints, between the piece under consideration and the pieces in contact with it.

III. *That the forces exerted on each of the parts into which each piece of the structure can be conceived to be divided shall balance each other.*—Suppose an ideal surface to divide any part of any one of the pieces of the structure from the remainder of the piece; the forces which act on the part so considered are—(1.) its weight, and (2.) (if it is at the external surface of the piece) the external force applied to it, if any, making together its *gross load;* (3.) the *stress*, or force, exerted at the ideal surface of division, between the part in question and the other parts of the piece.

89. **Stability, Strength, and Stiffness.** (*A. M.*, 132, 127).—It is necessary to the permanence of a structure, that the three foregoing conditions of equilibrium should be fulfilled, not only under one amount and one mode of distribution of load, but under all the

variations of the load as to amount and mode of distribution which can occur in the use of the structure.

Stability consists in the fulfilment of the *first* and *second* conditions of equilibrium of a structure under all variations of the load within given limits. A structure which is deficient in stability gives way by the displacement of its pieces from their proper positions.

When a structure, or one of its parts, is *flexible*, like the chain of a suspension bridge, or in any other way free to move, its stability consists in a tendency to recover its original figure and position after having been disturbed.

Strength consists in the fulfilment of the *third* condition of equilibrium of a structure for all loads not exceeding prescribed limits; that is to say, the greatest internal stress produced in any part of any piece of the structure, by the prescribed greatest load, must be such as the material can bear, not merely without immediate breaking, but without such injury to its texture as might endanger its breaking in the course of time.

A piece of a structure may be rendered unfit for its purpose, not merely by being broken, but by being stretched, compressed, bent, twisted, or otherwise strained out of its proper shape. It is necessary, therefore, that each piece of a structure should be of such dimensions that its alteration of figure under the greatest load applied to it shall not exceed given limits. This property is called *stiffness*, and is so connected with strength that it is necessary to consider them together.

SECTION II.—*Summary of the Principles of the Balance of Forces.*

90. (*A. M.*, 12, 13, 17 to 24).—A **Force** is an action between two bodies, either causing or tending to cause change in their relative rest or motion. **Equilibrium** or **Balance** is the condition of two or more forces which are so opposed that their combined action on a body produces no change in its rest or motion, and that each force merely *tends* to cause such change, without actually causing it.

In treatises on statics, the word *pressure* is often used to denote any balanced force; although, in the popular sense, that word is used to denote a force, of the nature of a thrust or push, distributed over a surface.

The relation of a force to one of the two bodies between which it acts, is determined, or made known, when the following three things are known respecting it:—first, the *place*, or part of the body to which it is applied; secondly, the *direction* of its action; thirdly, its *magnitude*.

I. The *place of the application* of a force to a body may be the whole or part of its internal mass; in which case the force is an *attraction* or a *repulsion*, according as it tends to move the bodies between which it acts towards or from each other; or the place of application may be the surface at which two bodies touch each other, or the bounding surface between two parts of the same body, in which case the force is a *tension or pull*, a *thrust or push*, or a *lateral stress*, according to circumstances.

Thus every force has its action distributed over a certain space, either a volume or a surface; and a force concentrated at a single point has no real existence. Nevertheless, it is necessary, in treating of the principles of statics, to begin by demonstrating the properties of such ideal forces, conceived to be concentrated at single points; for the conclusions so arrived at respecting *single forces* (as they may be called), are applicable to the distributed forces which really act in nature.

In reasoning respecting forces concentrated at single points, they are assumed to be applied to solid bodies which are *perfectly rigid*, or incapable of alteration of figure under any forces which can be applied to them. This also is a supposition not realized in nature; but its consequences may be applied to actual bodies, when their alterations of figure are insensible.

II. The *direction* of a force is that of the motion which it tends to produce. A straight line drawn through the point of application of a single force, and along its direction, is the *line of action* of that force.

III. The *magnitudes* of two forces are equal, when, being applied to the same body in opposite directions along the same line of action, they balance each other.

A single force may be represented on paper by an arrow-headed straight line; the commencement of the line indicating the point of application of the force,—the direction of the line, the direction of the force,—and the length of the line, the magnitude of the force, according to an arbitrary scale.

91. **Standard Unit of Weight.** (*A. M.*, 21).—The magnitude of a force is expressed arithmetically by stating in numbers its ratio to a certain *unit* or *standard* of force, which is usually the *weight* (or attraction towards the earth), at a certain latitude, and at a certain level, of a known mass of a certain material. Thus the British unit of force is the *standard pound avoirdupois;* which is the weight in the latitude of London, and near the level of the sea, of a certain piece of platinum kept in the Exchequer office. (See the Act 18 and 19 Vict., cap. 72; also a paper by Professor W. H. Miller, in the *Philosophical Transactions* for 1856.)

Amongst other units of force employed in Britain are,—

The grain = $\frac{1}{7000}$ of a pound avoirdupois.

The troy pound = 5,760 grains = $0{\cdot}8\dot{2}8571\dot{4}$ pound avoirdupois.

The hundredweight = 112 pounds avoirdupois.

The ton = 2,240 pounds avoirdupois.

The French standard unit of force is the *gramme*, which is the weight, in the latitude of Paris, of a cubic centimetre of pure water, measured at the temperature at which the density of water is greatest, viz., 3°·945 centigrade, or 39°·1 Fahrenheit, and under the pressure which supports a barometric column of 760 millimetres of mercury—that is, 29·922 inches.

A comparison of French and British measures of force and of size is given in a table at the end of this volume.

92. **Resultant of Forces Acting in One Straight Line.** (*A. M.*, 22).—The RESULTANT of any number of given forces applied to one body, is a single force capable of balancing that single force which balances the given forces; that is to say, the resultant of the given forces is equal and directly opposed to the force which balances the given forces; and is *equivalent* to the given forces so far as the balance of the body is concerned. The given forces are called *components* of their resultant.

The resultant of a set of balanced forces is nothing.

The resultant of any number of forces acting on one body in the same straight line of action, acts along that line, and is equal in magnitude to the sum of the component forces; it being understood, that when some of the component forces are opposed to the others, the word "*sum*" is to be taken in the algebraical sense; that is to say, that forces acting in the same direction are to be added to, and forces acting in opposite directions subtracted from each other.

When a system of forces acting along one straight line are balanced, the sum of the forces acting in one direction is equal to the sum of the forces acting in the opposite direction.

93. **Resultant and Balance of Inclined Forces.** (*A. M.*, 51 to 54).—The smallest number of inclined forces which can balance each other is three. Those three forces must act through one point, and in one plane. Their relation to each other depends on the following theorem, called the "PARALLELOGRAM OF FORCES," from which the whole science of statics may be deduced.

I. *If two forces whose lines of action traverse one point be represented in direction and magnitude by the sides of a parallelogram, their resultant is represented by the diagonal.*

For example, through the point O (fig. 60) let two forces act, represented in direction and magnitude by $\overline{OA}$ and $\overline{OB}$. The resultant or equivalent single force of those two forces is represented in direction and magnitude by the diagonal O C of the parallelogram O A C B. Its magnitude is given algebraically by the equation,

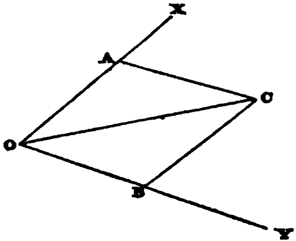

Fig. 60.

$$OC = \sqrt{\left\{ OA^2 + OB^2 + 2\, OA \cdot OB \cos AOB \right\}} \quad (1.)$$

To balance the forces $\overline{OA}$ and $\overline{OB}$, a force is required equal and directly opposed to their resultant $\overline{OC}$. This may be expressed by saying, that *if the directions and magnitudes of three forces be represented by the three sides of a triangle* (such as $\overline{OA}$, $\overline{AC}$, $\overline{CO}$), *then those three forces, acting through one point, balance each other*, or in other words, that three forces in the same plane balance each other at one point, when each is proportional to the sine of the angle between the other two.

The following corollary from the parallelogram of forces is called the "POLYGON OF FORCES:"—

II. *If a number of forces acting through the same point be represented by lines equal and parallel to the sides of a closed polygon, those forces balance each other.* To fix the ideas, let there be five forces acting through the point O (fig. 61), and represented in direction and magnitude by the lines F_1, F_2, F_3, F_4, F_5, which are equal and parallel to the sides of the closed polygon O A B C D O; viz.:—

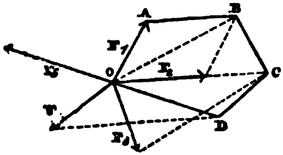

Fig. 61.

$$F_1 = \text{and} \parallel OA;\ F_2 = \text{and} \parallel AB;\ F_3 = \text{and} \parallel BC;$$

$$F_4 = \text{and} \parallel CD;\ F_5 = \text{and} \parallel DO.$$

Then, by the principle of the parallelogram of forces, the resultant of F_1 and F_2 is O B; the resultant of F_1, F_2, and F_3 is O C; the resultant of F_1, F_2, F_3, and F_4 is O D, equal and opposite to F_5, so that the final resultant is nothing.

The closed polygon may be either plane or "gauche"—that is, not in one plane.

III. *Principle of the Parallelopiped of Forces.*—The simplest gauche polygon is one of four sides. Let O A B C E F G H (fig. 62), be a parallelopiped whose diagonal is O H. Then any three successive edges so placed as to begin at O and end at H, form, together with the diagonal H O, a closed quadrilateral; consequently, if three forces F_1, F_2, F_3, acting through O, be represented by the three edges $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, of a parallelopiped, the diagonal $\overline{OH}$ represents their resultant, and a fourth force F_4 equal and opposite to $\overline{OH}$ balances them.

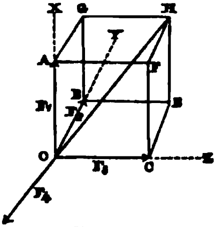

Fig. 62.

49. **Resolution of a Force.**—I. *Into two Components.* '(*A. M.*, 55, 56).—In order that a given single force may be resolvable into two components acting in given lines inclined to each other, it is necessary, *first*, that the lines of action of those components should intersect the line of action of the given force in one point; and *secondly*, that those three lines of action should be in one plane.

Returning then to Fig 60, let $\overline{OC}$ represent the given force, which it is required to resolve into two component forces, acting in the lines O X, O Y, which lie in one plane with O C, and intersect it in one point O.

Through C draw C A ‖ O Y, cutting O X in A, and C B ‖ O X, cutting O Y in B. Then will O A and O B represent the component forces required.

Two forces respectively equal to and directly opposed to $\overline{OA}$ and $\overline{OB}$ will balance $\overline{OC}$.

The magnitudes of the forces are in the following proportions:—

$$\text{OC} : \text{OA} : \text{OB}$$

$$:: \sin \text{AOB} : \sin \text{BOC} : \sin \text{AOC} \ldots\ldots\ldots\ldots (1.)$$

II. *Into three Components.*—In order that a given single force may be resolvable into three components acting in given lines inclined to each other, it is necessary that the lines of action of the components should intersect the line of action of the given force in one point.

Returning to Fig. 62, let $\overline{OH}$ represent the given force which it is required to resolve into three component forces, acting in the lines O X, O Y, O Z, which intersect O H in one point O.

Through H draw three planes parallel respectively to the planes Y O Z, Z O X, X O Y, and cutting respectively O X in A, O Y in B, O Z in C. Then will $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, represent the component forces required.

Three forces respectively equal to, and directly opposed to $\overline{\mathrm{O\,A}}$, $\overline{\mathrm{O\,B}}$, and $\overline{\mathrm{O\,C}}$, will balance $\overline{\mathrm{O\,H}}$.

III. *Rectangular Components.*—The rectangular components of a force are those into which it is resolved when the directions of their lines of action are at right angles to each other.

For example, in fig. 62, suppose O X, O Y, O Z, to be three axes of co-ordinates at right angles to each other. Then $\overline{\mathrm{O\,H}}$ is resolved into three rectangular components, O A, O B, O C, simply by letting fall from H perpendiculars on O X, O Y, O Z, cutting them at A, B, C, respectively.

Let the three rectangular components be denoted respectively by X, Y, Z, the resultant by R, and the angles which it makes with the components by α, β, γ, respectively; then the relations between the three rectangular components and their resultant are expressed by the following equations:—

$$X = R \cos \alpha; \;\; Y = R \cos \beta; \;\; Z = R \cos \gamma; \;\ldots\ldots\ldots (2.)$$

$$R^2 = X^2 + Y^2 + Z^2. \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

When the resultant is in the same plane with two of its components (as X and Y), the third component is null, and the equations 2 and 3 take the following form:—

$$X = R \cos \alpha = R \sin \beta; \;\; Y = R \cos \beta = R \sin \alpha; \;\; Z = 0; \ldots (4.)$$

$$R^2 = X^2 + Y^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

In using equations 2, 3, 4, and 5, it is to be remembered that cosines of obtuse angles are negative.

95. **Resultant and Balance of any number of inclined Forces Acting through one Point.**—To find this resultant by calculation, assume any three directions at right angles to each other as axes; resolve each force into three components (X, Y, Z) along those axes, and consider the components along a given axis which act in one direction as positive, and those which act in the opposite direction as negative; take the algebraical sums of the components along the three axes respectively ($\Sigma \cdot X$, $\Sigma \cdot Y$, $\Sigma \cdot Z$); these will be the *rectangular components of the resultant of all the forces;* and its magnitude and direction will be given by the following equations:—

$$R^2 = (\Sigma \cdot X)^2 + (\Sigma \cdot Y)^2 + (\Sigma \cdot Z)^2; \ldots\ldots\ldots\ldots\ldots (1.)$$

$$\cos \alpha = \frac{\Sigma \cdot X}{R}; \;\; \cos \beta = \frac{\Sigma \cdot Y}{R}; \;\; \cos \gamma = \frac{\Sigma \cdot Z}{R}. \;\ldots\ldots\ldots (2.)$$

If the forces all act in one plane, two rectangular axes in that plane are sufficient, and the terms containing Z disappear from the equations.

If the forces balance each other, the components parallel to each axis balance each other independently; that is to say, the three following conditions are fulfilled:—

$$\Sigma \cdot X = 0;\ \Sigma \cdot Y = 0;\ \Sigma \cdot Z = 0 \ldots\ldots\ldots\ldots\ldots (3.)$$

If the forces all act in one plane, these *conditions of equilibrium* are reduced to two.

96. **Resultant and Balance of Couples.** (*A. M.*, 25 to 37).—Two forces of equal magnitude applied to the same body in parallel and opposite directions, but not in the same line of action (such as F, F, in fig. 63), constitute what is called a "*couple*."

The *arm* or *leverage* of a couple (L, fig. 63) is the perpendicular distance between the lines of action of the two equal forces.

The tendency of a couple is to turn the body to which it is applied in the *plane* of the couple—that is, the plane which contains the lines of action of the two forces. (The plane in which a body turns is any plane parallel to those planes in the body whose position is not altered by the turning). The turning of a body is said to be *right-handed* when it appears to a spectator to take place in the same direction with that of the hands of a watch, and *left-handed* when in the opposite direction; and couples are designated as right-handed or left-handed according to the direction of the turning which they tend to produce. The couple represented in fig. 63 appears right-handed to the reader.

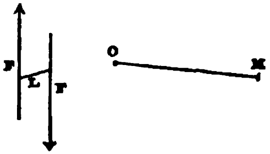

Fig. 63.

The *Moment* of a couple means the product of the magnitude of its force by the length of its arm (F L); and may be represented by the area of a rectangle whose sides are F and L. If the force be a certain number of pounds, and the arm a certain number of feet, the product of those two numbers is called the moment in *foot-pounds*, and similarly for other measures. The moment of a couple may also be represented by a single line on paper, by setting off upon its *axis* (that is, upon any line perpendicular to the plane of the couple) a length proportional to that moment (O M, fig. 63) in such a direction, that to an observer looking from M towards O the couple shall seem right-handed.

The following principle is the groundwork of the theory of couples. It may also be made the groundwork of the whole science of statics, instead of the principle of the parallelogram of forces; for each of those two principles is a necessary consequence of the other.

I. *If the moments of two couples acting in the same direction and in the same or parallel planes are equal, those couples are equivalent:* that is, their tendencies to turn the body to which they are applied are the same.

The following propositions are the chief consequences of the principle just stated.

II. The resultant of any number of couples acting in the same or parallel planes is equivalent to a couple whose moment is the algebraical sum of the moments of the combined couples.

III. Two opposite couples of equal moment in the same or parallel planes balance each other. Any number of couples in the same or parallel planes balance each other when the moments of the right-handed couples are together equal to the moments of the left-handed couples; in other words, when the resultant moment is nothing—a condition expressed algebraically by

$$\Sigma \cdot F\,L = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.)$$

IV. If the two sides of a parallelogram represent the axes and moments of two couples acting on the same body in planes inclined to each other, the diagonal of the parallelogram will represent the axis and moment of the resultant couple, which is equivalent to those two.

In other words, three couples represented by the three sides of a triangle balance each other.

V. If any number of couples acting on the same body be represented by a series of lines joined end to end, so as to form sides of a polygon, and if the polygon is closed, those couples balance each other.

These propositions are analogous to corresponding propositions relating to single forces; and couples, like single forces, can be resolved into components acting about two or three given axes.

97. **Resultant and Balance of Parallel Forces.** (*A. M.*, 38 to 47). —A balanced system of parallel forces consists either of pairs of directly opposed equal forces, or of couples of equal forces, or of combinations of such pairs and couples.

Hence the following propositions as to the relations amongst the *magnitudes* of systems of parallel forces.

I. In a balanced system of parallel forces the sums of the forces acting in opposite directions are equal; in other words, the algebraical sum of the magnitudes of all the forces taken with their proper signs is nothing.

II. The magnitude of the resultant of any combination of parallel forces is the algebraical sum of the magnitudes of the forces.

The relations amongst the *positions* of the lines of action of balanced parallel forces remain to be shown; and in this inquiry all pairs of directly opposed equal forces may be left out of consideration; for each such pair is independently balanced whatsoever its position may be; so that the question in each case is to be solved by means of the theory of couples.

The following is the simplest case:—

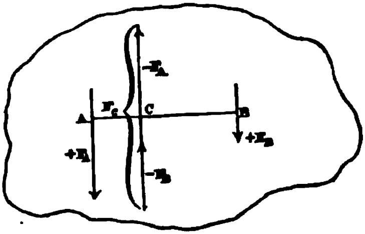

Fig. 64.

III. **Principle of the Lever.**—*If three parallel forces applied to one body balance each other, they must be in one plane; the two extreme forces must act in the same direction; the middle force must act in the opposite direction; and the magnitude of each force must be proportional to the distance between the lines of action of the other two.* Let a body (fig. 64) be maintained in equilibrio by two opposite couples acting in the same plane, and of equal moments,

$$F_A L_A = F_B L_B,$$

and let those couples be so applied to the body that the lines of action of two of those forces, $- F_A - F_B$, which act in the same direction, shall coincide. Then those two forces are equivalent to the single middle force $F_C = -(F_A + F_B)$, equal and opposite to the sum of the extreme forces $+ F_A$, $+ F_B$, and in the same plane with them; and if the straight line A C B be drawn perpendicular to the lines of action of the forces, then

$$\overline{AC} = L_A; \ \overline{CB} = L_B; \ \overline{AB} = L_A + L_B;$$

and consequently

$$F_A : F_B : F_C : : \overline{AC} : \overline{CB} : \overline{AB}; \ldots\ldots\ldots\ldots (1.)$$

This proposition holds also when the straight line A C B crosses the lines of action of the three forces obliquely.

IV. The *resultant* of any two of the three forces F_A, F_B, F_C, is equal and opposite to the third.

In order that two opposite parallel forces may have a single resultant, it is necessary that they should be unequal, the resultant being their difference. Should they be equal, they constitute a couple, which has no single resultant.

V. *Resultant of a Couple and a Single Force in Parallel Planes.*—

Let M denote the moment of a couple applied to a body (fig. 65); and at a point O let a single force F be applied, in a plane parallel to that of the couple. For the given couple substitute an equivalent couple, consisting of a force − F equal and directly opposed to F at O, and a force F acting through the point A, the arm $\overline{\mathrm{A\,O}}$ perpendicular to F being $= \frac{\mathrm{M}}{\mathrm{F}}$, and parallel to the plane of the couple M. Then the forces at O balance each other, and F acting through A is the resultant of the single force F applied at O, and the couple M; that is to say, that if with a single force F there be combined a couple M whose plane is parallel to the force, the effect of that combination is to shift the line of action of the force parallel to itself through a distance $\mathrm{O\,A} = \frac{\mathrm{M}}{\mathrm{F}}$;—to the left if M is right-handed—to the right if M is left-handed.

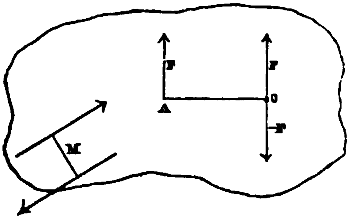

Fig. 65.

VI. *Moment of a Force with respect to an Axis.*—In fig. 66, let the straight line F represent a force. Let O X be any straight line perpendicular in direction to the line of action of the force, and not intersecting it, and let A B be the common perpendicular of those two lines. At B conceive a pair of equal and directly opposed forces to be applied in a line of action parallel to F, viz.: F′ = F, and − F′ = − F. The supposed application of such a pair of balanced forces does not alter the statical condition of the body. Then the original single force F, applied in a line traversing A, is equivalent to the force F′ applied in a line traversing B, the point in O X which is nearest to A, combined with the couple composed of F and − F′, whose moment is F · A B. This is called the *moment of the force* F *relatively to the axis* O X, and sometimes also, the *moment of the force* F *relatively to the plane* traversing O X, parallel to the line of action of the force.

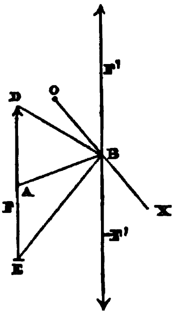

Fig. 66.

If from the point B there be drawn two straight lines B D and B E, to the extremities of the line F representing the force, the area of the triangle B D E being $= \frac{1}{2}$ F · $\overline{\mathrm{A\,B}}$, represents one-half of the moment of F relatively to O X.

VII. *Balance of any System of Parallel Forces in One Plane.*—

In order that any system of parallel forces whose lines of action are in one plane may balance each other, it is necessary and sufficient that the following conditions should be fulfilled:—

First—(As already stated) that the algebraical sum of the forces shall be nothing:—

Secondly—That the algebraical sum of the moments of the forces relatively to any axis perpendicular to the plane in which they act shall be nothing,

two conditions which are expressed symbolically as follows:— Let F denote any one of the forces, considered as positive or negative, according to the direction in which it acts; let y be the perpendicular distance of the line of action of this force from an arbitrarily assumed axis O X, y also being considered as positive or negative, according to its direction; then,

$$\Sigma \cdot \mathrm{F} = 0;\ \Sigma \cdot y\, \mathrm{F} = 0. \text{ ....}(2.)$$

In summing moments, right-handed couples are usually considered as positive, and left-handed couples as negative.

VIII. Let R denote the *resultant of any number of parallel forces in one plane*, and y_r, the distance of the line of action of that resultant from the assumed axis O X to which the positions of forces are referred; then,

$$\mathrm{R} = \Sigma \cdot \mathrm{F};$$
$$y_r = \frac{\Sigma \cdot y\, \mathrm{F}}{\Sigma \cdot \mathrm{F}}.$$

In some cases, the forces may have no single resultant, $\Sigma \cdot \mathrm{F}$ being $= 0$; and then, unless the forces balance each other completely, their resultant is a couple of the moment $\Sigma \cdot y\, \mathrm{F}$.

IX. *Balance of any System of Parallel Forces.*—In order that any system of parallel forces, whether in one plane or not, may balance each other, it is necessary and sufficient that the three following conditions should be fulfilled:—

First—(As already stated) that the algebraical sum of the forces shall be nothing:—

Secondly and Thirdly—That the algebraical sums of the moments of the forces, relatively to a pair of axes at right angles to each other, and to the lines of action of the forces, shall each be nothing,

two conditions which are expressed symbolically as follows:— Let O X and O Y denote the pair of axes; let F be the magnitude of any one of the forces; y its perpendicular distance from O X, and x its perpendicular distance from O Y; then,

$$\Sigma \cdot \mathrm{F} = 0;\ \Sigma \cdot y\, \mathrm{F} = 0;\ \Sigma \cdot x\, \mathrm{F} = 0; \text{..............}(3.)$$

X. Let R denote the *resultant of any system of parallel forces*, and x_r and y_r the distances of its line of action from two rectangular axes; then,

$$R = \Sigma \cdot F; \; x_r = \frac{\Sigma \cdot x F}{\Sigma \cdot F}; \; y_r = \frac{\Sigma \cdot y F}{\Sigma \cdot F} \; \ldots\ldots\ldots\ldots (4.)$$

In some cases the forces may have no single resultant, $\Sigma \cdot F$ being $= 0$; and then, unless the forces balance each other completely, their resultant is a couple, whose axis, direction, and moment, are found as follows:—

Let $$M_x = \Sigma . y F; \; M_y = -\Sigma . x F;$$

be the moments of the pair of partial resultant couples about the axes O X and O Y respectively. From O, along those axes, set off two lines representing respectively M_x and M_y; that is to say, proportional to those moments in length, and pointing in the direction from which those couples must respectively be viewed in order that they may appear right-handed. Complete the rectangle whose sides are those lines; its diagonal will represent the axis, direction, and moment of the final resultant couple. Let M_r be the moment of this couple; then,

$$M_r = \sqrt{\left\{ M_x^2 + M_y^2 \right\}}; \; \ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

and if θ be the angle which its axis makes with O X,

$$\cos \theta = \frac{M_x}{M_r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

98. The **Centre of Parallel Forces** (*A. M.*, 49, 50) is the single point referred to in the following principle. The forces to which that principle is applied are in general either weights or pressures; and the point in question is then called the *Centre of Gravity* or the *Centre of Pressure*, as the case may be.

If there be given a system of points, and the mutual ratios of a system of parallel forces applied to those points, which forces have a single resultant, then there is one point, and one only, which is traversed by the line of action of the resultant of every system of parallel forces having the given mutual ratios and applied to the given system of points, whatsoever may be the absolute magnitudes of those forces, and the angular position of their lines of action.

The position of that point is found as follows:—

Let O in fig. 67 be any convenient point, taken as the origin of co-ordinates, and O X, O Y, O Z, three axes of co-ordinates at right angles to each other, Let A be any one of the points to which the system of parallel forces in question are applied. From A draw x parallel to O X, and perpendicular to the plane Y Z, y parallel to O Y, and perpendicular to the plane Z X, and z parallel to O Z, and perpendicular to the plane X Y. x, y, and z are the rectangular co-ordinates of A, which, being known, the position of A is determined. Let F denote either the magnitude of the force applied at A, or any magnitude proportional to that magnitude. x, y, z, and F are supposed to be known for every point of the given system of points.

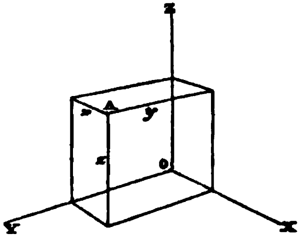

Fig. 67.

First, conceive all the parallel forces to act in lines parallel to the plane Y Z. Then the distance of their resultant, and of the centre of parallel forces from that plane is

$$x_r = \frac{\Sigma \cdot x F}{\Sigma \cdot F}. \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(1.)}$$

Secondly, conceive all the parallel forces to act in lines parallel to the plane Z X. Then the distance of their resultant, and of the centre of parallel forces from that plane is

$$y_r = \frac{\Sigma \cdot y F}{\Sigma \cdot F}. \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(2.)}$$

Thirdly, conceive all the parallel forces to act in lines parallel to the plane X Y. Then the distance of their resultant, and of the centre of parallel forces from that plane, is

$$z_r = \frac{\Sigma \cdot z F}{\Sigma \cdot F}. \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(3.)}$$

If the forces have no single resultant, so that $\Sigma \cdot F = 0$, there is no centre of parallel forces. This may be the case with pressures, but not with weights.

If the parallel forces applied to a system of points are all equal and in the same direction, it is obvious that the distance of the centre of parallel forces from any given plane is simply the mean of the distances of the points of the system from that plane.

99. **Resultant and Balance of any System of Forces in One Plane.** (*A. M.*, 59).—Let the plane be that of the axes O X and O Y in fig. 67; and in looking from Z towards O, let Y lie to the right of

X, so that rotation from X towards Y shall be right-handed. Let x and y be the co-ordinates of the point of application of one of the forces, or of any point in its line of action, relatively to the assumed origin and axes. Resolve each force into two rectangular components X and Y, as in Article 94, p. 137; then the rectangular components of the resultant are $\Sigma \cdot X$ and $\Sigma \cdot Y$; its magnitude is given by the equation

$$R^2 = (\Sigma \cdot X)^2 + (\Sigma \cdot Y), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.)$$

and the angle α_r which it makes with O X is found by the equations

$$\cos \alpha_r = \frac{\Sigma \cdot X}{R}; \ \sin \alpha_r = \frac{\Sigma \cdot Y}{R}. \quad \ldots\ldots\ldots\ldots\ldots \quad (2.)$$

This angle is acute or obtuse according as $\Sigma \cdot X$ is positive or negative; and it lies to the right or left of O X according as $\Sigma \cdot Y$ is positive or negative.

The resultant moment of the system of forces about the axis O Z is

$$M = \Sigma (x Y - y X), \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.)$$

and is right or left-handed according as M is positive or negative.

The perpendicular distance of the resultant force R from O is

$$L = \frac{M}{R}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.)$$

Let x_r and y_r be the co-ordinates of any point in the line of action of that resultant; then the equation of that line is

$$x_r \Sigma \cdot Y - y_r \Sigma \cdot X = M. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.)$$

If $M = 0$, the resultant acts through the origin O; if M has magnitude, and $R = 0$ (in which case $\Sigma \cdot X = 0$, $\Sigma \cdot Y = 0$) the resultant is a couple. The conditions of equilibrium of the system of forces are

$$\Sigma \cdot X = 0; \ \Sigma \cdot Y = 0; \ M = 0. \quad \ldots\ldots\ldots\ldots\ldots \quad (6.)$$

100. **Resultant and Balance of any System of Forces.** (*A. M.*, 60.) —To find the resultant and the conditions of equilibrium of any system of forces acting through any system of points, the forces and points are to be referred to three rectangular axes of co-ordinates.

As before, let O in fig. 67, p. 145, denote the origin of co-ordinates, and O X, O Y, O Z, the three rectangular axes; and let them be arranged so that in looking from

$$\left.\begin{matrix} X \\ Y \\ Z \end{matrix}\right\} \text{towards O, rotation from} \left\{\begin{matrix} Y \text{ towards } Z \\ Z \text{ towards } X \\ X \text{ towards } Y \end{matrix}\right\}$$

shall appear right-handed.

Let X, Y, Z, denote the rectangular components of any one of the forces; x, y, z, the co-ordinates of a point in its line of action.

Taking the algebraical sums of all the forces which act along the same axes, and of all the couples which act round the same axes, the six following quantities are found, which compose the resultant of the given system of forces:—

Forces.

$$\Sigma \cdot X;\ \Sigma \cdot Y;\ \Sigma \cdot Z;\ \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.)$$

Couples.

$$\left.\begin{array}{ll} \text{about O X;} & M_1 = \Sigma\,(yZ - zY); \\ \text{,,} \quad \text{O Y;} & M_2 = \Sigma\,(zX - xZ); \\ \text{,,} \quad \text{O Z;} & M_3 = \Sigma\,(xY - yX). \end{array}\right\} \ldots\ldots\ldots\ldots \quad (2.)$$

The three forces are equivalent to a single force

$$R = \sqrt{\left\{(\Sigma \cdot X)^2 + (\Sigma \cdot Y)^2 + (\Sigma \cdot Z)^2\right\}}, \ldots\ldots \quad (3.)$$

acting through O in a line which makes with the axes the angles given by the equations

$$\cos\alpha = \frac{\Sigma \cdot X}{R};\ \cos\beta = \frac{\Sigma \cdot Y}{R};\ \cos\gamma = \frac{\Sigma \cdot Z}{R}.\ \ldots\ldots \quad (4.)$$

The three couples M_1, M_2, M_3, are equivalent to one couple, whose magnitude is given by the equation

$$M = \sqrt{(M_1^2 + M_2^2 + M_3^2)}, \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.)$$

and whose axis makes with the axes of co-ordinates the angles given by the equations

$$\cos\lambda = \frac{M_1}{M};\ \cos\mu = \frac{M_2}{M};\ \cos\nu = \frac{M_3}{M}, \ldots\ldots\ldots\ldots \quad (6.)$$

$$\text{in which} \left\{\begin{matrix} \lambda \\ \mu \\ \nu \end{matrix}\right\} \begin{matrix} \text{denote respectively the angles} \\ \text{made by the axis of M with} \end{matrix} \left\{\begin{matrix} \text{O X} \\ \text{O Y} \\ \text{O Z} \end{matrix}\right\}.$$

The *Conditions of Equilibrium* of the system of forces may be expressed in either of the two following forms:—

$$\Sigma \cdot X = 0; \Sigma \cdot Y = 0; \Sigma \cdot Z = 0; M_1 = 0; M_2 = 0; M_3 = 0; \quad (7.)$$

or

$$R = 0; \ M = 0 \ldots\ldots\ldots\ldots \quad (8.)$$

When the system is not balanced, its resultant may fall under one or other of the following cases:—

Case I.—*When* M = 0, the resultant is the single force R acting through O.

Case II.—*When the axis of* M *is at right angles to the direction of* R,—a case expressed by the following equation:—

$$\cos \alpha \cos \lambda + \cos \beta \cos \mu + \cos \gamma \cos \nu = 0; \ \ldots\ldots \quad (9.)$$

the resultant of M and R is a single force equal and parallel to R, acting in a plane perpendicular to the axis of M, and at a perpendicular distance from O given by the equation

$$L = \frac{M}{R} \ldots\ldots\ldots\ldots \quad (10.)$$

Case III.—*When* R = 0, there is no single resultant; and the only resultant is the couple M.

Case IV.—*When the axis of* M *is parallel to the line of action of* R, that is, when either

$$\lambda = \alpha; \ \mu = \beta; \ \nu = \gamma, \ldots\ldots\ldots\ldots \quad (11.)$$

or

$$\lambda = -\alpha; \ \mu = -\beta; \ \nu = -\gamma; \ \ldots\ldots\ldots \quad (12.)$$

there is no single resultant; and the system of forces is equivalent to the force R and the couple M, being incapable of being farther simplified.

Case V.—*When the axis of* M *is oblique to the direction of* R, making with it the angle given by the equation

$$\cos \theta = \cos \lambda \cos \alpha + \cos \mu \cos \beta + \cos \nu \cos \gamma, \ldots \quad (13.)$$

the couple M is to be resolved into two rectangular components, viz:—

$$\left.\begin{array}{l} M \sin \theta \text{ round an axis perpendicular to R, and in} \\ \quad \text{the plane containing the direction of R and of} \\ \quad \text{the axis of M;} \\ \qquad M \cos \theta \text{ round an axis parallel to R.} \end{array}\right\} \quad (14.)$$

The force R and the couple M sin θ are equivalent, as in Case II., to a single force equal and parallel to R, whose line of action is in a plane perpendicular to that containing R and the axis of M, and whose perpendicular distance from O is

$$L = \frac{M \sin \theta}{R} \text{.} \qquad (15.)$$

The couple M cos θ, whose axis is parallel to the line of action of R, is incapable of further combination.

Hence it appears finally, that every system of forces which is not self-balanced, is equivalent either, (A); to a single force, as in Cases I. and II. (B); to a couple, as in Case III. (C); to a force, combined with a couple whose axis is parallel to the line of action of the force, as in Cases IV. and V. This can occur with inclined forces only; for the resultant of any number of parallel forces is either a single force or a couple.

101. **Parallel Projections or Transformations in Statics.** (*A. M.*, 61 to 66.)—If two figures be so related, that for each point in one there is a corresponding point in the other, and that to each pair of equal and parallel lines in the one there corresponds a pair of equal and parallel lines in the other, those figures are said to be PARALLEL PROJECTIONS of each other.

The relation between such a pair of figures is expressed algebraically as follows:—Let any figure be referred to axes of co-ordinates, whether rectangular or oblique; let x, y, z, denote the co-ordinates of any point in it, which may be denoted by A: let a second figure be constructed from a second set of axes of co-ordinates, either agreeing with, or differing from, the first set as to rectangularity or obliquity; let x', y', z', be the co-ordinates in the second figure, of the point A′ which corresponds to any point A in the first figure, and let those co-ordinates be so related to the co-ordinates of A, that for each pair of corresponding points, A, A′, in the two figures, the three pairs of corresponding co-ordinates shall bear to each other three constant ratios, such as

$$\frac{x'}{x} = a; \frac{y'}{y} = b; \frac{z'}{z} = c;$$

then are those two figures parallel projections of each other.

For example, all circles and ellipses are parallel projections of each other; so are all spheres, spheroids, and ellipsoids; so are all triangles; so are all triangular pyramids; so are all cylinders; so are all cones.

The following are the geometrical properties of parallel projections which are of most importance in statics:—

I. A parallel projection of a system of three points, lying in one straight line and dividing it in a given proportion, is also a system of three points, lying in one straight line and dividing it in the same proportion.

II. A parallel projection of a system of parallel lines whose lengths bear given ratios to each other, is also a system of parallel lines whose lengths bear the same ratios to each other.

III. A parallel projection of a closed polygon is a closed polygon.

IV. A parallel projection of a parallelogram is a parallelogram.

V. A parallel projection of a parallelopiped is a parallelopiped.

VI. A parallel projection of a pair of parallel plane surfaces, whose areas are in a given ratio, is also a pair of parallel plane surfaces, whose areas are in the same ratio.

VII. A parallel projection of a pair of volumes having a given ratio, is a pair of volumes having the same ratio.

The following are the mechanical properties of parallel projections in connection with the principles set forth in this section:—

VIII. If two systems of points be parallel projections of each other; and if to each of those systems there be applied a system of parallel forces bearing to each other the same system of ratios, then the *centres of parallel forces* for those two systems of points will be parallel projections of each other, mutually related in the same manner with the other pairs of corresponding points in the two systems.

IX. If a *balanced system of forces* acting through any system of points be represented by a system of lines, then will any parallel projection of that system of lines represent a balanced system of forces; and if any two systems of forces be represented by lines which are parallel projections of each other, the lines, or sets of lines, representing their *resultants*, are corresponding parallel projections of each other,—it being observed that *couples* are to be represented by pairs of lines, as pairs of opposite forces, or by areas, and not by single lines along their axes.

Section III.—*Of Distributed Forces.*

102. **Distributed Forces in General.** (*A. M.*, 67, 68.)—In Article 90, p. 133, it has already been explained, that the action of every real force is distributed throughout some volume, or over some surface. It is always possible, however, to find either a *single resultant*, or a *resultant couple*, or a *combination of a single force with a couple*, to which a given distributed force is equivalent, so far as it affects the equilibrium of the body, or part of a body, to which it is applied.

In the application of Mechanics to Structures, the only force distributed throughout the volume of a body which it is necessary to consider, is its *weight*, or attraction towards the earth; and the bodies considered are in every instance so small as compared with the earth, that this attraction may, without appreciable error, be held to act in parallel directions at each point in each body. Moreover, the forces distributed over surfaces are either parallel at each point of their surfaces of application, or capable of being resolved into sets of parallel forces; hence, *parallel distributed forces* have alone to be considered; and every such force is statically equivalent either to a single resultant, or to a resultant couple.

The *Intensity of a Distributed Force* is the ratio which the magnitude of that force, expressed in units of weight, bears to the space over which it is distributed, expressed in units of volume, or in units of surface, as the case may be. An *unit of Intensity* is an unit of force distributed over an unit of volume or of surface, as the case may be; so that there are two kinds of units of intensity. For example, *one pound per cubic foot* is an unit of intensity for a force distributed throughout a volume, such as weight; and *one pound per square foot* is an unit of intensity for a force distributed over a surface, such as pressure or friction.

103. **Weight—Specific Gravity.** (*A. M.*, 69.)—The *intensity of the weight* of a body is expressed either by stating how many units of weight are contained in an unit of volume (for example, pounds avoirdupois in a cubic foot, or in a cubic inch), or by stating the ratio which the weight of a given volume of the body bears to the weight of the same volume of a standard substance (pure water) under a standard pressure (the average atmospheric pressure of 14·7 lbs. on the square inch) and at a standard temperature (which in Britain is 62° Fahrenheit, and in France, the temperature at which water is most dense, or 39°·1 Fahr. = 3°·945 Cent). The last-mentioned ratio is called the "*Specific Gravity*" of the body. For the weight of a cubic foot, there is no single term in English: it might perhaps be called "HEAVINESS;" that being a word which at present is not appropriated to any scientific purpose. According to the French system of measures, there is no need for this distinction; because, as a litre (a cubic décimètre) of pure water at its maximum density weighs a kilogramme, the weight of a cubic décimètre of any substance in kilogrammes is its specific gravity, that of pure water being unity.

The weight of a cubic foot of pure water at 39°·1 Fahr. is

62·425 lbs. avoirdupois.

In rising from 39°·1 to 62° Fahr., pure water expands in the ratio of 1·001118 to 1, and has its density diminished in the ratio of ·998883 to 1,* hence the weight of a cubic foot of pure water at 62° Fahr. is

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* See Professor Miller's paper "On the Standard Pound," *Phil. Trans.*, 1856, Part I.

$$62{\cdot}425 \times {\cdot}998883 = 62{\cdot}355 \text{ lbs. avoirdupois;}$$

and for any other substance we have,

$$\left.\begin{array}{l}\text{Heaviness in lbs. avoirdupois per cubic foot} = \text{Specific} \\ \quad \text{Gravity} \times 62{\cdot}355. \ldots\ldots\ldots\ldots\ldots\ldots\end{array}\right\}\ldots(1.)$$

In a table at the end of this volume are given the specific gravity and heaviness of such materials as most commonly occur in structures. So far as that and similar tables relate to solid materials, they are approximate only; for the specific gravity of the same solid substance varies not only in different specimens, but frequently even in different parts of the same specimen; still the approximate values are sufficiently near the truth for practical purposes in the art of construction.

104. (*A. M.*, 70 to 85.)—The **Centre of Gravity** of a body, or of a system of bodies, is the point always traversed by the resultant of the weight of the body or system of bodies,—in other words, the *centre of parallel forces* for the weight of the body or system of bodies. (See Article 98.)

To *support* a body, that is, to balance its weight, the resultant of the supporting force must act through the centre of gravity.

When the centre of gravity of a *geometrical figure* is spoken of, it is to be understood to mean the point where the centre of gravity would be, if the figure were filled with a substance of uniform heaviness. The following are the most useful of the processes for finding centres of gravity.

I. If a body is *homogeneous*, or of equal specific gravity throughout, and so far *symmetrical* as to have a *centre of figure;* that is, a point within the body, which bisects every diameter of the body drawn through it, that point is also the centre of gravity of the body.

Amongst the bodies which answer this description are, the sphere, the ellipsoid, the circular cylinder, the elliptic cylinder, prisms whose bases have centres of figure, and parallelopipeds, whether right or oblique.

II. The *common centre of gravity of a set of bodies* whose several centres of gravity are known, is the *centre of parallel forces* for the weights of the several bodies, each considered as acting through its centre of gravity. (See Article 98, p. 144.)

III. If a homogeneous body be of a figure which is *symmetrical* on either side of a given plane, the centre of gravity is in that plane. If two or more such *planes of symmetry* intersect in one line, or *axis of symmetry,* the centre of gravity is in that axis. If three or more planes of symmetry intersect each other in a point, that point is the centre of gravity.

IV. To find the centre of gravity of a *homogeneous body of any figure*, assume three rectangular co-ordinate planes in any convenient position, as in fig. 67, p. 145.

To find the distance of the centre of gravity of the body from one of those planes (for example, that of Y Z), conceive the body to be divided into indefinitely thin plane layers parallel to that plane. Let s denote the area of any one of those layers, and $d\,x$ its thickness, so that $s\,d\,x$ is the volume of the layer, and

$$V = \int s\,d\,x,$$

the volume of the whole body, being the sum of the volumes of the layers. Let x be the perpendicular distance of the centre of the layer $s\,d\,x$ from the plane of Y Z. Then the perpendicular distance x_0 of the centre of gravity of the body from that plane is given by the equation

$$x_0 = \frac{\int x\,s\,d\,x}{V}. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1.)}$$

Find, by a similar process, the distances y_0, z_0, of the centre of gravity from the other two co-ordinate planes, and its position will be completely determined.

If the centre of gravity is previously known to be in a particular plane, it is sufficient to find by the above process its distances from *two* planes perpendicular to that plane and to each other.

If the centre of gravity is previously known to be in a particular line, it is sufficient to find its distance from *one* plane, perpendicular to that line.

V. *If the specific gravity of the body varies*, let w be the mean heaviness of the layer $s\,d\,x$, so that

$$W = \int w\,s\,d\,x,$$

is the weight of the body. Then

$$x_0 = \frac{\int w\,s\,d\,x}{W}. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.)}$$

VI. *Centre of Gravity found by Addition.*—When the figure of a body consists of parts, whose respective centres of gravity are known, the centre of gravity of the whole is to be found as in Case II.

VII. *Centre of Gravity found by Subtraction.*—When the figure of a homogeneous body, whose centre of gravity is sought, can be made by taking away a figure whose centre of gravity is known from a larger figure whose centre of gravity is known also, the following method may be used.

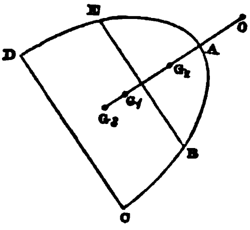

Fig. 68.

Let A C D be the larger figure, G_1 its known centre of gravity, W_1 its weight. Let A B E be the smaller figure, whose centre of gravity G_2 is known, W_2 its weight. Let E B C D be the figure whose centre of gravity G_3 is sought, made by taking away A B E from from A C D, so that its weight is

$$W_3 = W_1 - W_2.$$

Join $G_1 G_2$; G_3 will be in the prolongation of that straight line beyond G_1. In the same straight line produced, take any point O as origin of co-ordinates. Make $\overline{OG_1} = x_1$; $\overline{OG_2} = x_2$, $\overline{OG_3}$ (the unknown quantity) $= x_3$.

Then

$$x_3 = \frac{x_1 W_1 - x_2 W_2}{W_1 - W_2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

VIII. *Centre of Gravity Altered by Transposition.*—In fig. 69, let A B C D be a body of the weight W_0, whose centre of gravity G_0 is known. Let the figure of this body be altered, by transposing a part whose weight is W_1, from the position E C F to the position F D H, so that the new figure of the body is A B H E. Let G_1 be the original, and G_2 the new position of the centre of gravity of the transposed part. Then the centre of gravity of the whole body will be shifted to G_3, in a direction $G_0 G_3$ parallel to $G_2 G_1$, and through a distance given by the formula

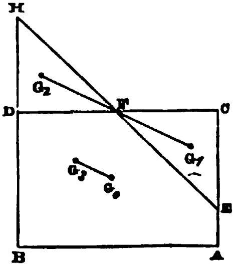

Fig. 69.

$$\overline{G_0 G_3} = \overline{G_1 G_2} \frac{W_1}{W_0}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

IX. *Centre of Gravity found by Projection or Transformation.*—If the figures of two homogeneous bodies are parallel projections of each other, the centres of gravity of those two bodies are corresponding points in those parallel projections.

To express this symbolically,—as in Article 101, let x, y, z, be the co-ordinates, rectangular or oblique, of any point in the figure of the first body; x', y', z', those of the corresponding point in the second body; x_o, y_o, z_o, the co-ordinates of the centre of gravity of the first body; x'_o, y'_o, z'_o those of the centre of gravity of the second body; then

$$\frac{x'_o}{x_o} = \frac{x'}{x}; \quad \frac{y'_o}{y_o} = \frac{y'}{y}; \quad \frac{z'_o}{z_o} = \frac{z'}{z}. \dots\dots\dots\dots (5.)$$

This theorem facilitates much the finding of the centres of gravity of figures which are parallel projections of more simple or more symmetrical figures.

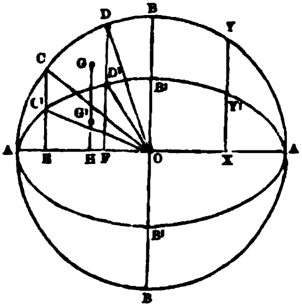

Fig. 70.

For example, let it be supposed that a formula is known (which will be given in p. 157) for finding the centre of gravity of a sector of a circular disc, and let it be required to find the centre of gravity of a sector of an elliptic disc. In fig. 70, let A B′ A B′ be the ellipse, A O A = 2 a, and B′ O B′ = 2 b, its axes, and C′ O D′ the sector whose centre of gravity is required. About the centre of the ellipse, O, describe the circle, A B A B, whose radius is the semi-axis major a. Through C′ and D′ respectively draw E C′ C and F D′ D, parallel to O B, and cutting the circle in C and D respectively; the circular sector C O D is the parallel projection of the elliptic sector C′ O D′. Let G be the centre of gravity of the sector of the circular disc, its co-ordinates being

$$\overline{\mathrm{O\,H}} = x_o; \; \overline{\mathrm{H\,G}} = y_o.$$

Then the co-ordinates of the centre of gravity G′ of the sector of the elliptic disc are

$$\left.\begin{array}{l} \overline{\mathrm{O\,H}} = x'_o = x_o; \\ \overline{\mathrm{H\,G'}} = y'_o = \dfrac{b\,y_o}{a} \end{array}\right\} \dots\dots\dots\dots\dots\dots\dots (6.)$$

X. *Centre of Gravity found Experimentally.*—The centre of gravity of a body of moderate size may be found approximately by experiment, by hanging it up successively by a single cord in two different positions, and finding the single point in the body which in both positions is intersected by the axis of the cord.

105. **Examples of Weights and Centres of Gravity.** (*A. M.*, 83.)—The following examples consist of formulæ for the weight, and the position of the centre of gravity, of homogeneous bodies of those forms which most commonly occur in practice. In each case w denotes the heaviness of the body, W, its weight, and x_o, &c., the co-ordinates of its centre of gravity, which in the diagrams is marked G, the origin of co-ordinates being marked O.

A.—Prisms and Cylinders with Parallel Bases.

The word *cylinder* is here to be taken in its most general meaning, as comprehending all solids traced by the motion of a plane curvilinear figure parallel to itself.

The examples here given apply to flat plates of uniform thickness.

In the formulæ for weights, the length or thickness is supposed to be *unity*.

The centre of gravity, in each case, is at the middle of the length (or thickness); and the formulæ give its situation in the plane figure which represents the cross section of the prism or cylinder, and which is specified at the commencement of each example.

I. *Triangle.*—(Fig. 71.) O, any angle. Bisect opposite side B C in D. Join ~~A~~ O D.

$$x_o = \overline{\mathrm{O\,G}} = \frac{2}{3}\overline{\mathrm{O\,D}}.$$

$$\mathrm{W} = \frac{w \cdot \overline{\mathrm{O\,D}} \cdot \overline{\mathrm{B\,C}} \cdot \sin. \angle \mathrm{O\,D\,C}}{2}.$$

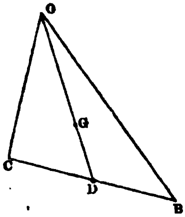

Fig. 71.

II. *Polygon.*—Divide it into triangles; find the centre of gravity of each; then find their common centre of gravity as in Article 104, Case II., p. 152.

III. *Trapezoid.*—(Fig. 72.) A B ∥ C E.

Greatest breadth, A B = B.
Least ,, C E = b.
Bisect A B in O, C E in D; join O D.

$$x_o = \overline{\mathrm{O\,G}} = \frac{\overline{\mathrm{O\,D}}}{2}\left(1 - \frac{1}{3}\cdot\frac{\mathrm{B}-b}{\mathrm{B}+b}\right).$$

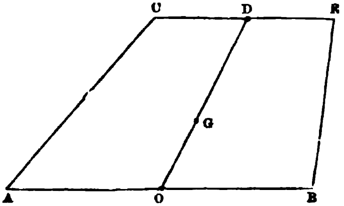

Fig. 72.

$$W = w \cdot \overline{OD} \cdot \frac{B+b}{2} \cdot \sin \angle DOE.$$

IV. *Trapezoid.*—(Second solution.)—(Fig. 73.) O, point where inclined sides meet. Let $\overline{OF} = x_1$, $\overline{OD} = x_2$, $\overline{OG} = x_o$.

$$x_o = \frac{2}{3} \cdot \frac{x_1^3 - x_2^3}{x_1^2 - x_2^2}$$

$$W = w \cdot \frac{x_1^2 - x_2^2}{2} \cdot \sin^2 \angle OFB.$$

$$(\text{cotan} \angle OAB + \text{cotan} \angle OBA).$$

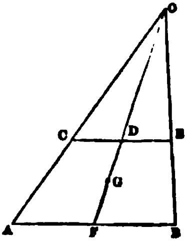

Fig. 73.

V. *Parabolic Half-Segment.*—(O A B, fig. 74.) O, vertex of diameter O X; $\overline{OA} = x_1$; $\overline{AB} = y_1$, ordinate ∥ tangent O C Y.

$$x_o = \frac{3}{5} x_1; \; y_o = \frac{3}{8} y_1.$$

$$W = \frac{2}{3} w \cdot x_1 y_1 \cdot \sin \angle XOY.$$

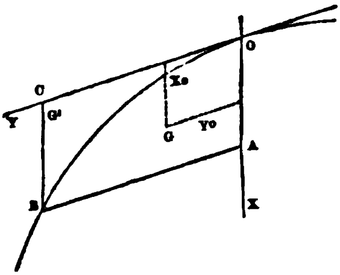

Fig. 74.

VI. *Parabolic Spandril.*—(O B C, fig. 74.) G', centre of gravity,

$$x_o = \frac{3}{10} x_1; \; y_o = \frac{3}{4} y_1; \; W = \frac{1}{3} w \cdot x_1 y_1 \cdot \sin \angle XOY.$$

VII. *Circular Sector.*—(O A C, fig. 75.) Let O X bisect the angle A O C; O Y ⊥ O X.

Radius $\overline{OA} = r$

Half-arc, to radius unity, $\frac{AC}{2AO} = \theta$.

$$\therefore \; x_o = \frac{2}{3} r \frac{\sin \theta}{\theta}; \; y_o = 0.$$

$$W = w r^2 \theta$$

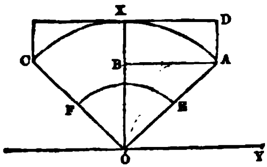

Fig. 75.

VIII. *Circular Half-Segment.*—(A B X, Fig. 75.)

$$x_0 = \frac{2}{3} r \cdot \frac{\sin^3\theta}{\theta - \sin\theta\cos\theta};\ y_0 = r \cdot \frac{4\sin^2\frac{\theta}{2} - \sin^2\theta\cos\theta}{3(\theta - \cos\theta\sin\theta)}.$$

$$W = \frac{1}{2} wr^2 (\theta - \cos\theta\sin\theta).$$

IX. *Circular Spandril.*—(A D X, Fig. 75.)

$$x_0 = \frac{1}{3} r \cdot \frac{\sin^3\theta}{2\sin\theta - \sin\theta\cos\theta - \theta}.$$

$$y_0 = \frac{1}{3} r \cdot \frac{3\sin^2\theta - 2\sin^2\theta\cos\theta - 4\sin^2\frac{\theta}{2}}{2\sin\theta - \sin\theta\cos\theta - \theta}.$$

$$W = wr^2 \cdot \left(\sin\theta - \frac{1}{2}\sin\theta\cos\theta - \frac{\theta}{2}\right).$$

X. *Sector of Ring.*—(A C F E, Fig. 75.) $\overline{OA} = r$; $\overline{OE} = r'$.

$$x_0 = \frac{2}{3} \cdot \frac{r^3 - r'^3}{r^2 - r'^2} \cdot \frac{\sin\theta}{\theta};\ y_0 = 0.$$

$$W = w(r^2 - r'^2)\,\theta.$$

XI. *Elliptic Sector, Half-Segment, or Spandril.*—Centre of gravity to be found by projection from that of corresponding circular figure, as in Article 104, Case IX., p. 154.

B.—WEDGES.

XII. *General Formulæ for Wedges.*—(Fig. 76.) All wedges may be divided into parts such as the figure here represented. O A Y, O X Y, planes meeting in the edge O Y; A X Y, cylindrical (or prismatic) surface perpendicular to the plane O X Y; O X A, plane triangle perpendicular to the edge O Y; O Z, axis perpendicular to X O Y. Let O X $= x_1$; X A $= z_1$. Then $z = \frac{z_1 x}{x_1}$;

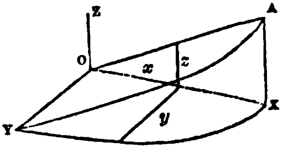

Fig. 76.

$$W = w \cdot \frac{z_1}{x_1} \int x y \cdot dx$$

$$x_0 = \frac{\int x^2 y \cdot dx}{\int x y \cdot dx};\ y_0 = \frac{\int x y^2 \cdot dx}{2\int x y \cdot dx};\ z_0 = \frac{z_1 x_0}{2 x_1}.$$

(This last equation denotes that G is in the plane which traverses O Y and bisects A X.)

In a symmetrical wedge, if O be taken at the middle of the edge, $y_0 = O$. Such is the case in the following examples, in each of which, length of edge $= 2\,y_1$.

XIII. *Rectangular Wedge.*—(= Triangular Prism.)—(Fig. 77.)

$$W = w \cdot x_1 y_1 z_1 ;\ x_0 = \frac{2}{3} x_1.$$

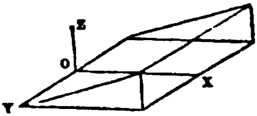

Fig. 77.

XIV. *Triangular Wedge.*—(= Triangular Pyramid.)—(Fig. 78.)

$$W = \frac{1}{3} w \cdot x_1 y_1 z_1 ;\ x_0 = \frac{1}{2} x_1.$$

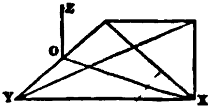

Fig. 78.

XV. *Semicircular Wedge.*—(Fig. 79.)

Radius $\overline{OX} = \overline{OY} = r$.

$$W = \frac{2}{3} w \cdot r^2 z_1 ;\ x_0 = \frac{3\pi}{16} r = \cdot 58905\, r.$$

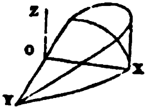

Fig 79.

XVI. *Annular, or Hollow Semicircular Wedge.*—(Fig. 80.)

External radius, r; internal r'.

$$W = \frac{2}{3}\ w \cdot \left(r^3 - r'^3\right) \frac{z^1}{r} ; x_0 = \frac{3\pi}{16} \cdot \frac{r^4 - r'^4}{r^3 - r'^3}$$

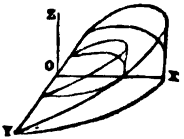

Fig. 80.

C.—CONES AND PYRAMIDS.

Let O denote the apex of the cone or pyramid, taken as the origin, and X the centre of gravity of a supposed flat plate whose middle section coincides with the base of the cone, or pyramid. The centre of gravity will lie in the axis O X.

Denote the area of the base by A, and the angle which it makes with the axis by θ.

XVII. *Complete Cone or Pyramid.*—Let the height $\overline{OX} = h$;

$$x_0 = \frac{3}{4} h; \; W = \frac{1}{3} w \cdot A h \sin \theta.$$

XVIII. *Truncated Cone or Pyramid.*—Height of portion truncated $= h'$.

$$x_0 = \frac{3}{4} \cdot \frac{h^4 - h'^4}{h^3 - h'^3}; \; W = \frac{1}{3} w A h \cdot \left(1 - \frac{h'^3}{h^3}\right) \sin \theta.$$

D.—PORTIONS OF A SPHERE.

XIX. *Zone or Ring of a Spherical Shell,* bounded by two conical surfaces having their common apex at the centre O of the sphere (fig. 81).

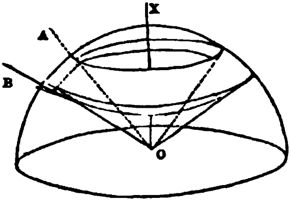

Fig. 81.

O X, axis of cones and zone.

$$\left.\begin{array}{l} r,\ \text{external radius} \\ r',\ \text{internal radius} \end{array}\right\}\ \text{of shell.}$$

$$\left.\begin{array}{l} \angle XOA = \alpha,\ \text{half-angle of less} \\ \angle XOB = \beta,\ \quad ,, \quad \text{greater} \end{array}\right\}\ \text{cone.}$$

$$x_0 = \frac{3}{4} \cdot \frac{r^4 - r'^4}{r^3 - r'^3} \cdot \frac{\cos \alpha + \cos \beta}{2}.$$

$$W = \frac{2 \pi w}{3} \left(r^3 - r'^3\right) \cdot \left(\cos \beta - \cos \alpha\right).$$

XX. *Sector of a Hemispherical Shell.*—(C X D, fig. 82.) O Y bisects angle D O C; $\frac{1}{2}$ D O C $= \theta$.

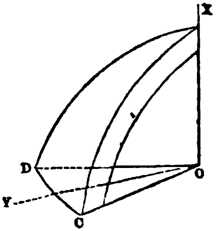

Fig. 82.

$$x_0 = \frac{3}{8} \cdot \frac{r^4 - r'^4}{r^3 - r'^3}; \; y_0 = \frac{3 \pi}{16} \cdot \frac{r^4 - r'^4}{r^3 - r'^3} \cdot \frac{\sin \theta}{\theta}.$$

$$W = \frac{2 \theta w}{3} \left(r^3 - r'^3\right).$$

106. **Stress—its Intensity, Resultant, Centre, and Moment.** (*A. M.*, 86 to 89.)—The word STRESS has been adopted as a general term to comprehend various forces which are exerted between contiguous bodies, or parts of bodies, and which are distributed over the surface of contact of the masses between which they act.

The INTENSITY of a stress is its amount in units of weight, divided by the extent of the surface over which it acts, in units of area.

The following table gives a comparison of various units in which the intensity of stress is expressed:—

	Pounds on the square foot.	Pounds on the square inch.
One pound on the square inch,.....	144	1
One pound on the square foot,......	1	$\frac{1}{144}$
One inch of mercury (that is, weight of a column of mercury at 32° Fahr., one inch high), ...	70·73	0·4912
One foot of water (at 39°·1 Fahr.),	62·425	0·4335
One inch of water (at 39°·1 Fahr.),	5·2021	0·036125
One foot of water (at 62° Fahr.),...	62·355	0·43302
One inch of water (at 62° Fahr.),...	5·19625	0·036085
One atmosphere, of 29·922 inches of mercury, or 760 millimètres,	2116·4	14·7
One foot of air, at 32° Fahr., and under the pressure of one atmosphere, ...	0·080728	0·0005606
One kilogramme on the square mètre, ...	0·20481	0·00142228
One kilogramme on the square millimètre,.........................	204810	1422·28
One millimètre of mercury,.........	2·7847	0·01934

The various kinds of stress may be thus classed:—

I. *Thrust*, or *Pressure*, is the force which acts between two contiguous bodies, or parts of a body, when each pushes the other from itself.

II. *Pull*, or *Tension*, is the force which acts between two contiguous bodies, or parts of a body, when each draws the other towards itself.

Pressure and tension may be either *normal* or *oblique*, relatively to the surface at which they act.

III. *Shear*, or *Tangential Stress*, is the force which acts between two contiguous bodies, or parts of a body, when each draws the other sideways, in a direction parallel to their surface of contact.

In expressing a Thrust and a Pull in parallel directions algebraically, if one is treated as positive, the other must be treated as negative. The choice of the positive or negative sign for either is a matter of convenience.

The word "*Pressure*," although, strictly speaking, equivalent to "*thrust*," is sometimes applied to *stress* in general; and when this is the case, it is to be understood that thrust is treated as positive.

The following are the processes for finding the *magnitude of the resultant* of a stress distributed over a plane surface, and the *centre of stress;* that is, the point where the line of action of that resultant cuts the plane surface:—

I. *If the stress is of uniform intensity*, the magnitude of its resultant is the product of that intensity and the area of the surface; and the centre of stress is at the centre of gravity of the surface. Or in symbols, let S be the area of the surface, p the intensity of the stress, P its resultant, then—

$$P = p\,S \quad \text{..............................} \quad (1.)$$

II. *If the stress is of varying intensity, but of one sign;* that is, all tension, or all pressure, or all shear in one direction.

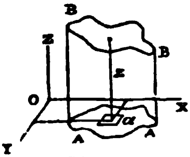

Fig. 83.

In fig. 83, let A A be the given plane surface at which the stress acts; O X, O Y, two rectangular axes of co-ordinates in its plane; O Z, a third axis perpendicular to that plane. Conceive a solid to exist, bounded at one end by the given plane surface A A, laterally by a cylindrical or prismatic surface generated by the motion of a straight line parallel to O Z round the outline of A A, and at the other end by a surface B B, of such a figure, that its ordinate z at any point shall be proportional to the intensity of the stress at the point a of the surface A A from which that ordinate proceeds, as shown by the equation

$$z = \frac{p}{w} \quad \text{..............................} \quad (2.)$$

Conceive the surface A A to be divided into an indefinite number of small rectangular areas, each denoted by $dx\,dy$, and so small that the stress on each is sensibly uniform; the entire area being

$$S = \int\!\!\int dx\,dy.$$

The volume of the ideal solid will be

$$V = \int\!\!\int z \cdot dx\,dy \quad \text{........................} \quad (3.)$$

So that if it be conceived to consist of a material whose heaviness is $w = \frac{p}{z}$, the amount of the stress will be equal to the weight of the solid; that is to say,

$$P = \int\int p\,d\,x\,d\,y = w\,V \text{.....................} (4.)$$

The *centre of stress* is the point on the surface A A perpendicularly opposite the centre of gravity of the ideal solid.

The simplest, and at the same time the commonest case of this kind is where the stress is *uniformly-varying;* that is, where its intensity at a given point is simply proportional to the perpendicular distance of that point from a given straight line in the plane of the surface A A. The ideal solid is now either a wedge, or a figure that can be made by adding and subtracting wedges; so that the resultant and centre of stress are to be found by the methods of Article 105, Cases XII. to XVI., and Article 104, Cases II. and VII. To express this symbolically, take the straight line in question for the axis O Y; conceive the surface to be divided into bands by lines parallel to O Y; let y denote the length of one of these bands, and $d\,x$ its breadth, so that $y\,d\,x$ is its area, and $S = \int y\,d\,x$ the area of the whole surface. Let x be the perpendicular distance of the centre of a band from the *line of no stress* O Y, and let the intensity of the stress there be

$$p = a\,x; \text{..............................} (5.)$$

a being a constant co-efficient; then the amount or resultant of the stress is

$$P = \int p\,y\,d\,x = a\int x\,y\,d\,x; \text{ ....} (6.)$$

and the perpendicular distance of the centre of stress from O Y is

$$x_0 = \frac{\int p\,x\,y\,d\,x}{\int p\,y\,d\,x} = \frac{a\int x^2\,y\,d\,x}{P} \text{.................} (7.)$$

Examples of this case will be given in treating of the pressure of water and of earth, and the stability of masonry.

III. *When the stress is of contrary signs;* for example, pressure at one part of the surface and tension at another, the resultants and centres of stress of the pressure and tension are to be found separately. Those partial resultants are then to be treated as a pair of parallel forces acting through the two respective centres of stress; their final resultant will be equal to their difference, if any, acting through a point found as in Article 97, Case IV., p. 141.

If the total pressure and total tension are equal to each other, they have no single resultant and no single centre of stress: their resultant being a couple, whose moment is equal to the total stress of either kind multiplied by the perpendicular distance between the resultant of the pressure and the resultant of the tension. Examples of this case will be given in treating of the strength of beams.

107. **Pressure and Balance of Fluids—Principles of Hydrostatics.** —*Fluid* is a term opposed to *solid*, and comprehending the liquid and gaseous conditions of bodies. The property common to the liquid and the gaseous conditions is that of *not tending to preserve a definite shape*, and the possession of this property by a body in perfection throughout all its parts, constitutes that body a *perfect fluid.*

A necessary consequence of that property is the following principle, which is the foundation of the whole science of hydrostatics:—

I. *In a perfect fluid, when still, the pressure exerted at a given point is normal to the surface on which it acts, and of equal intensity for all positions of that surface.*

The following are some of the most useful consequences of that principle:—

II. *A surface of equal pressure in a still fluid mass is everywhere perpendicular to the direction of gravity;* that is, horizontal throughout. In other words, the pressure at all points at the same level is of equal intensity.

III. *The intensity of the pressure at the lower of two points in a still fluid mass is greater than the intensity at the higher point, by an amount equal to the weight of a vertical column of the fluid whose height is the difference of elevation of the points, and base an unit of area.*

To express this symbolically, let p_0 denote the intensity of the pressure at the higher of two points in a fluid mass, and p_1 the intensity at a point whose vertical depth below the former point is x. Let w be the *mean heaviness* of the layer of fluid between those two points; then

$$p_1 = p_0 + w x. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

In a gas, such as air, w varies, being nearly proportional to p; but in a liquid, such as water, the variations of w are too small to be considered in practical cases.

For example, let the upper of the two points be the surface of a mass of water where it is exposed to the air; then p_0 is the atmospheric pressure; let the depth x of the second point below the surface be given in feet, and let the temperature be $39^\circ{\cdot}1$; then

$$p_1 \text{ in lbs. on the square foot} = p_0 + 62{\cdot}425\, x. \quad \ldots\ldots\ldots(2.)$$

In many questions relating to engineering, the pressure of the atmosphere may be left out of consideration, as it acts with sensibly equal intensity on all sides of the bodies exposed to it, and so balances its own action. The pressures calculated, in such cases, is the *excess* of the pressure of the water above the atmospheric pressure, which may be thus expressed,—

$$p' = p_1 - p_0 = 62{\cdot}4\, x \text{ nearly.} \quad\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

IV. The pressure of a liquid on a *floating* or *immersed body*, is equal to the weight of the volume of fluid displaced by that body; and the resultant of that pressure acts vertically upwards through the centre of gravity of that volume; which centre of gravity is called the "*centre of buoyancy.*"

V. The pressure of a liquid against *a plane surface immersed in it* is perpendicular to that surface in direction: its magnitude is equal to the weight of a volume of the liquid, found by multiplying the area of the surface by the depth to which its centre of gravity is immersed.

VI. The *centre of pressure* on such a surface, if the surface is horizontal, coincides with its centre of gravity; if the surface is vertical or sloping, the centre of pressure is always below the centre of gravity of the surface, and is found by considering that the pressure is an *uniformly-varying* stress, whose intensity at a given point varies as the distance of that point from the line where the given plane surface (produced if necessary) intersects the upper surface of the liquid.

To express the last two principles by symbols in the case in which the pressed surface is vertical or sloping, let the line where the plane of that surface cuts the upper surface of the liquid be taken as the axis O Y. Let θ denote the angle of inclination of the pressed surface to the horizon. Conceive that surface to be divided by parallel horizontal lines into an indefinite number of narrow bands. Let y be the length of any one of those bands, dx its breadth, x the distance of its centre from O Y; then $y\,dx$ is its area, $x \sin \theta$ the depth at which it is immersed; and if w be the weight of unity of volume of the fluid, the intensity of the pressure on that band is

$$p = w\,x \sin \theta. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

The whole area of the pressed surface, being the sum of the areas of all the bands, is $S = \int y\,dx$; the whole pressure upon it is

$$P = \int p\,y\,dx = w \sin \theta \int x\,y\,dx; \quad\ldots\ldots\ldots\ldots (5.)$$

The mean intensity of the pressure is

$$\frac{P}{S} = \frac{\int p\,y\,dx}{\int y\,dx} = w \sin \theta \frac{\int x\,y\,dx}{\int y\,dx}; \quad\ldots\ldots\ldots\ldots (6.)$$

and the distance of the centre of pressure from O Y is

$$x_0 = \frac{\int x p y \, d x}{\mathrm{P}} = \frac{\int x^2 y \, d x}{\int x y d x}. \qquad \text{(7.)}$$

For example, let the sloping pressed surface be rectangular, like a sluice, or the back of a reservoir-wall; and in the first instance, let it extend from the surface of a mass of water down to a distance x_1, measured along the slope, so that its lower edge is immersed to the depth $x_1 \sin \theta$. Then its centre of gravity is immersed to the depth $x_1 \sin \theta \div 2$, and the mean intensity of the pressure in lbs. on the square foot, is

$$\frac{\mathrm{P}}{\mathrm{S}} = \frac{62{\cdot}4 \, x_1 \sin \theta}{2}. \qquad \text{(8.)}$$

The breadth y is constant; so that the area of the surface is $\mathrm{S} = x_1 y$; and the total pressure is

$$\mathrm{P} = \frac{62{\cdot}4 \;\; x_1^2 \, y \sin \theta}{2}. \qquad \text{(9.)}$$

The distance of the centre of pressure from the upper edge is

$$x_0 = \frac{2}{3} \cdot x_1. \qquad \text{(10.)}$$

Next, let the upper edge, instead of being at the surface of the water, be at the distance x_2 from it, so as to be immersed to the depth $x_2 \sin \theta$. Then the centre of gravity of the pressed surface is immersed to the depth $(x_1 + x_2) \sin \theta \div 2$, and the mean intensity of the pressure upon it, in lbs. on the square foot, is

$$\frac{\mathrm{P}}{\mathrm{S}} = \frac{62{\cdot}4 \, (x_1 + x_2) \sin \theta}{2}; \qquad \text{(11.)}$$

the area of the surface is $(x_1 - x_2)\, y$, and the total pressure on it

$$\mathrm{P} = \frac{62{\cdot}4 \, (x_1^2 - x_2^2) \, y \sin \theta}{2}. \qquad \text{(12.)}$$

The distance of the centre of pressure from the line O Y is

$$x_0 = \frac{2}{3} \cdot \frac{x_1^3 - x_2^3}{x_1^2 - x_2^2} \qquad \text{(13.)}$$

108. **Compound Internal Stress of Solids.** (*A. M.*, 96 to 113.)—If a body be conceived to be divided into two parts by an ideal plane traversing it in any direction, the force exerted between those two parts at the plane of division is an *internal stress*.

According to the principles stated in the preceding article, the internal stress at a given point in a fluid is normal and of equal intensity for all positions of the ideal plane of division. In a solid body, on the other hand, the stress may be either normal, oblique, *or shearing*; and it may vary in direction and intensity, as the position of the ideal plane of division varies.

If the direction and intensity of the stress at a given point in a solid mass are given for three positions of the plane of division, they can be found for any position whatsoever. It is unnecessary in the present treatise to give the methods of solving this problem in all its generality. Certain particular cases only will be given, which are useful in the theories of the stability of earth and of the strength of materials.

I. *Conjugate Stresses—Principal Stresses.*—If two planes traverse a point in a body, and the direction of the stress on the first plane is parallel to the second plane, then the direction of the stress on the second plane is parallel to the first plane. Such a pair of stresses are said to be *conjugate;* and if they are both normal to their planes of application (and consequently perpendicular to each other) they are called *principal stresses.* Three conjugate stresses, or three principal stresses, may act through one point; but in the present treatise it is sufficient to consider two.

Fig. 84 represents a pair of conjugate oblique tensions acting in the directions X X and Y Y through a prismatic particle A B C D.

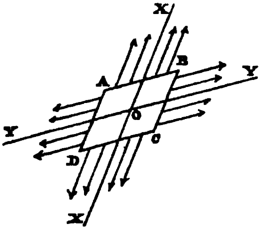

Fig. 84.

The rectangular directions in which principal stresses—that is, direct pulls and thrusts—act, through a given point in a solid, are called *axes of stress.*

In a fluid, the stress at a given point being of equal intensity in all directions, every direction has the property of an axis of stress. A solid *may* be in the same condition with a fluid as to stress; but it may also have the principal stresses at a given point of different intensities. In a mass of loose grains, the ratio of those intensities has a limit depending on friction, as will afterwards be more fully explained in treating of the stability of earth: —in a firm continuous solid, the principal stresses at a point may bear any ratio to each other, and may be either of the same or of opposite kinds.

II. *The Shearing Stress,* on two planes traversing a point in a solid at right angles to each other, is of equal intensity.

III. *A Pair of Equal and Opposite Principal Stresses*; that is, a pull and a thrust of equal intensity acting through a particle of a solid in directions at right angles to each other, are equivalent to a pair of shearing stresses of the same intensity on a pair of planes at right angles to each other, and making angles of 45° with the first pair of planes.

IV. *Combination of any Two Principal Stresses.*

Problem.—A pair of principal stresses of any intensities, and of the same or opposite kinds, being given, it is required to find the direction and intensity of the stress on a plane in any position at right angles to the plane parallel to which the two principal stresses act.

Let O X and O Y (figs. 85 and 86) be the directions of the two principal stresses; O X being the direction of the greater stress.

Let p_1 be the intensity of the greater stress;
and p_2 that of the less.

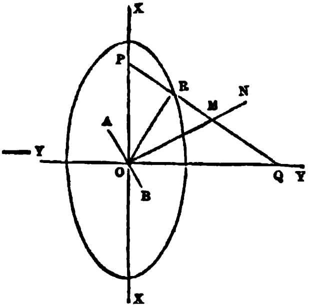

Fig. 85.

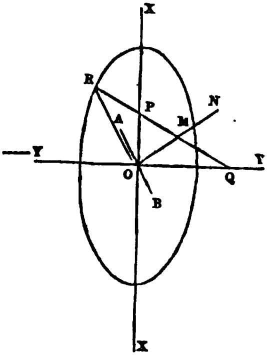

Fig. 86.

The kind of stress to which each of these belongs, pull or thrust, is to be distinguished by means of the algebraical signs. If a pull is considered as positive, a thrust is to be considered as negative, and *vice versâ.* It is in general convenient to consider that kind of stress as positive to which the greater principal stress belongs. Fig. 85 represents the case in which p_1 and p_2 are of the same kind; fig. 86 the case in which they are of opposite kinds. In all the following equations, the sign of p_2 is held to be *implied* in that symbol; that is to say, when p_2 is of the contrary kind to p_1, the sign applied to its arithmetical value, in computing by means of the equations, is to be reversed.

Let A B be the plane on which it is required to ascertain the direction and intensity of the stress, and O N a normal to that plane, making with the axis of greatest stress the angle

$$\angle \mathrm{X\,O\,N} = \widehat{xn}.$$

$p_1 = \frac{p_1+p_2}{2} + \frac{p_1-p_2}{2}$

$p_2 = \frac{p_1+p_2}{2} - \frac{p_1-p_2}{2}$

On O N take $\overline{\mathrm{OM}} = \frac{p_1+p_2}{2}$; this will represent a normal stress on A B of the same kind with the greater principal stress, and of an intensity which is a mean between the intensities of the two principal stresses.

Through M draw P M Q, making with the axes of stress the same angles which O N makes, but in the opposite direction; that is to say, take $\overline{\mathrm{MP}} = \overline{\mathrm{MQ}} = \overline{\mathrm{MO}}$. On the line thus found set off from M towards the axis of greatest stress, $\overline{\mathrm{MR}} = \frac{p_1-p_2}{2}$.

Join $\overline{\mathrm{OR}}$. Then will that line represent the direction and intensity of the stress on A B.

The algebraical expression of this solution is easily obtained by means of the formulæ of plane trigonometry, and consists of the two following equations:— $\overline{OR}^2 = \overline{OM}^2 + \overline{MR}^2 - 2\,\overline{OM}\cdot\overline{MR}\cos$ [illegible]

$$\text{Intensity, } \overline{\mathrm{OR}} \text{ or } p = \sqrt{\{p_1^2 \cdot \cos^2 \hat{xn} + p_2^2 \cdot \sin^2 \hat{xn}\}} \ldots (1.)$$

$$\text{Angle of obliquity, N O R or } \hat{nr}$$

$\frac{\sin nr}{\sin 2xn} = \frac{\frac{p_1-p_2}{2}}{p}$.

$$= \text{arc sin} \cdot \left(\sin 2\hat{xn} \cdot \frac{p_1 - p_2}{2p}\right). \ldots\ldots\ldots\ldots (2.)$$

This obliquity is always towards the axis of greatest stress.

In fig. 85, p_1 and p_2 are represented as being of the same kind; and $\overline{\mathrm{MR}}$ is consequently less than $\overline{\mathrm{OM}}$, so that $\overline{\mathrm{OR}}$ falls on the same side of O X with O N; that is to say, $\hat{nr} < \hat{xn}$. In fig. 86, p_1 and p_2 are of opposite kinds, $\overline{\mathrm{MR}}$ is greater than $\overline{\mathrm{OM}}$, and O R falls on the opposite side of O X to O M; that is to say, $\hat{nr} > \hat{xn}$.

The locus of the point M is a circle of the radius $\frac{p_1+p_2}{2}$, and that of the point R, an ellipse whose semi-axes are p_1 and p_2, and which may be called the Ellipse of Stress, because its semi-diameter in any direction represents the intensity of the stress in that direction.

V. *Deviation of Principal Stresses by a Shearing Stress.*

Problem.—Let p_x and p_y denote the original intensities of a pair of principal stresses acting at right angles to each other through one particle of a solid. Suppose that with these there is combined a shearing stress of the intensity q, acting in the same plane with the original pulls or thrusts; it is required to find the new intensities and new directions of the principal stresses.

To assist the conception of this problem, the original stresses referred to are represented in fig. 87, as acting through a particle of the form of a square prism. The principal stresses, both original and new, are represented as tensions, although any or all of them might be pressures. In the formulæ annexed, tensions are considered positive, pressures negative; angles lying to the right of A A are considered as positive, to the left as negative; and a shearing stress is considered as positive or negative according as it tends to make the upper right-hand and lower left-hand corner of the square particle acute or obtuse.

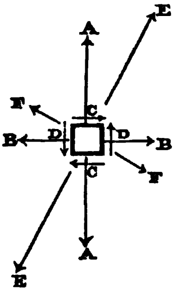

Fig. 87.

The arrows A A represent the greater original tension p_x; the ~~angles~~ arrows B B, the less original tension p_y; C, C, D, D, represent the positive shear of the intensity q, as acting at the four faces of the particle. The combination of this shear with the original tensions is equivalent to a new pair of principal tensions, oblique to the original pair. The greater new principal tension, p_1, is represented by the arrows E, E; it deviates to the right of p_x through an angle which will be denoted by θ. The less new principal tension p_2 is represented by the arrows F, F; it deviates through the same angle to the right of p_y.

Then the intensities of the new principal stresses are given by the equations,

$$\left.\begin{aligned} p_1 &= \frac{p_x + p_y}{2} + \sqrt{\left\{\frac{(p_x - p_y)^2}{4} + q^2\right\}}; \\ p_2 &= \frac{p_x + p_y}{2} - \sqrt{\left\{\frac{(p_x - p_y)^2}{4} + q^2\right\}}; \end{aligned}\right\} \text{.......} \quad (3.)$$

and the double of the angle of deviation by either of the following,

$$\tan 2\theta = \frac{2q}{p_x - p_y}; \text{ or } \operatorname{cotan} 2\theta = \frac{p_x - p_y}{2q}. \text{ ......} \quad (4.)$$

The greatest value of θ is 45°, when $p_x = p_y$.

The new principal stresses are to be conceived as acting normally on the faces of a new square prism.

109. **Parallel Projection of Distributed Forces.**—In applying the principles of parallel projection to distributed forces, it is to be borne in mind that those principles, as stated in Article 101, are applicable to lines representing the *amounts* or *resultants* of distributed forces, and *not their intensities.* The relations amongst the intensities of a system of distributed forces, whose resultants have been obtained by the method of projection, are to be arrived at by a subsequent process of dividing each projected resultant by the projected space over which it is distributed.

110. **Friction** (*A. M.*, 189, 190, 191) is that force which acts between two bodies at their surface of contact so as to resist their sliding on each other, and which depends on the force with which the bodies are pressed together. It is a kind of shearing stress. The following law respecting the friction of solid bodies has been ascertained by experiment:—

The friction which a given pair of solid bodies, with their surfaces in a given condition, are capable of exerting, is simply proportional to the force with which they are pressed together.

If a body be acted upon by a force tending to make it slide on another, then so long as that force does not exceed the amount fixed by this law, the friction will be equal and opposite to it, and will balance it.

There is a limit to the exactness of the above law, when the pressure becomes so intense as to crush or indent the parts of the bodies at and near their surface of contact. At and beyond that limit the friction increases more rapidly than the pressure; but that limit ought never to be attained in any structure. For some substances, especially those whose surfaces are sensibly indented by a moderate pressure, such as timber, the friction between a pair of surfaces which have remained for some time at rest relatively to each other, is somewhat greater than that between the same pair of surfaces when sliding on each other. That excess, however, of the *friction of rest* over the *friction of motion*, is instantly destroyed by a slight vibration; so that the *friction of motion* is alone to be taken into account, as contributing to the stability of a structure.

The friction between a pair of surfaces is calculated by multiplying the force with which they are directly pressed together, by a factor called the *co-efficient of friction*, which has a special value depending on the nature of the materials and the state of the surfaces. Let F denote the friction between a pair of surfaces; N, the force, in a direction perpendicular to the surfaces, with which they are pressed together; and f the co-efficient of friction; then

$$F = fN \dots\dots\dots\dots (1.)$$

The co-efficient of friction of a given pair of surfaces is the *tangent* of an angle called the *angle of repose*, being the greatest angle which an oblique pressure between the surfaces can make with a perpendicular to them, without making them slide on each other.

Let P denote the amount of an oblique pressure between two plane surfaces, inclined to their common normal at the angle of repose φ; then

$$F = fN = N \tan \varphi = P \sin \varphi = \frac{fP}{\sqrt{1+f^2}} \dots\dots\dots (2.)$$

The angle of repose is the steepest inclination of a plane to the horizon, at which a block of a given substance will remain balanced on it without sliding down.

The *intensity* of the friction between two surfaces bears the same proportion to the intensity of the pressure that the whole friction bears to the whole pressure.

The following is a table of the angle of repose φ, the co-efficient of friction $f = \tan \varphi$, and its reciprocal $1 : f$, for various materials—condensed from the tables of General Morin, and other sources, and arranged in a few comprehensive classes. The values of those constants which are given in the table have reference to the *friction of motion.*

SURFACES.	φ	f	$\frac{1}{f}$
Dry masonry and brickwork,	31° to 35°	0·6 to 0·7	1·67 to 1·43
Masonry and brickwork with wet mortar,	25½°	0·47	2·1
Masonry and brickwork, with slightly damp mortar,	36½°	0·74	1·35
Wood on stone,	22°	about 0·4	2·5
Iron on stone,	35° to 16⅔°	0·7 to 0·3	1·43 to 3·33
Masonry on dry clay,	27°	0·51	1·96
„ on moist clay,	18¼°	0·33	3
Earth on earth,	14° to 45°	0·25 to 1.0	4 to 1
„ „ dry sand, clay, and mixed earth,	21° to 37°	0·38 to 0·75	2·63 to 1·33
„ „ damp clay,	45°	1·0	1
„ „ wet clay,	17°	0·31	3·23
„ „ shingle and gravel,	35° to 48°	0·7 to 1·11	1·43 to 0·9
Wood on wood, dry,	14° to 26½°	·25 to ·5	4 to 2
„ „ soaped,	11½° to 2°	·2 to ·04	5 to 25
Metals on oak, dry,	26½° to 31°	·5 to ·6	2 to 1·67
„ „ wet,	13½° to 14½°	·24 to ·26	4·17 to 3·85
„ „ soapy,	11½°	·2	5
Metals on elm, dry,	11½° to 14°	·2 to ·25	5 to 4
Bronze on lignum vitæ, constantly wet,	3° ?	·05 ?	20 ?
Hemp on oak, dry,	28°	·53	1·89
„ „ wet,	18½°	·33	3
Leather on oak,	15° to 19½°	·27 to ·35	3·7 to 2·86
Leather on metals, dry,	29½°	·56	1·79
„ „ wet,	20°	·36	2·78
„ „ greasy,	13°	·23	4·35
„ „ oily,	8½°	·15	6·67
Metals on metals, dry,	8½° to 11½°	·15 to ·2	6·67 to 5
„ „ wet and clean,	16½°	·3	3·33
„ „ damp and slimy,	8°	·14	7·14
Smooth surfaces, occasionally greased,	4° to 4½°	·07 to ·08	14·3 to 12·5
„ „ continually greased,	3°	·05	20
Smoothest and best greased surfaces,	1¾° to 2°	·03 to ·036	33·3 to 27·6

SECTION IV.—*Balance and Stability of Frames, Chains, Ribs, and Blocks.*

(*A. M.*, 137 to 211.)

111. A **Frame** is a structure composed of bars, rods, links, or cords, attached together or supported by *joints*, such as occur in carpentry, in frames of metal bars, and in structures of ropes and chains, fixing the ends of two or more pieces together, but offering little or no resistance to change in the relative angular positions of those pieces. In a joint of this class, the *centre of resistance*, or point through which the resultant of the resistance to displacement of the pieces connected at the joint acts, is at or near the middle of the joint, and does not admit of any variation of position consistently with security.

The *Line of Resistance* of a frame is a line traversing the *centres of resistance* of the joints, and is in general a polygon, having its angles at these centres.

112. A **Single Bar** in a frame may act as a TIE, a STRUT, or a BEAM. (*A. M.*, 138 to 142.)

I. A *Tie* has equal and directly opposite forces applied to its two ends, acting outwards, or *from* each other. The bar is in a state of *tension*, and the stress exerted between any two divisions of it is a *pull*, equal and opposite to the applied forces. A *rope* or *chain* will answer the purpose of a tie.

The equilibrium of a moveable tie is stable; for if its angular position be deviated, the forces applied to its ends, which originally were directly opposed, now constitute a *couple* tending to restore the tie to its original position.

II. A *Strut* has equal and directly opposite forces applied to its two ends, acting inwards, or *towards* each other. The bar is in a state of compression, and the stress exerted between any two divisions of it is a *thrust* equal and opposite to the applied forces. It is obvious that a flexible body will *not* answer the purpose of a strut.

The equilibrium of a moveable strut is unstable; for if its angular position be deviated, the forces applied to its ends, which originally were directly opposed, now constitute a couple tending to make it deviate still farther from its original position.

In order that a strut may have stability, its ends must be prevented from deviating laterally. Pieces connected with the ends of a strut for this purpose are called *stays*.

III. A *Beam* is a bar supported at two points, and loaded in a direction perpendicular or oblique to its length.

CASE I.—Let the supporting pressures be parallel to each other and to the direction of the load; and let the load act *between* the points of support, as in fig. 88; where P represents the resultant of the gross load, including the weight of the beam itself; L, the point where the line of action of that resultant intersects the axis of the beam; R_1, R_2, the two supporting pressures or resistances of the props parallel to, and in the same plane with P, and acting through the points S_1, S_2, in the axis of the beam.

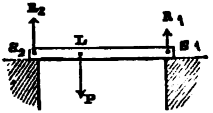

Fig. 88.

Then, according to the principle of the lever, Article 97, p. 141, each of those three forces is proportional to the distance between the lines of action of the other two; and the load is equal to the sum of the two supporting pressures; that is to say,

$$P : R_1 : R_2 : : \overline{S_1 S_2} : \overline{L S_2} : \overline{L S_1}; \text{ .... } (1.)$$

$$\text{and } P = R_1 + R_2 \text{ .... } (2.)$$

Case II.—Let the load act *beyond* the points of support, as in fig. 89, which represents a cantilever or projecting beam, held up by a wall or other prop at S_1, held down by a notch in a mass of masonry or otherwise at S_2, and loaded so that P is the resultant of the load, including the weight of the beam. Then the proportional equation (1.) remains exactly as before; but the load is equal to the difference of the supporting pressures; that is to say,

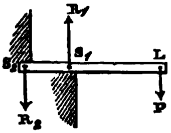

Fig. 89.

$$P = R_1 - R_2.$$

In these examples the beam is represented as horizontal; but the same principles would hold if it were inclined.

Case III.—Let the directions of the supporting forces R_1, R_2, be now inclined to that of the resultant of the load, P, as in fig. 90. This case is that of the equilibrium of three forces treated of in Article 93, p. 136, and consequently the following principles apply to it:—

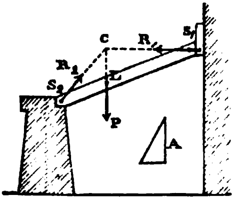

Fig. 90.

The lines of action of the supporting forces and of the resultant of the load must be in one plane.

They must intersect in one point (C, (fig. 90.

Those three forces must be proportional to the three sides of a triangle A, respectively parallel to their directions.

Problem.—Given, the resultant of the load in magnitude and position, P, the line of action of one of the supporting forces, R_1, and the centre of resistance of the other, S_2; required, the line of action of the second supporting force, and the magnitudes of both.

Produce the line of action of R_1, till it cuts the line of action of P at the point C; join C S_2; this will be the line of action of R_2; construct a triangle A with its sides respectively parallel to those three lines of action; the ratios of the sides of that triangle will give the ratios of the forces.

To express this algebraically, let i_1, i_2, be the angles made by the lines of action of the supporting forces with that of the resultant of the load; then

$$P : R_1 : R_2 :: \sin (i_1 + i_2) : \sin i_2 : \sin i_1. \quad \ldots\ldots\ldots (4.)$$

The same piece in a frame may act at once as a beam and tie, or as a beam and strut; or it may act alternately as a strut and as a tie, as the action of the load varies.

The load tends to break a tie by tearing it asunder, a strut by crushing it, and a beam by breaking it across. The power of materials to resist those tendencies will be considered in a later section.

113. **Distributed Loads.** (*A. M.*, 147.)—Before applying the principles of the present section to frames in which the load, whether external or arising from the weight of the bars, is distributed over their length, it is necessary to reduce that distributed load to an equivalent load, or series of loads, applied at the centres of resistance. The steps in this process are as follows:—

I. Find the resultant load on each single bar.

II. Resolve that load, as in Article 111, equation 1, p. 174, into two parallel components acting through the centres of resistance at the two ends of the bar.

III. At each centre of resistance where two bars meet, combine the component loads due to the loads on the two bars into one resultant, which is to be considered as the total load acting through that centre of resistance.

IV. When a centre of resistance is also a point of support, the component load acting through it, as found by step II. of the process, is to be left out of consideration until the supporting force required by the system of loads at the other joints has been determined; with this supporting force is to be compounded a force equal and opposite to the component load acting directly through the point of support, and the resultant will be the total supporting force.

In the following Articles of this section, all the frames will be supposed to be loaded only at those centres of resistance which are *not* points of support; and therefore, in those cases in which components of the load act directly through the points of support also, forces equal and opposite to such components must be combined with the supporting forces as determined in the following Articles, in order to complete the solution.

114. **Frames of Two Bars.** (*A. M.*, 145–6.)—Figures 91, 92, and 93, represent cases in which a frame, of two bars jointed to each at the point L, is loaded at that point with a given force, P, and is

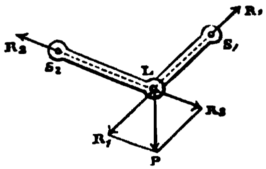

Fig. 91.

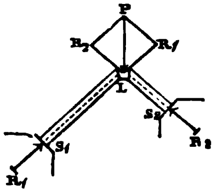

Fig. 92.

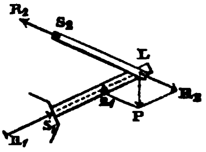

Fig. 93.

supported by the connection of the bars at their farther extremities, S_1, S_2, with fixed bodies. It is required to find the stress on each bar, and the supporting forces at S_1 and S_2.

Resolve the load P (as in Article 94, p. 137) into two components, R_1, R_2, acting along the respective lines of resistance of the two bars. Those components are the loads borne by the two bars respectively; to which loads the supporting forces at S_1, S_2, are equal and directly opposed.

The symbolical expression of this solution is as follows:—Let i_1, i_2, be the respective angles made by the lines of resistance of the bars with the line of action of the load; then

$$P : R_1 : R_2 : : \sin (i_1 + i_2) : \sin i_2 : \sin i_1.$$

The inward or outward direction of the forces acting along each bar indicates that the stress is a thrust or a pull, and the bar a strut or a tie, as the case may be. Fig. 91 represents the case of two ties; fig. 92, that of two struts (such as a pair of rafters abutting against two walls); fig. 93 that of a strut, L S_1, and a tie, L S_2 (such as the jib and the tie-rod of a crane).

A frame of two bars is *stable* as regards deviations in the plane of its lines of resistance.

With respect to *lateral* deviations of angular position, in a direction perpendicular to that plane, a frame of two ties is stable; so also is a frame consisting of a strut and a tie, when the direction of the load inclines *from* the line S_1 S_2, joining the points of support

A frame consisting of a strut and a tie, when the direction of the load inclines *towards* the line $S_1 S_2$, and a frame of two struts in all cases, are unstable laterally, unless provided with lateral stays.

These principles are true of *any pair of adjacent bars whose farther centres of resistance are fixed;* whether forming a frame by themselves, or a part of a more complex frame.

115. **Triangular Frames.** (*A. M.*, 148, 149.)—Let fig. 94 represent a frame, consisting of the three bars A, B, C, connected at the three joints 1, 2, 3,—viz., C and A at 1, A and B at 2, B and C at 3. Let a load P_1 be applied at the joint 1 in any given direction; let supporting forces, P_2, P_3, be applied at the joints 2, 3; the lines of action of those two forces must be in the same plane with that of P_1, and must either be parallel to it or intersect it in one point. The latter case is taken first, because its solution comprehends that of the former.

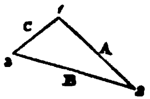

Fig. 94.

The three external forces balance each other, and are therefore proportional to the three sides of a triangle respectively parallel to their directions. In fig. 95, let A B C be such a triangle, in which

$\overline{\mathrm{C\,A}}$ represents P_1,
$\overline{\mathrm{A\,B}}$ ... P_2,
$\overline{\mathrm{B\,C}}$ ... P_3,

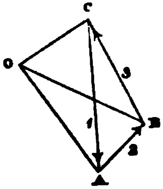

Fig. 95.

Draw C O parallel to the bar C, and A O parallel to the bar A, meeting in the point O, and join B O, which will be parallel to B.

The lengths of the three lines radiating from O will represent the stresses on the bars to which they are respectively parallel.

When the three external forces are parallel to each other, the triangle of forces A B C of fig. 95 becomes a straight line C A, as in fig. 96, divided into two segments by the point B. Let straight lines radiate from O to A, B, C, respectively parallel to the bars of the frame; then if the load C A be applied at 1 (fig. 94), A B applied at 2, and B C applied at 3, are the supporting forces required to balance it; and the radiating lines O A, O B, O C, represent the stresses on the bars A, B, C, respectively, as before.

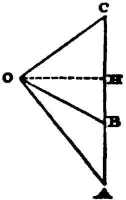

Fig. 96.

From O let fall O H perpendicular to C A, the common direction of the external forces. Then that line will represent a component of the stress, which is of equal amount in each bar. When C A, as is usually the case, is vertical, O H is horizontal; and the force represented by it is called the "*horizontal thrust*" of the frame. *Horizontal Stress* or *Resistance* would be a more precise term; because the force in question is a pull in some parts of the frame, and a thrust in others.

In fig. 94, A and C are *struts*, and B a *tie*. If the frame were exactly inverted, all the forces would bear the same proportions to each other; but A and C would be *ties*, and B a *strut*.

The trigonometrical expression of the relations amongst the forces acting in a triangular frame, under parallel forces, is as follows:—

Let a, b, c, denote the respective angles of inclination of the bars A, B, C, to the line O H (that is, in general, to a horizontal line); then

$$\text{Horizontal Stress O H} = \frac{\text{load C A}}{\tan c \pm \tan a} \text{ .... (1.)}$$

$$\begin{matrix}\text{Supporting} \\ \text{Forces}\end{matrix} \left\{ \begin{matrix} \text{A B} = \text{O H} \cdot (\tan a \mp \tan b); \\ \text{B C} = \text{O H} \cdot (\tan b \pm \tan c). \end{matrix} \right\} \text{.... (2.)}$$

The sign $\left\{ \begin{matrix} + \\ - \end{matrix} \right\}$ is to be used when the two inclinations are in $\left\{ \begin{matrix} \text{opposite directions} \\ \text{the same direction.} \end{matrix} \right.$

$$\text{Stresses} \left\{ \begin{matrix} \text{O A} = \text{O H} \cdot \sec a \\ \text{O B} = \text{O H} \cdot \sec b \\ \text{O C} = \text{O H} \cdot \sec c \end{matrix} \right\} \text{ .... (3.)}$$

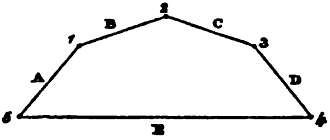

Fig. 97.

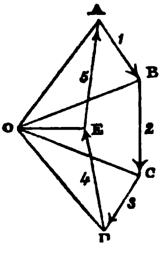

Fig. 98.

116. **Polygonal Frame.** (*A. M.*, 150, 153.)—In fig. 97, let A, B, C, D, E, be the lines of resistance of the bars of a frame, connected together at the joints, whose centres of resistance are, 1 between A and B, 2 between B and C, 3 between C and D, 4 between D and E, and 5 between E and A. In the figure, the frame consists of five bars; but the principle is applicable to any number. From a point O, in fig. 98, (which may be called the *Diagram of Forces*), draw radiating lines O A, O B, O C, O D, O E, parallel respectively to the lines of resistance of the bars; and on those radiating lines take any lengths whatsoever, to represent the stresses on the several bars, which may have any magnitudes within the limits of strength of the material. Join the points thus found by straight lines, so as to form a closed polygon A B C D E A; then the sides of the polygon will represent a system of forces, which, being applied to the joints of the frame, will balance each other; each such force being applied to the joint between the bars whose lines of resistance are parallel to the pair

of radiating lines that enclose the side of the polygon of forces representing the force in question.

When the external forces are parallel to each other, the polygon of forces of fig. 98 becomes a straight line A D, as in fig. 99, divided into segments by the radiating lines; and each segment represents the external force which acts at the joint of the bars whose lines of resistance are parallel to the radiating lines that bound the segment. Moreover, the segment of the line A D which is intercepted between the radiating lines parallel to the lines of resistance of *any two bars whether contiguous or not*, represents the resultant of the external forces which act at points *between the bars.*

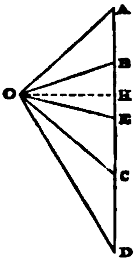

Fig. 99.

Thus, A D represents the total load, consisting of the three portions A B, B C, C D, applied at 1, 2, 3, respectively. D A represents the total supporting force, equal and opposite to the load, consisting of the two portions D E, E A, applied at 4 and 5 respectively. A C represents the resultant of the load applied between the bars A and C; and similarly for any other pair of bars.

From O draw O H perpendicular to A D; then that line represents a component of the stress, whose amount is the same in each bar of the frame. When the load, as is usually the case, is vertical, that component is called the "*horizontal thrust*" of the frame, and, as in Article 114, might more correctly be called *horizontal stress* or *resistance*, seeing that it is a pull in some of the bars and a thrust in others.

The trigonometrical expression of those principles is as follows:—

Let the force O H be denoted simply by H.

Let i, i', denote the inclinations to O H of the lines of resistance of *any two bars*, contiguous or not.

Let R, R′, be the respective stresses which act along those bars.

Let P be the resultant of the external forces acting through the joint or joints between those two bars.

Then

$$P = H(\tan i \pm \tan i'); \dots\dots\dots\dots\dots\dots (1.)$$

$$R = H \cdot \sec i;\ R' = H \cdot \sec i'. \dots\dots\dots\dots\dots\dots (2.)$$

The $\left\{\begin{matrix}\text{sum}\\ \text{difference}\end{matrix}\right\}$ of the tangents of the inclinations is to be used, according as they are $\left\{\begin{matrix}\text{opposite}\\ \text{similar}\end{matrix}\right\}$.

117. **Open Polygonal Frame.** (*A.M.*, 151, 154.)—When the frame, instead of being closed, as in fig. 97, is converted into an open frame, by the omission of one bar, such as E, the corresponding modification

is made in the diagram of inclined forces, fig. 98, by omitting the lines O E, D E, E A, and in the diagram of parallel forces, fig. 99, by omitting the line O E. Then, in both diagrams, D O and O A represent the *supporting forces* respectively, equal and directly opposed to the stresses along the extreme bars of the frame, D and A, which must be exerted by the supports (called in this case *abutments*), at the points 4 and 5, against the ends of those bars, in order to maintain the equilibrium.

In the case of parallel loads, the following formulæ give the horizontal stress and supporting pressures.

Let i_d and i_a denote the angles of inclination of the bars D and A respectively.

Let $R_d = \text{O D}$ and $R_a = \text{O A}$ be the stresses along them.

Let $\Sigma \cdot P = \text{A D}$ denote the total load on the frame; then,

$$H = \frac{\Sigma \cdot P}{\tan i_d + \tan i_a}; \text{ ....(1.)}$$

$$R_d = H \cdot \sec i_d;\ R_a = H \cdot \sec i_a \text{ ....(2.)}$$

118. **Polygonal Frame—Stability.** (*A. M.*, 152.)—The stability or instability of a polygonal frame depends on the principles stated in Article 112, p. 173, viz., that if a bar be free to change its angular position, then if it is a tie it is stable, and if a strut, unstable; and that a strut may be rendered stable by fixing its ends.

For example, in the frame of fig. 97, E is a tie, and stable; A, B, C, and D, are struts, free to change their angular position, and therefore unstable.

But these struts may be rendered stable in the plane of the frame by means of stays; for example, let two stay-bars connect the joints 1 with 4, and 3 with 5; then the points 1, 2, and 3, are all fixed, so that none of the struts can change their angular positions. The same effect might be produced by two stay-bars connecting the joint 2 with 5 and 4.

The frame, as a whole, is unstable, as being liable to overturn laterally, unless provided with lateral stays, connecting its joints with fixed points.

Now, suppose the frame to be exactly inverted, the loads at 1, 2, and 3, and the supporting forces at 4 and 5, being the same as before. Then E becomes a strut; but it is stable, because its ends are fixed in position; and A, B, C, and D becomes ties, and are stable without being stayed.

An open polygon consisting of ties, such as is formed by A, B, C, and D, when inverted, is called by mathematicians, a *funicular polygon*, because it may be made of ropes.

It is to be observed, that the stability of an *unstayed* polygon of ties is of the kind which admits of *oscillation* to and fro about the position of equilibrium. That oscillation may be injurious in practice, and stays may be required to prevent it.

119. **Bracing of Frames.** (*A. M.*, 155.)—A *brace* is a stay-bar on which there is a permanent stress. When the external forces applied to a frame, although balancing each other as an entire system, are distributed in a manner not consistent with the equilibrium of each bar separately; then, in the diagram of forces, upon attempting to construct a polygon of external forces, having its angles on the radiating lines, gaps will be left in the outline of that polygon. The lines necessary to fill up those gaps will indicate the forces to be supplied by means of the resistance of braces. These may be either struts or ties, connecting two or more joints together, and supplying by their resistances, at the joints which they connect, the forces wanting to produce equilibrium of each bar.

The resistance of a brace introduces a pair of equal and opposite forces, acting along the line of resistance of the brace, upon the pair of joints which it connects. It therefore does not alter the *resultant* of the forces applied to that pair of joints in amount nor in position, but only the *distribution* of the components of that resultant on the pair of joints.

To exemplify the use of braces, and the mode of determining the stresses on them, let fig. 100 represent a frame such as frequently occurs in iron roofs, consisting of two struts or rafters, A and E, and three tie-bars, B, C, and D, forming a polygon of five sides, jointed at 1, 2, 3, 4, 5, loaded vertically at 1, and supported by the vertical resistance of a pair of walls at 2 and 5. The joints 3 and 4 having no loads applied to them, are connected with 1 by the braces 1 4 and 1 3. It is required to find the stresses on those braces, and on the other pieces of the frame,

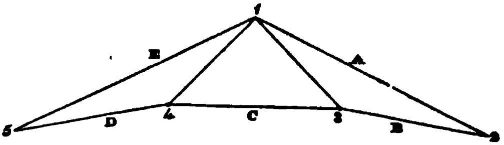

Fig. 100.

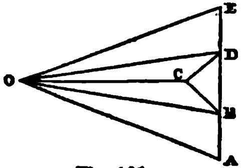

Fig. 101.

To make the diagram of forces, fig. 101, draw as before from a point O, radiating lines O E, O D, O C, O B, O A, parallel to the bars forming the external polygon of the frame; and across those lines draw the vertical line E A, to represent the direction of the load and of the supporting forces.

The two extreme segments of this line, A B and D E, are to be taken to represent the supporting forces at 2 and 5; and their sum, E D + B A, will represent the *actual* load at 1. E A represents what the load at 1 *ought to be*, to fulfil the conditions of equilibrium of the bars A and E; therefore D B represents the *deficiency* of the actual load at 1, or the additional downward force, to be supplied at 1 by means of the bracing. Also B D, equal and opposite to D B, represents the resultant deficiency of supporting pressure at the joints 3 and 4, where the actual external force applied is nothing; and this has to be supplied by means of the bracing.

In the diagram of forces, draw D C parallel to the brace 1 4, and C B parallel to the brace 1 3, intersecting the radiating line O C in C. Then will D C represent the pull along the brace 1 4, acting obliquely upwards at 4, and obliquely downwards at 1; and C B will represent the pull along the brace 1 3, acting obliquely upwards at 3, and obliquely downwards at 1. The resultant of the additional forces thus applied at 1 is D B, a vertical downward force, equal to the deficiency before mentioned; and the oblique upward forces at 3 and 4 complete a polygon of forces A B C D E A, whose angles are on the radiating lines parallel to the bars of the frame, and which therefore fulfils the conditions of Article 116, p. 178.

The stresses on the external bars of the frame are represented as before, by the lengths of the radiating lines in fig. 101.

As another and a very simple example of bracing, very common in framework, may be taken the frame in fig. 102. A and B are two struts, forming the two halves of one straight bar; C and D are two equal tie-rods; E, a strut-brace. A vertical load P is applied at the joint 1, between A and B; two vertical supporting pressures, each denoted by R = P ÷ 2, act at the joints 4 and 2. The joint 3 has no external load.

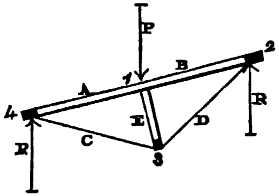

Fig. 102.

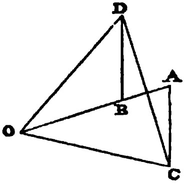

Fig. 103.

Fig. 103 is the diagram of forces, constructed as follows:—Through a point O draw O B A parallel to A and B, O C parallel to C, and O D parallel to D. Make O D = O C; join C D; this line will be parallel to the brace E, and perpendicular to O A.

Through D and C draw vertical lines D B, C A; these, being equal to each other, are to be taken to represent the two supporting pressures R; and their sum D B + A C will represent the load P. The equal tensions on C and D will be represented by O C and O D, and the thrusts along A, B, and E, by O A, O B, and C D.

The polygon of external forces in this case is the crossed quadrilateral A C D B, in which C A and B D represent (as already stated) the supporting pressures, and D C and A B the components of the load P respectively parallel and perpendicular to the brace E. When A and B are horizontal, and E vertical, A B in fig. 103 vanishes, and B D and C A coincide with the two halves of C D.

120. **Rigidity of a Truss.** (*A. M.*, 156, 157.)—The word *truss* is applied in carpentry to a triangular frame, and to a polygonal frame to which rigidity is given by staying and bracing, so that its figure shall be incapable of alteration by turning of the bars about their joints. If each joint were *absolutely* of the kind described in Article 111, that is, like a hinge, incapable of offering any resistance to alteration of the relative angular position of the bars connected by it, it would be necessary, in order to fulfil the condition of rigidity, that every polygonal frame should be divided by the lines of resistance of stays and braces into triangles and other polygons, so arranged that every polygon of four or more sides should be surrounded by triangles on all but two sides and the included angle at farthest. For every unstayed polygon of four sides or more, with flexible joints, is flexible, unless all the angles except one be fixed by being connected with triangles.

Sometimes, however, a certain amount of stiffness in the joints of a frame, and sometimes the resistance of its bars to bending, is relied upon to give rigidity to the frame, when the load upon it is subject to small variations only in its mode of distribution. For example, in the truss of fig. 104, the tie-beam A A is made in one piece, or in two or more pieces so connected together as to act like one piece; and part of its weight is suspended from the joints C, C, by the rods C B, C B. These rods also serve to make the resistance of the tie-beam C C to being bent act so as to prevent the struts A C, C C, C A, from deviating from their proper angular positions, by turning on the joints A, C, C, A. If A B, B B, and B A, were three distinct pieces, with flexible joints at B B, it is evident that the frame might be disfigured by distortion of the quadrangle B C C B.

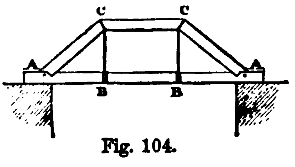

Fig. 104.

The object of stiffening a truss by braces is to enable it to sustain loads variously distributed; for were the load always distributed in one way, a frame might be designed of a figure exactly suited to that load, so that there should be no need of bracing.

The variations of load produce variations of stress on all the pieces of the frame, but especially on the braces; and each piece must be suited to withstand the greatest stress to which it is liable.

Some pieces, and especially braces, may have to act sometimes as struts and sometimes as ties, according to the mode of distribution of the load.

121. **Secondary and Compound Trussing.** (*A. M.*, 158 to 160.)—A *secondary truss* is a truss which is supported by another truss.

When a load is distributed over a great number of centres of resistance, it may be advantageous, instead of connecting all those centres by one polygonal frame, to sustain them by means of several small trusses, which are supported by larger trusses, and so on, the whole structure of secondary trusses resting finally on one large truss, which may be called the *primary truss.* In such a combination the same piece may often form part of different trusses; and then the stress upon it is to be determined according to the following principle:—

When the same bar forms at the same time part of two or more different frames, the stress on it is the resultant of the several stresses to which it is subject by reason of its position in the several frames.

In a *Compound Truss*, several frames, without being distinguishable into primary and secondary, are combined and connected in such a manner that certain pieces are common to two or more of them, and require to have their stresses determined by the principle above stated.

Examples of secondary and compound trusses will be given in treating of structures in timber and iron.

122. **Resistance of a Frame at a Section.** (*A. M.*, 161.)—The labour of calculating the stress on the bars of a frame may sometimes be abridged by the application of the following principle:—

If a frame be acted upon by any system of external forces, and if that frame be conceived to be completely divided into two parts by an ideal surface, the stresses along the bars which are intersected by that surface, balance the external forces which act on each of the two parts of the frame.

In most cases which occur in practice, the lines of resistance of the bars, and the lines of action of the external forces, are all in one vertical plane, and the external forces are vertical. In such cases the most convenient position for an assumed plane of section is vertical, and perpendicular to the plane of the frame. Take the vertical line of intersection of these two planes for an axis of co-ordinates,—say for the axis of y, and any convenient point in it for the origin O; let the axis of x be horizontal, and in the plane of the frame, and the axis of z horizontal, and in the plane of section.

The external forces applied to the part of the frame at one side of the plane of section (either may be chosen) being combined, as in Article 99, p. 146, give three data—viz., the total force along $x = \Sigma \cdot X$; the total force along $y = \Sigma \cdot Y$; and the moment of the couple acting round $z = M$; and the bars which are cut by the plane of section must exert resistances capable of balancing those two forces and that couple. If not more than three bars are cut by the plane of section, there are not more than three unknown quantities, *and three* relations between them and given quantities, so that the problem is determinate; if more than three bars are cut by the plane of section, the problem is or may be indeterminate.

The formulæ to which this reasoning leads are as follows:—Let x be positive in a direction from the plane of section towards the part of the structure which is considered in determining $\Sigma \cdot X$, $\Sigma \cdot Y$, and M; let $+y$ be measured upwards; let angles measured from Ox towards $+y$, that is, upwards, be positive; and let the lines of resistance of the three bars cut by the plane of section make the angles i_1, i_2, i_3, with x. Let n_1, n_2, n_3, be the perpendicular distances of those three lines of resistance from O, distances lying $\left.\begin{matrix}\text{upwards}\\ \text{downwards}\end{matrix}\right\}$ from Ox being considered as $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right\}$.

Let R_1, R_2, R_3, be the resistances, or total stresses, along the three bars, pulls being positive, and thrusts negative. Then we have the following three equations:—

$$\left.\begin{aligned}\Sigma \cdot X &= R_1 \cos i_1 + R_2 \cos i_2 + R_3 \cos i_3;\\ \Sigma \cdot Y &= R_1 \sin i_1 + R_2 \sin i_2 + R_3 \sin i_3;\\ -M &= R_1 n_1 + R_2 n_2 + R_3 n_3;\end{aligned}\right\} \ldots\ldots(1.)$$

from which the three quantities sought, R_1, R_2, R_3, can be found.

Speaking with reference to the given plane of section, $\Sigma \cdot X$ may be called the *normal stress*, $\Sigma \cdot Y$, the *shearing stress*, and M, the *moment of flexure*, or *bending stress;* for it tends to bend the frame at the section under consideration. M is to be considered as $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right\}$ according as it tends to make the frame become concave $\left\{\begin{matrix}\text{upwards}\\ \text{downwards.}\end{matrix}\right\}$

Examples of the application of this method will be given in treating of lattice-beams of timber and iron.

123. **Balance of a Chain or Cord.**—A loaded chain may be looked upon as a polygonal frame whose pieces and joints are so numerous that its figure may without sensible error be treated as a continuous

curve. The following are the principles respecting the equilibrium of loaded chains and cords which are of most importance in practice.

I. *Balance of a Chain in general.*—Let D A C, in fig. 105, represent a flexible cord or chain supported at the points C and D, and loaded by forces in any direction, constant or varying, distributed over its whole length with constant or varying intensity.

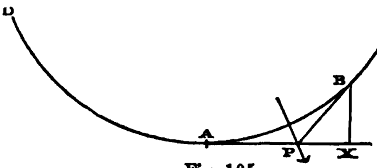

Fig. 105.

Let A and B be any two points in this chain; from those points draw tangents to the chain, A P and B P, meeting in P. The load acting on the chain between the points A and B is balanced by the pulls along the chain at those two points respectively; those pulls must respectively act along the tangents A P, B P; hence the resultant of the load between A and B acts through the point of intersection of the tangents at A and B; and that load, and the tensions on the chain at A and B, are respectively proportional to the sides of a triangle parallel to their directions.

II. *Chain under Vertical Load—Curve of Equilibrium.*—If the direction of the load be everywhere parallel and vertical, draw a vertical straight line, C D, fig. 106, to represent the total load, and from its ends draw C O and D O, parallel to two tangents at the points of support of the chain, and meeting in O; those lines will represent the tensions on the chain at its points of support.

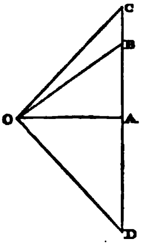

Fig. 106.

Let A, in fig. 105, be the lowest point of the chain. In fig. 106, draw the horizontal line O A; this will represent the horizontal component of the tension of the chain at every point, and if O B be parallel to a tangent to the chain at B (fig. 105) A B will represent the portion of the load supported between A and B, and O B the tension at B.

To express this algebraically, let

$H = O A =$ *horizontal tension* along the chain at A;

$R = O B =$ pull along the chain at B;

$P = A B =$ load on the chain between A and B;

$i = \angle X P B$ (fig 105) $= \angle A O B$ (fig. 106) = inclination of chain at B;

then,

$$P = H \tan i; \; R = \sqrt{(P^2 + H^2)} = H \sec i \text{.........} (1.)$$

To deduce from these formulæ an equation by which the form of the curve assumed by the chain can be determined when the distribution of the load is known, let that curve be referred to rectangular horizontal and vertical co-ordinates, measured from the lowest point A, fig. 105, the co-ordinates of B being, A X $= x$, X B $= y$; then

$$\tan i = \frac{d\,y}{d\,x} = \frac{P}{H}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.)}$$

a differential equation which enables the form assumed by the cord (or "*curve of equilibrium*") to be determined when the distribution of the load is known.

124. To Draw a Curve of Equilibrium approximately.

PROBLEM.—Let H and K, fig. 107, be the two points of suspension of a chain under a vertical load; let the distribution of the load be given, and the direction of a tangent H L to the chain at one of the points of suspension; it is required to draw approximately the figure of the chain.

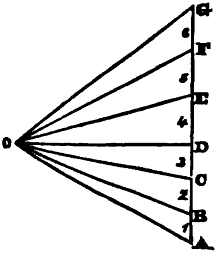

Fig. 107.

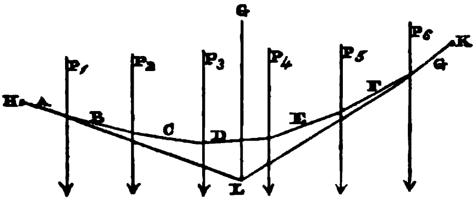

Fig. 108.

Find the centre of gravity of the entire load, and let G L be a vertical line passing through it, cutting the tangent H L in L. Join K L; this will be the tangent to the chain at the other point of suspension.

Conceive the load to be divided into any convenient number of portions: the more numerous these are, the closer will be the approximation to the required curve. Find the centres of gravity of those portions, and let P_1, P_2, &c., be vertical lines passing through those centres of gravity.

In fig. 108, draw the vertical line, or *scale of loads*, A G, whose whole length represents the entire load; divide it into parts, A B or 1, B C or 2, &c., representing the several portions of the load. Through A draw A O parallel to L H, and through G draw G O parallel to K L, cutting each other in O. From O draw radiating lines O B, O C, &c., to the points of division of the scale of loads.

Then, in fig. 107, from the point of intersection of A, or H L, with P_1, draw B parallel to O B, cutting P_2; from the point of intersection of B and P_2, draw C parallel to O C, cutting P_3, and so on, until the "funicular polygon" A B C, &c., is completed; that polygon is composed of tangents to the required curve of equilibrium, to which an approximation may be drawn by sketching a curve so as to touch the sides of the polygon.

125. **Chain under an Uniform Vertical Load—Suspension Bridge.** (*A. M.*, 169, 170.)—By an uniform vertical load is meant a load uniformly distributed along a horizontal straight line; so that if

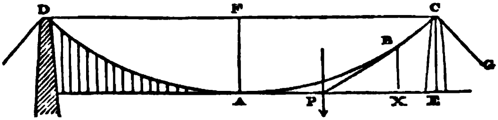

Fig. 109.

A, fig. 109, be the lowest point of the rope or cord, the load suspended between A and B shall be proportional to $\overline{AX} = x$, the horizontal distance between those points, and capable of being expressed by the equation

$$P = p\,x; \qquad (1.)$$

where p is a constant quantity, denoting the *intensity of the load in units of weight per unit of horizontal length*: in pounds per lineal foot, for example.

In this case, because the load between A and B is uniformly distributed, its resultant bisects A X; also, the tangent B P bisects A X; and the curve assumed by the chain is a PARABOLA whose vertex is at A.

The proportions of the load, and the horizontal and oblique tensions are as follows:—

$$\left.\begin{array}{ll} P : H : R & \overline{BX} : \overline{XP} : \overline{PB} :: y : \frac{x}{2} : \sqrt{\left(y^2 + \frac{x^2}{4}\right)} \\ & :: p\,x : \frac{p\,x^2}{2\,y} : p\,x \cdot \sqrt{\left(1 + \frac{x^2}{4\,y^2}\right)}. \end{array}\right\} (2.)$$

The focal distance of the parabola is

$$m = \frac{x^2}{4\,y} = \frac{H}{2\,p} \qquad (3.)$$

These equations are applicable, with sufficient accuracy for practical purposes, to most examples of *Suspension Bridges with vertical rods;* for although in a bridge of that class the load is not continuous, the platform being hung by rods from a certain number of points in each cable or chain; nor uniformly disturbed, the load arising from the weight of the cables or chains and of the suspending rods being more intense near the piers; yet, in most cases which occur in practice, the condition of each cable or chain approaches sufficiently near to that of a cord continuously and uniformly loaded to enable the preceding equations to be applied without material error.

The following solutions of some useful problems are deduced from these equations:—

PROBLEM FIRST.—*Given the elevations,* y_1, y_2, *of the two points of support of the chain above its lowest point, and also the horizontal distance, or span* a, *between those points of support: it is required to find the horizontal distances,* x_1, x_2, *of the lowest point from the two points of support: also the focal distance* m.

$$x_1 = a \cdot \frac{\sqrt{y_1}}{\sqrt{y_1} + \sqrt{y_2}}; \; x_2 = a \cdot \frac{\sqrt{y_2}}{\sqrt{y_1} + \sqrt{y_2}} \text{.........} (4.)$$

$$m = \frac{a^2}{4\,y_1 + 4\,y_2 + 8\,\sqrt{y_1\,y_2}} \text{................} (5.)$$

When the points of support are at the same level,

$$y_1 = y_2; \; x_1 = \frac{a}{2}; \; m = \frac{a^2}{16\,y_1} \text{................} (6.)$$

PROBLEM SECOND.—*Given the same data, to find the inclinations* i_1, i_2, *of the chain at the points of support.*

$$\tan i_1 = \frac{2\,y_1}{x_1} = \frac{2\,y_1 + 2\,\sqrt{y_1 y_2}}{a}; \; \tan i_2 = \frac{2\,y_2}{x_2} = \frac{2\,y_2 + 2\,\sqrt{y_1 y_2}}{a} \; (7.)$$

when $$y_1 = y_2, \; \tan i_1 = \tan i_2 = \frac{4\,y_1}{a} \text{..............} (8.)$$

PROBLEM THIRD.—*Given the same data, and the load* p *per unit of length: required the horizontal tension* H, *and the tensions* R_1, R_2, *at the points of support.*

$$H = 2\,p\,m = \frac{p\,a^2}{2\,y_1 + 2\,y_2 + 4\,\sqrt{y_1\,y_2}}; \text{.........} (9.)$$

$$R_1 = H\,\sqrt{\left(1 + \frac{4\,y_1^2}{x_1^2}\right)}; \; R_2 = H\,\sqrt{\left(1 + \frac{4\,y_2^2}{x_2^2}\right)} \text{....} (10.)$$

When $y_1 = y_2$, those equations become

$$H = \frac{p\,a^2}{8\,y_1};\ R_1 = R_2 = H\sqrt{\left(1 + \frac{16\,y_1^2}{a^2}\right)}. \ \ldots\ldots \ (11.)$$

Problem Fourth.—*Given the same data as in Problem First, to find the length of the chain.*

The following are two well-known formulæ for the length of a parabolic arc, commencing at the vertex, one being in terms of the co-ordinates x and y of the farther extremity of the arc, and the other in terms of the focal distance m, and the inclination i of the farther extremity of the arc to a tangent at the vertex.

$$s = \sqrt{\left(y^2 + \frac{x^2}{4}\right)} + \frac{x^2}{4\,y} \cdot \text{hyp. log.} \frac{y + \sqrt{\left(y^2 + \frac{x^2}{4}\right)}}{\frac{x}{2}}$$

$$= m\left\{\tan i \cdot \sec i + \text{hyp. log. } (\tan i + \sec i)\right\} \ \ldots\ldots \ (12.)$$

The *length of the chain* is $s_1 + s_2$, where s_1 is found by putting x_1 and y_1 in the first of the above formulæ, or i_1 in the second, and s_2 by putting x_2 and y_2 in the first formula, or i_2, in the second.

The following *approximate formula* for the length of a parabolic arc is in many cases sufficiently near the truth for practical purposes;

$$s = x + \frac{2\,y^2}{3\,x} \text{ nearly}; \ \ldots\ldots\ldots\ldots\ldots\ldots \ (13.)$$

which gives the total length of the cord,

$$s_1 + s_2 = a + \frac{2}{3}\left(\frac{y_1^2}{x_1} + \frac{y_2^2}{x_2}\right) \text{nearly}, \ \ldots\ldots\ldots\ldots \ (14.)$$

and when $y_1 = y_2$, this becomes

$$2\,s_1 = a + \frac{8}{3} \cdot \frac{y_1^2}{a} \text{ nearly}; \ldots\ldots\ldots\ldots\ldots\ldots \ (15.)$$

Problem Fifth.—*Given the same data, to find, approximately, the small elongation of the chain* d $(s_1 + s_2)$ *required to produce a given small depression* d y *of the lowest point* A, *and conversely.*

$$\frac{d\,(s_1 + s_2)}{d\,y} = \frac{4}{3}\left(\frac{y_1}{x_1} + \frac{y_2}{x_2}\right) \ldots\ldots\ldots\ldots\ldots\ldots \ (16.)$$

When $y_1 = y_2$, this equation becomes

$$\frac{2\,d\,s_1}{d\,y} = \frac{16\,y_1}{3\,a} \quad\text{.........................}\quad (17.)$$

These formulæ serve to compute the depression which the middle point of a suspension bridge undergoes in consequence of a given elongation of the cable or chain, whether caused by heat or by tension.

PROBLEM SIXTH.—*To find the pressure on the top of each pier.*

When the piers of a suspension bridge are slender and vertical (as is usually the case), the resultant pressure of the chain or cable on the top of the pier ought to be vertical also. Thus, let C E, in fig. 109, represent the vertical axis of a pier, and C G the portion of the chain or cable behind the pier, which either supports another division of the platform, or is made fast to a mass of rock, or of masonry, or otherwise. If the chain or cable passes over a curved plate on the top of the pier called a *saddle*, on which it is free to slide, the tensions of the portions of the chain or cable on either side of the saddle will be equal; and in order that those tensions may compose a vertical pressure on the pier, their inclinations must be equal and opposite. Let i be the common value of those inclinations; R the common value of the two tensions; then the vertical pressure on the pier is

$$V = 2\,R \sin i = 2\,H \tan i = 2\,p\,x; \quad\text{.........}\quad (18.)$$

that is, twice the weight of the portion of the bridge between the pier and the lowest point, A, of the curve C B A D.

But if the two divisions of the chain or cable D A C, C G, which meet at C, be *made fast* to a sort of truck, which is supported by rollers on a *horizontal* cast iron platform on the top of the pier, then the pressure on the pier will be vertical, whether the inclinations of the two divisions of the chain or cable be equal or unequal; and it is only necessary that the *horizontal components* of their tension should be equal; that is to say, let i, i', be the inclinations of the two divisions of the chain or cable in opposite directions at C, and R, R′, their tensions, then

$$R = H \sec i;\; R' = H \sec i';$$

$$V = R \sin i + R' \sin i' = H\,(\tan i + \tan i') \text{......} (19.)$$

126. **Suspension Bridge with Sloping Rods.** (*A. M.*, 172.)—Let the uniformly-loaded platform of a suspension bridge be hung from the chains by parallel sloping rods, making an uniform angle j with the vertical. The condition of a chain thus loaded is the same with that of a chain loaded vertically, except in the direction of the

load; and the form assumed by the chain is a parabola, having its axis parallel to the direction of the suspension-rods.

In fig. 110, let C A represent a chain, or portion of a chain, supported or fixed at C, and horizontal at A, its lowest point. Let A H be a horizontal tangent at A, representing the platform of the bridge; and let the suspension rods be all parallel to C E, which makes the angle ∠ E C H $= j$ with the vertical. Let B X represent any rod, and suppose a vertical load v to be supported at the point X. Then, by the principles of the equilibrium of a *frame of two bars*, this load will produce a *pull*, p, on the rod X B, and a *thrust*, q, on the platform between X and H; and the three forces v, p, q, will be proportional to the sides of a triangle parallel to their directions, such as the triangle C E H; that is to say,

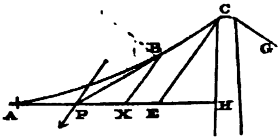

Fig. 110.

$$v : p : q :: \overline{CH} : \overline{CE} : \overline{EH} :: 1 : \sec j : \tan j \ldots\ldots (1.)$$

Next, instead of considering the load on one rod B X, consider the entire vertical load V between A and X.

Let P represent the amount of the pull acting on the rods between A and X, and Q the total thrust on the platform at the point X; then,

$$V : P : Q :: \overline{CH} : \overline{CE} : \overline{EH} :: 1 : \sec j : \tan j \ldots (2.)$$

The *oblique load* $P = V \sec j$ is what hangs from the chain between A and B. Being uniformly distributed, its resultant bisects A X in P, which is also the point of intersection of the tangents A P, B P; and the ratio of the oblique load P, the horizontal tension H along the chain at A, and the tension R along the chain at B, is that of the sides of the triangle B X P; that is to say,

$$P : H : R :: \overline{BX} : \overline{XP} = \frac{\overline{AX}}{2} : \overline{BP} \ldots\ldots\ldots (3.)$$

The curve C B A is a parabola having its axis parallel to the inclined suspension rods; and its equation referred to oblique co-ordinates, with the origin at A, is as follows. Let $AX = x$, $XB = y$; then,

$$y = \frac{x^2 \cdot \cos^2 j}{4m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

where m, as in Article 125, denotes the focal distance of the parabola, given by the equation

$$m = \frac{x^2 \cdot \cos^2 j}{4 y} \text{ .... } (5.)$$

x and y being the co-ordinates of any *known* point in the curve. The length of the tangent $\overline{\mathrm{B\,P}} = t$ is given by the following equation:—

$$t = \sqrt{\left(\frac{x^2}{4} + y^2 + x y \cdot \sin j\right)} \text{ .... } (6.)$$

Hence are deduced the following formulæ for the relations amongst the forces which act in a suspension bridge with inclined rods:—Let v now be taken to denote the *intensity* of the vertical load per unit of length of horizontal platform—per foot, for example; p the intensity of the oblique load; q the rate at which the thrust along the platform increases from A towards H. Then

$$\left.\begin{aligned} \mathrm{V} &= v x; \\ \mathrm{P} &= p x = v x \cdot \sec j; \\ \mathrm{Q} &= q x = v x \cdot \tan j; \end{aligned}\right\} \text{ .... } (7.)$$

$$\mathrm{H} = \frac{x \mathrm{P}}{2y} = \frac{p x^2}{2y} = \frac{2 p m}{\cos^2 j} = 2 v m \cdot \sec^3 j \text{ .... } (8.)$$

$$\mathrm{R} = \frac{t \mathrm{P}}{y} = \frac{2 t \mathrm{H}}{x} = \frac{p t x}{y} = \frac{v t x \sec j}{y} \text{ .... } (9.)$$

The horizontal pull H at the point A may be sustained in three different ways, viz.:—

I. The chain may be *anchored* or made fast at A to a mass of rock or masonry.

II. It may be attached at A to another equal and similar chain, similarly loaded by means of oblique rods, sloping at an equal angle in the direction opposite to that of the rods B X, &c., so that A may be in the middle of the span of the bridge.

III. The chain may be made fast at A to the horizontal platform A H, so that the pull at A shall be balanced by an equal and opposite thrust along the platform, which must be strong enough and stiff enough to sustain that thrust. In this case, the total thrust at any point, X, of the platform is no longer simply $\mathrm{Q} = q x$, but

$$\begin{aligned} \mathrm{H} + \mathrm{Q} &= \left(\frac{\mathrm{P}}{2 y} + q\right) x \\ &= v\,(2 m \cdot \sec^3 j + x \cdot \tan j) \text{ .... } (10.) \end{aligned}$$

The *length of the parabolic arc*, A B, is given exactly by the following formulæ:—Let i denote the inclination of the parabola at the point B to a line perpendicular to its axis. Then

$$i = \text{arc} \cdot \cos \left(\frac{x}{2t} \cdot \cos j \right) \text{..................(11.)}$$

which, when B coincides with A, becomes simply $i = j$. Then from the known formulæ for the lengths of parabolic arcs, we have

$$\text{parabolic arc A B} = m \left\{ \tan i \sec i - \tan j \sec j \right.$$

$$\left. + \text{hyp. log.} \frac{\tan i + \sec i}{\tan j + \sec j} \right\} \text{..................(12.)}$$

In most cases which occur in practice, however, it is sufficient to use the following approximate formula:—

$$\text{arc A B} = x + y \cdot \sin j + \frac{2}{3} \cdot \frac{y^2 \cdot \cos^2 j}{x + y \cdot \sin j}, \text{ nearly......(13.)}$$

The formulæ of this Article are applicable to Mr. Dredge's suspension bridges, in which the suspending rods are inclined, and although not exactly parallel, are nearly so.

127. **Deflection of a Flexible Tie.** (*A. M.*, 171.)—Let a vertical load, P, be applied at A, fig. 111, and sustained by means of a horizontal strut, A B, abutting against a pier at B, and a sloping rope or chain, or other flexible tie, A D C, fixed to the top of the pier at C. The weight of the strut, A B, is supposed to be divided into two components, one of which is supported at B, while the other is *included* in the load P. The weight, W, of the flexible tie, A D C, is supposed to be known, and to be considered separately; and with these data there is proposed the following

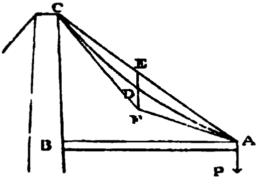

Fig. 111.

Problem.—W *being small compared with* P, *to find approximately the vertical depression* E D *of the flexible tie below the straight line* A C, *the pulls along it at* A, D, *and* C, *and the horizontal thrust along* A B.

Because W is small compared with P, the curvature of the tie will be small, and the distribution of its weight along a horizontal line may be taken as *approximately* uniform; therefore its figure will be *nearly* a parabola; the tangent at D will be sensibly parallel to A C, and the tangents at A and C will meet in a point which will be near the vertical line E D F, which line bisects A C, and is bisected in D. Hence the following solution is in general sufficiently near the truth for practical purposes. Let R_a, R_d, R_c be the tensions of the tie at A, D, C, respectively, and H the horizontal thrust; then

$$\left.\begin{aligned}
H &= \left(P + \frac{W}{2}\right)\frac{\overline{AB}}{\overline{BC}};\\
R_a &= \sqrt{\left(H^2 + P^2\right)};\\
R_d &= \sqrt{\left\{H^2 + \left(P + \frac{W}{2}\right)^2\right\}};\\
R_c &= \sqrt{\left\{H^2 + (P + W)^2\right\}};\\
\overline{DE} &= \tfrac{1}{8}\,\overline{BC}\;\frac{W}{P + \frac{W}{2}}.
\end{aligned}\right\}\text{ ....(1.)}$$

The *difference of length* between the curve A D C and the straight line A E C is found very nearly by the following formula:—

$$\overline{ADC} - \overline{AEC} = \frac{8}{3}\cdot\frac{\overline{AB}^2\cdot\overline{DE}^2}{\overline{AC}^3} = \frac{1}{24}\cdot\frac{\overline{AB}^2\cdot\overline{BC}^2}{\overline{AC}^3}\cdot\left\{\frac{W}{P + \frac{W}{2}}\right\}^2 \text{ .... (2.)}$$

If E F be made = 2 D E, F C and F A will be approximately tangents to the chain at C and A.

128. The **Catenary** (*A. M.*, 175), in the most general sense of the word, is the curve formed by a chain when loaded in any manner; but when used without qualification, its application is usually restricted to the case of a chain of uniform section and material, loaded with its own weight only. As thus defined, the catenary has the following properties:—

I. All catenaries are similar.

II. The *figure* of the catenary is expressed algebraically by the following equation. (See fig. 112.) Let A be the *vertex*, or lowest point of the catenary, where it is horizontal. A O is a vertical line, called the *parameter*, or *modulus* of the catenary, on which all its dimensions depend; let the length of that line be denoted by m. Take O for the origin of co-ordinates. Let B be any other point in the catenary, whose abscissa,

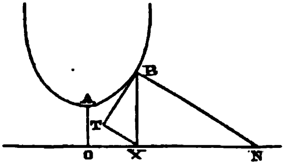

Fig. 112.

or horizontal distance from O, is O X $= x$, and vertical height above the same point X B $= y$. Then

$$\text{the ordinate X B} = y = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right);^{*}$$

$$\text{the arc A B} = s = \frac{m}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right) = \sqrt{y^2 - m^2};$$

the abscissa in terms of the ordinate,

$$x = m \cdot \text{hyp. log.}\left(\frac{y}{m} + \sqrt{\frac{y^2}{m^2} - 1}\,\right); \tag{1.}$$

In terms of y and s $x = m . \log \frac{y+s}{m}$

$$\text{the area, A O X B} = \int y\,dx = \frac{m^2}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right) = m s;$$

the rate of slope at the point B,

$$\frac{dy}{dx} = \tan i = \frac{1}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right) = \frac{s}{m};$$

(i being the angle of inclination of the curve at B to the horizon). The radius of curvature at the same point is,

$$m \sec^2 i = \rho = \frac{y^2}{m} = \frac{m}{4}\left(e^{\frac{2x}{m}} + e^{-\frac{2x}{m}} + 2\right);$$

at the point A, $\rho = m$.

On B X as a hypothenuse construct the right-angled triangle X T B, in which X T $=$ O A $= m$. Then T B will be a tangent to the catenary at B, and will be equal in length to the arc A B $= s$.

Through B draw B N perpendicular to B T, cutting O X produced in N; B N is equal to the radius of curvature at the point B. Since BN : BX :: BX : XT ∴ BN $= \frac{y^2}{m}$.

III. The *mechanical* properties of the catenary are as follows:—

Let p be the weight of an unit of length of the chain (as one foot);

---

[*] The functions $e^{\frac{x}{m}}$ and $e^{-\frac{x}{m}}$, or the *Naperian Anti-logarithm* of $\frac{x}{m}$ and its reciprocal, are most easily calculated by means of a table of Naperian or hyperbolic anti-logarithms, or, in its absence, by a table of hyperbolic logarithms. But should a table of common logarithms or anti-logarithms alone be at hand, the following formula is to be used:—

$$e^{\pm\frac{x}{m}} = 10^{\pm\,\cdot 4343\,\frac{x}{m}} \text{ nearly.}$$

H, the horizontal tension at A; P, the vertical load between the points A and B; R, the tension at B. Then

$$H = p\,m;\ P = p\,s;\ R = \sqrt{H^2 + P^2} = p\,y;\ \ldots\ldots\ (2.)$$

so that the parameter represents a length of chain whose weight is equal to the horizontal tension; and the ordinate X B $= y$ at any point represents a length of chain whose weight is equal to the tension at that point.

IV. Problem.—*Given two points in a catenary, and the length of chain between them; required the remainder of the curve.*

Let k be the horizontal distance between the two points, v their difference of level, l the length of chain between them. Those three quantities are the data.

The unknown quantities may be expressed in the following manner:—Let x_1, y_1, be the co-ordinates of the higher given point, and s_1 the arc terminating at it, all measured from the yet unknown vertex of the catenary, and x_2, y_2, s_2, the corresponding quantities for the lower given point.

Then the parameter m is to be found by a series of approximations from the following equation:— A.M. h 179

$$m\left(e^{\frac{k}{2m}} - e^{-\frac{k}{2m}}\right) = \sqrt{l^2 - v^2};\ \ldots\ldots\ldots\ldots\ (3.)$$

the position of the vertex horizontally, by either of the equations,

$$x_1 = \frac{1}{2}\left(m \cdot \text{hyp. log.} \frac{l+v}{l-v} + k\right);\ x_2 = \frac{1}{2}\left(m \cdot \text{hyp. log.} \frac{l+v}{l-v} - k\right);\ (4.)$$

and the position of the vertex vertically by calculating y from x and m for either of the given points.

The part of a catenary in the neighbourhood of the vertex differs but little in figure from a parabola whose focal distance is $m \div 2$, half the modulus of the catenary; and in calculations for practical purposes within certain limits, the parabola may be used instead of the true catenary, its equation being more simple.

To show the amount of the difference between those curves, the following comparison is given, in which, instead of the finite equation of the catenary, an infinite converging series is substituted. The ordinate is supposed to be measured from the point O in fig. 113, at the distance m below the vertex.

$$\text{Ordinate of the} \begin{cases} \text{Catenary;}\ y = m\left(1 + \dfrac{x^2}{2\,m^2} + \dfrac{x^4}{24\,m^4} + \dfrac{x^6}{720\,m^6} + \text{\&c.}\right); \\ \text{Parabola;}\ y = m\left(1 + \dfrac{x^2}{2\,m^2}\right); \end{cases}$$

$$\text{Slope of the}\begin{cases}\text{Catenary; } \dfrac{d\,y}{d\,x}=\dfrac{x}{m}\left(1+\dfrac{x^2}{6\,m^2}+\dfrac{x^4}{120\,m^4}+\&\text{c.}\right); \\ \text{Parabola; } \dfrac{d\,y}{d\,x}=\dfrac{x}{m};\end{cases}$$

$$\text{Area of the}\begin{cases}\text{Catenary; } \int y\,d\,x=m\,x\left(1+\dfrac{x^2}{6\,m^2}+\dfrac{x^4}{120\,m^4}+\&\text{c.}\right); \\ \text{Parabola; } \int y\,d\,x=m\,x\left(1+\dfrac{x^2}{6\,m^2}\right);\end{cases}$$

$$\text{Length of the}\begin{cases}\text{Catenary; } s=x\left(1+\dfrac{x^2}{6\,m^2}+\dfrac{x^4}{120\,m^4}+\&\text{c.}\right); \\ \text{Parabola; } s=x\left(1+\dfrac{x^2}{6\,m^2}-\dfrac{x^4}{40\,m^4}+\&\text{c.}\right).\end{cases}$$

It is to be borne in mind that the quantity denoted by m in these formulæ is *double* of that denoted by m in Article 125.

The following table exemplifies their results for the case $x=m \div 3$:—

	Ordinate $=m\,\times$	Slope.	Area $=m\,x\,\times$	Length $=x\,\times$
Catenary,.....	1·0561	0·3395	1·0186	1·0186
Parabola, ...	1·0556	0·3333	1·0185	1·0182
Difference,...	0·0005	0·0062	0·0001	0·0004

The **Catenary of Uniform Strength** is the figure assumed by a chain loaded in any manner, whose sectional area at each point is proportional to the tension. The figure assumed by such a chain, when loaded with its own weight only, was investigated by Mr. Davies Gilbert, in a paper published in the *Philosophical Transactions* for 1824. The Reverend Canon Moseley, in his *Mechanics of Engineering and Architecture,* has investigated the figure of the catenary of equal strength when the chain is loaded with suspending rods and a platform, as well as with its own weight. The resulting equations are of great complexity when in their exact form; but Mr. Moseley shows that in those cases which occur in practice the parabola forms a close approximation to the true curve, as it does in the case of the common catenary.

Under the head of "Structures in Iron," it will be shown how far it is useful in practice to take into account the peculiarities of the catenary of uniform strength.

129. **Centre of Gravity of a Flexible Structure.** (*A. M.*, 176.)—In every case in which a perfectly flexible structure, such as a cord, a chain, or a funicular polygon, is loaded with weights only, the figure of stable equilibrium in the structure is that which corresponds to the lowest possible position of the centre of gravity of the entire load. This principle enables all problems respecting the equilibrium of vertically loaded flexible structures to be solved by means of the "Calculus of Variations;" but it has not hitherto been much applied to practical questions.

130. **Transformation of Frames and Chains.** (*A.M.*, 166.)—The principle explained in Article 101, p. 150, of the transformation of a set of lines representing one balanced system of forces into another set of lines representing another system of forces which is also balanced, by means of what is called "PARALLEL PROJECTION," being applied to the theory of frames, takes the following form:—

If a frame whose lines of resistance constitute a given figure, be balanced under a system of external forces represented by a given system of lines, then will a frame whose lines of resistance constitute a figure which is a parallel projection of the original figure, be balanced under a system of forces represented by the corresponding parallel projection of the given system of lines; and the lines representing the stresses along the bars of the new frame will be the corresponding parallel projections of the lines representing the stresses along the bars of the original frame.

This theorem enables the conditions of equilibrium of any unsymmetrical frame which happens to be a parallel projection of a symmetrical frame, to be deduced from the conditions of equilibrium of the symmetrical frame.

The principle of transformation by parallel projection is applicable to continuously loaded chains as well as to polygonal frames. For instance, the bridge-chain with sloping rods of Article 126, p. 191, might be treated as a parallel projection of a bridge-chain with vertical rods, made by substituting oblique for rectangular co-ordinates.

The algebraical expressions for the alterations made by parallel projection in the co-ordinates of a loaded chain or cord, and in the forces applied to it, are as follows:—

In the original figure, let y be the vertical co-ordinate of any point, and x the horizontal co-ordinate. Let P be the vertical load applied between any point B of the chain and its lowest point A; let $p = \frac{d\,\mathrm{P}}{d\,x}$ be its intensity per horizontal unit of length; let H be the horizontal component of the tension; let R be the tension at the point B.

Suppose that in the transformed figure, the vertical ordinate y', and the vertical load P′, which is represented by a vertical line, are unchanged in length and direction, so that we have

$$y' = y;\ \mathrm{P}' = \mathrm{P}; \ldots\ldots\ldots\ldots\ldots(1.)$$

but for each horizontal co-ordinate x, let there be substituted a horizontal or oblique co-ordinate x', inclined at the angle j to the horizon (which may be $= 0$), and altered in length by the constant ratio $\frac{x'}{x} = a$. Then for the horizontal tension H, there will be substituted a horizontal or oblique tension H′, parallel to x', and altered in the same proportion with that co-ordinate; that is to say,

$$x' = a\,x;\ \mathrm{H}' = a\,\mathrm{H} \ldots\ldots\ldots\ldots\ldots(2.)$$

The original tension at B is the resultant of the vertical load P and the horizontal tension H. Let R be its amount, and i its inclination to H; then

$$\mathrm{R} = \sqrt{\mathrm{P}^2 + \mathrm{H}^2}; \ldots\ldots\ldots\ldots\ldots(3.)$$

and the ratios of those three forces are expressed by the proportion

$$\mathrm{P} : \mathrm{H} : \mathrm{R} :: \tan i : 1 : \sec i :: \sin i : \cos i : 1 \ldots\ldots(4.)$$

Let R′ be the amount of the tension at the point B in the new structure, corresponding to B, and let i' be its inclination to the horizontal or oblique co-ordinate x'; then

$$\mathrm{R}' = \sqrt{(\mathrm{P}'^2 + \mathrm{H}'^2 \pm 2\,\mathrm{P}'\,\mathrm{H}' \sin j)} \ldots\ldots\ldots\ldots(5.)$$

$$\mathrm{P}' : \mathrm{H}' : \mathrm{R}' :: \sin i' : \cos(i' \pm j) : \cos j \ldots\ldots\ldots(6.)$$

The alternative signs $\pm$ are to be used according as i' and j $\left\{\begin{matrix}\text{agree}\\ \text{differ}\end{matrix}\right\}$ in direction.

The *intensity* of the load in the transformed structure *per unit of length* measured along $d\,x'$, whether horizontally or obliquely, is

$$p' = \frac{d\,\mathrm{P}'}{d\,x'} = \frac{p}{a}; \ldots\ldots\ldots\ldots\ldots(7.)$$

and if x' be oblique, and the intensity of the load be estimated *per unit of horizontal length*, it becomes

$$p' \sec j = \frac{p}{a \cdot \cos j} \ldots\ldots\ldots\ldots\ldots(8.)$$

131. The **Transformed Catenary** furnishes a good example of the transformations of chains, being derived by parallel projection from the common catenary. It has already been stated (see Article 128, equations 1) that in the common catenary the area O A B X,

fig. 113, is proportional to the arc A B, being equal to a rectangle whose sides are respectively the modulus $m = \text{O A}$, and a straight line equal to the arc A B. Hence the common catenary is the curve of equilibrium for a chain supporting a load which, whether arising from its own weight alone or from other weights also, is proportional upon any given arc A B of the chain, to the area enclosed between that arc, the two ordinates A O and B X, and the *directrix* O X, which is at the depth m below the vertex; the *intensity* of the load at any point B being proportional to the ordinate $y = \text{B X}$. This condition of the chain may be represented to the mind by conceiving the *whole* load to consist of the weight of an uniformly thick sheet of some uniformly heavy substance, bounded above by the catenary and below by the straight line O X. Let w denote the weight of an unit of area (say a square foot) of that sheet; then in the *Common Catenary,*

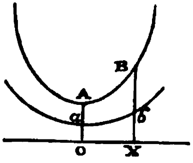

Fig. 118.

$$\left.\begin{aligned}
&\text{the horizontal tension } \mathrm{H} = w \cdot \mathrm{O\,A}^2 = w\,m^2;\\
&\text{the intensity of the load at B} = p = w\,y = \frac{w\,m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right);\\
&\text{the load between A and B} = \mathrm{P} = w \cdot \mathrm{O\,A\,X\,B} = w \int y\,dx\\
&\qquad = \frac{w\,m^2}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right);\\
&\text{the tension at B} = \sqrt{\mathrm{P}^2 + \mathrm{H}^2} = w\,m\,y = \frac{w\,m^2}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right).
\end{aligned}\right\} (1.)$$

Now suppose a curve to be made, such as is represented by $a\,b$ in fig. 114, by preserving the horizontal abscissa of each point in the chain, but altering its vertical ordinate in a constant ratio; so that

$$\left.\begin{aligned}
&\mathrm{O\,A} : \mathrm{O}\,a :: \overset{X}{\mathrm{Q}}\mathrm{B} : \overset{Y}{\mathrm{Q}}\,b;\\
&\text{or denoting O } a \text{ by } y_o, \text{ and } \cancel{\mathrm{Q}}\,b \text{ by } y'\\
&m : y_o :: y : y';
\end{aligned}\right\} \ldots\ldots. (2.)$$

Then the new curve, or TRANSFORMED CATENARY, $a\,b$, is the form of equilibrium for a chain so loaded that the load on any arc $a\,b$ is proportional to the area O $a\,b$ X, and the intensity at the point b to the ordinate X b. In the transformed catenary all the horizontal forces remain the same as in the original catenary; while all the vertical forces are altered in the ratio $y_o : m$; that is to say,

$$\left.\begin{aligned}
&\text{The horizontal tension } \mathrm{H}' = \mathrm{H} = w\, m^2;\\
&\text{the intensity of the load at } b = p' = w\, y' = \frac{w\, y_\circ}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right);\\
&\text{the load between } a \text{ and } b = \mathrm{P}' = w \int y'\, d\, x\\
&\qquad = \frac{w\, m\, y_\circ}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right);\\
&\text{the tension at } b = \sqrt{\mathrm{P}'^2 + \mathrm{H}^2}.
\end{aligned}\right\}\text{(3.)}$$

In the course of the application of these principles, the following problem may occur:—*given, the directrix* O X, *the vertex* a *of the chain, and a point of support* b; *it is required to complete the figure of the chain.* For this purpose it is necessary and sufficient to find the modulus m, which is done by means of the following formula; let $y_\circ =$ O a be the ordinate at a, y' the ordinate at the point of support, x the horizontal distance O X; then,

$$m = \frac{x}{\text{hyp. log.}\left(\frac{y'}{y_\circ} + \sqrt{\frac{y'^2}{y_\circ^2} - 1\cdot}\right)} \ldots\ldots\ldots\ldots\text{(4.)}$$

The principal use of the transformed catenary is as a figure for arches. (See the next Article.)

132. **Linear Arches or Ribs in general—Their Transformation.** (*A. M.*, 178.)—Conceive a cord or chain to be exactly inverted, so that the load applied to it, unchanged in direction, amount, and distribution, shall act inwards instead of outwards; suppose, further, that the cord or chain is in some manner stayed or stiffened, so as to enable it to preserve its figure and to resist a thrust; it then becomes a *linear arch* or *equilibrated rib;* and for the pull at each point of the original chain is now substituted an exactly equal *thrust* along the rib at the corresponding point.

Linear arches do not actually exist; but the propositions respecting them are applicable to the lines of resistance of real arches and arched ribs, in a manner which will be explained in treating of masonry.

All the propositions and equations of the preceding Articles, respecting cords or chains, are applicable to linear arches, substituting only a *thrust* for a *pull,* as the stress along the line of resistance.

The principles of Article 123, p. 185, are applicable to linear arches in general, with external forces applied in any direction.

The principles of Articles 124, 125, 128, 130, and 131, pp. 187 to

202, are applicable to linear arches under *vertical loads;* and in such arches, the quantity denoted by H in the formulæ represents a *constant thrust*, in a direction perpendicular to that of the load.

The form of equilibrium for a linear arch under an uniform load is a *parabola*, similar to that described in Article 125, p. 188.

In the case of a linear arch under a vertical load, the word *intrados* is used to denote the figure of the arch itself, and the word *extrados*, to denote a line traversing the *upper* ends of ordinates, drawn *upwards* from the intrados, of lengths proportional to the intensities of the load.

The figure of equilibrium for a linear arch with a horizontal extrados is either a catenary or a transformed catenary inverted; and the equations of Article 131 are applicable to the determination of its figure and of the forces which act in it, *w* being taken to denote the weight of so much of the loading material as is contained in one square foot of the area between the extrados O X, fig. 113, and the intrados A B or *a b*. This is what is called by most mechanical writers, an "equilibrated arch."

The principles of Article 130, relative to the transformation of cords and chains, are applicable also to linear arches or ribs. This subject will be further considered in the sequel.

133. **Circular Rib for Fluid Pressure.** (*A. M.*, 179.)—A linear arch, to resist an uniform normal pressure from without, should be circular.

In fig. 114, let A B A B be a circular linear arch, rib, or ring,

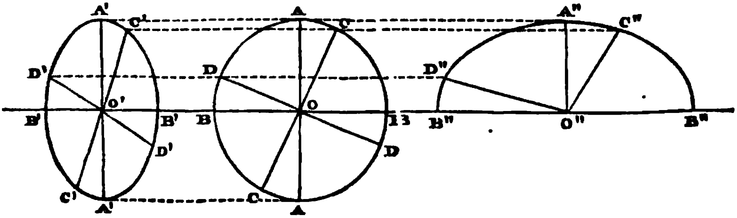

Fig. 114.

whose centre is O, pressed upon from without by a normal pressure of uniform intensity.

In order that the intensity of that pressure may be conveniently expressed in units of force per unit of area, conceive the ring in question to represent a vertical section of a cylindrical shell, whose length, in a direction perpendicular to the plane of the figure, is one foot. Let *p* denote the intensity of the external pressure, in lbs. on the square foot; *r* the radius of the ring in feet; T the thrust exerted round it, which, because its length is one foot, is a thrust in lbs. per foot of length of the cylinder; then,

$$T = p\,r \text{.............................} (1.)$$

that is to say:—*the thrust round a circular ring under an uniform normal pressure is the product of the pressure on an unit of circumference by the radius.*

The uniform normal pressure p, if not actually caused by the thrust of a fluid, is similar to fluid pressure; and it is equivalent to a pair of conjugate pressures in any two directions at right angles to each other, of equal intensity. For example, let x be vertical, y horizontal, and let p_x, p_y, be the intensities of the vertical and horizontal pressure respectively; then

$$p_x = p_y = p; \text{.........................} (2.)$$

and the same is true for any pair of rectangular pressures; and if P be the total vertical pressure, and H the total horizontal pressure, exerted upon one quadrant A B of the circle, we have

$$H = P = T = p\,r \text{........................} (3.)$$

134. **Elliptical Ribs for Uniform Pressures.** (*A. M.*, 180.)—If a linear arch has to sustain the pressure of a mass in which the pair of conjugate thrusts at each point are uniform in amount and direction, but not equal to each other, all the forces acting parallel to any given direction will be altered from those which act in a fluid mass, by a given constant ratio; so that they may be represented by *parallel projections* of the lines which represent the forces that act in a fluid mass. Hence the figure of a linear arch, which sustains such a system of pressures as that now considered, must be a parallel projection of a circle; that is, an *ellipse*. To investigate the relations which must exist amongst the dimensions of an elliptic linear arch under a pair of conjugate pressures of uniform intensity, let A′ B′ A′ B′, B″ A″ B″, in fig. 114, represent elliptic ribs, transformed from the circular rib A B A B by parallel projection, the vertical dimensions being unchanged, and the horizontal dimensions either expanded (as B″ B″), or contracted (as B′ B′), in a given uniform ratio denoted by c; so that r shall be the vertical and $c\,r$ the horizontal semi-axis of the ellipse; and if x, y, be respectively the vertical and horizontal co-ordinates of any point in the circle, and x', y', those of the corresponding point in the ellipse, we shall have

$$x' = x; \; y' = c\,y \text{......................} (1.)$$

If C C, D D, be any pair of diameters of the circle at right angles to each other, their projections will be a pair of conjugate diameters of the ellipse, as C′ C′, D′ D′; that is, diameters each of which is parallel to a tangent at the end of the other.

Let P′ be the total vertical pressure, and H′ the total horizontal pressure, on one quadrant of the ellipse, as A′ B′, or A″ B″; P′ is also the vertical thrust on the rib at B′ or B″, and H′ the horizontal thrust at A′ or A″.

Then, by the principle of transformation,

$$\left.\begin{array}{r} P' = P = T = p\,r; \\ H' = c\,H = c\,T = c\,p\,r; \end{array}\right\} \dots\dots\dots\dots\dots\dots \text{(2.)}$$

or, *the total thrusts are as the axes to which they are parallel,*

Further, let P″ be the total pressure, parallel to any semidiameter of the ellipse (as O′ D′ or O″ D″) on the quadrant D′ C′ or D″ C″, which force is also the thrust of the rib at C′ or C″, the extremity of the diameter conjugate to O′ D′ or O″ D″; and let O′ D′ or O″ D″ $= r'$; then

$$P'' = \frac{r'}{r} P = p\,r'; \dots\dots\dots\dots\dots\dots\dots\dots \text{(3.)}$$

or, *the total thrusts are as the semidiameters to which they are parallel.*

Next, let p'_x, p'_y, be the *intensities* of the conjugate horizontal and vertical pressures on the elliptic arch; that is, of the "*principal stresses.*" (Articles 109, 112.) Each of those intensities being found by dividing the corresponding total pressure by the area of the plane to which it is normal, they are given by the following equation:—

$$p'_x = \frac{P'}{c r} = \frac{p}{c};\; p'_y = \frac{H'}{r} = c\,p; \dots\dots\dots\dots\dots \text{(4.)}$$

so that *the intensities of the principal pressures are as the squares of the axes of the elliptic rib to which they are parallel.*

Hence, to adapt an elliptic rib to uniform vertical and horizontal pressures, *the ratio of the axes of the rib must be the square root of the ratio of the intensities of the principal pressures;* that is,

$$\frac{O\,B'}{O\,A'} = c = \sqrt{\frac{p'_y}{p'_x}}. \dots\dots\dots\dots\dots\dots\dots \text{(5.)}$$

The external pressure on any point D′ or D″, of the elliptic rib is directed towards the centre, O′ or O″, and its intensity, per unit of area of the plane to which it is conjugate (O′ C′ or O″ C″), is given by the following equation, in which r' denotes the semidiameter (O′ D′ or O″ D″) parallel to the pressure in question, and r'' the conjugate semidiameter (O′ C′ or O″ C″):—

$$p' = \frac{P''}{r''} = p \cdot \frac{r'}{r''}; \dots\dots\dots\dots\dots\dots\dots \text{(6.)}$$

that is, *the intensity of the pressure in the direction of a given diameter is directly as that diameter and inversely as the conjugate diameter.*

Let p'' be the intensity of the external pressure in the direction of the semidiameter r''. Then it is evident that

$$p' : p'' :: r'^2 : r''^2; \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.)$$

that is, *the intensities of a pair of conjugate pressures are to each other as the squares of the conjugate diameters of the elliptic rib to which they are respectively parallel.*

135. **Distorted Elliptic Rib.** (*A. M.*, 181.)—To adapt an elliptic rib to the sustaining of the pressure of a mass in which, while the state of stress is uniform, the pressure conjugate to a vertical pressure is not horizontal, but inclined at a given angle j to the horizon, the figure of the ellipse must be derived from that of a circle by the substitution of inclined for horizontal co-ordinates.

In fig. 115, let B A C be a semicircular arch on which the external pressures are normal and uniform, and of the intensity p, as before; the radius being r, and the thrust round the arch, and load on a quadrant, being as before, $P = H = T = p\,r$. Let D be any point in the circle, whose co-ordinates are vertical, $O\,E = x$, horizontal, $E\,D = y$. Let B′ A′ C′ be a semi-elliptic arch, in which the vertical ordinates are the same with those of the circle, while for each horizontal ordinate is substituted an ordinate inclined to the horizon by the constant angle j, and bearing to the corresponding horizontal ordinate of the circle the contant ratio c; that is to say, let

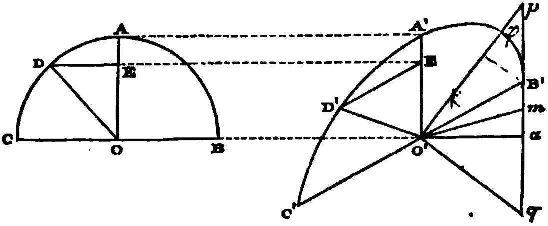

Fig. 115.

$$\overline{O'E'} = x' = x; \ \overline{E'D'} = y' = c\,y; \ \angle E\,E'\,D' = j \ldots (1.)$$

Then for the vertical semidiameter of the circle $O\,A = r$, will be substituted the equal vertical semidiameter of the ellipse $O'\,A' = r$; and for the horizontal semidiameter of the circle $C\,B = r$, will be substituted the inclined semidiameter of the ellipse $C' B' = c\, r$, which is *conjugate* to the vertical semidiameter.

The forces applied to the elliptic arch are to be resolved into vertical and *inclined* components, parallel to O′ A′ and C′ B′, instead of vertical and horizontal components. Let P′ denote the total vertical pressure, and H′ the total inclined pressure, on either of the elliptic quadrants, C′ A′, A′ B′; H′ is also the inclined thrust of the arch at A′, and P′ the vertical thrust at B′ or C′. Then

$$\left.\begin{array}{r} P' = P = p\, r; \\ H' = c\, H = c\, P = c\, p\, r; \end{array}\right\} \dots\dots\dots\dots\dots\dots (2.)$$

that is to say, those forces are, as before, *proportional to the diameters to which they are parallel.*

Let p'_x be the intensity of the vertical pressure on the elliptic arch per unit of area of the inclined plane to which it is conjugate, C′ B′; let p'_y be the intensity of the inclined pressure per unit of area of the vertical plane to which it is conjugate; then

$$p'_x = \frac{P'}{c r} = \frac{p}{c};\ p'_y = \frac{H'}{r} = c\, p;\ c = \sqrt{\frac{p'_y}{p'_x}} \dots\dots\dots (3).$$

so that, as before, *the intensities of the conjugate pressures are as the squares of the diameters to which they are parallel.*

The thrust of the arch at any point D′ is as before, proportional to the diameter conjugate to O′ D′.

It is sometimes convenient to express the intensity of the vertical pressure per unit of area of the *horizontal projection* of the space over which it is distributed; this is given by the equation

$$p'_x \cdot \sec j = \frac{p}{c \cdot \cos j}; \dots\dots\dots\dots\dots\dots\dots\dots (4.)$$

It is to be borne in mind that this is not the pressure on unity of area of a horizontal plane (which pressure is inversely as the horizontal diameter of the ellipse, and directly as the diameter conjugate to that diameter, to which latter diameter it is parallel), but the pressure on that area of a plane inclined at the angle j, whose horizontal projection is unity.

The following geometrical construction serves to determine the major and minor axes of the ellipse B′ A′ C′.

Draw $O' a \perp$ and $= O' A'$; join $\overline{B' a}$, which bisect in m; in $B' a$ produced both ways take $m\, p = m\, q = O'\, m$; join $O' p$, $O' q$; these lines, which are perpendicular to each other, are the *directions* of the axes of the ellipse, and the *lengths* of those axes are respectively equal to the segments of the line $p\, q$, viz., $\overline{B' p} = \overline{a\, q}$, $\overline{B' q} = \overline{a\, p}$.

$A+B = 2\,Om = r\sqrt{(1+c^2+2c\cdot\cos j)}$

$A-B = \overline{B'a} = r\sqrt{(1+c^2-2c\cdot\cos j)}$

$A^2+B^2=r^2+c^2r^2$

$A^2+B^2+2AB=r^2+c^2r^2+$ [illegible]

The following is the algebraical expression of this solution:—Let A denote the major and B the minor semi-axis of the ellipse. Then

$$\left.\begin{array}{l} \mathrm{A}=\dfrac{r}{2}\left\{\sqrt{(1+c^2+2\,c\cdot\cos j)}+\sqrt{(1+c^2-2\,c\cdot\cos j)}\right\}; \\ \mathrm{B}=\dfrac{r}{2}\left\{\sqrt{(1+c^2+2\,c\cdot\cos j)}-\sqrt{(1+c^2-2\,c\cdot\cos j)}\right\}; \end{array}\right\}\text{(5.)}$$

The angle $\angle$ B′ O′ p, $\overset{=k}{\wedge}$ which the nearest axis makes with the diameter C′ B′, is found by the equation

$$> \sin \mathrm{B'O'}p=\frac{\mathrm{B}}{cr}\sqrt{\left(\frac{\mathrm{A}^2-c^2r^2}{\mathrm{A}^2-\mathrm{B}^2}\right)} \text{ or } \frac{\mathrm{A}}{cr}\sqrt{\left(\frac{\mathrm{B}^2-c^2r^2}{\mathrm{A}^2-\mathrm{B}^2}\right)};\ \text{(6.)}$$

according as that axis is the longer —— the shorter.

136. **Ribs for Normal Pressure—Hydrostatic Arch.** (*A. M.*, 182, 183, 319 A.)—The condition of a linear arch of any figure at any point where the pressure is normal, is similar to that of a circular rib of the same curvature under a pressure of the same intensity; and hence the following principle:—*the thrust at any normally pressed point of a rib is the product of the radius of curvature by the intensity of the pressure;* that is, denoting the radius of curvature by ϱ, the normal pressure per unit of length of curve by p, and the thrust by T,

$$\mathrm{T}=p\,\varrho \text{ ....(1.)}$$

It is further evident, that *if the pressure be normal at every point of the rib*, the thrust must be constant at every point; for it can vary only by the application of a tangential pressure to the arch; and *the radius of curvature must be inversely as the pressure.*

This is the case in the HYDROSTATIC ARCH, which is a linear arch or rib suited for sustaining normal pressure at each point proportional, like that of a liquid in repose, to the depth below a given horizontal plane.

The radius of curvature at a given point in the hydrostatic arch being inversely proportional to the intensity of the pressure, is also inversely proportional to the depth below the horizontal plane at which vertical ordinates representing that intensity commence.

In fig. 116, let Y O Y represent the level surface from which the pressure increases at an uniform rate downwards, so as to be similar to the pressure of a liquid having its upper surface at Y O Y. Let A be the crown of the hydrostatic arch, being the point where it is nearest the level surface, and consequently horizontal. Let co-ordinates be measured from the point O in the level surface, directly above the crown of the arch; so that O X = Y C = x shall be the

$= \overline{E}c^2 + \overline{C}b = A^2\cos^2\varphi + B^2\sin^2\varphi =$ [illegible]

[illegible] $= B\sin\varphi \therefore \sin\varphi =$

$A^2 - B^2$

vertical ordinate, and O Y = X C = y the horizontal ordinate, of any point, C, in the arch. Let O A, the least depth of the arch below

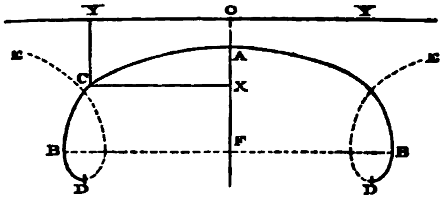

Fig. 116.

the level surface, be denoted by x_0, the radius of curvature at the crown by ϱ_0, and the radius of curvature at any point, C, by ϱ.

Let w be the weight of an unit of volume of the liquid, to whose pressure the load on the arch is equivalent. Then the intensities of the external normal pressure at the crown A, and at any point, C, are expressed respectively by

$$p_0 = w\,x_0;\ p = w\,x. \qquad (2.)$$

The thrust along the rib, which is a constant quantity, is given by the equation

$$T = p_0\,\varrho_0 = w\,x_0\,\varrho_0 = p\,\varrho = w\,x\,\varrho; \qquad (3.)$$

from which follows the following geometrical equation, being that which characterizes the figure of the arch:—

$$x\,\varrho = x_0\,\varrho_0. \qquad (3.)$$

When x_0 and ϱ_0 are given, the property of having the radius of curvature inversely proportional to the vertical ordinate from a given horizontal axis enables the curve to be drawn approximately, by the junction of a number of short circular arcs, as in fig. 117; the radius of each short arc being inversely as the mean depth of that arc below O Y. The curve is found to present some resemblance to a trochoid (with which, however, it is by no means identical). At a certain point, B, it becomes vertical, beyond which it continues to turn, until at D it becomes horizontal; at this point its depth below the level surface is greatest, and its radius of curvature least. Then ascending, it forms a loop, crosses its former course, and proceeds towards E to form a second arch similar

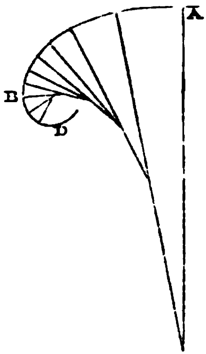

Fig. 117.

to B A B. Its coils, consisting of alternate arches and loops, all similar, follow each other in an endless series.

It is obvious that only one coil or division of this curve, viz., from one of the lowest points, D, through a vertex, A, to a second point, D, is available for the figure of an arch; and that the portion B A B, above the points where the curve is vertical, is alone available for supporting a load.

Let x_1, y_1, be the co-ordinates of the point B. The vertical load above the semi-arch A B is represented by

$$P_1 = w \int_0^{y_1} x\, d y; \text{......................} (4.)$$

and this being sustained by the thrust T of the arch at B, must be equal to that thrust; whence follows the equation

$$x \varrho = x_0 \varrho_0 = \int_0^{y_1} x\, d y. \text{ ....} (5.)$$

The vertical load above any point, C, is

$$P = w \int_0^{y} x\, d y = T \sin i; \text{ ....} (6.)$$

i being the inclination of the arch to the horizon.

The horizontal external pressure against the semi-arch from B to A is the same with that on a vertical plane, A F, immersed in a liquid of the specific gravity w, with its upper edge at the depth x_0 below the surface (see Article 107, p. 166); and it is balanced by the thrust T at the crown of the arch, so that its amount is

$$H = w \int_{x_0}^{x_1} p\, d x = w \cdot \frac{x_1^2 - x_0^2}{2} = T = p \varrho. \text{ ....} (7.)$$

Equation 7 gives for the value of the vertical tangent ordinate at B,

$$x_1 = \sqrt{x_0^2 + 2 x_0 \varrho_0}. \text{ ....} (8.)$$

The horizontal external pressure between B and any point, C, is equal to the pressure of a liquid of the specific gravity w on a vertical plane X F with its upper edge immersed to the depth x, so that its amount is

$$w \int_{x}^{x_1} p\, d x = w \cdot \frac{x_1^2 - x^2}{2} = T \cos i. \text{ ....} (9.)$$

The various geometrical properties of the figure of the hydrostatic arch expressed by the preceding equations are thus summed up in one formula,—

$$x_0 \rho_0 = x \rho = \int_0^{y_1} x\, d\, y = \frac{\int_0^y x\, d\, y}{\sin i} = \frac{x_1^2 - x_0^2}{2} = \frac{x_1^2 - x^2}{2 \cos i} \quad (10.)$$

To obtain exact expressions for the horizontal co-ordinate y, whose maximum value is the half-span y_1, and also for the lengths of arcs of the curve, it is necessary to use elliptic functions. Those functions are so little studied that their use will not be further adverted to here. The reader is therefore referred, for further information on that point, to the papers of M. Yvon-Villarceaux, in the *Mémoires des Savans étrangers*, vol. xii., and in the *Revue de l'Architecture* for 1845, and to *A Manual of Applied Mechanics*, p. 193.

For practical purposes, the following approximation is in general sufficient:—

PROBLEM.—Given the *rise* F A = a and *half-span* F B = y_1, of a proposed hydrostatic arch: it is required to find the depth of load x_0 at the crown, and the radii of curvature, ρ_0, ρ_1, at the crown A and springing B, to draw the arch, and to compute its load and thrust.

A close approximation to x_0 is given as follows:—

$$\left.\begin{array}{c} \text{Let } b = y_1 + \dfrac{y_1^2}{30\, a}; \text{ then} \\ x_0 = a \cdot \dfrac{a^3}{b^3 - a^3}. \end{array}\right\} \ldots\ldots\ldots\ldots (11.)$$

Then observing that $x_1 = \text{O F} = x_0 + a$, we find, from equation 10,

$$\left.\begin{array}{l} \rho_0 = \dfrac{x_1^2 - x_0^2}{2\, x_0} = a + \dfrac{a^2}{2\, x_0}; \\ \rho_1 = \dfrac{x_1^2 - x_0^2}{2\, x_1} = a - \dfrac{a^2}{2 (x_0 + a)}. \end{array}\right\} \ldots\ldots\ldots\ldots (12.)$$

These radii being known, the figure of the arch can be drawn approximately by small circular arcs, as in fig. 117, already described.

The load on the half-arch, and the thrust, which are equal to each other, are now to be computed by equation 7, p. 210.

A mechanical mode of drawing a hydrostatic arch is based on the fact, that its figure is identical with one of the "elastic curves" or forms assumed by an uniformly stiff spring when bent. (*A. M.*, 319A.)

The accuracy of figure and uniformity of stiffness of a spring are to be ascertained by the two following tests:—

First, the spring when unstrained should be exactly straight:

Secondly, when bent into a hoop by pinching the two ends together, it should form an exact circle.

A spring A (fig. 118), fulfilling these conditions, is to have its ends fixed to two bars at B and D, and the other ends of those bars, C and E, are to be pulled directly asunder. Then the straight line C E in which the forces so pulling the bars are exerted, will represent the upper surface of the loading material, and the spring A will assume the figure of the corresponding hydrostatic arch. Any proportion of rise to span can be obtained by varying the tension on the ends of the bars, and the proportion which their lengths bear to the length of the spring.

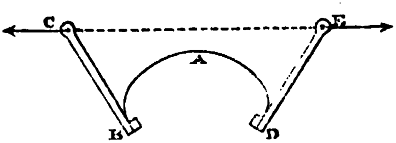

Fig. 118.

137. **Transformed Hydrostatic, or Geostatic Arch.** (*A. M.*, 184.)—It has been proposed, by this term, to denote a linear arch of a figure suited to sustain a pressure similar to that of earth, which (as will be shown in the sequel) consists, in a given vertical plane, of a pair of conjugate pressures, one vertical and proportional to the depth below a given plane, horizontal or sloping, and the other parallel to the horizontal or sloping plane, and bearing to the vertical pressure a certain constant ratio, depending on the nature of the material, and other circumstances to be explained in the sequel. In what follows, the horizontal or sloping plane will be called the *conjugate plane*, and ordinates parallel to its line of steepest declivity, when it slopes, or to any line in it, when it is horizontal, *conjugate ordinates*. The intensity of the vertical pressure will be estimated per unit of area of the *conjugate plane;* and the pressure parallel to the line of steepest declivity of that plane, when it slopes, or to any line in it, when it is horizontal, will be called the *conjugate pressure*, and its intensity will be estimated per unit of area of a vertical plane.

Let p_x denote the intensity of the vertical pressure, and p_y that of the conjugate pressure, at any given point. Construct the figure of a hydrostatic arch suited to sustain fluid pressure of the intensity p_x. Then the transformed arch is to be drawn by preserving all the vertical co-ordinates of the hydrostatic arch, and changing the horizontal co-ordinates into conjugate co-ordinates, having their lengths altered in the constant ratio.

$$c = \sqrt{\frac{p_y}{p_x}}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

exactly as in the case of transforming a circular into an elliptic rib, in Article 134 and 135.

The radius of curvature at the springing is altered in the ratio $\frac{1}{c}$, and that at the crown in the ratio $c^2 : 1$.

Let P, P′, be the total vertical loads on one-half of the original and transformed half-arch respectively; H = P and H′, their respective conjugate thrusts, of which the former is horizontal, and the latter may be horizontal, or inclined at the angle j.

Then the bulk of the transformed arch with its load is altered in the ratio of $c \cos j : 1$; and if the new and transformed arches be of the same material, we find,

$$P' = c \cos j \cdot P;\ H' = c P' = c^2 \cos j \cdot P. \ldots\ldots (2.)$$

138. **A Linear Arch or Rib of any Figure** (*A. M.*, 185, 187), under a *Vertical load distributed in any manner*, being given, it is always possible to determine a system of horizontal or sloping pressures, which, being applied to that rib, will keep it in equilibrio. These last may be called the *Conjugate Pressures.*

The only case which will here be given in detail is that in which the conjugate pressures are horizontal, and the load symmetrically distributed on each side of the crown of the arch, A, fig 119.

PROBLEM I.—To find the total horizontal pressure against the rib below a given point. The following is the graphic solution of this problem :—Let C be any point in the rib.

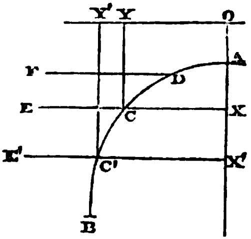

Fig. 119.

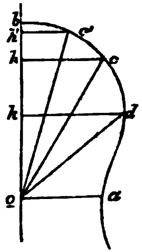

Fig. 120.

In the diagram of forces, fig. 120, draw $o\,c$ parallel to a tangent to the rib at C. Draw the vertical line $o\,b$ as a *scale of loads*, on which take $o\,h$ = P to represent the vertical load supported on the arc A C. Through h draw the horizontal line $h\,c$, cutting $o\,c$ in c; then $o\,c = \mathrm{T}$ will be the thrust along the rib at C, and $h\,c = \mathrm{H}$, the horizontal component of that thrust, will be the *total horizontal pressure which must be exerted against* C B, *the part of the rib below* C.

This solution is expressed algebraically as follows:—As the origin of co-ordinates in fig. 119, take any convenient point O in the vertical line O A traversing the crown of the arched rib; let $\mathrm{O\,X} = \mathrm{Y\,C} = x$ and $\mathrm{O\,Y} = \mathrm{X\,C} = y$ be the co-ordinates of the point C; so that if i is the inclination of the arch at C (and of the line $o\,c$ in fig. 120) to the horizon, $d\,y \div d\,x = \text{cotan}\ i$.

Then,

$$\mathrm{H} = \mathrm{P}\frac{d\,y}{d\,x} = \mathrm{P}\ \text{cotan}\ i;\ \mathrm{T} = \sqrt{\mathrm{P}^2 + \mathrm{H}^2} = \mathrm{P}\ \text{cosec}\ i. \quad (1.)$$

PROBLEM II.—To find the thrust at the crown of the rib.

The preceding process fails to give any result for the crown of the rib A; but the principle of Article 133, p. 204, shows, that if p_0 be the value of $d\,\mathrm{P} \div d\,y$, the intensity of the load, at that point, the horizontal thrust is

$$\mathrm{T}_0 = \mathrm{P}_0\ \rho_0; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

ρ_0 being the radius of curvature of the rib at its crown.

PROBLEM III.—*To find the mean intensity of the horizontal pressure required in a given layer of the spandril;* that is, of the mass of material touching the convex side of the rib. (Fig. 119.)

Let C′ (fig. 119) be a point in the arch a short way below C, whose co-ordinates are $x + d\,x$, $y + d\,y$, so that $d\,x$ is the depth of the horizontal layer C E E′ C′. In the diagram of forces (fig. 120), draw $o\,c'$ parallel to a tangent to the rib at C′; on the vertical scale of loads take $o\,h' = \mathrm{P} + d\,\mathrm{P}$ to represent the vertical load on the arc A C′; draw the horizontal line $h'\,c'$ cutting $o\,c'$ in c'. Then $o\,c' = \mathrm{T}'$ is the thrust along the rib at C′; and $h'\,c' = \mathrm{H}'$, the horizontal component of that thrust, is the horizontal pressure which must be exerted against the part of the rib below C′; so that

$$h\,c - h'\,c' = \mathrm{H} - \mathrm{H}' = -\,d\,\mathrm{H}, \ldots\ldots\ldots\ldots (3.)$$

is the horizontal pressure to be exerted through the layer C E E′ C′ and

$$p_x = -\frac{d\,\mathrm{H}}{d\,x} = -\frac{d}{d\,x}\left(\mathrm{P}\frac{d\,y}{d\,x}\right) \ldots\ldots\ldots\ldots (4.)$$

the intensity of that pressure.

The negative sign prefixed to $d\,H$ denotes that if H *diminishes in going downwards*, as in the example given, pressure is required through the layer. Through those layers at which H *increases in going downwards*, either *tension from without*, or *pressure from within*, is required to keep the rib in equilibrio.

PROBLEM IV.—To find the greatest horizontal thrust, and the "point of rupture," and "angle of rupture."

First Solution.—By a graphic process. Through o in fig. 120, draw a number of radiating lines, such as $o\,c$, $o\,c'$, &c., parallel to the rib at various points, as C, C′, &c., and find as in Problem I. and III., the lengths of those lines so as to represent the thrust along the rib at the several points C, C′, &c. The length of the horizontal line $o\,a$, representing the thrust at the crown, is to be calculated as in Problem II. Through the points a, c, c', &c., thus found, draw a curve. Find the point d in that curve which is furthest from the scale of loads $o\,b$; then the horizontal line $d\,k = \mathrm{H}_0$ will represent the maximum horizontal thrust.

Join $o\,d$, and find the point D in fig. 119, at which the rib is parallel to $o\,d$; this is the "point of rupture," or point at which the horizontal thrust attains a maximum; and the "angle of rupture" is the inclination of the rib at that point, or $\angle\, d\,o\,a$ in fig. 120, which will be denoted in the sequel by i_0.

The horizontal plane D F is the upper boundary of that part of the spandril which exerts the maximum horizontal pressure H_0.

Second Solution.—By arithmetical trials. Compute, as in Problem I., the values of H for some points in the arch. Between the point which gives the greatest value of H in the first set of trials, and the two on either side of it, introduce intermediate points, for which compute the values of H, and repeat the process until the point of rupture is found with the desired degree of exactness.

Third Solution.—By the differential calculus and the solution of an equation. If the relations between x, y, and P can be expressed by equations, make the expression for the *intensity* of the horizontal pressure in equation 4 equal to 0; and by solving the equation so obtained, deduce the position of D and the values of i_0 and H_0. The equation to be solved has, in most cases, two roots, one of which corresponds to the crown of the arch A, and the other to the required point D; but it is easy to distinguish between them. If there are more than two roots, they indicate a set of points at each of which $p_y = 0$, and which are alternately points of $\left\{\begin{matrix}\text{maximum}\\ \text{minimum}\end{matrix}\right\}$ horizontal thrust, according as $\frac{d^2\,\mathrm{H}}{d\,x^2}$ is $\left\{\begin{matrix}\text{negative}\\ \text{positive}\end{matrix}\right\}$; that is, according as $\frac{d\,p_y}{d\,x}$ is $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right\}$. Cases of this kind are of rare occurrence in practice.

If there is but one root, it corresponds to the crown of the rib; the hydrostatic arch (Article 136), is an example of this, in which the crown is the point of greatest horizontal thrust. In the Catenary (Article 128), and Transformed Catenary (Article 131), and other "curves of equilibrium" for vertical loads (Article 123), H is constant, and $p_y = 0$ for every point in the rib.

If the rib rises *vertically* from its springing-point, as at B, the whole of the horizontal pressure which sustains it is distributed through the layers of the spandril. (The term "*complete*" has lately been introduced to denote such a rib.) If the arch rises obliquely from such a springing-point as C, part at least of its greatest horizontal thrust consists of the horizontal component of the thrust along the rib at that point. Such a rib is said to be "*segmental*."

Let i_1 denote the inclination to the horizon of the rib at its springing-point, and P_1 the whole vertical load from the crown to the springing-point: then the horizontal component in question is P_1 cotan i_1; so that $H_0 - P_1$ cotan i_1 is the part of the greatest horizontal thrust which is distributed through the spandril.

PROBLEM V.—To find the position of the resultant of the maximum horizontal thrust.

From the point of rupture D down to the springing, conceive the spandril to be divided into horizontal layers. Let $d\,x$ denote the depth of any one of those layers; p_y the intensity of the horizontal pressure exerted by it against the rib;

x, the depth of its centre below O Y, fig. 119;
x_0, the depth of the joint of rupture below O Y;
x_1, the depth of the springing-point below O Y;

Then, x_H, the required depth of the resultant below O Y, may be expressed in either of the following forms:—

$$x_H = \frac{\int_0^{H_0} x\,d\,H}{H_0} = \frac{\int_{x_0}^{x_1} x\,p_y\,d\,x + x_1\,P_1 \text{ cotan } i_1}{H_0} \quad \ldots\ldots(5.)$$

Example I.—In the *Catenary* and *Transformed Catenary*, and other ribs equilibrated under vertical loads,

$$H_0 = P_1 \text{ cotan } i_1;\ x_H = x_1. \quad \ldots\ldots\ldots\ldots\ldots\ldots(6.)$$

Example II.—In a *Semicircular Rib* of the radius r under *uniform normal pressure* of the intensity p; let the origin of co-ordinates be at the crown of the arch. [illegible] $H_0 = T = pr$.

[illegible] $= r$ [illegible] $x_H =$ [illegible] $= \frac{pr^2}{[illegible]} = \frac{r}{2}$

$$\mathrm{H}_0 = p\,r\,;\ x_{\mathrm{H}} = \frac{r}{2}. \quad \text{.....................(7.)}$$

As to semi-elliptic arches under conjugate uniform pressures, see Article 134, p. 205.

Example III.—In the *Hydrostatic Arch*, as in fig. 116, Article 136, p. 209, let the origin of co-ordinates be in the extrados above the crown; then

$$\mathrm{H}_0 = w \cdot \frac{x_1^2 - x_0^2}{2};\ x_{\mathrm{H}} = \frac{2}{3} \cdot \frac{x_1^3 - x_0^3}{x_1^2 - x_0^2}. \quad \text{..........(8).}$$

In the transformed hydrostatic or geostatic arch, x_{H} is the same as in the hydrostatic arch. As to the thrust, see Article 137, p. 213.

Example IV.—In a *Semicircular Rib* with a horizontal extrados, let r be the radius of the rib; let the origin of co-ordinates be at the *crown of the arch*; let $m\,r$ be the height of the extrados above the crown; and let w be the weight of each unit of vertical area of the load.

The intensity of the horizontal pressure through a given layer of the spandril is,

$$p_y = w\,r \left(1 + m - \cos i - \frac{i - \cos i \sin i}{2 \sin^3 i}\right) \text{.........(9.)}$$

The angle of rupture i_0 is found by solving the transcendental equation,

$$p_y = 0, \quad \text{...........................(10.)}$$

This is to be done by successive approximations; and as a first approximation may be taken

$$i_0 = \operatorname{arc} \cdot \cos \frac{1 + 3\,m}{2} \text{ approximately.} \quad \text{......(10 A.)}$$

The maximum thrust is given by the formula

$$\mathrm{H}_0 = w\,r^2 \left\{(1 + m) \cos i_0 - \frac{\cos^2 i_0}{2} - \frac{i_0 \operatorname{cotan} i_0}{2}\right\};\ \text{(11.)}$$

and the depth of its resultant below the crown of the arch by the formula

$x_H = \int_{i_0}^{90} \frac{b_y\,x\,dx}{H_0}$

$$x_{\mathrm{H}} = \frac{r^3}{\mathrm{H}_0} \cdot \int_{i_0}^{90^\circ} p_y \sin i\,(1 - \cos i)\,d\,i \quad \text{........(12.)}$$

Example V.—In a *Circular Segmental Rib* with a *horizontal* extrados, let i_1 be the inclination of the arch at the springing, and P_1 the vertical load at the springing, and let the rest of the notation be as in the last example.

Let the angle of rupture be found as before.

Case 1.—$i_0 >$ *or* $= i_1$. Then

$$H_0 = P_1 \operatorname{cotan} i_1\,;\ x_H = r\,(1 - \cos i_1) \text{........} (13.)$$

Case 2.—$i_0 < i_1$. Find H_0 and p_y as in Example IV.; then

$$x_H = \frac{1}{H_0}\left\{r^2 \int_{i_0}^{i_1} p_y \sin i\,(1 - \cos i)\,d\,i + r\,P_1 \operatorname{cotan} i_1\,(1 - \cos i_1)\right\} \quad (14.)$$

Example VI.—*Semi-elliptic Rib, with a horizontal extrados.* Conceive a semicircular rib whose radius is equal to the rise of the semi-elliptic rib, and extrados at the same height above the crown, and find H_0 and x_H for the semicircular rib as in Example IV. x_H for the semi-elliptic rib will be the same; and the thrust is to be found by the principle of transformation, as in Article 134, p. 205.

The best form, however, for oval complete ribs is that of the hydrostatic arch, which sufficiently resembles the semi-ellipse to be substituted for it.

139. **Pointed Rib.**—If a linear arch, as in fig. 121, consists of two arcs, B C, C B, meeting in a point at C, it is necessary to equilibrium that there should be concentrated at the point C a load equal to that which would have been distributed over the two arcs A C, C A, extending from the point C to the respective crowns, A, A, of the curves of which two portions form the pointed arch.

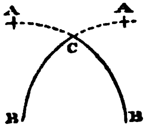

Fig. 121.

Under the head of "Masonry" it will be shown how and under what circumstances that concentration of load becomes unnecessary in stone arches.

140. **Stability of Blocks.** (*A. M.*, 205, 206.)—The conditions of stability of a single block supported upon another body at a plane joint may be thus summed up:—

In fig. 122, let A A represent the upper block, B B part of the supporting body, *e* E the joint, C its centre of pressure, P C the resultant of the whole pressure distributed over the joint, N C, T C, its components perpendicular and parallel to the joints respectively. Then the conditions of stability are the following:—

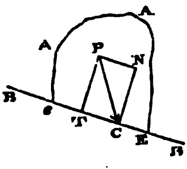

Fig. 122.

I. *In order that the block may not slide, the* *obliquity of the pressure must not exceed the angle of repose* (see Article 110, p. 171), that is to say,

$$\angle \mathrm{P\,C\,N} \leqq \varphi \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

II. *In order that the block may be in no danger of overturning, the ratio which the deviation of the centre of pressure from the centre of figure of the joint bears to the length of the diameter of the joint traversing those two centres, must not exceed a certain fraction.* The value of that fraction varies, according to circumstances, which will be explained in treating of Masonry, from one-eighth to three-eighths.

The first of these conditions is called that of *stability of friction*, the second, that of *stability of position*.

In a structure composed of a series of blocks, or of a series of courses so bonded that each may be considered as one block, which blocks or courses press against each other at plane joints, the two conditions of stability must be fulfilled at each joint.

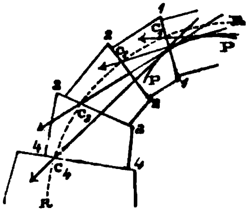

Fig. 123.

Let fig. 123 represent part of such a structure, 1. 1, 2, 2, 3, 3, 4, 4, being some of its plane joints.

Suppose the centre of pressure C_1 of the joint 1. 1, to be known, and also the amount and direction of the pressure, as indicated by the arrow traversing C_1. With that pressure combine the weight of the block 1, 2, 2, 1, together with any other external force which may act on that block; the resultant will be the total pressure to be resisted at the joint 2, 2, which will be given in magnitude, direction, and position, and will intersect that joint in the centre of pressure C_2. By continuing this process there are found the centres of pressure C_3, C_4, &c., of any number of successive joints, and the directions and magnitudes of the resultant pressures acting at those joints.

The magnitude and position of the resultant pressure at any joint whatsoever, and consequently the centre of pressure at that joint, may also be found simply by taking the resultant of all the forces which act on one of the parts into which that joint divides the structure.

The centres of pressure at the joints are sometimes called *centres of resistance*. A line traversing all those centres of resistance, such as the dotted line R R, in fig. 122, has received from Mr. Moseley the name of the "*line of resistance;*" and that author has also shown how in many cases the equation which expresses the form of that line may be determined, and applied to the solution of useful problems.

The straight lines representing the resultant pressures may be all parallel, or may all lie in the same straight line, or may all intersect in one point. The more common case, however, is that in which those straight lines intersect each other in a series of points, so as to form a polygon. A curve, such as P P, in fig. 95, touching all the sides of that polygon, is called by Mr. Moseley the "*line of pressures.*"

The properties which the line of resistance and line of pressures must have, in order that the conditions of stability may be fulfilled, are the following:—

To insure stability of position, *the line of resistance must not deviate from the centre of figure of any joint by more than a certain fraction of the diameter of the joint, measured in the direction of deviation.*

To insure stability of friction, *the normal to each joint must not make an angle greater than the angle of repose with a tangent to the line of pressures drawn through the centre of resistance of that joint.*

The moment of stability of a body or structure supported at a given plane joint is the moment of the couple of forces which must be applied in a given vertical plane to that body or structure in addition to its own weight, in order to transfer the centre of resistance of the joint to the limiting position consistent with stability, and is equal to the product of the weight of the body or structure by the horizontal distance of a vertical line traversing its centre of gravity from the limiting position of the centre of resistance of the base or supporting joint. The applied couple usually consists of the thrust of a frame, or an arch, or the pressure of a fluid, or of a mass of earth, against the structure, together with the equal, opposite, and parallel, but not directly opposed, resistance of the joint to that lateral force.

141. **Transformation of Blockwork Structures.** (*A. M.*, 208, 209.) —If a structure composed of blocks have stability of position when acted on by forces represented by a given system of lines, then will a structure whose figure is a parallel projection of the original structure have stability of position when acted on by forces represented by the corresponding parallel projection of the original system of lines; also, the centres of pressure in the new structure will be the corresponding projections of the centres of pressure in the original structure.

The question, whether the new structure obtained by transformation will possess *stability of friction* is an independent problem.

The application of the principles of the stability of blockwork structures will be illustrated under the head of Masonry.

SECTION V.—*Of the Strength of Materials in General.*

142.—**Strain, Stress, Strength, Working Load.** (*A. M.*, 244.)—The present section contains a summary of the principles of the strength of materials so far as they relate to questions which arise in designing structures. The rules are given without demonstration, in as small compass as possible, in order to save the necessity of referring, in ordinary cases, to more bulky treatises; and are almost all abstracted and abridged from the treatise already referred to on *Applied Mechanics*, Part II., Chapter III.

The *load*, or combination of external forces, which is applied to any piece in a structure produces *strain*, or alteration of the volumes and figures of the whole piece, and of each of its particles, which is accompanied by *stress* amongst the particles of the piece, being the combination of forces which they exert in resisting the tendency of the load to disfigure and break the piece. If the load is continually increased, it at length produces either *fracture*, or (if the material is very tough and ductile) such a disfigurement of the piece as is practically equivalent to fracture, by rendering it useless.

The **Ultimate Strength or Breaking Load** of a body is the load required to produce fracture in some specified way. The **Proof Strength or Proof Load** is the load required to produce the greatest strain of a specific kind consistent with safety; that is, with the retention of the strength of the material unimpaired. A load exceeding the proof strength of the body, although it may not produce instant fracture, produces fracture eventually by long-continued application and frequent repetition.*

The **Working Load** on each piece of a structure is made less than the proof strength in a certain ratio determined by practical experience, in order to provide for unforeseen contingencies.

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* A series of experiments on the effect of the frequent application and removal of a load, were made by the Commissioners on the Application of Iron to Railway Structures, the general results of which were as follows:—

When cast iron bars were exposed to successive transverse blows, each blow producing *one-third* of the ultimate deflection (or deflection immediately before breaking), they bore 4,000 such blows without having their strength impaired; but when the force of each blow produced *one-half* of the ultimate deflection, every bar broke before receiving the 4,000th blow.

When cast iron bars were exposed to successive deflections by means of a cam, of *one-third* of the ultimate deflection, they bore 100,000 such deflections without having their strength impaired; but when each deflection was *one-half* of the ultimate deflection, the bars broke with fewer than 900 deflections.

"In wrought iron bars, no perceptible effect was produced by 10,000 successive deflections, by means of a revolving cam, each deflection being due to half the weight, which, when applied statically, produced a large permanent deflection."

A new series of experiments on the effect of vibratory action and long-continued

Each solid has as many different kinds of strength as there are different ways in which it can be strained or broken, as shown in the following classification:—

Application of Load.	Strain.	Fracture.
Longitudinal.......	Extension..............	Tearing.
	Compression...........	Crushing.
Transverse.........	Distortion..............	Shearing.
	Twisting................	Wrenching.
	Bending.................	Breaking across.

143. **Factors of Safety** (*A. M.*, 247) are of three kinds, viz.:—the ratio in which the *breaking load* exceeds the *proof load*, the ratio in which the *breaking load* exceeds the *working load*, and the ratio in which the *proof load* exceeds the *working load*.

When not otherwise specified, a factor of safety is, in general, to be understood in the second of those senses—viz., the ratio in which the breaking load exceeds the working load.

The following table gives examples of the values of those factors which occur in actual structures:—

	Ult. Strength. ÷ Proof Strength.	Ult. Strength. ÷ Working Load.	Proof Strength. ÷ Working Load.
Strongest steel, ...	1½	...	...
Ordinary steel and wr. iron, steady load,	2	3	1½
" " moving load,	2	4 to 6	2 to 3
Wrought iron rivetted structures,.....	3	6	2
Cast iron, steady load,...................	2 to 3	3 to 4	about 1½
" moving load,................	3	6 to 8	2 to 2⅔
Timber, average,........................	3	10	3⅓
Stone and brick, ...	about 2	4 to10,av.abt.3	av. about 4

Almost all the experiments hitherto made on the strength of materials give the *ultimate strength* only. In using those data for

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changes of load on wrought iron girders, by Mr. Fairbairn, has for some time been in progress. Those experiments (so far as they had then been carried) were communicated to the British Association at Oxford, in June, 1860; and the following is a summary of the results:—

The beam experimented on was a rivetted wrought iron plate girder.

When the load applied was about *one-fourth* of the breaking weight, the beam withstood 596,790 successive applications of it, without visible alteration.

The load was then increased to *two-sevenths* of the breaking weight, and applied 403,210 times, when the beam showed a slight increase of permanent set.

The load was further increased to *two-fifths* of the breaking weight, when the beam broke with the 5,175th application.

The successive applications of the load were accompanied with considerable strains from vibration and impact.

the designing of structures, the most convenient process of calculation is to multiply the intended *working load* of a piece by the proper factor, so as to find the *breaking load*, and to make the ultimate strength of the piece equal to that breaking load.

144. The **Proof** or **Testing** by experiment of the strength of a piece of material is to be conducted in two different ways, according to the object in view.

I. If the piece is to be *afterwards used*, the testing load must be so limited that there shall be no possibility of its impairing the strength of the piece; that is, it must not exceed the proof strength, being from one-third to one-half of the ultimate strength. About double of the working load is in general sufficient. Care should be taken to avoid vibrations and shocks when the testing load approaches near to the proof strength.

II. If the piece is to be *sacrificed* for the sake of ascertaining the strength of the material, the load is to be increased by degrees until the piece breaks, care being taken, especially when the breaking point is approached, to increase the load by small quantities at a time, so as to get a sufficiently precise result.

The *proof strength* requires much more time and trouble for its determination than the ultimate strength. One mode of approximating to the proof strength of a piece is to apply a moderate load and remove it, apply the same load again and remove it, two or three times in succession, observing at each time of application of the load, the *strain* or alteration of figure of the piece when loaded, by stretching, compression, bending, distortion, or twisting, as the case may be. If that alteration does *not sensibly increase* by repeated applications of the same load, the load is within the limit of proof strength. The effects of a greater and a greater load being successively tested in the same way, a load will at length be reached whose successive applications produce increasing disfigurements of the piece; and this load will be greater than the proof strength, which will lie between the last load and the last load but one in the series of experiments.

It was formerly supposed that the production of a *set*, that is, a disfigurement which continues after the removal of the load, was a test of the proof strength being exceeded; but Mr. Hodgkinson showed that supposition to be erroneous, by proving that in most materials a set is produced by almost any load, how small soever.

The strength of bars and beams to resist breaking across, and of axles to resist twisting, can be tested by the application of known weights either directly or through a lever.

To test the tenacity of rods, chains, and ropes, and the resistance of pillars to crushing, more powerful and complex mechanism is required. The apparatus most commonly employed is the hydraulic press. In computing the stress which it produces, no reliance ought to be placed on the load on the safety valve, or on a weight hung to the pump handle, as indicating the intensity of the pressure, which should be ascertained by means of Bourdon's gauge. This remark applies also to the proving of boilers by water pressure.

From experiments made by Messrs. More of Glasgow, and by the Author, it appears, that in experiments on the tension and compression of bars, about *one-tenth* should be *deducted* from the pressure in the hydraulic press for the friction of the press plunger.

The measurement of tension and compression by means of the hydraulic press is but a rough approximation at the best. It may be sufficient for an immediate practical purpose; but for the exact determination of general laws, although the load may be applied at one end of the piece to be tested by means of a hydraulic press, it ought to be resisted and measured at the other end by means of a combination of levers, such as that used at Woolwich Dockyard, and described by Mr. Barlow in his work *On the Strength of Materials.*

145. **Co-efficients or Moduli of Strength** are quantities expressing the *intensity* of the stress under which a piece of a given material gives way when strained in a given manner; such intensity being expressed in units of weight for each unit of sectional area of the layer of particles at which the body first begins to yield. In Britain, the ordinary unit of intensity employed in expressing the strength of materials is the *pound avoirdupois on the square inch.* As to other units, see Article 106, p. 161.

Co-efficients of strength are of as many different kinds as there are different ways of breaking a body. Their use will be explained in the sequel. Tables of their values are given at the end of the volume.

Co-efficients of strength, when of the same kind, may still vary according to the direction in which the stress is applied to the body. Thus the tenacity, or resistance to tearing, of most kinds of wood, is much greater against tension exerted along than across the grain.

146. **Stiffness or Rigidity, Pliability, their Moduli or Co-efficients.** —Rigidity or stiffness is the property which a solid body possesses, of resisting forces tending to change its figure. It may be expressed as a quantity, called a *modulus or co-efficient of stiffness*, by taking the ratio of the intensity of a given stress of a given kind, to the strain, or alteration of figure, with which that stress is accompanied:—that strain being expressed as a quantity by dividing the alteration of some dimension of the body by the original length of

that dimension. In most substances which are used in construction, the moduli of stiffness, though not exactly constant, are nearly constant for stresses not exceeding the proof strength.

The reciprocal of a modulus of stiffness may be called a "*modulus of pliability;*" that is to say,

$$\text{Modulus of Stiffness} = \frac{\text{Intensity of Stress}}{\text{Strain}};$$

$$\text{Modulus of Pliability} = \frac{\text{Strain}}{\text{Intensity of Stress}}.$$

The use of specific moduli of stiffness will be explained in the sequel. Tables of their values are given at the end of the volume.

147. The **Elasticity of a Solid** (*A. M.*, 236 to 238, 243, 248 to 263) consists of stiffness, or resistance to change of figure, combined with the power of recovering the original figure when the straining force is withdrawn. If that recovery is perfect and exact, the body is said to be "*perfectly elastic;*" if there is a "*set,*" or permanent change of figure, after the removal of the straining force, the body is said to be "imperfectly elastic." The elasticity of no solid substance is absolutely perfect, but that of many substances is nearly perfect when the stress does not exceed the proof strength, and may be made sensibly perfect by restricting the stress within small enough limits.

Moduli or *Co-efficients of Elasticity* are the values of moduli of stiffness when the stress is so limited that the value of each of those moduli is sensibly constant, and the elasticity of the body sensibly perfect. It can be shown that in a homogeneous solid, there may be *twenty-one* independent co-efficients of elasticity,* which, in a solid that is equally elastic in all directions, are reduced to *two*—viz., the co-efficient of direct elasticity, or resistance to direct lengthening and shortening, and the co-efficient of resistance to distortion.

The *General Problem of the Internal Equilibrium of an Elastic Solid* is this:—Given the free form of a solid, the values of its co-efficients of elasticity, the attractions acting on its particles, and the stresses applied to its surface; to find its change of form, and the strains of all its particles.† This problem is to be solved, in general, by the aid of an ideal division of the solid into molecules, rectangular in their free state, and referred to rectangular co-ordinates. Some particular cases are most readily solved by means of spherical, cylindrical, or otherwise curved co-ordinates. The general problem is of extreme complexity, and its complete solution has not yet been obtained in a practically available form; but the cases which occur in practice, and to which the remainder of this section relates, can be solved with sufficient accuracy by comparatively simple approximate methods. Most of those approximate methods are analogous to the "method of sections" described in its application to framework in Article 122, p. 184. The body under consideration is conceived to be divided into two parts by an ideal plane of section; the external forces and couples acting on one of those two parts are computed; and they must be equal and opposite to the forces and couples resulting from the *entire* stress at the ideal sectional plane, which is so found. Then as to the *distribution* of that stress, direct and shearing, some law is assumed, which, if not exactly true, is known either by experiment or by theory, or by both combined, to be a sufficiently close approximation to the truth.

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* See *Phil. Trans.*, 1856-7.

† See Lamé, *Leçons sur la Théorie Mathématique de l'Elasticité des Corps Solides.*

Except in a few comparatively simple cases, the strict method of investigation, by means of the equations of internal equilibrium, has hitherto been used only as a means of determining whether the ordinary approximative methods are sufficiently close.

148. **Resilience,** or **Spring** (*A. M.*, 244), is the quantity of *mechanical work* required to produce the proof-stress on a given piece of material, and is equal to the product of the *proof strain*, or alteration of figure, into the mean load which acts during the production of that strain; that is to say, in general, very nearly one-half of the proof load.

149. **Resistance of Bars to Stretching and Tearing.** (*A. M.*, 265 to 269.)—The ultimate strength or breaking load of a bar exposed to direct and uniform tension is the product of the area of cross-section of the bar into the *tenacity* of the material. Therefore let

P denote the breaking load, in pounds;
S the area of section, in square inches;
f the tenacity, in pounds on the square inch; then

$$P = fS; \; S = \frac{P}{f} \ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

The *elongation* of the bar under any load P′ not exceeding the proof load is found as follows:—

Let x denote the original length of the bar, Δx the elongation, and

$$\alpha = \frac{\Delta x}{x},$$

the *proportion* which that elongation bears to the original length of the bar, being the numerical measure of the strain.

Let $p = \mathrm{P} \div \mathrm{S}$ denote the intensity of the stress, and E, the *modulus of direct elasticity*, or resistance to stretching. Then

$$\alpha = \frac{p}{\mathrm{E}} \text{ .... (2.)}$$

Let f' denote the *proof tension* of the material, so that $f'\,\mathrm{S}$ is the proof load of the bar; then the *proof strain*, or proportionate elongation under the proof load, is $f' \div \mathrm{E}$.

The **Resilience** or **Spring** of the bar, or the work performed in stretching it to the limit of proof strain, is computed as follows:—x being the length, as before, the elongation of the bar under the proof load is $f'\,x \div \mathrm{E}$. The force which acts through this space has for its least value 0, for its greatest value $\mathrm{P} = f'\,\mathrm{S}$, and for its mean value $f'\,\mathrm{S} \div 2$; so that the work performed in stretching the bar to the proof strain is

$$\frac{f'\,\mathrm{S}}{2} \cdot \frac{f'\,x}{\mathrm{E}} = \frac{f'^2}{\mathrm{E}} \cdot \frac{\mathrm{S}\,x}{2} \text{ .... (3.)}$$

The co-efficient $f'^2 \div \mathrm{E}$, by which one-half of the volume of the bar is multiplied in the above formula, is called the MODULUS OF RESILIENCE.

A *sudden pull* of $f'\,\mathrm{S} \div 2$, or *one-half of the proof load*, being applied to the bar, will produce the *entire proof strain* of $f' \div \mathrm{E}$, which is produced by the *gradual* application of the proof load itself; for the work performed by the action of the constant force $f'\,\mathrm{S} \div 2$, through a given space, is the same with the work performed by the action, through the same space, of a force increasing at an uniform rate from 0 up to $f'\,\mathrm{S}$. Hence a bar, to resist with safety the sudden application of a given pull, requires to have twice the strength that is necessary to resist the gradual application and steady action of the same pull.

Tables of the tenacity and of the modulus of direct elasticity of various substances are given at the end of the volume.

150. **Cylindrical Boilers and Pipes.** (*A. M.*, 271.)—Let r denote the radius of a thin hollow cylinder, such as the shell of a high pressure boiler;

t the thickness of the shell;

f the tenacity of the material, in pounds per square inch;

p the intensity of the pressure, in pounds per square inch, required to burst the shell. This ought to be taken at SIX TIMES the effective working pressure—*effective pressure* meaning the excess of the pressure from within above the pressure from without, which last is usually the atmospheric pressure, of 14·7 lbs. on the square inch or thereabouts.

Then

$$p = \frac{f\,t}{r}; \text{ ....(1.)}$$

and the proper proportion of thickness to radius is given by the formula,—

$$\frac{t}{r} = \frac{p}{f}. \text{ ....(2.)}$$

151. **Spherical Shells,** such as the ends of "egg-ended" cylindrical boilers, the tops of steam domes, &c., are *twice as strong* as cylindrical shells of the same radius and thickness.

Suppose a shell of the figure of a segment of a sphere to have a circular flange round its base, through which it is bolted to a flange upon a cylindrical shell, or upon another spherical shell.

Let r denote the radius of the sphere, in inches;

r', the radius of the circular base of the segmental shell, in inches;

p, the bursting pressure, in lbs. on the square inch;

then the number and dimensions of the bolts by which the flange is held should be such, that the load required to tear them asunder all at once shall be

$$3{\cdot}1416\, r'^2 p; \text{ ....(1.)}$$

and the flange itself should require, in order to crush it, the following thrust in the direction of a tangent to it:—

[illegible]

[illegible]

$$\frac{1}{2} p\, r' \cdot \sqrt{r^2 - r'^2}. \text{ ....(2.)}$$

If the segment is a complete hemisphere, $r' = r$, and the last expression becomes $= 0$.

152. **Thick Hollow Cylinder.** (*A. M.*, 273.)—The assumption that the tension in a hollow cylinder is uniformly distributed throughout the thickness of the shell is approximately true only when the thickness is small as compared with the radius.

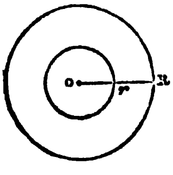

Fig. 124.

Let R represent the external and r the internal radius of a thick hollow cylinder, such as a hydraulic press, the tenacity of whose material is f, and whose bursting pressure is p. Then we must have

$$\frac{R^2 - r^2}{R^2 + r^2} = \frac{p}{f};\dots\dots\dots\dots(1.)$$

and, consequently,

$$\frac{R}{r} = \sqrt{\left(\frac{f+p}{f-p}\right)};\dots\dots\dots\dots(2.)$$

by means of which formula, when r, f, and p, are given, R may be computed.

153. **Thick Hollow Sphere.** (*A. M.*, 275.)—In this case, using the same symbols as in the last Article, the following formulæ give the ratios of the bursting pressure to the tenacity, and of the external to the internal radius:—

$$\frac{p}{f} = \frac{2\,R^3 - 2r^3}{R^3 + 2r^3};\dots\dots\dots\dots(1.)$$

$$\frac{R}{r} = \sqrt[3]{\left(\frac{2f + 2p}{2f - p}\right)}\dots\dots\dots\dots(2.)$$

154. **Lateral Pliability, Cubic Compressibility.**—When the side surfaces of a bar are free, the application of tension to its ends causes it to contract in thickness as well as to extend in length; and the application of pressure to its ends causes it to expand in thickness as well as to contract in length. This property, which may be called "Lateral Pliability," exists to the greatest possible extent in perfect fluids, whose parts yield laterally to the slightest longitudinal stress, and is least in those solids which are least capable of changes of figure.

If a solid bar has the alteration of its transverse dimensions prevented or resisted by any means, it yields less longitudinally to a longitudinal stress than it does when it is free to yield laterally; in other words, its direct or longitudinal stiffness may be increased; and that according to laws whose mathematical expression will presently be given. Its strength is increased also; but in what proportion is not yet known precisely.

Let p denote the intensity of a longitudinal stress, not exceeding the limit below which moduli of stiffness are constant; and when the bar is free to alter its lateral dimensions, let,

$\alpha = \frac{p}{E}$ denote the fraction of its original length by which its length is altered, and

$\beta = \frac{p}{D}$, the fraction of its original diameter by which its diameter is altered in the contrary direction to its length; then we have

$$p = A\,\alpha - B\,\beta;\dots\dots\dots\dots(1.)$$

A and B being two co-efficients whose relations to E and D are expressed by the two following pairs of equations:—

$$\left.\begin{aligned} A &= \frac{E\,D\,(D-E)}{D^2 - E\,D - 2\,E^2} = E + \frac{2\,E^3}{D^2 - E\,D - 2\,E^2}; \\ B &= \frac{E^2\,D}{D^2 - E\,D - 2\,E^2};\ \frac{B}{A} = \frac{E}{D-E}; \end{aligned}\right\} \quad (2.)$$

$$E = A - \frac{2\,B^2}{A+B};\ D = A - 2\,B + \frac{A^2}{B};\ \frac{D}{E} = \frac{A+B}{B}. \quad (3.)$$

The co-efficient A represents the resistance of the bar to direct elongation or compression when $\beta = 0$; that is, when the transverse dimensions of the bar are prevented from changing.*

The resistance to alteration of volume bears the following relations to the before-mentioned co-efficients. Let p denote the uniform intensity of a pressure or tension applied *all over the surface* of a body, and δ the fraction of the original volume by which the volume is diminished or increased; then the *cubic elasticity* is

$$\frac{p}{\delta} = \frac{A + 2\,B}{3} = \frac{3\,D - 6\,E}{D\,E}; \ldots\ldots\ldots\ldots\ldots (4.)$$

and the reciprocal of this is the *cubic compressibility.*

The values of A and B have been ascertained for a few substances only. For brass and crystal, according to M. Wertheim's experiments (*Annales de Chimie*, third series, vol. xxiii.), the following ratios hold very nearly:—

$$A \div B = 2;\ D \div E = 3;\ A \div E = \frac{3}{2};\ B \div E = \frac{3}{4}; \ldots (5.)$$

and consequently, for those substances,

$$\frac{p}{\delta} = \frac{2}{3}\,A = E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

155. **Heights of Moduli of Stiffness and Strength.**—The term "*Height*," as applied to a given modulus, whether of stiffness or

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* A is the co-efficient of elasticity deduced from experiments on the velocity of sound in a solid body of large transverse dimensions, by means of the following formula, in which v is the velocity of sound in feet per second, and w the weight of a cubic foot of the body;

$$\text{A in lbs. on the square foot} = \frac{w\,v^2}{32 \cdot 2}.$$

When the transverse dimensions of the body are narrow, this formula gives results lying between the value of A and that of E.

of strength, means the height of an imaginary column of the substance to which the modulus belongs, whose weight would cause a pressure on its base, equal in intensity to the stress expressed by the given modulus. Hence,

Height of a modulus in feet

$$= \frac{\text{Modulus in lbs. on the square foot}}{\text{Heaviness of substance in lbs. to the cubic foot}}$$

$$= \frac{\text{Modulus in lbs. on the square inch}}{\text{Weight of 12 cubic inches of substance}};$$

Height of a modulus in inches

$$= \frac{\text{Modulus in lbs. on the square inch}}{\text{Weight of a cubic inch of substance}}.$$

156. **Resistance to Shearing and Distortion.** (*A. M.*, 278 to 281.)—In structures, many cases occur in which the principal pieces, such as plates, links, bars, or beams, being themselves subjected to a direct pull, are connected with each other at their joints by fastenings, such as rivets, bolts, pins, keys, or screws, which are under the action of a shearing force, tending to make them give way by the sliding of one part over another.

The present Article refers to those cases only in which the shearing stress on a body is uniform in direction and in intensity. The effects of shearing stress varying in intensity will be considered under the head of Resistance to Bending, which is in general accompanied by such a stress; and the effects of shearing stress varying in direction as well as in intensity under the head of Resistance to Torsion.

To insure uniform distribution of the stress, it is necessary that the rivet or other fastening should fit so tight in its hole or socket, that the friction at its surface may be at least of equal intensity to the shearing stress. When this condition is fulfilled, the intensity of that stress is represented simply by $F \div S$, F being the shearing force, and S the area which resists it.

In consequence of the relation between shearing stress and direct stress, stated in Article 108, Division II., p. 167, it appears that a body may give way to a shearing stress either by actual shearing, at a plane parallel to the direction of the shearing force, or by tearing in a direction making an angle of 45° with that force.

When a shearing stress does not exceed the limit within which moduli of stiffness are sensibly constant, it produces distortion of the body on which it acts. Let q denote the intensity of a shearing stress applied to the four lateral faces of an originally square prismatic particle, so as to distort it; and let ν be the ***distortion***, expressed by the ***tangent of the difference between each of the distorted angles of the prism and a right angle;*** then

$$\frac{q}{\nu} = \mathrm{C}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

is the ***modulus of transverse elasticity***, or ***resistance to distortion.***

One mode of expressing the distortion of an originally square prism is as follows:—Let α denote the proportionate elongation of one of the diagonals of its end, β the proportionate shortening of the other; then the distortion is

$$\nu = \alpha + \beta.$$

The co-efficient C is necessarily related to those mentioned in Article 154, pp. 229, 230, in the following manner:—

$$\mathrm{C} = \frac{\mathrm{A} - \mathrm{B}}{2} = \frac{\mathrm{E\,D}}{2\,(\mathrm{D} + \mathrm{E})}. \ldots\ldots\ldots\ldots\ldots(2.)$$

For brass and crystal, according to M. Wertheim's experiments, we have

$$\mathrm{C} = \frac{\mathrm{A}}{4} = \frac{\mathrm{B}}{2} = \frac{3\,\mathrm{E}}{8} = \frac{\mathrm{D}}{8}. \ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

The co-efficient C expresses the quality of rigidity, or resistance to change of figure, which distinguishes solids from fluids.

In a perfect fluid, the following relations hold:—

$$\left.\begin{array}{l}\mathrm{C} = 0;\ \mathrm{E} = 0;\ \mathrm{D} = 0;\\ \mathrm{A} = \mathrm{B} = \dfrac{p}{\delta}\ (\text{the cubic elasticity}).\end{array}\right\} \ldots\ldots\ldots\ldots(4.)$$

The general effect of heat on solid bodies is to diminish C and increase $p \div \delta$.

157. **Resistance to Compression and Direct Crushing.** (*A. M.*, 282 to 286.)—Resistance to *Longitudinal Compression*, when the proof stress is not exceeded, is sensibly equal to the resistance to stretching, and is expressed by the same modulus. When that limit is exceeded, it becomes irregular.

Crushing, or breaking by compression, is not a simple phenomenon like tearing, but is more or less complex and varied, according to the nature of the substance.

The present Article has reference to direct crushing only, and is limited to those cases in which the pillars, blocks, struts, or rods, along which the pressure acts are not so long in proportion to their diameter as to have a sensible tendency to give way by bending sideways. Those cases comprehend—

Stone and brick pillars and blocks of ordinary proportions;

Pillars, rods, and struts of cast iron, in which the length is not more than five times the diameter, approximately;

Pillars, rods, and struts of wrought iron, in which the length is not more than ten times the diameter, approximately;

Pillars, rods, and struts of dry timber, in which the length is not more than about twenty times the diameter.

Let P denote the *crushing load* of the piece;

S the area of its transverse section in square inches;

f the resistance of the material to crushing, in lbs. on the square inch; then *if the load is uniformly distributed*,

$$P = f S \text{ ....} (1.)$$

A table of the resistance of materials to direct crushing, in lbs. on the square inch, is given at the end of the volume.

If the load is not uniformly distributed over the transverse section of the pillar, the strength of the pillar is diminished in the same ratio in which the mean intensity of the stress is less than the maximum intensity. To find that ratio, it is sufficiently near the truth for practical purposes to consider the stress as "*uniformly varying.*" (See Article 106, Division II., p. 163, equations 5, 6, 7.) Suppose the pillar to be cylindrical, square, or of a regular polygonal figure in cross-section. Let x_0 be the greatest deviation of the centre of pressure from the centre of figure in any cross-section; that is, the greatest deviation of the line of action of the load from the axis of the pillar.

Let x_1 be the distance of the point of greatest stress from the axis of the pillar; that is, the semidiameter of the pillar in the direction in which the load deviates from the axis.

Let $I = \int x^2 y \, d x$ denote what is called the "moment of inertia" of the cross-section of the pillar.

Then the crushing load is,

$$P = \frac{f S}{1 + \frac{x_0 x_1 S}{I}} \text{ ....} (2.)$$

The following are some of the values of $\frac{x_1 S}{I}$ in the preceding formula: the "*neutral axis*" meaning the diameter to which the deviation x_0 is perpendicular.

FIGURE OF CROSS-SECTION.	$\frac{x_1 S}{I}$.
I. Rectangle, hb; b, neutral axis, II. Square, h^2, ...	$\frac{6}{h}$.
III. Ellipse: neutral axis, b; other axis, h; IV. Circle: diameter, h, ...	$\frac{8}{h}$.
V. Hollow rectangle: outside dimensions, h, b; inside dimensions, h', b'; neutral axis, b,	$\frac{6h(hb - h'b')}{h^3 b - h'^3 b'}$.
VI. Hollow square, $h^2 - h'^2$, ...	$\frac{6h}{h^2 + h'^2}$.
VII. Circular ring: diameter, outside, h; inside, h',	$\frac{8h}{h^2 + h'^2}$.

It is often advisable, especially in masonry, so to limit the deviation of the centre of pressure from the axis of the pillar, that there shall be *no tension* on any part of it. This condition is fulfilled when the least pressure is positive, or nothing, and the greatest stress not more than double of the mean stress, so that $P \leq fS \div 2$; and consequently, when

$$x_0 \leq \frac{I}{x_1 S}, \qquad (3.)$$

the reciprocal of the quantity of whose values examples have just been given.

The modulus of resistance to direct crushing, as the tables show, often differs considerably from the tenacity. The nature and amount of those differences depend mainly on the modes in which the crushing takes place. These may be classed as follows:—

I. *Crushing by splitting* (fig. 121), into a number of nearly prismatic fragments, separated by smooth surfaces whose general direction is nearly parallel to the direction of the load, is characteristic of hard homogeneous substances of a glassy texture, such as vitrified bricks.

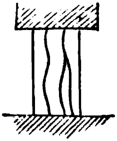

Fig. 125.

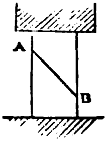

Fig. 126.

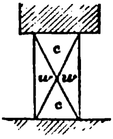

Fig. 127.

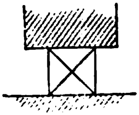

Fig. 128.

II. *Crushing by shearing or sliding* of portions of the block along oblique surfaces of separation is characteristic of substances of a granular texture, like cast iron, and most kinds of stone and brick. Sometimes the sliding takes place at a single plane surface, like AB in fig. 126; sometimes two cones or pyramids are formed, like c, c, in fig. 127, which are forced towards each other, and split or drive outwards a number of wedges surrounding them, like *w*, *w*, in the same figure. Sometimes the block splits into four wedges, as in fig. 128.

The surfaces of shearing make an angle with the direction of the crushing force, which Mr. Hodgkinson (who first fully investigated those phenomena) found to have values depending on the kind and quality of material. For different qualities of cast iron, for example, that angle ranges from 42° to 32°. The greatest intensity of shearing stress is on a plane making an angle of 45° with the direction of the crushing force; and the deviation of the plane of shearing from that angle shows that the resistance to shearing is not purely a cohesive force, independent of the normal pressure at the plane of shearing, but consists partly of a force analogous to friction, increasing with the intensity of the normal pressure.

Mr. Hodgkinson considers that in order to determine the true resistance of substances to direct crushing, experiments should be made on blocks in which the proportion of length to diameter is not less than that of 3 to 2, in order that the material may be free to divide itself by shearing. When a block which is shorter in proportion to its diameter is crushed, the friction of the flat surfaces between which it is crushed has a perceptible effect in *holding its parts together*, so as to resist their separation by shearing; and thus the apparent strength of the substance is increased beyond its real strength.

In all substances which are crushed by splitting and by shearing, the resistance to crushing considerably exceeds the tenacity, as the tables show. The resistance of cast iron to crushing, for example, was found by Mr. Hodgkinson to be somewhat more than *six* times its tenacity.

III. *Crushing by bulging*, or lateral swelling and spreading of the block which is crushed, is characteristic of ductile and tough materials, such as wrought iron. Owing to the gradual manner in which materials of this nature give way to a crushing force, it is difficult to determine their resistance to that force exactly. That resistance is in general less, and sometimes considerably less, than the tenacity. In wrought iron, the resistance to the direct crushing of short blocks, as nearly as it can be ascertained, is from $\frac{2}{3}$ to $\frac{4}{5}$ of the tenacity.

IV. *Crushing by buckling or crippling* is characteristic of fibrous substances, under the action of a thrust along the fibres. It consists in a lateral bending and wrinkling of the fibres, sometimes accompanied by a splitting of them asunder. It takes place in timber, in plates of wrought iron, and in bars longer than those which give way by bulging. The resistance of fibrous substances to crushing is in general considerably less than their tenacity, especially where the lateral adhesion of the fibres to each other is weak compared with their tenacity. The resistance of most kinds of timber to crushing, when dry, is from $\frac{1}{2}$ to $\frac{2}{3}$ of the tenacity. Moisture in the timber weakens the lateral adhesion of the fibres, and reduces the resistance to crushing to about one-half of its amount in the dry state.

158. **Crushing by Cross-breaking** is the mode of fracture of columns and struts in which the length greatly exceeds the diameter. Under the breaking load, they yield sideways, and are broken across like beams under a transverse load.

The laws of this mode of breaking were investigated experimentally by Mr. Hodgkinson. The following are his formulæ for cast iron cylindrical pillars:—

When the length is not less than thirty times the diameter; for solid pillars, let h be the diameter *in inches*, L the length *in feet*, and A a constant multiplier; then

$$\text{Breaking load} = \mathrm{A}\, h^{3.6} \div \mathrm{L}^{1.7}; \ldots\ldots\ldots\ldots\ldots (1.)$$

for hollow pillars, let h be the external and h' the internal diameter, in inches; then

$$\text{Breaking load} = \mathrm{A}\, (h^{3.6} - h'^{3.6}) \div \mathrm{L}^{1.7}. \ldots\ldots\ldots (2.)$$

The values of the co-efficient A are as follows:—

	Tons.
for solid pillars with rounded ends,	14·9
„ „ „ fixed ends,	44·16
for hollow pillars with rounded ends,	13·0
„ „ „ fixed ends,	44·3.

The strength of a pillar with one end fixed and the other rounded, is nearly a mean between the strength of two pillars of the same dimensions, one with both ends fixed, and the other with both ends rounded.

When the length is less than thirty times the diameter; let b denote the breaking load of the pillar, computed by the preceding formulæ:—

Let c be the crushing load of a short block of the same sectional area, = 49 tons × sectional area in square inches; then

$$\text{Actual breaking load of pillar} = \frac{4\,b\,c}{4\,b + 3\,c}. \quad \ldots\ldots.(3.)$$

The following are formulæ deduced by Mr. Lewis Gordon from Mr. Hodgkinson's experiments:—

Let P be the crushing load of a long rod or pillar, in lbs.;
S the sectional area of material in it, in square inches;
l, its length,
h, its least external diameter, } both in the same units of measure.
Then, approximately—

$$P = \frac{f\,S}{1 + a \cdot \frac{l^2}{h^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

The following are the values of f and a, for pillars fixed at both ends, by having flat capitals and bases:—

	f, lbs. per inch.	a.
Wrought iron,	36,000	$\frac{1}{3{,}000}$
Cast iron,	80,000	$\frac{1}{400}$

For a pillar rounded or jointed at both ends,

$$P = \frac{f\,S}{1 + 4\,a\,\frac{l^2}{h^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.)$$

In using the formulæ, the ratio $\frac{l}{h}$ is generally fixed beforehand, to a degree of approximation sufficient for the purposes of the calculation.

In wrought iron framework and machinery, the bars which act as struts, in order that they may have sufficient stiffness, are made of various forms in cross-section, well known as "angle iron," "channel iron," "T-iron," "double T-iron," &c. In each of these forms, the line to be considered as represented by h in the formulæ is the diameter, in that direction in which the bar is most flexible, of a triangle or rectangle circumscribed about its section.

Wrought iron cells are rectangular tubes (generally square) usually composed of four plate iron sides, rivetted to angle iron bars at the corners. The *ultimate resistance* of a single square cell to crushing by the buckling or bending of its sides, when the thickness of the plates is *not less than one-thirtieth of the diameter of the cell,* as determined by Mr. Fairbairn and Mr. Hodgkinson, is

27,000 lbs. per square inch section of iron;

but when a number of cells exist side by side, their stiffness is increased, and their ultimate resistance to a thrust may be taken at

33,000 to 36,000 lbs. per square inch section of iron.

The latter co-efficients apply also to cylindrical cells.

The following is Mr. Hodgkinson's formula for the breaking load of posts and struts of oak and red pine, when exposed to crushing by cross breaking:—

$$P = A \frac{h^2}{l^2} S; \qquad (6.)$$

S being the sectional area in square inches, $h:l$ the ratio of the least diameter to the length, and $A = 3{,}000{,}000$ lbs. per square inch.

The *factor of safety* for the working load of timber is 10.

For square posts and struts, the formula becomes

$$P = A \frac{h^4}{l^2} \qquad (7.)$$

If the strength of a timber post be computed both by this formula and by the formula for direct crushing, viz.:—

$$P = f S, \qquad (8.)$$

the *lesser* value should be adopted as the true strength.

159. **Resistance to Collapsing.**—When a thin hollow cylinder is pressed from without, it gives way by *collapsing,* under a pressure whose intensity has been found by Mr. Fairbairn (*Philos. Trans.,* 1858)* to vary nearly according to the following laws:—

Inversely as the length;

Inversely as the diameter;

Directly as a function of the thickness, which is very nearly the power whose index is 2·19; but which for ordinary practical purposes may be treated as sensibly equal to the *square* of the thickness.

The following formula gives approximately the *collapsing pressure* p, in lbs. on the square inch, of a plate iron flue with butt-joints, whose length l, diameter d, and thickness t, are all expressed in *the same units of measure*:—

---

* See also *Useful Information for Engineers,* second series, 1860.

$$p = 9{,}672{,}000 \frac{t^2}{l\,d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

Let t and d be expressed in inches, and let L be the length in feet; the above formula becomes

$$p = 806{,}000 \frac{t^2}{\mathrm{L}\,d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

Mr. Fairbairn having strengthened tubes by rivetting round them rings of T-iron, or angle iron, at equal distances apart, finds that their strength is that corresponding to the length *from ring to ring*.

Mr. Fairbairn finds that the collapsing pressure of a tube of an elliptic form of cross-section is found approximately by substituting for d, in the preceding formulæ, the diameter of the osculating circle at the flattest part of the ellipse; that is, let a be the greater, and b the less *semi-axis* of the ellipse; then we are to make

$$d = \frac{2\,a^2}{b} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

160. **Action of a Transverse Load on a Beam.** (*A. M.*, 288.)—It has already been shown, in Article 112, p. 174, how to determine the proportions between the resultant of the gross load of a beam and the two forces which support it. In the present Article those cases alone will be considered in which the loading and supporting forces are parallel to each other, and in one plane.

In Article 122, p. 184, it has been shown how to determine the resistances exerted by the pieces of a frame which are cut by an ideal sectional plane, in terms of the forces and couples which act on one of the portions into which that plane of section divides the frame.

The method followed in determining the effect of a transverse load on a continuous beam is similar; except that the resistance at the plane section, which is to be determined, does not consist of a finite number of forces acting along the axes of certain bars, but of a distributed stress, acting with various intensities, and, it may be, in various directions, at different points of the section of the beam.

In what follows, the load of the beam will be conceived to consist of weights acting vertically downwards, and the supporting forces will also be conceived to be vertical. The longitudinal axis of the beam being perpendicular to the applied forces, will accordingly be horizontal. The conclusions arrived at are applicable to cases in which the axis of the beam and the direction of the applied forces are inclined, so long as they are perpendicular to each other.

Let any point in the longitudinal axis be taken as the origin of co-ordinates; and at a given horizontal distance x' from that origin, conceive a vertical section perpendicular to the longitudinal axis to divide the beam into two parts.

Let F denote the resultant of all the vertical forces, whether loading or supporting, which act on the part of the beam to the left of the vertical plane of section, and let x'' be the horizontal distance of the line of action of that resultant from the origin.

If the beam is strong enough to sustain the forces applied to it, there will be a *shearing stress* equal and opposite to F, distributed (in what manner will afterwards appear) over the given vertical section; and that shearing stress, or vertical resistance, will constitute, along with the resultant applied force F, a couple whose moment is

$$M = F\,(x'' - x') \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

This is called the *bending moment* or *moment of flexure* of the beam at the vertical section in question; it is resisted by the direct stress at that section, in a manner to be explained in the sequel; and it tends to make the originally straight longitudinal axis of the beam become concave in the direction towards which the resultant applied force F acts.

The mathematical process for finding F and M at any given cross-section of a beam, though always the same in principle, may be varied considerably in detail. The following is on the whole the most convenient way of conducting it.

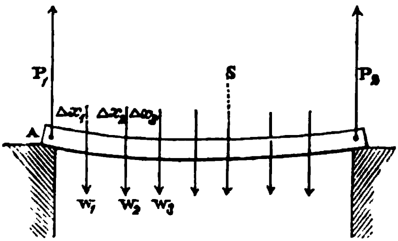

Fig. 129.

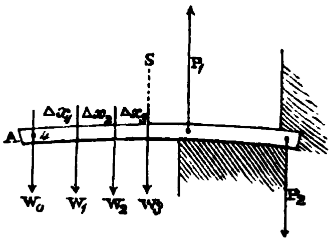

Fig. 130.

Fig. 129 represents a beam *supported* at both ends, and loaded between them. Fig. 130 represents a beam *supported* and *fixed* at one end, and loaded on a projecting portion. P_1, P_2, represent in each case the supporting forces; in fig. 129, W_1, W_2, W_3, &c., represent portions of the load; in fig. 130, W_0 represents the endmost portion of the load, and W_1, W_2, W_3, other portions; in both figures, Δx_1, Δx_2, Δx_3, &c., denote the lengths of the parts into which the lines of action of the portions of the load divide the horizontal axis of the beam.

The figures represent the load as applied at detached points; but when it is continuously distributed, the length of any indefinitely short portion of the beam will be denoted by $d\,x$, the intensity of the load upon it *per unit of length* by w, and the amount of the load upon it by $w\,d\,x$.

The process to be gone through will then consist of the following steps:—

STEP I. *To find the Supporting Forces.*—Assume any convenient point in the horizontal axis as origin of co-ordinates, and find the distance x_0 of the resultant of the load from it, by the method of Article 97, Division VIII, p. 143, for forces acting through detached points, and by the method of Article 104, Division V., p. 153, for a continuously distributed load; that is to say,

$$\left.\begin{aligned} x_0 &= \frac{\Sigma \cdot x\,W}{\Sigma \cdot W}; \text{ or} \\ x_0 &= \frac{\int x\,w\,d\,x}{\int w\,d\,x}. \end{aligned}\right\} \quad \text{.......................(2.)}$$

Then, as in Article 112, p. 174, find the two supporting forces, P_1 and P_2; that is to say, if x_1 and x_2 be the distances of the points of support from the origin, with their proper signs, make

$$\left.\begin{aligned} P_1 &= \frac{x_2 - x_0}{x_2 - x_1}\,\Sigma \cdot W \left(\text{or} \int w\,d\,x\right); \\ P_2 &= \frac{x_0 - x_1}{x_2 - x_1}\,\Sigma \cdot W \left(\text{or} \int w\,d\,x\right). \end{aligned}\right\} \quad \text{...........(3.)}$$

STEP II. *To find the Shearing Forces at a Series of Sections.*—In what position soever the origin of co-ordinates may have been during the previous step, assume it now, in a beam supported at both ends, to be at one of the points of support (as A, fig. 129), and in a beam fixed at one end, to be at the loaded point farthest from the fixed end (as A, fig. 130). Consider upward forces as positive, downward as negative.

Then the shearing force at any given cross-section of the beam is the resultant of all the forces acting on the beam from the origin to that cross-section; so that at the series of points where P_1, W_1, W_2, W_3, &c., act in fig. 129, and where W_0, W_1, W_2, W_3, &c., act in fig. 130, it has the series of values,

In Fig. 129.	In Fig. 130.
$F_0 = P_1$;	$-F_0 = W_0$;
$F_1 = P_1 - W_1$;	$-F_1 = W_0 + W_1$;
$F_2 = P_1 - W_1 - W_2$;	$-F_2 = W_0 + W_1 + W_2$;
$F_3 = P_1 - W_1 - W_2 - W_3$;	$-F_3 = W_0 + W_1 + W_2 + W_3$;
&c.;	&c.;
and generally,	and generally
$F = P_1 - \Sigma \cdot W$;...(4.)	$-F = \Sigma \cdot W$; ....(5.)

so that the shearing forces at a series of sections can be computed by successive subtractions or successive additions, as the case may be.

For a continuously distributed load, these equations became respectively,

$$\text{In a beam supported at both ends, } F = P_1 - \int_0^{x'} w\,dx; \text{(6.)}$$

$$\text{In a beam fixed at one end } -F = \int_0^{x'} w\,dx; \text{.....(7.)}$$

in which expressions, x' denotes the distance from the origin A to the plane of section.

The symbol $-F$ denotes that the shearing force is downward.

The **Greatest Shearing Force** acts in a beam supported at both ends, close to one or other of the points of support, and its value is either P_1 or P_2. In a beam fixed at one end the greatest shearing force on the projecting part acts close to the outer point of support, and its value is equal to the entire load.

In a beam supported at both ends, the **Shearing Force vanishes** at some intermediate section, whose position may be found from equation 4 or equation 6, by making $F = 0$.

STEP III. *To find the Bending Moments at a Series of Sections.*—At the origin A there is no bending moment. Multiply the length of each of the divisions Δx of the longitudinal axis of the beam by the shearing force F, which acts at the outer end of that division; the first of the products so obtained is the bending moment at the inner end of the first division; and by adding to it the other products successively, there are obtained successively the bending moments at the inner ends of the other divisions in succession.*

That is to say,—bending moment,

---

* This process is substantially the same with that employed by Mr. Herbert Latham, in his work *On Iron Bridges*, to compute the stress in a half-lattice girder.

at the origin $A = 0$;
at the line of action of W_1; $M_1 = F_0 \cdot \Delta x_1$;
" " " W_2; $M_2 = F_0 \cdot \Delta x_1 + F_1 . \Delta x_2$;
" " " W_3; $M_3 = F_0 \cdot \Delta x_1 + F_1 . \Delta x_2 + F_2 \cdot \Delta x_3$;
&c. &c.

$$\text{and generally, } M = \Sigma \cdot F \,\Delta\, x. \quad \text{.............(8.)}$$

If the divisions $\Delta\, x$ *are of equal length*, this becomes

$$M = \Delta\, x \cdot \Sigma\, F; \quad \text{......................(9.)}$$

and for a continuously distributed load,

$$M = \int_0^x F\, d\, x. \quad \text{.......................(10.)}$$

The three preceding equations, 8, 9, and 10, are applicable to beams whether supported at both ends or fixed at one end. By substituting for F in equation 10 its values as given by equations 6 and 7 respectively, we obtain the following results:—

For a beam supported at both ends,

$$M = P_1\, x' - \int_0^{x'} \int_0^x w\, d\, x$$

$$= P_1\, x' - \int_0^{x'} (x' - x)\, w\, d\, x; \quad \text{............(11.)}$$

For a beam fixed at one end,

$$-M = \int_0^{x'} \int_0^x w\, d\, x = \int_0^{x'} (x' - x)\, w\, d\, x; \quad \text{.......(12.)}$$

in the latter of which equations, the symbol $-M$ denotes that the bending moment acts downwards.

The **Greatest Bending Moment** acts, in a beam fixed at one end, at the outer point of support; and in a beam supported at both ends, at the section where the shearing force vanishes, found, as already stated in Step II., from the equation $F = 0$.

When the load on a beam supported at both ends is symmetrically distributed relatively to its middle section, the Greatest Bending Moment acts at that section; and it is sometimes convenient to assume a point in that section as the origin of co-ordinates.

STEP IV. *To find the effect of combining several Loads on one Beam, whose separate actions are known;*—for each cross-section, the shearing force is the sum of the shearing forces, and the bending moment the sum of the bending moments, which the loads would produce separately.

STEP V. *To deduce the Shearing Force and Bending Moment in one Beam from those in another Beam similarly supported and loaded.*—This is done by the aid of the following principles:—

When beams differing in length and in the amounts of the loads upon them are similarly supported, and have their loads similarly distributed, the shearing forces at corresponding sections in them vary as the total loads, and the bending moments as the products of the loads and lengths.

This principle may be expressed by symbols in either of the two following ways:—

First, Let l, l', denote the lengths of two beams, similarly supported; let W, W', denote their total loads, similarly distributed; let F, F', be the shearing forces, and M, M', the bending moments, at sections similarly situated in the two beams; then

$$\mathrm{W} : \mathrm{W}' :: \mathrm{F} : \mathrm{F}'; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.)$$

$$l\,\mathrm{W} : l'\,\mathrm{W}' :: \mathrm{M} : \mathrm{M}'. \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14.)$$

Secondly, Let k and m be two numerical factors, depending on the way in which a beam is supported, the mode of distribution of its load, and the position of the cross-section under consideration; then

$$\mathrm{F} = k\,\mathrm{W}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15.)$$

$$\mathrm{M} = m\,\mathrm{W}\,l. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16.)$$

Loads upon beams are stated either in pounds, hundredweights, or tons; lengths of beams, in feet, or in inches; and according to the units of load and length employed, the unit of bending moment is a *foot-pound*, an *inch-pound*, a *foot-hundredweight*, an *inch-hundredweight*, a *foot-ton*, or an *inch-ton*, as the case may be.

As the transverse dimensions of beams are expressed in inches, and their moduli of strength in pounds on the square inch, the most convenient units are, the pound, the inch, and the inch-pound.

The following is a comparison of different *unit of bending moment*.

Inch-lbs.
12 = 1 Ft.-lb.
112 = 9⅓ = 1 Inch-cwt.
1344 = 112 = 12 = 1 Foot-cwt.
2240 = 186⅔ = 20 = 1⅔ = 1 Inch-ton.
26880 = 2240 = 240 = 20 = 12 = 1 Foot-ton.

161. Examples of the Action of a Transverse Load on a Beam.

TABLE OF EXAMPLES.

CASES.	F_l	x'_1	k	M_O	x'_0	m
A. **Beams fixed at one end.**						
I. Loaded at extreme end with W,...............	$-W$	anywhere	-1	$-Wx'$	l	-1
II. Uniform load of intensity $w = W \div l$	$-wx'$	l	-1	$-\frac{wx'^2}{2}$	l	$-\frac{1}{2}$
III. Uniform load of intensity w, and additional load W′ at extreme end,.........	$-W' - wx'$	l	-1	$-W'x' - \frac{wx'^2}{2}$	l	$-\frac{W' + \frac{wl}{2}}{W' + wl}$
B. **Beams supported at both ends.**						
IV. Single load W, in the middle;						
half of beam next origin,	$\frac{W}{2}$	0	$\frac{1}{2}$	$\frac{Wx'}{2}$	$\frac{l}{2}$	$\frac{1}{4}$
farther half, ...	$-\frac{W}{2}$	l	$-\frac{1}{2}$	$\frac{W(l-x')}{2}$		

TABLE OF EXAMPLES—*continued.*

CASES.	F	x'_1	k	M	x'_0	m
V. Single load W, applied at x'';						
between x'' and origin;	$\frac{l-x''}{l}W$	anywhere	$\frac{l-x''}{l}$	$\frac{x'(l-x'')}{l}W$	x''	$\frac{x''(l-x'')}{l^2}$
beyond x'';	$-\frac{x''W}{l}$	anywhere	$-\frac{x''}{l}$	$\frac{(l-x')x''}{l}W$		
VI. Uniform load of intensity $w = W \div l$,	$w\left(\frac{l}{2}-x'\right)$	0 and l	$\pm\frac{1}{2}$	$\frac{wx'(l-x')}{2}$	$\frac{l}{2}$	$\frac{1}{8}$
VII. Partial load of uniform intensity $w = W \div x''$ from 0 to x''; remainder unloaded;						
between x'' and origin;	$w\left(x''-\frac{x''^2}{2l}-x'\right)$	0	$1-\frac{x''}{2l}$	$w\left\{\left(x''-\frac{x''^2}{2l}\right)x' - \frac{x'^2}{2}\right\}$	$x''-\frac{x''^2}{2l}$	$\frac{x''}{2l}\left(1-\frac{x''}{2l}\right)^2$
beyond x''..............	$-\frac{wx''^2}{2l}$			$\frac{wx''^2}{2l}(l-x')$		

—In each example in the preceding table, l denotes, for a beam fixed at one end, the length measured from the outer point of support to the farthest projecting loaded point, and for a beam supported at both ends, the length or *span* between the points of support; W denotes the total load; F and M denote the shearing force and bending moment at any cross-section situated at the distance x' from the origin (which, as in the preceding article, is the point where $M = 0$); F_1 denotes the greatest shearing force; x'_1 the position of the section where it occurs; M_0 the greatest bending moment; x'_0 the position of the cross-section where it occurs; k, m, the two factors employed in equations 15 and 16, for the sections of greatest shearing stress and greatest bending moment respectively; that is to say,

$$k = F_1 \div W; \; m = M_0 \div W\,l. \quad \ldots\ldots\ldots\ldots\ldots (1.)$$

To transform the expressions in the preceding table, Cases IV. to VII., which are suited for co-ordinates measured from one point of support of a beam supported at both ends, into expressions suited for co-ordinates measured from the middle of the beam, let c be the *half-span*, and substitute $2\,c$, $x' - c$, $x'_1 - c$, $x'_0 - c$, and $x'' - c$, respectively, for l, x', x'_1, x'_0, and x'', throughout the whole of that part of the table.

In the following example, a beam supported at both ends is supposed to be loaded at a series of detached points, which divide the length of the beam into N equal divisions, so that the length of one of those divisions is $l \div N$. The origin of co-ordinates being at a point of support, the plane of section in each example is supposed to be immediately *beyond* the n^{th} division from that point.

CASE.	F	x'_1	k	M	x'_0	m
VIII. Each intermediate point loaded with w; total load $(N - 1)\,w$.	$\left(\frac{N-1}{2} - n\right) . w$	0 or $N - 1$.	$\pm \frac{1}{2}$	$\frac{n\,(N - n)}{2\,N}\,l\,w$	$\frac{l}{2}$	(N even) $\frac{1}{8}$ (N odd) $\frac{N^2 - 1}{8\,N^2}$

CASE IX. *Travelling Load.*—A beam of the span l is supported at the two ends; a permanent load of the uniform intensity of w lbs. per lineal foot is distributed over it. An additional load, such as the weight of a railway train, of w' lbs. per lineal foot, gradually rolls on to the beam from one end, covering it at last from end to end, and then rolls off again at the other end. It is required to find the greatest shearing force at any given section, and also the greatest bending moment.

The *Greatest Shearing Force* at a given cross-section occurs when the longer of the two segments into which it divides the beam is loaded with the travelling load as well as with the permanent load, and the shorter loaded with the permanent load only. Let F' denote that force, and x the distance of the section in question from the nearer end of the beam; then

$$F' = w\left(\frac{l}{2} - x\right) + \frac{w'(l-x)^2}{2l} \text{..............} (2.)$$

This formula may be rendered somewhat more symmetrical by taking the *middle* of the beam, instead of one of the ends, as origin of co-ordinates. Let x', then, denote the distance of the section in question from the middle of the beam, and $c = l \div 2$, its half-span; then $x = c - x'$; and

$$F' = w\,x' + \frac{w'(c+x')^2}{4c} \text{. ....} (3.)$$

The *Greatest Bending Moment* at each section occurs when the travelling load extends over the whole length of the beam; so that in this respect no difference exists between the present case and case VI. of the preceding table; that is to say,

$$M = \frac{(w+w')\,x\,(l-x)}{2} = \frac{(w+w')\,(c^2 - x'^2)}{2} \text{. ....} (4.)$$

The preceding principles are represented graphically by the following construction. In fig. 131, let A B represent the span of the beam; bisect it in C. Through A draw D A E perpendicular to A B, and take A D to represent *half the greatest permanent load*, that is, A D $= w\,l \div 2$, and A E to represent on the same scale *half the greatest travelling load*, that is, A E $= w'\,l \div 2$. Draw the straight line C D; also the parabola B L E, having its vertex at B, touching A B. About C describe the semicircle A F B, and consider C F^2, or the square of the radius of that semicircle perpendicular to A B, as representing the *greatest bending moment at the middle of the beam*, or $(w + w')\,l^2 \div 8$. Let G represent the position of any cross-section of the beam. Draw H G I K perpendicular to A B; then H I will represent the greatest

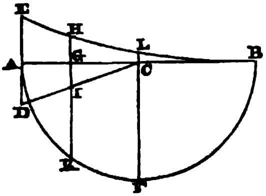

Fig. 131.

shearing force at that section, and $G K^2$ the greatest bending moment.

The ordinate $C L$ of the parabola at the middle of the span, being one-fourth of $A E$, represents the greatest shearing force in the middle of the beam, which is one-eighth of the greatest travelling load, or $w' l \div 8$.

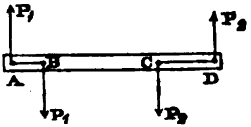

Fig. 132.

CASE X.—If a beam has *equal and opposite couples* applied to its two ends, —for example, if the beam in fig. 132 has the couple of equal and opposite forces P_1 applied at A and B, and the couple of equal and opposite forces P_2 at C and D, and if the opposite moments,

$$P_1 \cdot A B = P_2 \cdot C D = M, \ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

are equal, then each of the endmost divisions of the beam, A B and C D, is in the condition of a beam fixed at one end and loaded at the other (Case I.); and the middle division B C is subject to the *uniform moment of flexure* M, and to no shearing force.

162. **Resistance to Cross-Breaking—Transverse Strength.** (*A. M.*, 293 to 295, 309, 310.)—The bending moment at each cross-section of a beam bends the beam so as to make any originally plane layer of the beam, perpendicular to the direction of the load, become concave in the direction towards which the moment acts, and convex in the opposite direction. Thus, fig. 133 represents a side view of a short portion of a beam supported at the ends and loaded at intermediate points; C C′ is a layer, originally plane, and perpendicular to the direction of the load, which is now bent so as to become concave above and convex below.

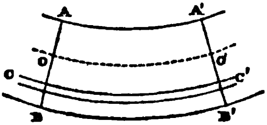

Fig 133.

The layers at and near the concave side of the beam A A′ are shortened, and the layers near the convex side B B′ lengthened, by the bending action of the load. There is one intermediate surface O O′ which is neither lengthened nor shortened; it is called the "*neutral surface.*" The particles at that surface are not necessarily, however, in a state devoid of strain; for, in common with the other particles of the beam, they are compressed and extended in a pair of diagonal directions, making angles of 45° with the surface, by the shearing action of the load, when such action exists.

The condition of the particles of a beam, produced by the combined bending and shearing actions of the load, is illustrated by fig. 134, which represents a vertical longitudinal section of a rectangular beam, supported at the ends, and loaded at intermediate points. It is covered with a network consisting of two sets of curves cutting each other at right angles. The curves convex upwards are *lines of direct thrust;* those convex downwards are *lines of direct tension.* A pair of tangents to the pair of curves which traverse any particle, are the *axes of stress* of that particle. (See Art. 108, p. 167.) The *neutral surface* is cut by both sets of curves at angles of 45°. At that vertical section of the beam where the shearing force vanishes, and the bending moment is greatest, both sets of curves become horizontal.

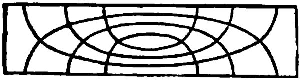

Fig. 134.

Except in cases to be afterwards specified, it is unnecessary to consider the shearing action of the load on a beam.

When a beam breaks under the bending action of its load, it gives way either by the crushing of the compressed side, A A′, or by the tearing of the stretched side, B B′.

In fig. 135, A represents a beam of a granular material, like cast iron, giving way by the crushing of the compressed side, out of which a sort of wedge is forced. B represents a beam giving way by the tearing asunder of the stretched side.

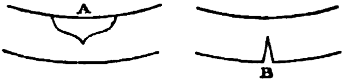

Fig. 135.

The *resistance* of a beam to bending and cross-breaking at any given cross-section is the moment of the couple, consisting of the thrust along the longitudinally-compressed layers, and the equal and opposite tension along the longitudinally-stretched layers.

It has been found by experiment, that in most cases which occur in practice, the longitudinal stress of the layers of a beam may, without material error, be assumed to be *uniformly-varying* (Article 106, p. 163), its intensity being simply proportional to the distance of the layer from the neutral surface.

Let fig. 136 represent a cross-section of a beam (such as that represented in fig. 133), A the compressed side, B the extended side, C any layer, and O O the *neutral axis* of the section, being the line in which it is cut by the neutral surface. Let p denote the intensity of the stress along the layer C, and y the distance of that layer from the neutral axis; because the stress is uniformly varying, $p \div y$ is a constant quantity. Let that constant be denoted for the present by a.

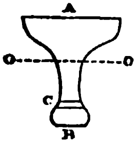

Fig. 136.

Let z be the breadth of the layer C, and $d\,y$ its thickness; Then the amount of the stress along it is

$$p\,z\,d\,y = a\,y\,z\,d\,y;$$

the amount of the stress along all the layers at the given cross-section is

$$a\int y\,z\,d\,y;$$

and this amount must be nothing,—in other words, the total thrust and total tension at the cross-section must be equal,—because the forces applied to the beam are wholly transverse; from which it follows, that

$$\int y\,z\,d\,y = 0, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

and the *neutral axis traverses the centre of gravity of the cross-section.* This principle enables the neutral axis to be found by the aid of the methods explained in Articles 104 and 105, pp. 152 to 158; it being borne in mind that the process of finding the centre of gravity of a given plane figure is the same with that of finding the centre of gravity of a homogeneous uniformly thick flat plate of that figure.

To find the greatest value of the constant $p \div y$ consistent with the strength of the beam at the given cross-section, let y_a be the distance of the compressed side, and y_b that of the extended side from the neutral axis; f_a the greatest thrust, and f_b the greatest tension, which the material can bear in the form of a beam; compute $f_a \div y_a$, and $f_b \div y_b$, and adopt the *less* of those two quantities as the value of $p \div y$, which may now be denoted by $f \div y_1$; f being f_a or f_b, and y_1 being y_a or y_b, according as the beam is liable to give way by crushing or by tearing.

The moment relatively to the neutral axis, of the stress exerted along any given layer of the cross-section, is

$$y\,p\,z\,d\,y = \frac{f}{y_1}\,y^2\,z\,d\,y;$$

and the sum of all such moments, being the

Moment of Resistance of the given cross-section of the beam to breaking across, is given by the formula,

$$\mathrm{M}_0 = \int p\,y\,z\,d\,y = \frac{f}{y_1}\int y^2\,z\,d\,y; \ldots\ldots\ldots\ldots(2.)$$

or making $\int y^2 z \, d y = \mathrm{I}$,

$$\mathrm{M}_0 = \frac{f\mathrm{I}}{y_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\ \text{A.})$$

When the *breaking* load is in question, the co-efficient f is what is called the MODULUS OF RUPTURE of the material. It does not always agree with the resistance of the same material to direct crushing or direct tearing, but has a special value, which can be found by experiments on cross-breaking only. One of the causes of this phenomenon is probably the fact, already stated in Article 154, p. 229, that the resistance of a material to a direct stress is increased by preventing or diminishing the alteration of its transverse dimensions; and another cause may be the fact, that the strength of masses of metal, especially when cast, is greater in the external layer, or *skin*, than in the interior of the mass. When a bar is directly torn asunder, the strength indicated is that of the weakest part of the mass, which is in the centre; when it is broken across, the strength indicated is that either of the skin, which is the strongest part, or of some part near the skin.

When the *proof load* or *working load* is in question, the co-efficient f is the modulus of rupture divided by a suitable *factor of safety*, as to which see Article 143, p. 222.

The factor denoted by I in the preceding equation is what is conventionally called the "*moment of inertia*" of the cross-section of the beam. For sections whose figures are similar, or are parallel projections of each other, the moments of inertia are to each other as the breadths, and as the cubes of the depths of the sections; and the values of y_1 are as the depths. If, therefore, b be the breadth and h the depth of the rectangle circumscribing the cross-section of a given beam at the point where the moment of flexure is greatest, we may put

$$\mathrm{I} = n' b h^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

$$y = m' h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

n' and m' being numerical factors depending on the form of section; and making $n' \div m' = n$, the moment of resistance may be thus expressed,—

$$\mathrm{M}_0 = n f b h^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

Hence it appears, that the *resistances of similar cross-sections to cross-breaking are as their breadths and as the squares of their depths.*

The relation between the load and the dimensions of a beam is found by equating the value of the greatest bending moment in terms of the load and length of the beam, as given in Article 160, equations 10, 11, 12, and 16, pp. 243, 244, to the value of the moment of resistance of the beam, at the cross-section where that greatest bending moment acts, as given in equation 5 of this Article; that is to say,

$$M_0 = m\,W\,l = n\,f\,b\,h^2 \ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

m being the co-efficient depending on the mode of distribution of the load, as defined by equation 1 of Article 161, p. 247, and given for particular cases in the tables and examples of Article 161.

In using the above equation 6, it is to be understood that the same unit of measure is to be employed for all the dimensions of the beam; and inasmuch as the values of the "modulus of rupture" f given in tables are generally stated in *pounds on the square inch*, so that the *inch* is the proper unit for the transverse dimensions b and h, the length or span l ought to be expressed in inches also, so that the bending moment will be computed in *inch-pounds*.

In finding the value of the moment of inertia I of cross-sections of complex figure, the following rules are useful:—

If a complex cross-section is made up of a number of simple figures, conceive the centre of gravity of each of those figures to be traversed by a neutral axis parallel to the neutral axis of the whole section. Find the moment of inertia of each of the component figures relatively to its own neutral axis; multiply its area by the square of the distance between its own neutral axis and the neutral axis of the whole section; and add together all the results so found, for the moment of inertia of the whole section. To express this in symbols, let A' be the area of any one of the component figures, y' the distance of its neutral axis from the neutral axis of the whole section, I' its moment of inertia relatively to its own neutral axis; then the moment of inertia of the whole section is

$$I = \Sigma \cdot I' + \Sigma \cdot y'^2\,A'. \ldots\ldots\ldots\ldots\ldots\ldots (7.)$$

When the figure of the cross-section can be made by *taking away* one simpler figure from another, so that the centre of gravity of the whole figure is found *by subtraction* (as in p. 154), both the area and the moment of inertia of the subtracted figure are to be considered as negative, and so treated, in making use of equation 7.

163. **Examples of Moment of Resistance.**—The following table contains examples of the values of the factors n', m', and n, of equations 3, 4, and 5:—

FORM OF CROSS SECTIONS.	$n'=\frac{I}{bh^3}$.	$m'=\frac{y_1}{h}$	$n=\frac{M_0}{fbh^2}$.
I. Rectangle bh, (including square)	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{1}{6}$
II. Ellipse— Vertical axis h, Horizontal axis b, (including circle)	$\frac{\pi}{64}=\frac{1}{20\cdot4}$ $=0\cdot0491$	$\frac{1}{2}$	$\frac{\pi}{32}=\frac{1}{10\cdot2}$ $=0\cdot0982$
III. Hollow rectangle, $bh-b'h'$; also I-formed section, where b' is the sum of the breadths of the lateral hollows,	$\frac{1}{12}\left(1-\frac{b'h'^3}{bh^3}\right)$	$\frac{1}{2}$	$\frac{1}{6}\left(1-\frac{b'h'^3}{bh^3}\right)$
IV. Hollow square— $h^2-h'^2$	$\frac{1}{12}\left(1-\frac{h'^4}{h^4}\right)$	$\frac{1}{2}$	$\frac{1}{6}\left(1-\frac{h'^4}{h^4}\right)$
V. Hollow ellipse,	$\frac{1}{20\cdot4}\left(1-\frac{b'h'^3}{bh^3}\right)$	$\frac{1}{2}$	$\frac{1}{10\cdot2}\left(1-\frac{b'h'^3}{bh^3}\right)$
VI. Hollow circle,	$\frac{1}{20\cdot4}\left(1-\frac{h'^4}{h^4}\right)$	$\frac{1}{2}$	$\frac{1}{10\cdot2}\left(1-\frac{h'^4}{h^4}\right)$
VII. Isosceles triangle; base b, height h; y_1 measured from summit,	$\frac{1}{36}$	$\frac{2}{3}$	$\frac{1}{24}$

The following examples are not well suited for introduction into the tables:—

EXAMPLE VIII.—T-formed section.

	Areas.	Depths.
Flange or table,	A_1	h_1
Vertical web,	A_2	h_2

Totals, $A_1+A_2=A$; $h_1+h_2=h$.

Exact Solution.—Distance of the neutral axis from the edge of the vertical web,—

$$y_1=\frac{A_1\left(h_2+\frac{h_1}{2}\right)+A_2\frac{h_2}{2}}{A}=\frac{A_1(2h_2+h_1)+A_2h_2}{2A}=\frac{h_2}{2}+\frac{A_1h}{2A}$$

[redacted]..t. ax. fr. cen. of m. ... $= \frac{A_1 h}{2A}$

″ ″ ″ ″ ″ ″ Large $= \frac{h}{2} - \frac{A_1 h}{2A} = \frac{A_2 h}{2A}$

$$y_1 = \frac{h}{2} + \frac{(h_1 + h_2)\,A_1 \overset{-h_1 A}{\cancel{+ h_1 A_2}}}{2\,A};$$

Moment of Inertia of whole section,—

$$I = \frac{A_1 h_1^2 + A_2 h_2^2}{12} + \frac{A_1 A_2 (h_1 + h_2)^2}{4\,A};$$

Moment of Resistance, as before,—

$$M_0 = \frac{fI}{y_1}. \qquad \ldots\ldots(1.)$$

$I = \frac{A_1 h_1^2}{12} + A_1 \frac{h_2^2}{4A^2}$

$+ \frac{A_2 h_2^2}{12} + \frac{A_1^2 h^2}{4A^2}$

Approximate Solution.—When h_1 is small compared with h_2, make $h' = h_2 + \frac{1}{2} h_1$; then, the following are nearly correct:—

$$y_1 = \frac{h'}{2}\left(1 + \frac{A_1}{A}\right);\ I = h'^2\left(\frac{A_2}{12} + \frac{A_1 A_2}{4A}\right);$$

$$M_0 = \frac{fI}{y_1} = \frac{fh'}{6} \cdot \frac{(3\,A_1 + A_2)\,A_2}{A + A_1};$$

$$= \frac{fh'}{6} \cdot \frac{A_2\,(A_2 + 4\,A_1)}{A_2 + 2\,A_1}. \qquad \ldots\ldots\ldots(2.)$$

Example IX.—Double T-formed section. The beam is assumed to give way by the tearing of the lower flange or table B. Let the areas and depths of the parts of which the section consists be denoted as follows:—

	Areas.	Depths.
Upper flange or table............	A_1,	h_1.
Vertical web, ...	A_2,	h_2.
Lower flange or table,............	A_3,	h_3.

Totals,......$A_1 + A_2 + A_3 = A$, $h_1 + h_2 + h_3 = h$.

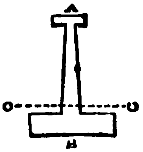

Fig. 137.

Exact Solution.—The height of the neutral axis above the lower side of this section is

$$y_b = \frac{h}{2} - \frac{(h_2 + h_3)\,A_3 - (h_1 + h_2)\,A_1 - (h_3 - h_1)\,A_2}{2\,A};$$

The Moment of Inertia of the section,—

$$I = \frac{A_1 h_1^2 + A_2 h_2^2 + A_3 h_3^2}{12} + \frac{1}{4\,A}\left\{A_1 A_3 (h_1 + h_3 + 2\,h_2)^2 + A_1 A_2 (h_1 + h_2)^2 + A_2 A_3 (h_2 + h_3)^2\right\};$$

and the Moment of Resistance,—

$$M_0 = \frac{f_b I}{y_b}; \qquad (3.$$

Approximate Solution.—When h_1 and h_3 are small compared with h_2, make

$$\left.\begin{aligned} h' &= h_2 + \frac{h_1 + h_3}{2}; \\ &\text{then the following formulæ are nearly correct:—} \\ y_b &= \frac{h'}{2}\left(1 - \frac{A_3 - A_1}{A}\right) = \frac{h'}{2} \cdot \frac{A_2 + 2\,A_1}{A}; \\ I &= h'^2 \left\{ \frac{A_2}{12} + \frac{A_2\,A_3 + A_1\,A_2 + 4\,A_1\,A_3}{4\,A} \right\}; \\ M_0 &= \frac{f_b I}{y_b} = \frac{f_b h'}{6} \cdot \frac{A_2\,(A_2 + 4\,A_1 + 4\,A_2) + 12\,A_1\,A_3}{A_2 + 2\,A_1}; \end{aligned}\right\} \text{(4.)}$$

Another Approximate Solution, when A_2 is very small, or when there is an open frame instead of a vertical web—

$$y_b = \frac{h' A_1}{A};\ I = \frac{h'^2 \cdot A_1 A_3}{A};\ M_0 = f_b\,h' A_3 \ldots\ldots\ldots\ldots \text{(5.)}$$

164. **Cross-section of Equal Strength.**—The use of the T-shaped and double-T-shaped cross-sections mentioned in the last Article, is to economize any material whose resistances to cross-breaking by crushing and by tearing are different, by so adjusting the position of the neutral axis, that the tendencies of the beam to break across by crushing and by tearing shall be as nearly as possible equal. The following are the rules for effecting that adjustment:—

Let f_a be the modulus of rupture by crushing;
" f_b " " " by tearing;
" y_a the distance from the neutral axis to the compressed side;
" y_b " " " extended side;
$y_a + y_b = h$ being the depth of the beam;

then the neutral axis should be so placed as to divide that depth in the following proportion:—

$$\left.\begin{aligned} &f_a + f_b : f_a : f_b \\ &:: \quad h \quad : y_a : y_b \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1.)}$$

Let A_2, as before, denote the area of the cross-section of the vertical web of a beam, *measured from centre to centre* of the top and bottom flanges; A_1 the area of the compressed flange, A_3 that of the extended flange.

The following solutions are to the same degree of approximation with those in equations 2 and 4 of Article 163.

CASE I. *f_a greater than f_b, beam T-shaped.*—Here the flange is required at the stretched side of the girder; and its area must be as follows:—

$$A_3 = \frac{f_a - f_b}{2 f_b} A_2 \text{..........................} (2.)$$

The Moment of Resistance of this form of cross-section is (see Article 163, equation 2, p. 255)—

$$M_0 = \frac{(2f_a - f_b)\, h' A_2}{6} \text{......................} (3.)$$

CASE II. *f_a greater than f_b, beam double T-shaped.*—The area of the *compressed* flange being A_1, that of the *stretched* flange should be as follows:—

$$A_3 = \frac{f_a}{f_b} A_1 + \frac{f_a - f_b}{2 f_b} A_2; \text{......................} (4.)$$

when the Moment of Resistance will become

$$M_0 = h' \left\{ f_a A_1 + (2f_a - f_b) \frac{A_2}{6} \right\}$$

$$= h' \left\{ f_b A_3 - (f_a - 2 f_b) \frac{A_2}{6} \right\} \text{..............} (5.)$$

In designing a beam to bear a given bending moment, the depth h' and area A_2 of the vertical web are to be fixed by considerations of practical convenience, when equation 5 will enable the areas of either or both of the flanges to be computed.

Example.—Suppose that for a certain sort of cast iron,

f_a = 80,000 lbs. in the square inch;
f_b = 20,000 „ „ „

so that in a well-proportioned section,

$$5 : 4 : 1 :: h : y_a : y_b.$$

Then in a T-shaped section,

$$\left.\begin{aligned} A_3 &= \frac{4-1}{2} A_2 = \frac{3}{2} A_2; \text{ and} \\ M_0 &= \frac{140{,}000\ h'\ A_2}{6}; \end{aligned}\right\} \text{...........} (6.)$$

and in a double T-shaped section,

$$A_3 = 4\, A_1 + \frac{3}{2} A_2;\text{*} \text{ and}$$

---

[*] This result agrees nearly with the proportions which Mr. Hodgkinson found experimentally to be the best for cast iron beams.

$$M_0 = h'\left\{80{,}000\,A_1 + 140{,}000\frac{A_2}{6}\right\}$$

$$= h'\left\{20{,}000\ \ A_3 - 40{,}000\,\frac{A_2}{6}\right\}\ldots\ldots\ldots\ldots\ldots(7.)$$

CASE III. *f_a less than f_b, beam T-shaped.*—Here the flange is required at the *compressed* side of the beam, and its area should be,

$$A_1 = \frac{f_b - f_a}{2f_a}\cdot A_2.\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8.)$$

The Moment of Resistance of this cross-section is

$$M_0 = \frac{(2f_b - f_a)\,h'\,A_2}{6}\ldots\ldots\ldots\ldots\ldots\ldots(9.)$$

CASE IV. *f_a less than f_b, beam double T-shaped.*—The area of the *stretched* flange being A_3, that of the *compressed* flange should be as follows:—

$$A_1 = \frac{f_b}{f_a}A_3 + \frac{f_b - f_a}{2f_a}A_2;\ \ldots\ldots\ldots\ldots\ldots(10.)$$

when the Moment of Resistance will become

$$M_0 = h'\left\{f_b\,A_3 + (2f_b - f_a)\,\frac{A_2}{6}\right\}$$

$$= h'\left\{f_a\,A_1 + (2f_a - f_b)\,\frac{A_2}{6}\right\}\ \ldots\ldots\ldots\ldots(11.)$$

In designing a beam to bear a given bending moment, the depth h' and area A_2 of the vertical web are to be fixed by considerations of practical convenience, when equation 11 will enable the area of either or both of the flanges to be computed.

Example.—Suppose that for a certain sort of wrought iron,

$f_a = 36{,}000$ lbs. on the square inch
$f_b = 60{,}000$ „ „ „ „

so that in a well-proportioned section

$$8 : 3 : 5 :: h : y_a : y_b.$$

Then in a T-shaped section,

$$\left.\begin{aligned} A_1 &= \frac{5-3}{6}\,A_2 = \frac{A_2}{3}; \\ M_0 &= \frac{84{,}000\,h'\,A_2}{6}; \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots(12.)$$

and in a double T-shaped section

$$A_1 = \frac{5}{3}\,A_3 + \frac{1}{3}\,A_2;$$

$$M_0 = h'\left\{60{,}000\,A_3 + 84{,}000\,\frac{A_2}{6}\right\}$$

$$= h'\left\{36{,}000\,A_1 + 12{,}000\,\frac{A_2}{6}\right\}.^{*}\ldots\ldots\ldots\ldots(13.)$$

165. **Longitudinal Sections of Uniform Strength for Beams** (*A. M.*, 299) are those in which the dimensions of the cross-section are varied in such a manner that its ultimate moment of resistance bears at each point of the beam the same proportion to the bending moment of the load. That moment of resistance, for figures of the same kind, being proportional to the breadth and to the square of the depth, can be varied either by varying the breadth, the depth, or both. The law

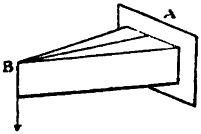

Fig. 138.

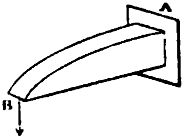

Fig. 139.

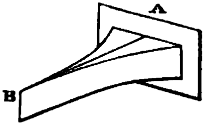

Fig. 140.

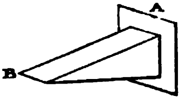

Fig. 141.

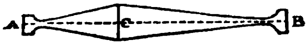

Fig. 142.

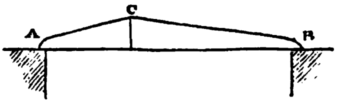

Fig. 143.

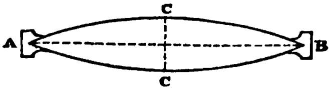

Fig. 144.

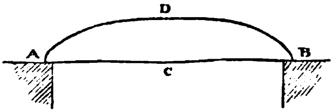

Fig. 145.

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* The first theoretical solution of Cases II. and IV. was contained in a paper by Mr. Callcott Reilly, read to the British Association at Oxford in 1860.

of variation depends upon the mode of variation of the moment of flexure of the beam from point to point, and this depends on the distribution of the load and of the supporting forces, in a way which has been exemplified in Articles 160 and 161. When the depth of the beam is made uniform, and the breadth varied, the vertical longitudinal section is rectangular, and the plan is of a figure depending on the mode of variation of the breadth. When the breadth of the beam is made uniform, and the depth varied, the plan is rectangular, and the vertical longitudinal section is of a figure depending on the mode of variation of the depth. The following table gives examples of the results of those principles:—

Mode of Loading and Supporting.	$b h^2$ proportional to	Depth h constant; Figure of Plan.	Breadth b constant; Figure of Vertical Longitudinal Section
I. (Figs. 138, 139). Fixed at A. loaded at B, ...	Distance from B.	Triangle, apex at B, fig. 138.	Parabola, vertex at B, fig. 139.
II. (Figs. 140, 141). Fixed at A, uniformly loaded,...	Square of distance from B.	Pair of parabolas, vertices touching each other at B, fig. 140.	Triangle, apex at B, fig. 141.
III. (Figs. 142, 143). Supported at A and B, loaded at C, ...	Distance from adjacent point of support.	Pair of triangles, common base at C, apices at A and B, fig. 142.	Pair of parabolas, vertices at A and B, meeting at C, fig. 143.
IV. (Figs. 144, 145). Supported at A and B, uniformly loaded, ...	Product of distances from points of support.	Pair of parabolas, vertices at C, C, in middle of beam; common base A B, fig. 144.	Ellipse A D B, fig. 145.

The formulæ for a *constant depth* are applicable, approximately, to the breadths of the flanges of the T-shaped and double T-shaped girders, described in Article 164. In applying the principles of this Article, it is to be borne in mind, that the *shearing force* has not yet been taken into account; and that, consequently, the figures described in the above table require, at and near the places where they taper to edges, some additional material to enable them to withstand that force. In figs. 142 and 144, such additional material is shown, disposed in the form of projections or palms at the points of support, which serve both to resist the shearing force, and to give lateral steadiness to the beams. As to the greatest intensity of the shearing stress, see Article 168.

166. **Modulus of Rupture of Cast Iron Beams.** (*A. M.*, 297.)—Mr. William Henry Barlow, in a paper read to the Royal Society (see *Phil. Trans.*, 1855), showed that the modulus of rupture of cast iron beams has various values, ranging from the mere direct tenacity of the iron up to about two-and-a-quarter times that tenacity, according to the figure of the cross-section of the beam. This was proved by experiments on beams, which were, in some cases, of a solid rectangular section, and in other cases, of an open work rectangular section. So far as those experiments went, they were in accordance with the following empirical formula:—

$$f = f_0 + f' \cdot \frac{\mathrm{H}}{h}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

where f is the modulus of rupture of the beam in question; f_0, the direct tenacity of the iron of which it is made; f', a co-efficient determined empirically; and $\frac{\mathrm{H}}{h}$, the ratio which the *depth of solid metal* H in the cross-section of the beam bears to the *total depth of section h*. The following were the values of the constants for the cast iron experimented on:—

$$\left.\begin{array}{r}\text{Direct tenacity, } f_0 = 18{,}750 \text{ lbs. per square inch;}\\ f' = 23{,}000 \text{ lbs. per square inch;}\\ = 1\tfrac{1}{4} f_0 \text{ nearly.}\end{array}\right\} \ldots\ldots\ldots\ldots (2.)$$

Mr. Barlow afterwards made further experiments on cast iron beams of various forms of section, and also experiments on wrought iron beams, showing, though not so conclusively, variations in the modulus of rupture of wrought iron analogous to those which have been proved to exist in the case of cast iron.

167. **Allowance for Weight of Beam—Limiting Length of Beam.** (*A. M.*, 314, 315.)—When a beam is of great span, its own weight may bear a proportion to the load which it has to carry, sufficiently great to require to be taken into account in determining the dimensions of the beam. The following is the process to be performed for that purpose, when the load is uniformly distributed, and the beam of uniform cross-section. Let W′ be the external working load, s_1 its factor of safety, s_2 a factor of safety suited to a steady load, like the weight of the beam.

Let b' denote the breadth of any part of the beam, as computed by considering the *external breaking load alone*, s_1 W′. Compute the weight of the beam from that *provisional* breadth, and let it be denoted by B′. Then $\frac{s_1 W'}{s_1 W' - s_2 B'}$, is the proportion in which the *gross* breaking load exceeds the external part of that load. Consequently, if for the *provisional* breadth b' there be substituted the *exact* breadth,

$$b = \frac{b' s_1 W'}{s_1 W' - s_2 B'}, \qquad \text{(1.)}$$

the beam will now be strong enough to bear both the proposed external load W′, and its own weight, which will now be

$$B = \frac{B' s_1 W'}{s_1 W' - s_2 B'}; \qquad \text{(2.)}$$

and the true gross breaking load will be

$$W = s_1 W' + s_2 B = \frac{s_1^2 W'^2}{s_1 W' - s_2 B'}. \qquad \text{(3.)}$$

As the factor of safety for a steady load is in general one-half of that for a moving load, s_1 may be made $= 2 s_2$; in which case the preceding formulæ become

$$b = \frac{2 b' W'}{2 W' - B'}; \qquad \text{(4.)}$$

$$B = \frac{2 B' W'}{2 W' - B'}; \qquad \text{(5.)}$$

$$W = \frac{2 s_1 W'^2}{2 W' - B'}, \qquad \text{(6.)}$$

In all these formulæ, both the external load and the weight of the beam are treated as if uniformly distributed—a supposition which is sometimes exact, and always sufficiently near the truth for the purposes of the present Article.

The gross load of beams of similar figures and proportions, varying as the breadth and square of the depth directly, and inversely as the length, is proportional to the square of a given linear dimension. The weights of such beams are proportional to the cubes of corresponding linear dimensions. Hence the weight increases at a faster rate than the gross load; and for each particular figure of a beam of a given material and proportion of its dimensions, there must be a certain size at which the beam will bear its own weight only, without any additional load.

To reduce this to calculation, let the uniformly distributed gross breaking load of a beam of a given figure be expressed as follows:—

$$W = s_1 W' + s_2 B = \frac{8\,n\,f\,h\,A}{l}; \text{..............(7.)}$$

l, h, and A being the length, depth, and sectional area of the beam, f the modulus of rupture, and n a factor depending on the form of cross-section. The weight of the beam will be expressed by

$$B = k\,w'\,l\,A; \text{..........................(8.)}$$

w' being the weight of an unit of volume of the material, and k a factor depending on the figure of the beam. Then the ratio of the weight of the beam multiplied by its proper factor of safety to the gross breaking load is

$$\frac{s_2 B}{W} = \frac{s_2\,k\,w'\,l^2}{8\,n\,f\,h}; \text{........................(9.)}$$

which increases in the simple ratio of the length, if the proportion $l \div h$ is fixed. When this is the case, the length L of a beam, whose weight (treated as uniformly distributed) is its working load, is given by the condition $s_2 B = W$; that is,

$$L = \frac{8\,n\,f\,h}{s_2\,k\,w'\,l}; \text{........................(10.)}$$

This *limiting length* having once been determined for a given class of beams, may be used to compute the ratios of the gross breaking load, weight of the beam, and external working load to each other, for a beam of the given class, and of any smaller length, l, according to the following proportional equation:—

$$L : \frac{l}{s_2} : \frac{L - l}{s_1} : : W : B : W'; \text{................(11.)}$$

In all the following examples, the factors of safety employed are $s_1 = 6$; $s_2 = 3$:—

EXAMPLE I.—Let the beams in question be plain *rectangular cast iron beams*, so that $n = \frac{1}{6}$, $k = 1$, $w' = 0{\cdot}257$ lb. per cubic inch; let $f = 40{,}000$ lbs. per square inch; and let $\frac{h}{l} = \frac{1}{15}$. Then

$$L = 4{,}612 \text{ inches} = 384 \text{ feet nearly}; \text{...........(12.)}$$

also, l being expressed in inches—

$$4{,}612 : \frac{l}{3} : \frac{4612 - l}{6} : : W : B : W'. \text{ ....(13.)}$$

Example II.—Cast iron beam of uniform single T-shaped section of equal strength (as in Article 164, Case I., p. 257).

$$A_3 = \frac{3}{2} A_2 ; \therefore A = \frac{5}{2} A_2 \text{ and } A_2 = \frac{2}{5} A.$$

$$M_0 = \frac{W\,l}{8} = \frac{7 f_a h' A_2}{24} = \frac{14 f_a h' A}{120} ; \therefore W = \frac{14 f_a h' A}{15\, l}.$$

As before, $B = 0{\cdot}257\, l\, A$;

Let $f_a = 80{,}000$ lbs. on the square inch; $\frac{h'}{l} = \frac{l}{15}$; then

$$\left.\begin{array}{r} \dfrac{s_2 B}{W} = \dfrac{l \text{ in inches}}{6{,}456}, \text{ and} \\ L = 6{,}456 \text{ inches} = 538 \text{ feet;} \end{array}\right\} \ldots\ldots\ldots\ldots (14.)$$

also,

$$6{,}456 : \frac{l}{3} : \frac{6{,}456 - l}{6} : : W : B : W'. \ldots\ldots\ldots\ldots (15.)$$

Example III.—Cast iron beam of uniform double T-shaped section of equal strength (as in Article 164, Case II., p. 257).

$$A_3 = \frac{3}{2} A_2 + 4\ A_1 ; \therefore A = \frac{5}{2} A_2 + 5\ A_1.$$

$$M_0 = \frac{W\,l}{8} = f_a h' \left(\frac{7}{24} A_2 + A_1\right) ; \therefore W = \frac{f_a h'}{l} \left(\frac{7}{3} A_2 + 8\ A_1\right).$$

In order to obtain a definite result, some proportion must be assumed between the area of the upper flange A_1, and that of the vertical web A_2. For the sake of illustration, let $A_1 = A_2 \div 2$; which proportion is not unusual in practice. Then

$$A = 5\ A_2 = 10\ A_1 = \frac{10}{7} A_3 ; \text{ and}$$

$$W = \frac{f_a h' A}{l} \left(\frac{7}{15} + \frac{4}{5}\right) = \frac{19}{15} \cdot \frac{f_a h' A}{l}.$$

As before, $B = 0{\cdot}257\, l\, A$. Let $f_a = 80{,}000$ lb. on the square inch; $\frac{h'}{l} = \frac{1}{15}$; then

$$\left.\begin{array}{r} \dfrac{s_2 B}{W} = \dfrac{l \text{ in inches}}{8{,}762}; \text{ and} \\ L = 8{,}762 \text{ inches} = 730 \text{ feet nearly;} \end{array}\right\} \ldots\ldots\ldots (16.)$$

also,

$$8{,}762 : \frac{l}{3} : \frac{8{,}762 - l}{6} : : \mathrm{W} : \mathrm{B} : \mathrm{W}' \ldots\ldots\ldots (17.)$$

EXAMPLE IV.—Wrought iron beam of uniform single T-shaped section of equal strength (as in Article 164, Case III., p. 258).

$$\mathrm{A}_1 = \frac{1}{3}\,\mathrm{A}_2;\ \therefore\ \mathrm{A} = \frac{4}{3}\,\mathrm{A}_2 \text{ and } \mathrm{A}_2 = \frac{3}{4}\,\mathrm{A}_1$$

$$\mathrm{M}_0 = \frac{\mathrm{W}\,l}{8} = \frac{7\,f_b\,h'\,\mathrm{A}_2}{30} = \frac{7\,f_b\,h'\,\mathrm{A}}{40};\ \therefore\ \mathrm{W} = \frac{7\ f_b\,h'\,\mathrm{A}}{5\,l}.$$

In this case, $\mathrm{B} = 0{\cdot}277\ l\ \mathrm{A} = \frac{5}{18}\,l\ \mathrm{A}$.

Let $f_b = 60{,}000$ lbs. on the square inch; $\frac{h'}{l} = \frac{1}{15}$; then

$$\left.\begin{array}{l} \dfrac{s_2\,\mathrm{B}}{\mathrm{W}} = \dfrac{l \text{ in inches}}{6{,}720};\ \text{and} \\ \mathrm{L} = 6{,}720 \text{ inches} = 560 \text{ feet nearly}; \end{array}\right\} \ldots\ldots\ldots (18.)$$

also,

$$6{,}720 : \frac{l}{3} : \frac{6{,}720 - l}{6} : : \mathrm{W} : \mathrm{B} : \mathrm{W}' \ldots\ldots\ldots\ldots (19.)$$

EXAMPLE V.—Wrought iron beam of uniform double T-shaped section of equal strength (as in Article 164, Case IV., p. 258).

$$\mathrm{A}_1 = \frac{1}{3}\,\mathrm{A}_2 + \frac{5}{3}\,\mathrm{A}_3;\ \therefore\ \mathrm{A} = \frac{4}{3}\,\mathrm{A}_2 + \frac{8}{3}\,\mathrm{A}_3;$$

$$\mathrm{M}_0 = \frac{\mathrm{W}\,l}{8} = f_b\,h'\left(\frac{7\,\mathrm{A}_2}{30} + \mathrm{A}_3\right);\ \therefore\ \mathrm{W} = \frac{f_b\,h'}{l}\left(\frac{28}{15}\,\mathrm{A}_2 + 8\,\mathrm{A}_3\right).$$

In order to obtain a definite result, let $\mathrm{A}_3 = \mathrm{A}_2$; which proportion is not unusual in practice. Then $\mathrm{A} = 4\,\mathrm{A}_2$; and

$$\mathrm{W} = \frac{f_b\,h'\,\mathrm{A}}{l}\left(\frac{7}{15} + 2\right) = \frac{37}{15}\cdot\frac{f_b\,h'\,\mathrm{A}}{l}.$$

As before, let $\mathrm{B} = \frac{5}{18}\,l\ \mathrm{A}$; $f_b = 60{,}000$; $\frac{h'}{l} = \frac{1}{15}$; then

$$\left.\begin{array}{l} \dfrac{s_2\,\mathrm{B}}{\mathrm{W}} = \dfrac{l \text{ in inches}}{11{,}840};\ \text{and} \\ \mathrm{L} = 11{,}840 \text{ inches} = 987 \text{ feet nearly}; \end{array}\right\} \ldots\ldots\ldots (20.)$$

also,

$$11{,}840 : \frac{l}{3} : \frac{11{,}840 - l}{6} :: \mathrm{W} : \mathrm{B} : \mathrm{W}' \text{..........(21.)}$$

168. **Distribution of Shearing Stress in Beams.** (*A. M.*, 309.)—It has already been shown in Article 160, Division II., how to find the greatest amount of the shearing action of the load at a given cross-section of a beam. Let F denote that amount, A the area of the cross-section at which it acts; then

$$\textit{Mean intensity} \text{ of shearing stress} = \frac{\mathrm{F}}{\mathrm{A}} \text{............(1.)}$$

The *distribution* of that stress over the cross-section is such that *its intensity is greatest at the neutral axis*, and gradually diminishes towards the upper and lower surfaces of the beam, where it vanishes. That *greatest intensity* is found by the following process:—Conceive, as in fig. 136, Article 162, p. 250, the cross-section to be divided into thin horizontal layers, such as C; let z be the breadth of any layer, $d\,y$ its depth, y its distance from the neutral axis; also let z_0 be the breadth of the cross-section at the neutral axis; I the "*moment of inertia*" of the cross-section, as defined in Article 162, p. 252; y_1 the distance from the neutral axis to *either the upper or the under* surface of the beam; q_0 the required greatest intensity of the shearing stress. Then

$$q_0 = \frac{\mathrm{F}}{\mathrm{I}\,z_0} \int_0^{y_1} y\,z\,d\,y. \text{....................(2.)}$$

The symbol $\int_0^{y_1}$ denotes that the integration or summation of the products $y\,z\,d\,y$ of the area of each layer into its distance from the neutral axis, is to extend from the neutral axis to either the upper or the lower surface of the beam, that integration being thus performed for one only of the two parts into which the neutral axis divides the cross-section. It is a matter of convenience only which of those parts is chosen, as the same result is arrived at in either case.

The maximum intensity of the shearing stress at the given cross-section exceeds the mean intensity in the following proportion:—

$$\frac{q_0\,\mathrm{A}}{\mathrm{F}} = \frac{\mathrm{A}}{\mathrm{I}\,z_0} \int_0^{y_1} y\,z\,d\,y; \text{..................(3.)}$$

a ratio depending solely on the figure of the cross-section.

The following table gives some of its values:—

	FIGURE OF CROSS-SECTION.	$\frac{q_0 A}{F}$.
I.	Rectangle, $z_0 = b$, ...	$\frac{3}{2}$.
II.	Ellipse and Circle,......................	$\frac{4}{3}$.
III.	Hollow Rectangle— $A = bh - b'h'$; $z_0 = b - b'$. This includes I-shaped sections, ...	$\frac{3}{2} \cdot \frac{(bh - b'h') \cdot (bh^2 - b'h'^2)}{(b - b') \cdot (bh^3 - b'h'^3)}$.
IV.	Hollow square, $h^2 - h'^2$,.................	$\frac{3}{2}\left(1 + \frac{hh'}{h^2 + h'^2}\right)$.
V. VI.	Hollow ellipse and hollow circle; the numerical factor $\frac{4}{3}$; the symbolical factor, the same as for the hollow rectangle and hollow square respectively.	
VII.	Single T-shaped section; flange A_1; web A_2; $A_1 + A_2 = A$,...............	$\frac{3}{2} \cdot \frac{4AA_1 + A_2^2}{A_2(4A_1 + A_2)}$.
VIII.	Double T-shaped section; flanges A_1, A_3; web A_2; $A_1 + A_2 + A_3 = A$,	$\frac{A(24A_1A_3 + 12A_1A_2 + 12A_2A_3) + 3A_2^2}{A_2(24A_1A_3 + 8A_1A_2 + 8A_2A_3 + 2A_2^2)}$

When A_1 and A_3, in Case VIII., are large compared with A_2, —that is to say, when a beam consists of strong upper and lower flanges or horizontal bars, connected by a thin vertical web, the shearing force may be treated as if it were entirely borne by the vertical web, and uniformly distributed.

The smallest cross-section of a beam is generally fixed by reasons of convenience, independent of the shearing force to which it is exposed, and is generally much greater than is necessary in order to bear that force. But when it is practicable to adapt the least cross-section of the beam accurately to the shearing force, the preceding formulæ and table furnish the means of doing so, by making

$$q_0 = \frac{f'}{s}; \quad \text{..............................(4.)}$$

where f' is the modulus of rupture by shearing, and s a factor of safety. This equation gives for the least sectional area,

$$A = \frac{q_0 A}{F} \cdot \frac{F}{q_0} = \frac{q_0 A}{F} \cdot \frac{s F}{f'}; \text{ ....} (5.)$$

in which formula, $q_0 A \div F$ is to be found by means of equation 3, or of the preceding table of examples.

169. **Deflection of Beams.** (*A. M.*, 300 to 304.)—By the term "*Deflection*," when not otherwise specified, is understood the *greatest displacement* of any point of a loaded beam from its position when the beam is unloaded. Three cases may be distinguished,—that of *Deflection under any load*, that of *Proof Deflection*, or deflection under the greatest load which does not impair the strength of the beam, and that of *Ultimate Deflection*, or deflection immediately before breaking. When the load does not exceed the proof load, the deflection of a given beam, under a load distributed in a given manner, is very nearly proportional to the load: when the proof load is exceeded, the deflection increases in general faster than the load, and in an irregular manner, so that the ultimate deflection is not capable of exact computation. The remainder of this Article will therefore relate to deflections under loads not exceeding the proof load.

The determination of the deflection of a beam under a transverse load is a process which consists of three steps, by which are found successively, the *curvature* at any cross-section, the *slope* at any cross-section, and the *deflection*.

STEP I.—To find the *curvature* at a given cross-section,—divide the bending moment at that cross-section (as found in Article 160, Division III., p. 242) by the "moment of inertia" of that section (as found in Article 162, p. 252), and by the modulus of direct elasticity of the material. The result is the curvature,—that is, the reciprocal of the radius of curvature of a longitudinal line in the beam, which was straight when the beam was unloaded. Denote that radius by r, and the other quantities by the symbols already employed; then

$$\frac{1}{r} = \frac{M}{E I} \text{ ....} (1.)$$

The positive or negative sign of this expression will show whether the curvature is concave upwards or downwards.

When the beam is under its proof load, and the given cross-section is that of greatest stress, let M_0 denote the bending moment of that section, and I_0 the moment of inertia; then, as has been shown in Article 162, equations 2 A, 3, 4, and 5, we have

$$M_0 = \frac{f I_0}{y_1}$$

(where f' is the *modulus of proof strength*, or, for most materials, from one-half to one-third of the modulus of rupture, see Article 143, p. 222); so that, in the case now considered, equation 1 becomes

$$\frac{1}{r_0} = \frac{M_0}{E I_0} = \frac{f'}{E y_1} = \frac{f'}{E m' h} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

(m' having the meaning explained in Article 162, p. 252).

This formula gives the *sharpest curvature* which the beam can bear without injury; and as $f' \div E$ is the *proof strain* of the material, that curvature depends on the proof strain, the depth h, and the form of section only.

When the dimensions are all given in inches, the bending moment in inch-pounds, and the moduli of proof strength and elasticity in pounds on the square inch, the radius of curvature will be computed in inches.

The denominator E I in equation 1, expresses the *transverse stiffness or resistance of the beam to bending* at any given cross-section; and as I may be expressed in the form $n' b h^3$ (Article 162, equation 3, p. 252), the resistances of similar cross-sections of beams of the same material are to each other as their breadths, and as the cubes of their depths; and consequently,—

The curvatures of beams of the same material at sections of similar figures, under equal bending moments, *are inversely as their breadths, and inversely as the cubes of their depths.*

Equation 2 also shows that,—

The curvatures of beams of the same material, at sections of similar figures, under their respective proof bending moments, *are inversely as their depths simply.*

In the case of a *cross-section of equal strength* (such as those described in Article 164), equation 2 may be put in the following form:—let f'_a and f'_b be the moduli of proof resistance to cross-breaking by compression and by tearing respectively; then

$$\frac{1}{r_0} = \frac{f'_a + f'_b}{E h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{ A.})$$

The curved form assumed by an originally straight longitudinal line in a beam might be drawn approximately by the aid of equation 1, were it not that the great lengths of the radii of curvature, and the smallness of the ordinates of the curve, in all cases which occur in practice, render it neither practicable nor useful to draw the figure of that curve in its natural proportions. But the following process, invented, so far as I am aware, by Mr. C. H. Wild, enables a diagram to be drawn, which represents, with a near approach to accuracy, that curve, *with its vertical dimensions exaggerated*, so as to show conspicuously the slopes and ordinates.

Compute, by equation 1, the radii of curvature for a series of equi-distant points in the beam. Diminish all those radii in any proportion which may be convenient, and draw a curve composed of small circular arcs with the diminished radii. Then in the same ratio that the radii, as compared with the horizontal scale of the drawing, are diminished, will the vertical scale of the drawing, according to which the ordinates are shown, be exaggerated.

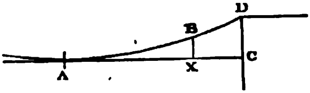

Fig. 146.

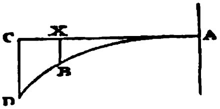

Fig. 147.

STEP II.—To find the *slope*, or inclination of an originally straight longitudinal line in a beam to its original position. The solution of this problem depends on the principle, that the *difference of slope at two points in that line, is the product of the distance between those points into the mean curvature of the portion of the line between them.* That is to say, in symbols, let dx denote the length of a portion of the line, $1 \div r$ its mean curvature, di the difference of slope at the two ends of that portion; then

$$di = \frac{dx}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

Let i_0 be the slope of the beam at the point taken as the origin of co-ordinates; i, the slope at a point whose distance from that origin is x'; conceive the distance x' to be divided into an indefinite number of small parts, the length of each being dx; compute by equation 1 the curvature of each of those parts, and by equation 3 the successive differences of slope; sum or integrate those results, and the final result will be the whole difference between the slopes at the origin and at the point x'; that is to say,

$$i = i_0 + \int_0^{x'} \frac{dx}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

When the beam is supported and loaded in such a way that it is known to have *no slope* at a certain point, that point should be taken as the origin. This occurs in two cases; that of a beam fixed at one end and loaded on the projecting portion (fig, 147), which has no slope at the fixed end A; and that of a beam supported at both ends and symmetrically loaded (half shown in fig.

146), which has no slope at the middle point A. In these cases, let the tangent A X C at the point of no slope be taken as the axis of abscissæ, along which x' is to be measured; then

$$i_0 = 0; \text{ and } i = \int_0^{x'} \frac{d\,x}{r} \text{.................} (5.)$$

These are the most common cases in practice. In other cases, the slope i_0 at the origin must remain indeterminate until the third step of the solution is performed.

The following principles are the consequences of equation 3, when applied to *similar beams of the same material, under loads similarly distributed:—*

The slopes at corresponding points are as the lengths and curvatures; and therefore,

Under equal loads, *the slopes at corresponding points are directly as the lengths, and inversely as the breadths and cubes of the depths;*

Under the proof loads, *the slopes at corresponding points are directly as the lengths, and inversely as the depths.*

The following formulæ express these principles, as applied to the finding of the *steepest slope* in a given beam, which is in general at the point most distant from the point of no slope; for example, at D, in figs. 146 and 147.

Under a *given load* W;

$$\text{steepest slope } i'_1 = \frac{m''' \, W \, c^2}{E \, n' \, b \, h^3}; \text{................} (6.)$$

Under the *proof load,*

$$\text{steepest slope } i_1 = \frac{m'' \, f' \, c}{E \, m' \, h}; \text{.................} (7.)$$

And in sections of equal strength,

$$i_1 = \frac{m'' \, (f'_a + f'_b) \, c}{E \, h} \text{..................} (7\,\text{A.})$$

For beams fixed at one end, $c = l$; for beams supported at both ends, $c = l \div 2$.

m' and n' are the factors already explained in Article 162, equations 3 and 4, p. 252, and of which values have been given in the table, p. 254.

m'' and m''' are factors depending on the distribution of the load, the mode of support, and the longitudinal section of the beam. Examples of their values will be given in a table further on. They bear the following relations to each other:—

$$\left.\begin{array}{l} \text{in beams fixed at one end } m''' = m \, m''; \\ \text{in beams supported at both ends } m''' = 2 \, m \, m''; \end{array}\right\} \text{....} (8.)$$

m being the factor already explained in Article 161, equation 1, p. 247, and of which values have been given in the tables of pp. 245 and 246.

STEP III.—To find the *Deflection.* By this term is to be understood the *depression of the lowest point below the highest point of an originally straight horizontal longitudinal line in the beam.*

Let $d\,x$ be the distance between two points in that line, i the mean slope of the line between them, and $d\,v$ their difference of level; then

$$d\,v = i\,d\,x \text{.........................(9.)}$$

Assume any convenient point in the line in question as the origin of co-ordinates; let x' be the distance of another point from it; conceive that distance to be divided into an indefinite number of small parts, the length of each being $d\,x$; compute, by the second step of the process, the slope of each of those divisions, and by equation 9, the successive differences of elevation of their ends; the sum or integral of those results will be the elevation or depression of the point x' relatively to the origin, according as it is positive or negative; that is to say,

$$v = \int_0^{x'} i\,d\,x \text{.........................(10.)}$$

This equation finally determines the figure assumed by an originally straight longitudinal line in the beam.

In the two cases represented by figs. 146 and 147—that is, when the beam is symmetrically loaded, or fixed at one end—the most convenient point for the origin is still the point of no slope A, and the *deflection* sought is the difference of elevation between that point and the furthest point D, whose distance from it is, in a symmetrically loaded beam, the *half-span,* $l \div 2$, and in a beam fixed at one end, the length of the projecting part, l. Hence, denoting the deflection by v_1,

$$\left.\begin{array}{l}\text{In a symmetrically loaded beam, } v_1 = \int_0^{\frac{l}{2}} i\,d\,x; \\ \text{in a beam fixed at one end, } v_1 = \int_0^{l} i\,d\,x.\end{array}\right\} \text{(11.)}$$

In other cases, the most convenient point for the origin of co-ordinates is in general one of the points of support; the fixity of the other point of support, for which $v = 0$, will give an equation from which i_0 in equation 4 may be found, and the positions of the most elevated and depressed points are to be found by the condition that for them the slope $i = 0$. Examples of such problems will be given in the sequel.

The following principles are the consequences of equation 9, when applied to *similar beams of the same material, under loads similarly distributed*:—

Under equal loads, *the deflections are directly as the squares of the lengths, and inversely as the breadths and cubes of the depths.*

Under the proof loads, *the deflections are directly as the squares of the lengths, and inversely as the depths.*

The following formulæ express these principles:—

Deflection under a given load W,

$$v'_1 = \frac{n''' \mathrm{W} c^3}{\mathrm{E} n' b h^3}; \qquad \text{(12.)}$$

Proof Deflection,

$$v_1 = \frac{n'' f c^2}{\mathrm{E} m' h}. \qquad \text{(13.)}$$

And in sections of equal strength,

$$v_1 = \frac{n'' (f_a + f_b) c}{\mathrm{E} h}. \qquad \text{(13 A.)}$$

For beams fixed at one end, $c = l$; for beams supported at both ends, $c = l \div 2$.

m' and n' are the factors already explained in Article 162, equations 3 and 4, p. 252, and of which values have been given in the table, p. 254.

n'' and n''' are factors depending on the distribution of the load, the mode of support, and the longitudinal section of the beam. They bear the following relations to each other:—

$$\left.\begin{array}{l}\text{in beams fixed at one end, } n''' = m\, n''; \\ \text{in beams supported at both ends, } n''' = 2\, m\, n''\end{array}\right\} \ldots \text{(14.)}$$

The following tables give examples of the values of the factors in equations 6, 7, 7 A, 12, 13, and 13 A:—

Case.	Proof Load. Factors for		Any Load. Factors for	
	Slope. m''	Deflection. n''	Slope. m'''	Deflection. n'''
A. UNIFORM CROSS-SECTION.				
I. Constant Moment of Flexure,	1	$\frac{1}{2}$		
II. Fixed at one end, loaded at other,	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
III. Fixed at one end, uniformly loaded,	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$
IV. Supported at both ends, loaded in middle,	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$
V. Supported at both ends, uniformly loaded,	$\frac{2}{3}$	$\frac{5}{12}$	$\frac{1}{6}$	$\frac{5}{48}$
B. UNIFORM STRENGTH AND UNIFORM DEPTH. (See Article 165, pp. 259, 260.)				
(The curvature of these is uniform).				
VI. Fixed at one end, loaded at other,	1	$\frac{1}{2}$	1	$\frac{1}{2}$
VII. Fixed at one end, uniformly loaded,	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
VIII. Supported at both ends, loaded in middle,	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
IX. Supported at both ends, uniformly loaded,	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
C. UNIFORM STRENGTH AND UNIFORM BREADTH. (See Article 165, pp. 259, 260.)				
X. Fixed at one end, loaded at other,	2	$\frac{2}{3}$	2	$\frac{2}{3}$
XI. Fixed at one end, uniformly loaded,	infinite	1	infinite	$\frac{1}{2}$
XII. Supported at both ends, loaded in middle,	2	$\frac{2}{3}$	1	$\frac{1}{3}$
XIII. Supported at both ends, uniformly loaded,	1·5708	0·5708	0·3927	0·1427

The values of m'' and n'' for beams of uniform strength, as given in the above table, are greater than those which occur in practice, because, in computing the table, no account has been taken of the additional material which is placed at the ends of such beams, in order to give sufficient resistance to shearing (see p. 267).

The error thus arising applies chiefly to m'', the factor for the maximum slope. For the factor for the deflection, n'', the error is inconsiderable, as experiment has shown.

170. The **Proportion of the Greatest Depth of a Beam to the Span** (*A. M.*, 302,) is so regulated, that the proportion of the greatest deflection to the span shall not exceed a limit which experience has shown to be consistent with convenience. That proportion, from various examples, appears to be—

$$\text{For the working load, } \frac{v_1}{l} = \text{from } \frac{1}{600} \text{ to } \frac{1}{1{,}500}.$$

$$\text{For the proof load, ... } \frac{v_1}{l} = \text{from } \frac{1}{200} \text{ to } \frac{1}{600}.$$

The determination of the proportion, $h_0 \div l$, of the greatest depth of the beam to the span, so as to give the required stiffness, is effected by the aid of equation 13 of Article 169, p. 273, from which we obtain

$$\frac{v_1}{l} = \frac{n'' f c}{2\,\mathrm{E} m' h_0} = \frac{n'' f\, l}{4\,\mathrm{E} m' h_0};$$

consequently the required ratio is given by the equation

$$\frac{h_0}{l} = \frac{n'' f}{4\, m' \mathrm{E}} \cdot \frac{l}{v_1}, \text{ ....(1.)}$$

an expression consisting of three factors: a factor, $n'' \div 4\,m'$, depending on the distribution of the load and the figure of the beam; a factor, $l \div v_1$, being the prescribed ratio of the span to the deflection; and a factor, $f \div \mathrm{E}$, being the *proof* strain, or the *working* strain, of the material, as the case may be. When the cross-section is one of equal strength, as in Article 164, equation 1 may be put in the following form:—

$$\frac{h_0}{l} = \frac{n''(f'_a + f'_b)\, l}{4\,\mathrm{E}\, v_1}. \text{ ....(2.)}$$

EXAMPLE I.—Let the beam be under its *working load*, uniformly distributed, on a beam of uniform section, alike above and below. Then $n'' = \frac{5}{12}$, $m' = \frac{1}{2}$. Let $\frac{l}{v_1} = 1{,}000$ be the prescribed ratio of the span to the working deflection. Let the material be wrought iron, for which $\frac{1}{3,000}$ is a safe value for the working strain $\frac{f}{\mathrm{E}}$. Then

$$\frac{h_0}{l}=\frac{5}{24}\cdot\frac{1,000}{3,000}=\frac{5}{72}=\frac{1}{14\cdot 4};$$

which is very nearly the average proportion of depth to span adopted for wrought iron girders in practice.

Example II.—Let the beam still be under its working load, uniformly distributed; let the cross-section be of equal strength, and let the longitudinal section be one of *uniform strength and uniform depth*. (See Article 165, Case IV., p. 260.) In this case, $n''=\frac{1}{2}$. Let $l \div v_1$ be still $=1,000$. The material being wrought iron, and the factor of safety about 6, let $f'_a = 6,000$; $f'_b = 10,000$; and let $\mathrm{E} = 29,000,000$. Then

$$\frac{h_0}{l}=\frac{1}{8}\cdot\frac{16,000\times 1,000}{29,000,000}=\frac{1}{14\cdot 5};$$

being nearly the same as in the preceding example.

Example III.—As in Example II., let the beam be under its working load, uniformly distributed; let the cross-section be of equal strength, and the longitudinal section of uniform strength and uniform depth. Then $n''=\frac{1}{2}$. Let the material be cast iron; let the factor of safety be 6, and let $f'_a = 13,333$, $f'_b = 3,333$, $\mathrm{E} = 16,666,000$. The following are the proportions of greatest depth to length for two different values of the proportion of the greatest deflection to the length:—

$$\text{for } \frac{l}{v_1}=600,\ \frac{h_0}{l}=\frac{1}{13\cdot 3}$$

$$\text{for } \frac{l}{v_1}=800,\ \frac{h_0}{l}=\frac{1}{10}.$$

171. **Summary of the Process of Designing a Beam.**—In designing a beam of a given material, and of a given span, to support a given load, distributed in a given way, the process to be followed may be thus summed up:—

I. Decide to what class of figures the cross-section shall belong; for example, whether it is to be rectangular, similar above and below, T-shaped, double T-shaped, of equal strength, and so forth (see Article 164, pp. 256 to 259); also, of what kind the longitudinal section is to be: as to which, see Article 165, pp. 259 to 261.

II. Determine the greatest depth, by the considerations mentioned in Article 170, p. 275.

III. Find the shearing force and bending moment at a sufficient number of cross-sections, and the greatest shearing force and bending moment, as in Articles 160, 161, pp. 239 to 249.

IV. Multiply the greatest bending moment by a proper factor of safety; which, for a travelling or otherwise moving load, will, in general, be *six*. This gives the breaking moment for rupture by cross-breaking. In like manner, the greatest shearing force, multiplied by a proper factor of safety, gives the ultimate resistance to shearing at the section where the shearing force is greatest.

V. Determine *provisionally* the product of the extreme breadth and square of the depth at the section of greatest bending moment, by dividing that moment (M_0) by the modulus of rupture of the material (f), and by the proper factor (n). (See Article 162, equation 5, p. 252; also the table of the values of n, p. 254.) That is to say, make

$$b' h^2 = \frac{M_0}{n f} \text{..........................} (1.)$$

Divide this by the square of the depth, already found: the result will be the *provisional extreme breadth, b'*.

In some cases, such as those of T-shaped and double T-shaped sections of equal strength, the above process may not be convenient; and then the *provisional sectional areas* of the different parts of the beam are to be deduced from the required moment of resistance M_0, and the already fixed depth h, by the aid of equations 1, 2, 3, 4, or 5, of Article 163, pp. 255, 256, or of the formulæ of Article 164, pp. 256 to 259.

VI. From the extreme depth and the extreme breadth, or sectional area (as the case may be), at the section of greatest bending moment, find all the other required transverse dimensions of the beam.

VII. Thence compute its weight. If this is a small fraction of the external load, the results already obtained are sufficient.

VIII. But if the weight of the beam forms a considerable part of the load, the results already obtained are *provisional* only, and the breadths (and therefore the sectional areas) are to be increased everywhere in the proportion given by Article 167, equation 1, p. 262. The weight of the beam also will be increased in the same proportion.

By means of equation 2 of the same Article, p. 262, the ratio of the weight of the beam to the external load may be found approximately so soon as the extreme depth, and the diameter of the cross and longitudinal sections have been fixed; and then the breaking load may be found approximately by equation 3 of the same Article, and used in computing the required ultimate resistance to cross-breaking and to shearing; whence the true breadths and areas of the beam may be found at once. But when this method is followed, the exact weight of the beam should afterwards be computed from the dimensions, to test whether the approximate value is sufficiently near the truth.

IX. The method of Article 168, pp. 266 to 268, may, if necessary, be employed to test whether the cross-section at the points of greatest shearing force is sufficient to resist that force.

172. **Suddenly-applied Load—Swiftly-rolling Load.** (*A. M.*, 306.)—A suddenly-applied transverse load, like the suddenly-applied pull of Article 149, p. 227, produces at first double the maximum stress, and double the strain, which the application of a load gradually increasing from nothing to the amount of the given load would produce.

The action of the rolling load to which a railway bridge is subjected is intermediate, in those cases which occur in practice, between that of an absolutely sudden load and a perfectly gradual load. It has been investigated mathematically by Mr. Stokes, and experimentally by Captain Galton, and the results are given in the Report of the Commissioners on the Application of Iron to Railway Structures.

The additional strain arising, whether from the sudden application or swift motion of the load, is sufficiently provided for in practice by the method already so frequently referred to, of making the factor of safety for the travelling part of the load about double of the factor of safety for the fixed part.

173. The **Resilience or Spring of a Beam** (*A. M.*, 305,) is the *work performed* in bending it to the proof deflection;—in other words, the *energy of the greatest shock* which the beam can bear without injury; such energy being expressed by the product of a weight into the height from which it must fall to produce the shock in question. This, if the load is concentrated at or near one point, is the product of half the proof load into the proof deflection; that is to say, let W be the proof load; then the resilience is

$$\frac{\mathrm{W}v_1}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

If the load is distributed, the length of the beam is to be divided into a number of small elements, and half the proof load on each element multiplied by the distance through which that element is moved during the proof deflection of the beam. Let u be that distance; then for beams fixed at one end,

$$\left.\begin{array}{r} u = v; \\ \text{and for beams supported at both ends,} \\ u = v_1 - v. \end{array}\right\} \dots\dots\dots\dots\dots\dots (2.)$$

Let $d\,x$ be the length of an element of the beam; w the intensity of the load on it, per unit of length; then the resilience is

$$\frac{1}{2} \int u\,w \cdot d\,x \dots\dots\dots\dots\dots\dots (3.)$$

The cases in which the determination of resilience is most useful in practice are those in which the load is applied at one point.

Let the beam be fixed at one end and loaded at the other, c being the length of its projecting part. Then

$$Resilience = \frac{W v_1}{2} = \frac{n\,n''}{2\,m'} \cdot \frac{f'^2}{E} \cdot c\,b\,h \dots\dots\dots\dots (4.)$$

This expression consists of three factors, viz.:—

(1.) The volume of the prism circumscribed about the beam, $c\,b\,h$.

(2.) A *Modulus of Resilience*, $\frac{f'^2}{E}$, of the kind already mentioned in Article 149, p. 227.

(3.) A numerical factor, $\frac{n\,n''}{2\,m'}$; in which n and m' (see p. 252) depend on the form of cross-section of the beam, and n'' (see p. 273), on the form of longitudinal section and of plan. The following are values of this compound factor for a *rectangular cross-section*, for which $n = \frac{1}{6}$, $m' = \frac{1}{2}$, and therefore $\frac{n\,n''}{2\,m'} = \frac{n''}{6}$:—

	$\frac{n''}{6}$.
I. Uniform breadth and depth, …………………………………	$\frac{1}{18}$.
II. Uniform strength, uniform depth, ………………………	$\frac{1}{12}$.
III. Uniform strength, uniform breadth, ……………………	$\frac{1}{9}$.

If a beam be supported at both ends and loaded in the middle, its length being $l = 2\,c$, its proof deflection is the same with that of a beam of the same transverse dimensions and of the length c, fixed at one end and loaded at the other; and its proof load is double of that of the latter beam; therefore, its resilience is double of that of the latter beam. Consequently, for rectangular beams of the half span c, supported at both ends and loaded in the middle, we have the following values for the numerical factor of the resilience:—

	$\frac{n''}{6}$.
IV. Uniform breadth and depth,....................................	$\frac{1}{9}$.
V. Uniform strength, uniform depth, ...	$\frac{1}{6}$.
VI. Uniform strength, uniform breadth,.........................	$\frac{2}{9}$.

174. **Effect of Twisting on a Beam.** (*A. M.*, 320 to 325.)—A full account of the theory of resistance to twisting and wrenching would be out of place in the present treatise, as engineering structures are never subjected to any considerable strain of that kind. For the solution of such questions as commonly occur in practice respecting such structures, the following principles are sufficient:—

I. The *Moment of Torsion* or *Twisting Moment* of a load, means the moment of the pair of equal and opposite couples, which, being applied at different points in the length of a bar, tend to twist the intermediate portion, and, if great enough, to break it by wrenching.

II. The *Ultimate Moment of Resistance* of a bar to wrenching ranges from about once and a-half to twice its Moment of Resistance to cross-breaking.

III. Suppose that the resultant load on a beam, W, and the supporting pressures, act in a plane which, instead of coinciding with the middle longitudinal vertical section of the beam, lies to one side of that section, and parallel to it, at the distance L. Then besides the *bending moment* on each cross-section of the beam (M), found as in Article 160, there is a *Twisting Moment* whose value is,

$$T = P_1\,L \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

P_1 being the greatest supporting pressure.

In finding the Moment of Resistance (M_1) required to give the beam sufficient strength, the following formula is near enough *to* the truth for practical purposes:—

$$M_1 = \sqrt{\left\{\frac{M^2}{4} + T^2\right\}} + \frac{M}{2}; \quad \text{..................} (2.)$$

AM, 3

and the dimensions of the beam are to be computed as if this quantity, instead of M, were the bending moment of the load.

175. The **Expansion and Contraction** of long beams (*A. M.*, 309), which arise from the changes of atmospheric temperature, are usually provided for by supporting one end of each beam on rollers of steel or hardened cast iron. The following table shows the proportion in which the length of a bar of certain materials is increased by an elevation of temperature from the melting point of ice (32° Fahr., or 0° Centigrade) to the boiling point of water under the mean atmospheric pressure (212° Fahr., or 100° Centigrade); that is, by an elevation of 180° Fahr., or 100° Centigrade:—

METALS.

Brass,	·00216
Bronze,	·00181
Copper,	·00184
Gold,	·0015
Cast iron,	·00111
Wrought iron and steel,	·00114 to ·00125
Lead,	·0029
Platinum,	·0009
Silver,	·002
Tin,	·002 to ·0025
Zinc,	·00294

EARTHY MATERIALS.

(The expansibilities of stone from the experiments of Mr. Adie.)

Brick, common,	·00355
„ fire,	·0005
Cement,	·0014
Glass, average of different kinds,	·0009
Granite,	·0008 to ·0009
Marble,	·00065 to ·0011
Sandstone,	·0009 to ·0012
Slate,	·00104

TIMBER.

(Expansion along the grain, when dry, according to Mr. Joule, *Proceed. Roy. Soc.*, Nov. 5, 1857.)

Baywood,	·000461 to ·000566
Deal,	·000428 to ·000438

Mr. Joule found that moisture diminishes, annuls, and even reverses, the expansibility of timber by heat, and that tension increases it.

176. **Beam Fixed at Both Ends.** (*A. M.*, 307.)—The particular problems respecting beams, which have been solved in the preceding Articles, have all reference to cases in which the beam is either firmly fixed at one end and loaded on the projecting portion, or simply supported at the two ends and loaded between them, and in which, consequently, the determination of the shearing force and bending moment at each point, and of the curvature, slope, and deflection, are simple and direct processes, proceeding step by step from the determination of one quantity to that of another. In this and the following Articles, problems will be considered in which the shearing force and bending moment depend, to a greater or less extent, on the curvature, slope, and deflection; and in which, consequently, the algebraical process of elimination is often required, two or more unknown quantities having to be determined at once by solving an equal number of equations at the same time.

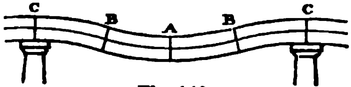

Fig. 148.

A beam is *fixed*, as well as supported at both ends, when a pair of equal and opposite couples are made to act on the vertical sectional planes at its points of support, of magnitude sufficient to maintain its longitudinal axis horizontal there, and so to diminish the deflection, slope, and curvature of its middle portion. This is generally accomplished by making the beam form part of one continuous girder with several points of support, or by making it project on either side beyond its points of support, and so fastening or loading the projecting portions, that their loads, or the resistance of their fastenings, shall give the required pair of couples.

In fig. 148, let C B A B C represent a beam supported at the points, C, C, loaded along its intervening portion, and so fixed or loaded beyond these points, that at them its longitudinal axis is horizontal, instead of having the slope i'_1, which it would have if the beam were simply *supported* at C, C, and not fixed. At each of the vertical sections above the points of support, C, C, there is an *uniformly-varying horizontal stress*, being a pull above and a thrust below the neutral axis; and the moment of that pair of stresses is that of the pair of equal and opposite couples which maintain the beam horizontal at the points of support. It is required to find,—in the first place, that resisting moment at the vertical planes of support (from which the stress on the material there may at once be found); and secondly, the effect of that moment on the curvature, slope, deflection, and strength of the beam.

The general method of solution of this question is as follows:—Compute, by equation 5 of Article 169, p. 271, i'_1, the slope which the neutral surface of the beam would have at the points, C, C, if it were simply supported there, and not fixed. Then, by the expression $\mathrm{E\,I}\,i'_1 \div c$, find the *uniform* moment of flexure, which if it acted on the beam in such a manner as to make it become convex upwards, would produce a slope at the points, C, C, *equal and contrary* to i'_1. This will be the required moment of resistance at the vertical sections C, C. It will afterwards appear that this is the greatest moment of resistance in the beam; so that by putting it instead of M_0 in the formulæ of Articles 162, 163, and 164, pp. 251 to 259, the conditions of strength of the beam are determined. Denote this moment by $-\mathrm{M}_1$, the negative sign denoting that it tends to produce convexity upwards, while the load on the beam tends to produce convexity downwards.

Let M be what the moment of flexure at any point of the beam *would be*, if it were simply supported at C, C. Then the actual moment of flexure is

$$\mathrm{M} - \mathrm{M}_1,$$

and by substituting this for M in the equations of Article 169, pp. 268 to 273, the curvature, slope, and deflection, with the proof load, or with any load, are found.

Where M is the greater, as at A, the beam is convex downwards. Where M is the less, as at C, the beam is convex upwards. There are a pair of points, B, B, at which $\mathrm{M} = \mathrm{M}_1$, so that the moment of flexure, and consequently the curvature, vanish, and the beam is subjected to a shearing force alone; these are called the *points of contrary flexure;* and they divide the middle part of the beam, which is convex downwards, from the two endmost parts, which are convex upwards.

EXAMPLE I.—*Symmetrical load on a beam of uniform section, in general.* By Article 169, equation 6, p. 271, observing that $l = 2\,c$, we have

$$i'_1 = \frac{2\,m''\,m}{n'} \cdot \frac{\mathrm{W}\,c^2}{\mathrm{E}\,b\,h^3};$$

And by the table in the same Article, p. 274, Case I.

$$\mathrm{M}_1 = \frac{\mathrm{E\,I}\,i'_1}{c} = \frac{n'\,\mathrm{E}\,b\,h^3\,i'_1}{c};$$

$$= 2\,m''\,m\,\mathrm{W}\,c = m'' \cdot m\,\mathrm{W}\,l = m'' \cdot \mathrm{M}_0 . \ldots\ldots\ldots (1.)$$

M_0 being what the bending moment at A *would have been*, had the beam been simply supported.

The values of m'' are given in Article 169, p. 274.

Let M'_0 be the actual bending moment at A. Then

$$M'_0 = (1 - m'')\, M_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

The greatest moment of flexure must be either at A or C, or at both, if the moments of these sections be equal and opposite. But for beams of uniform section, m'' is never less greater than $\frac{1}{2}$; therefore the greatest moment of flexure is at C, or both at C and A, and never at A alone.

The *ultimate strength* or greatest moment of resistance of the beam is expressed by the following formula, obtained by putting M_1 instead of $m\,W\,l$, in equation 6 of Article 162, p. 253:—

$$M_1 = m''\, m\, W\, l = n\, f\, b\, h^2; \ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

W being the breaking load, and f the modulus of rupture.

Hence it appears, that *by fixing the ends of an uniform beam so that they shall be horizontal, its strength is increased in the ratio* $1 : m''$.

The *deflection* is found by subtracting that due to the uniform moment M_1, from that which the load would produce if the beam were simply supported at C and C.

The result is as follows:—

$$v_1 = \left(\frac{n''}{m''} - \frac{1}{2}\right) \cdot \frac{M_1\, c^2}{E\, I} = \left(n'' - \frac{m''}{2}\right) \cdot \frac{M_0\, c^2}{E\, I} \ldots\ldots (4.)$$

For values of n'' see the table already referred to, p. 274. From the last of those expressions, it appears that by fixing the ends horizontal, an uniform beam is made stiffer *under a given load* in the ratio

$$n'' : \left(n'' - \frac{m''}{2}\right).$$

If in the first expression for the deflection, M_1 be considered to represent the moment of resistance corresponding to the proof or limiting safe stress at the section C, we may make $M_1 \div I = f' \div m'\, h$; so as to obtain the following expression for the *deflection under the proof load:*—

$$v_1 = \left(\frac{n''}{m''} - \frac{1}{2}\right) \frac{f\, c^2}{E\, m'\, h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

being less than the proof deflection of a beam simply supported, as given by equation 13, Article 169, p. 273, in the ratio

$$\left(\frac{n''}{m''}-\frac{1}{2}\right):n''.$$

The points of contrary flexure are to be found in each particular case by solving the equation

$$\mathrm{M}-\mathrm{M}_1=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.)$$

Examples II. and III. are particular cases of the general problem in Example I.

Example II.—*Uniform section, loaded in the middle.*

$$\left.\begin{aligned}&m=\frac{1}{4};\ m''=\frac{1}{2};\ n''=\frac{1}{3};\\&\mathrm{M}'_0=\mathrm{M}_1=\frac{1}{2}\,\mathrm{M}_0=\frac{1}{8}\,\mathrm{W}\,l=\frac{1}{4}\,\mathrm{W}\,c=n\,f\,b\,h^2;\\&v_1=\frac{1}{6}\cdot\frac{f\,c^2}{\mathrm{E}\,m'\,h};\ \left(\frac{n''}{m''}-\frac{1}{2}\right)\div n''=\frac{1}{2}.\end{aligned}\right\}\ldots(7.)$$

The points of contrary flexure are midway between A and C.

Example III.—*Uniform section, uniformly loaded.*

$$\left.\begin{aligned}&\mathrm{W}=w\,l=2\,c\,w\\&m=\frac{1}{8};\ m''=\frac{2}{3};\ n''=\frac{5}{12};\\&\mathrm{M}_1=\frac{2}{3}\,\mathrm{M}_0=\frac{1}{12}\,\mathrm{W}\,l=\frac{1}{6}\,\mathrm{W}\,c=n\,f\,b\,h^2;\\&\mathrm{M}'_0=\frac{1}{2}\,\mathrm{M}_1=\frac{1}{3}\,\mathrm{M}_0=\frac{1}{24}\,\mathrm{W}\,l;\\&v_1=\frac{1}{8}\cdot\frac{f\,c^2}{\mathrm{E}\,m'\,h};\ \left(\frac{n''}{m''}-\frac{1}{2}\right)\div n''=\frac{3}{10}.\end{aligned}\right\}\ldots(8.)$$

The points of contrary flexure are each at the following distance from A, the middle point of the beam:—

$$\frac{c}{\sqrt{3}}=0{\cdot}577\,c=0{\cdot}289\,l;\ldots\ldots\ldots\ldots\ldots\ldots(9.)$$

Example IV.—*Uniform strength, uniform depth, uniform load.*

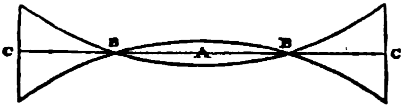

Fig. 149.

In this case the uniformity of strength is attained by making the breadth at each point proportional to the moment of flexure, as shown in the plan, fig. 149, preserving, at the points of contrary flexure B, B, a sufficient thickness only to resist the shearing force.

The curvature of the beam is uniform in amount, changing in direction only at the points of contrary flexure. Therefore, in fig. 148, C B and B A, at each side of the beam, are two arcs of circles of equal radii, horizontal at A and C, and touching each other at B; therefore those arcs are of equal length; therefore each point of contrary flexure B is midway between the middle of the beam A and the point of support C.

It is evident also, that the proof deflection of the beam must be double of that of an uniformly curved beam of half the span, supported at the ends without being fixed; that is to say, one-half of that of an uniformly curved beam of the same span, supported but not fixed; or symbolically

$$v_1 = \frac{1}{4} \cdot \frac{f c^2}{E m' h} \ldots\ldots\ldots\ldots(10.)$$

The actual moment of flexure at A must be the same as in an uniformly loaded beam, with the same intensity of load $w = W \div 2c$, supported, but not fixed at B, B; that is to say,

$$M'_0 = \frac{W c}{16} = \frac{W l}{32} = \frac{M_0}{4} \ldots\ldots\ldots\ldots(11.)$$

and therefore, the moment of flexure at C is

$$n f b_1 h^2 = M_1 = M_0 - M'_0 = \frac{3 M_0}{4} = \frac{3 W c}{16} = \frac{3 W l}{32}; \; (12.)$$

b_1 being the breadth of the beam at C, which is three times the breadth b_0 at A.

The breadth b, at any other point, whose distance from A is x, is given by the equation

$$b = \frac{1}{3}\left(1 - \frac{4 x^2}{c^2}\right) b_1 = \left(1 - \frac{4 x^2}{c^2}\right) b_0 \ldots\ldots\ldots(13.)$$

In using this equation, the positive or negative sign of the result merely indicates the direction of the curvature.

According to this equation, the figure of the beam in plan (fig. 149) consists of two parabolas, having their vertices at A, and intersecting each other in the points of contrary flexure, B, B, for which $x = \pm \frac{c}{2}$.

177. **A Beam Fixed at One End and Supported at Both** (*A. M.*, 308) is sensibly in the same condition with the part C B A B of the beam in fig. 148, extending from one of the fixed points C to the *farther* point of contrary flexure, which now represents a point *supported, but not fixed.* Hence if a continuous girder be supported on a series of piers, the span of each of the endmost bays should be to the span of each intermediate bay in the ratio $c + x_0 : 2\,c$, where x_0 is the distance A B from the lowest point to a point of contrary flexure.

178. **Beams imperfectly fixed at the Ends—Continuous Girders.** —The continuity of a girder over the piers which support it is not sufficient to fix it horizontal at the points of support, unless all the spans are alike loaded. Hence the full strengthening and stiffening effect of such continuity, as investigated in Article 176, is to be reckoned upon as regards the *fixed part of the load* only, and not as regards the travelling part.

In determining the moment of resistance required in the parts of such a girder which are directly over the piers, the whole girder is to be supposed loaded with the travelling load as well as the fixed load, and the method of Article 176 applied in order to find M_1 (p. 283).

But the same method will not in this case give a sufficiently great value to M'_0, the moment of resistance required at the middle of each span; for the greatest bending moment at that point is produced when one span is loaded with the travelling load as well as the fixed load, and the adjoining spans loaded with the fixed load only.

Greater or less precision may be used in finding M'_0 according to circumstances. The following approximate method errs on the safe side, and is the best to employ when the fixed load is light compared with the travelling load.

METHOD I.—Calculate that part of M'_0 which arises from the fixed load as for a girder fixed horizontal at the ends (Article 176, equation 2, p. 284), and that part which arises from the travelling load as for a girder merely supported at the ends, but not fixed. (Article 161, Case VI., p. 246). In other words, compute the bending moment which would be produced by the entire load, fixed and travelling, on a beam merely supported at the ends, and subtract from the result the already computed moment of flexure M_1 at the points of support, due to the fixed load alone.

That is to say, in symbols, let w be the intensity of the fixed load, in lbs. on the lineal foot, and w' that of the travelling load; then

$$M'_0 = \frac{(w+w')\,l^2}{8} - M_1 \ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

For example, for a *beam of uniform section, uniformly loaded,*

$$\left.\begin{aligned} M_1 &= \frac{w\,l^2}{12};\ \text{and} \\ M'_0 &= \left(\frac{w}{3} + w'\right)\frac{l^2}{8} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

To find the deflection, calculate that produced by the total load in a beam merely supported at the ends, and subtract that due to the uniform moment M_1.

That process gives the following result, for an uniform beam under a given load, $c = l \div 2$ as usual, being the half span:—

$$v'_1 = (w + 5\,w')\,\frac{c^4}{24\,E\,I} \ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

and supposing, as is usually the case, that M'_0 is equal to or greater than M_1 (that is, if w' is equal to or greater than $w \div 3$), so that the greatest stress is in the middle, the *proof deflection* is

$$v_1 = \frac{w + 5\,w'}{w + 3\,w'} \cdot \frac{f\,c^2}{4\,E\,m'\,h} \ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

If, on the other hand, M_1 is the greater (w' being less than $w \div 3$),

$$v_1 = \frac{w + 5\,w'}{2\,w} \cdot \frac{f\,c^2}{4\,E\,m'\,h} \ldots\ldots\ldots\ldots\ldots\ldots(5.)$$

Method II.—When the fixed load is greater than, equal to, or not much less than, the travelling load on a given length of girder, the following method may be used:—

Let fig. 150 represent a viaduct of several spans, consisting of a continuous girder resting at C, C, C, &c., on a series of equidistant piers. The endmost span C E is smaller than the rest; the principle upon which it is to be determined will be afterwards explained.

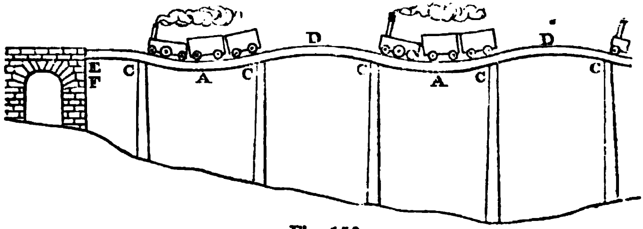

Fig. 150.

For the present, the bridge is to be conceived to consist of an indefinitely long series of equal spans, each alternate span only being uniformly loaded from end to end with the greatest possible travelling load.

Let w be the intensity, per lineal foot, of the fixed part of the load; w', that of the travelling part; so that $w + w'$ is the intensity of the load on the more heavily loaded spans, A, A, &c., and w that of the load on the intermediate spans, D, D, &c.

Let $-M_1$ denote the yet unknown negative moment of flexure at the points of support over the piers, C, C, C, &c.

In any heavily loaded division, let horizontal distances denoted by x be measured from the central point A.

In any lightly loaded division, let distances denoted by x' be measured from the central point D.

Let $c = l \div 2$ denote the half-span of each bay.

Let the beam be supposed of uniform section, the moment of inertia being I, and the depth h, as before.

Then the following are the results of the processes in Article 169:—

	Lightly loaded Division.	Heavily loaded Division.
Bending Moment M=	$\frac{w}{2}\left(c^2 - x'^2\right) - M_1$	$\frac{w+w'}{2}\left(c^2 - x^2\right) - M_1$
Slope $i = \int \frac{M}{EI} dx =$	$\frac{1}{EI}\left\{\frac{w}{2}\left(c^2 x - \frac{x'^3}{3}\right) - M_1 x'\right\}$	$\frac{1}{EI}\left\{\frac{w+w'}{2}\left(c^2 x - \frac{x^3}{3}\right) - M_1 x\right\}$

The condition of continuity of the beam above the points of support is, that for $x = c$, and $x' = -c$, the slope i shall be the same. This gives the following equation:—

$$-\frac{w c^3}{3} + M_1 c = \frac{(w + w') c^3}{3} - M_1 c;$$

whence we obtain the following value of the negative moment of flexure above each pier:—

$$-M_1 = \frac{(2w + w') c^2}{6} = \frac{(2w + w') l^2}{24}. \quad \text{............(6.)}$$

Introducing this into the expressions for bending moments, and slope, and proceeding with the processes of Article 169, we obtain the following results:—

	Lightly loaded Division.	Heavily loaded Division.
Bending Moment M =	$\frac{w-w'}{6}c^2-\frac{w}{2}x^2$	$\frac{w+2w'}{6}c^2-\frac{w+w'}{2}x^2$
Slope i .... =	$\frac{1}{EI}\left\{\frac{w-w'}{6}c^2x'-\frac{w}{6}x'^3\right\}$	$\frac{1}{EI}\left\{\frac{w+2w'}{6}c^2x-\frac{w+w'}{6}x^3\right\}$
Deflection v at any point .... =	$\frac{1}{EI}\left\{\frac{w-2w'}{24}c^4-\frac{w-w'}{12}c^2x'^2+\frac{w}{24}x'^4\right\}$	$\frac{1}{EI}\left\{\frac{w+3w'}{24}c^4-\frac{w+2w'}{12}c^2x^2+\frac{w+w'}{24}x^4\right\}$

The following cases of these equations are the most important in practice:—

Bending Moments—

$$\left.\begin{array}{l}\text{At the centre D of a lightly loaded span,}\\ \qquad M'_0=\frac{w-w'}{6}c^2=\frac{w-w'}{24}l^2;\\ \text{At the centre A of a heavily loaded span,}\\ \qquad M_0=\frac{w+2w'}{6}c^2=\frac{w+2w'}{24}l^2.\end{array}\right\}\text{..............(7.)}$$

The *greatest moment of flexure* will be either M_0 at A, or $-M_1$ at C, according as the intensity of the travelling load w', or that of the fixed load w, is the greater.

Central Deflexions under any load—

$$\left.\begin{array}{l}\text{Of a lightly loaded division, } v'_1=(w-2w')\frac{c^4}{24\,EI};\\ \text{Of a heavily loaded division, } v_1=(w+3w')\frac{c^4}{24\,EI}.\end{array}\right\}\text{(8.)}$$

If w is less than $2w'$, the first of these becomes an *elevation*, being negative.

Central Deflexion of a heavily loaded Division under the Proof Load—

$$\left.\begin{array}{l}\text{If } w' > w, \text{ so that } \mathrm{M}_0 > -\mathrm{M}_1, \\ \qquad v_1 = \dfrac{f' c^2}{4\,\mathrm{E}\,m' h} \cdot \dfrac{w + 3\,w'}{w + 2\,w'}; \\ \text{if } w > w', \text{ so that } -\mathrm{M}_1 > \mathrm{M}_0; \\ \qquad v_1 = \dfrac{f' c^2}{4\,\mathrm{E}\,m' h} \cdot \dfrac{w + 3\,w'}{2\,w + w'}\end{array}\right\}\ldots\ldots\ldots\ldots(9.)$$

The corresponding deflections of a lightly loaded division are found by multiplying these expressions by $\frac{w - 2\,w'}{w + 3\,w'}$.

Points of no curvature occur at the following distances from the centre of each division:—

$$\left.\begin{array}{l}\text{In a lightly loaded division, at } x' = \pm\, c\sqrt{\dfrac{w - w'}{3\,w}}; \\ \text{In a heavily loaded division, at } x = \pm\, c\sqrt{\dfrac{w + 2\,w'}{3\,(w + w')}}.\end{array}\right\}(10.)$$

When those points occur in pairs, they are *points of contrary flexure;* and this is always the case in a heavily loaded span; but in a lightly loaded span, if $w' = w$, there is but one point of no curvature, which is at the middle of the division, and is not a point of contrary flexure; and if $w' > w$, there is no such point in that span.

C E represents a division of the girder, at the end of the viaduct, of such a length that when it is unsupported at E its weight may be at least sufficient to produce the proper moment of flexure $-\mathrm{M}_1$ above the nearest pier C. In order that this may be the case, its length $\mathrm{C\,E} = l'$ should be at least sufficient to fulfil the following condition:—

$$\frac{w\,l'^2}{2} = -\mathrm{M}_1 = \frac{2\,w + w'}{6}\,c^2;$$

and consequently, the least limit of that length is given by the following formula:—

$$\mathrm{C\,E} = l' \geqq c\sqrt{\frac{2\,w + w'}{3\,w}}\ldots\ldots\ldots\ldots(11.)$$

($\geqq$ means "not less than," and $\leqq$ "not greater than.")

The division C E should not extend farther from C than the farthest point of contrary flexure, when that division has the travelling load on it; that is to say, the greatest limit of its length is

$$C\,E = l' \leqq c\left(1 + \sqrt{\frac{w + 2w'}{3(w + w')}}\right) \ldots\ldots\ldots\ldots (12.)$$

In order that the fulfilment of these conditions may be possible, the expression (12) must not be less than the expression (11). When the end E of the girder is not supported by the action of the travelling load, it rests on the abutment F.

Throughout the whole of the preceding calculations in this Article, it is to be understood that the *same factor of safety is employed both for the fixed and the travelling parts of the load.* If it is considered advisable that the factor of safety for the ordinary working travelling load should be double that for the fixed load (for example, that the former should be 6, while the latter is 3); then w and w' are to be held to represent the intensities of the two parts of the *proof* load, each being one-third of the corresponding portion of the load which would break the beam if divided in the same manner; so that M_0 or $-M_1$, as the case may be, is one-third of the breaking moment. In the course of the ordinary traffic upon the bridge, the intensity of the fixed load w will continue the same as before, while that of the greatest travelling load will be reduced to one-half of that of the travelling proof load; that is to say, $w' \div 2$.

179. **Rafter, or Sloping Beam with an Abutment.**—In fig. 151, A B represents a straight beam, loaded with weights, and having an abutment at A. The supporting pressures at A and B are to be found by the process explained in Article 112, Case III., p. 174.

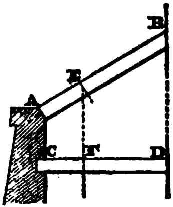

Fig. 151.

Resolve the load and the supporting pressures respectively into components parallel and perpendicular to the beam, or, as they may be called, longitudinal and transverse components. The strain on the beam is compounded of longitudinal compression, produced by the longitudinal forces, and of bending, produced by the transverse forces.

For example, let the load be uniformly distributed, and let w be its intensity in lbs. per lineal inch of the span *measured horizontally;* that is to say, if l denotes the length of the sloping beam between the points of support A, B, i its angle of inclination, and W the total load, the value of that intensity is

$$w = W \div l \cos i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

The supporting pressure at B is horizontal; that at A is inclined at the angle whose tangent is 2 tan i; and their values are respectively—

$$\left.\begin{array}{l}\text{At B, } \mathrm{H} = \mathrm{W} \div 2 \tan i \\ \text{At A, } \sqrt{\mathrm{H}^2 + \mathrm{W}^2} \ldots\end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

The longitudinal components of the load and supporting pressures are as follows:—

$$\left.\begin{array}{l}\text{Of the load, } -\mathrm{W} \sin i = -w\,l \cos i \sin i; \\ \text{Of the pressure at B; } -\mathrm{H} \cos i = -\dfrac{\mathrm{W} \cos^2 i}{2 \sin i}; \\ \text{Of the pressure at A; } \mathrm{H} \cos i + \mathrm{W} \sin i \\ \qquad = \dfrac{\mathrm{W}}{2}\left(\dfrac{1}{\sin i} + \sin i\right);\end{array}\right\} \ldots (3.)$$

the negative signs in the first two expressions denoting downward action. The transverse components are,—

$$\left.\begin{array}{ll}\text{Of the load, } \ldots\ldots\ldots\ldots\ldots\ldots & -\mathrm{W} \cos i = -w\,l \cos^2 i; \\ \left.\begin{array}{l}\text{Of each of the supporting} \\ \text{pressures} \ldots\ldots\ldots\ldots\ldots\ldots\end{array}\right\} & \dfrac{\mathrm{W} \cos i}{2}\end{array}\right\} \ldots (4.)$$

Let A denote the area of a given transverse section of the beam at E, whose distance from B is denoted by x'. Then there is at that section a longitudinal thrust whose intensity, found by dividing its amount by the area A, is as follows:—

$$p' = \left(\frac{\mathrm{W} \cos^2 i}{2 \sin i} + w\,x \cos i \sin i\right) \div \mathrm{A} \ldots\ldots\ldots\ldots (5.)$$

The bending moment M at the same cross-section is the same as in a beam of the span l, loaded with $w \cos^2 i$ lbs. on the lineal inch *measured along the beam*, and is to be found from these data by the formula of Article 161, Case VI., p. 246, or by the method of Article 176, p. 282, according as the beam is merely fixed in position at A and B, or fixed in *direction* as well as in position.

It may here be remarked that if C D be a horizontal beam of the span $l \cos i$ (being the horizontal projection of the span of A B), loaded with the same uniformly distributed load W as A B, and supported or fixed at the ends in the same manner, the moment of flexure at F, the cross-section corresponding to E, will be the same as at E.

Let I be "the moment of inertia" of the cross-section at E (see

p. 252), and $m'h$ the distance from the neutral axis to the concave side of the beam. Then the moment of flexure M produces an additional thrust at that side of the intensity,

$$p'' = \frac{M m' h}{I} \text{.........................(6.)}$$

so that the greatest intensity of thrust at that cross-section, and the condition that it shall not exceed a safe limit of intensity (f') are expressed as follows:—

$$p' + p'' \leqq f' \text{.........................(7.)}$$

In the best practical examples, the beam is fixed in direction at A and B; and in that case, the greatest moment of flexure, and the greatest longitudinal thrust, both occur at the abutting joint A. The value of the bending moment being taken from Article 176, equation 8, p. 285, the greatest intensity of thrust is found to be

$$p' + p'' = \frac{W}{2A}\left(\frac{1}{\sin i} + \sin i\right) + \frac{W l \cos i}{12} \cdot \frac{m'h}{I} \text{......(7 A.)}$$

In designing a sloping beam, the depth h may be fixed in the first place, as in Article 170, p. 275. The kind of cross-section adopted will then fix the ratios m', and $I \div m'h^2 A$, I and A themselves being still indeterminate. Let the last of these ratios be denoted by q. Then equation 7 A may be put in the following form:—

$$p' + p'' = \frac{W}{A}\left\{\frac{1}{2}\left(\frac{1}{\sin i} + \sin i\right) + \frac{l \cos i}{12 q h}\right\} \text{.......(7 B.)}$$

whence is deduced the following formula for computing the required sectional area:—

$$A = \frac{W}{f'}\left\{\frac{1}{2}\left(\frac{1}{\sin i} + \sin i\right) + \frac{l \cos i}{12 q h}\right\} \text{............(8.)}$$

Table of Values of $q = I \div m'h^2 A \left(= \frac{nbh}{A}\right.$, n having the values given in Article 163, p. $\left.254\right)$.

FORM OF CROSS-SECTION.	q.
I. Rectangle, ...	$\frac{1}{6}$.
II. Ellipse and Circle, ...	$\frac{1}{8}$.

III. Hollow Rectangle, $A = bh - b'h'$; also I-formed section, b' being the sum of the breadths of the lateral hollows,............. $\frac{1}{6}\left(1 - \frac{b'h'^3}{bh^3}\right) \div \left(1 - \frac{b'h'}{bh}\right)$.

VI. Hollow Square, $A = h^2 - h'^2$......... $\frac{1}{6}\left(1 + \frac{h'^2}{h^2}\right)$.

V. Hollow Ellipse,........................ $\frac{1}{8}\left(1 - \frac{b'h'^3}{bh^3}\right) \div \left(1 - \frac{b'h'}{bh}\right)$.

VI. Hollow Circle, ... $\frac{1}{8}\left(1 + \frac{h'^2}{h^2}\right)$.

VII. T-formed Section; approximate solution as in Article 163, equation 2, p. 255,—

(Flange A_1; web A_2) .... $\frac{A_2(A_2 + 4A_1)}{6(A_2 + A_1)(A_2 + 2A_1)}$.

VIII. Double T-formed section; approximate solution as in Article 163, equation 4, p. 256

(Flanges A_1, A_3; web A_2; the beam supposed to give way by crushing the flange A_1)

$$\frac{A_2(A_2 + 4A_1 + 4A_3) + 12A_1A_3}{6(A_2 + 2A_3)(A_1 + A_2 + A_3)}.$$

XI. Double T-formed section, alike above and below ($A_3 = A_1$);...... $\frac{1}{6}\left(1 + \frac{4A_1}{A_2 + 2A_1}\right)$.

When the deflection of the sloping beam A B is compared with that of the horizontal beam C D of equal horizontal span, and under the same load, it appears, from the principle of Article 169, p. 273, that *if those beams are of equal and similar cross-section*, their deflections at corresponding points being as the cubes of the lengths, and as the loads producing deflection, which are inversely as the lengths, are to each other as the squares of the lengths; that is

Deflection of A B : deflection of C D : : 1 : $\cos^2 i$......(9.)

Also, the *vertical components* of the deflections are as the lengths simply, or

Vertical component of deflection of A B,... : deflection of C D : : 1 : $\cos i$...(10.)

But if A B *be increased in breadth*, as compared with C D in the ratio of 1 : cos i, or sec i : 1, *the vertical components of their deflections will be equal.* This principle will be referred to in the next article.

179 A. To **Deduce the Greatest Stress in a Beam from the Deflection.**—This is done by means of a formula deduced from equation 13 of Article 169, p. 273, as follows:—

Let h be the depth of the beam at the section of greatest stress, and $m' h$ the distance from the neutral axis of that section to that surface of the beam at which the greatest stress is required; m', a factor explained in Article 162, p. 252, depending on the form of cross-section:—

c, the half-span of a beam supported at both ends, or the length of the loaded part of a beam supported at one end;

n'', the factor for proof deflection, explained and exemplified in Article 169, pp. 273, 274;

E, the modulus of elasticity of the material;

v, the observed deflection;

then the intensity of the greatest stress is

$$p_1 = \frac{\mathrm{E}\, m' h\, v}{n'' c^2} \text{..........................} (1.)$$

To the values of the factor n'' given in the table, p. 274, may be added the following, which are taken from Articles 176 and 177, pp. 284, 285, 286, 288, and 289.

CASES.	Factors.
XIV. Beam fixed at both ends, section uniform, load in the middle, ...	$\frac{1}{6}$
XV. Beam fixed at both ends, section uniform, load uniform, ...	$\frac{1}{8}$
XVI. Beam fixed at both ends, depth uniform, load uniform, strength uniform, ...	$\frac{1}{4}$
XVII. Beam imperfectly fixed at both ends, section uniform, load uniform, the dead load w being small compared with the rolling load w', and the greatest stress in the middle, ...	$\frac{w + 5 w'}{4 w + 12 w'}$
XVIII. Beam imperfectly fixed at both ends, section uniform, load uniform, the dead load w being considerable compared with the rolling load w', the lesser of the two following factors (see p. 291),	$\frac{w + 3 w'}{4 w + 8 w'}$ $\frac{w + 3 w'}{8 w + 4 w'}$

180. **Strength and Stiffness of an Arched Rib under Vertical Loads.**—Fig. 152 represents an arched rib, springing from a pair of

abutments, and supposed to be under a vertical load. Let B A C D B′ be a curve traversing the centres of gravity of all the cross-sections of the rib: this may be called the *neutral curve;* and it represents the figure of a "linear arch," or indefinitely thin rib, whose conditions of equilibrium are the same with those of the actual arch. Those conditions have been explained in Article 123, Case II., pp. 186, 187; Article 124, pp. 187, 188; Article 125, pp. 188 to 191; Article 128, pp. 195 to 198; Articles 130, 131, and 132, pp. 199 to 203.

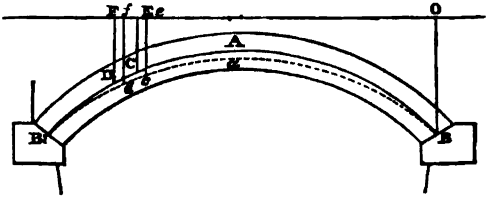

Fig. 152.

When a vertical load is distributed over the arch, agreeably to the conditions of equilibrium of the neutral curve, each particle of the arch is compressed, in a direction parallel to a tangent at the nearest point of the neutral curve; and but for the circumstance to be stated presently, that compression would be uniform throughout each cross-section of the rib, so that the neutral curve would be the "line of resistance."

But the compression depresses the whole arch, so that the neutral curve assumes some new figure, such as B *a c d* B′, in which its curvature at each point differs from the original curvature; and hence, even under a load distributed as for an equilibrated or linear arch, there is a bending action combined with the direct compression. When the distribution of the load differs from that suited to the neutral curve as a linear arch, the bending action varies in its amount and distribution.

In either case the arch acts in the double capacity of a rib under direct compression, and a beam under a transverse load; and its strain and stress at each point are the resultants of the strains and stresses arising from the directly compressive action of the load, and from its bending action.

PROBLEM FIRST. *General Case.*—In solving problems which relate to this subject, it is in general most convenient to measure co-ordinates from a point such as O, in the same vertical line with one end, B, of the neutral curve.

C being any point in the curve, let

$$x = \mathrm{O\,E} \text{ be its horizontal distance from O;}$$

$$y = \mathrm{E\,C} \text{ its vertical depth below O;}$$

Let l = B B′ be the span of the neutral curve, and k its rise.

Let w be the whole intensity of the vertical load, whether constant or variable, in lbs. per inch of horizontal distance, so that

$$\int_0^l w\,d\,x \text{ is the whole load on the arch.}$$

The load $w\,d\,x$ on each small portion of the arch may be conceived to consist of two parts,

$w_1\ d\ x$, producing direct compression alone, being distributed according to the laws of the equilibrium of a linear arch,—that is, in such a manner that $w_1 = \mathrm{H}\frac{d^2 y}{d x^2}$ (H being the still undetermined horizontal thrust of the arch), and

$$(w - w_1)\,d\,x = \left(w - \mathrm{H}\,\frac{d^2 y}{d\,x^2}\right) d\,x, \text{ ....(1.)}$$

producing bending.

Having formed the preceding expression, by putting for w and $\frac{d^2 y}{d\,x^2}$ their proper values, proceed as follows:—

The vertical component of the shearing force at any point, such as C, is (see p. 242)—

$$\mathrm{F} = \mathrm{F}_0 - \int_0^x w\,d\,x + \mathrm{H}\left(\frac{d\,y}{d\,x} - \frac{d\,y_0}{d\,x_0}\right) \text{ ....(2.)}$$

F_0 being the still undetermined vertical component of the shearing force at B, and $\frac{d\,y_0}{d\,x_0}$ the slope of the neutral curve at that point.

The bending moment at C is (see p. 243)—

$$\left.\begin{array}{c} \mathrm{M} = \mathrm{M}_0 + \int_0^x \mathrm{F}\,d\,x = \mathrm{M}_0 + \mathrm{F}_0\,x - \int_0^x\int_0^x w\,d\,x^2 - \\ \mathrm{H}\left(y_0 - y + x\frac{d y_0}{d x_0}\right) \end{array}\right\} \text{...(3.)}$$

M_0 being the still undetermined bending moment at B.

The *alteration of curvature* produced in the neutral curve at C by the bending action is $-\mathrm{M} \div \mathrm{E\,I}$; the negative sign being prefixed to denote that downward curvature is to be considered as positive; and the *alteration of slope* is expressed as follows:—

$$i = \frac{d\,v}{d\,x} = i_0 - \int_0^x \frac{\mathrm{M}}{\mathrm{E\,I}}\sqrt{1 + \frac{d\,y^2}{d\,x^2}}\cdot d\,x; \text{........(4.)}$$

i_0 being the still undetermined alteration of the slope at B.

The vertical deflection at C is expressed thus,—

$$v = \int_0^x i \, d x. \text{ ....} (5.)$$

The bending action of the load is thus expressed by the four equations, 2, 3, 4, 5, containing four indeterminate constants, H, F_0, M_0, i_0. If, in each of those equations, x be made $= l$, expressions are obtained applicable to the further end of the span, B′. These expressions may be denoted by F_1, M_1, i_1, v_1.

Let $d\,s = \text{C D} = \sqrt{d\,x^2 + d\,y^2}$ denote the length of an indefinitely short arc of the neutral curve. That arc is not altered in length by the bending action of the load; but it is altered by the direct compression in the proportion given by the following equation:—

$$\frac{d\,t}{d\,s} = - \frac{\text{H} \frac{d\,s}{d\,x}}{\text{E A}}; \text{ ....} (6.)$$

in which A denotes the sectional area of the rib at C, and the negative sign indicates compression.

To find the combined effect of the bending action and the compressive action on the figure of the neutral curve, proceed as follows:—

Let u denote the positive horizontal displacement of a point in it, such as C. For example, C D being the original position of an indefinitely short arc, and $c\,d$ its altered position, let

$$\begin{array}{ll} \text{O E} = x; & \text{O F} = x + d\,x; \\ \text{E C} = y; & \text{F D} = y + d\,y; \\ \multicolumn{2}{c}{\text{C D} = d\,s;} \\ \text{O}\,e = x + u; & \text{O}\,f = x + u + d\,x + d\,u; \\ e\,c = y + v; & f\,d = y + v + d\,y + d\,v; \\ \multicolumn{2}{c}{c\,d = d\,s + d\,t.} \end{array}$$

Then from the two equations,

$$d\,s^2 = d\,x^2 + d\,y^2;$$

$$(d\,s + d\,t)^2 = (d\,x + d\,u)^2 + (d\,y + d\,v)^2;$$

The following is deduced:—

$$2\,d\,s \cdot d\,t + d\,t^2 = 2\,d\,x \cdot d\,u + d\,u^2 + 2\,d\,y \cdot d\,v + d\,v^2;$$

and from this the terms $d\,t^2$, $d\,u^2$, $d\,v^2$, may be rejected, as inappreciably small compared with the other terms, reducing it to the following:—

$$d s \cdot d t = d x \cdot d u + d y \cdot d v;$$

whence is obtained the following expression for the *horizontal displacement of* D *relatively to* C:—

$$d u = \frac{d s}{d x} d t - \frac{d y}{d x} d v. \quad \text{.................(7.)}$$

For $d t$ put its value according to equation 6, and make $\frac{d s^2}{d x^2} = 1 + \frac{d y^2}{d x^2}$, and $d v = i\, d x$; then

$$d u = - \frac{\mathrm{H}}{\mathrm{E\,A}} \cdot \left(1 + \frac{d y^2}{d x^2}\right)^{\frac{3}{2}} d x - i \frac{d y}{d x} d x; \quad \text{...(7 A.)}$$

which being integrated, gives for the horizontal displacement of C relatively to B and in a direction away from it,

$$u = - \int_0^x \left\{ \frac{\mathrm{H}}{\mathrm{E\,A}} \left(1 + \frac{d y^2}{d x^2}\right)^{\frac{3}{2}} + i \frac{d y}{d x} \right\} d x; \quad \text{.......(8.)}$$

an expression containing the same four indeterminate constants that have already been mentioned; and if x be made $= l$, there is obtained the *alteration of the span* B B′, which may be denoted by u_1.

If the abutments are absolutely immoveable, $u_1 = 0$. If they yield, u_1 may be found by experiment. Hence, as a *first equation* of condition for finding the indeterminate constants, we have

$$u_1 = 0, \text{ or a given quantity.} \quad \text{.................(9.)}$$

A second equation of condition expresses the immobility in a vertical direction of B′, the further end of the rib, and is as follows:—

$$v_1 = 0. \quad \text{.............................(10.)}$$

The ends of the arched rib are either fixed or not fixed in direction. In the former case, $i_0 = 0$; and in the latter, $\mathrm{M}_0 = 0$; so that in either case, the number of indeterminate constants is reduced to three. One more equation of condition is therefore required; and it is one or other of the following:—

If the ends are fixed in direction, $i_1 = 0$; ....(11.)

if they are not fixed in direction, $\mathrm{M}_1 = 0$. .....(11 A.)

The values of the three constants being found by elimination from the three equations of condition, are to be introduced into the expressions for the moment of flexure (3) and the deflection (5), which will now become formulæ for calculation.

If thrust be treated as positive, and tension as negative, the greatest intensity of stress at any given cross-section is to be computed by the formula,

$$p_1 = \frac{\mathrm{H}\frac{d\,s}{d\,x}}{\mathrm{A}} \pm \frac{\mathrm{M}\,m'\,h}{\mathrm{I}};\ldots\ldots\ldots\ldots\ldots\ldots(12.)$$

the positive or negative sign being used according as the moment M acts towards or from the edge of the rib under consideration, whose distance from the neutral curve is $m'\,h$.

From the expression 12 may be deduced the position of the point where the stress is greatest for a given arrangement of load, the arrangement of load which makes that stress an absolute maximum, and the corresponding value of the stress.

The *vertical deviation* of the *line of resistance* from the neutral curve at any point is given by the expression

$$\mathrm{M} \div \mathrm{H};\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13.)$$

and its *perpendicular* or *normal deviation* by the expression

$$\mathrm{M} \div \mathrm{H}\frac{d\,s}{d\,x};\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14.)$$

and these deviations take place in the direction towards which M acts.

When the deflection is found by direct experiment, the following formula may be used to compute the greatest stress from it:—

$$p_1 = \frac{\mathrm{H}\frac{d\,s}{d\,x}}{\mathrm{A}} \pm \frac{4\,\mathrm{E}\,m'\,h\,v}{n''\,l^2};\ldots\ldots\ldots\ldots\ldots\ldots(15.)$$

the second term being similar to the expression in Article 179 A, p. 296.

The preceding is a general method, applicable to all cases in which the load is vertical. The following particular cases are the most useful in practice:—

PROBLEM SECOND. *Rib of Uniform Stiffness.*—If the depth and figure of the cross-section of an arched rib are uniform, and its breadth is at each point proportional to the secant of the inclination of the rib to the horizon at that point; that is, to

$$\frac{d\,s}{d\,x} = \sqrt{1 + \frac{d\,y^2}{d\,x^2}};$$

so that if A_1 be the sectional area, and I_1 the moment of inertia, of the rib at the crown, and A and I the corresponding quantities at any other point, we have

$$A = A_1 \sqrt{1 + \frac{d\,y^2}{d\,x^2}};\ I = I_1 \sqrt{1 + \frac{d\,y^2}{d\,x^2}};\ldots\text{(16.)}$$

then the intensity of the direct thrust along the rib is everywhere equal, and the vertical deflection at each point is the same with that of an uniform straight horizontal beam of the same section with the arched rib at its crown, and acted upon by the same bending moments.

This is expressed symbolically by introducing the preceding expressions into equations 4, 6, 8, 12, and 15, which now take the following form:—

$$i = \frac{d\,v}{d\,x} = i_0 - \frac{1}{E\,I_1}\int_0^x M\,d\,x;\ \ldots\ldots\ldots\ldots\text{(4 A.)}$$

$$\frac{d\,t}{d\,s} = -\frac{H}{E\,A_1};\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(6 A.)}$$

$$u = -\frac{H}{E\,A_1}\int_0^x \left(1 + \frac{d\,y^2}{d\,x^2}\right) d\,x - \int_0^x i\,\frac{d\,y}{d\,x}\,d\,x;\ \text{(8 A.)}$$

In the present case, as well as in all cases in which the depth and figure of section are uniform, it is convenient to express the moment of inertia of the cross-section in terms of its area and depth, as in Article 178, p. 294, by the aid of a factor q, as follows:—

$$I = q\,m'\,h^2\,A;\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(17.)}$$

(see the table of values of q, pp. 294, 295); for thus E A_1 is rendered a common divisor in the expression (8 A.) for the change of span, which becomes

$$u_1 = \frac{1}{E\,A_1}\left\{ -H\int_0^l \left(1 + \frac{d\,y^2}{d\,x^2}\right) d\,x + \frac{1}{q\,m'\,h^2}\int_0^l \frac{d\,y}{d\,x} \int_0^x M \cdot d\,x^2 \right\};\ \text{(18.)}$$

while equation 12, for the greatest stress at a given cross-section, becomes

$$p_1 = \frac{1}{A'} \left\{ \mathrm{H} \pm \frac{\mathrm{M}}{q\,h} \right\}; \ldots\ldots\ldots\ldots\ldots\ldots (12\ \text{A.})$$

affording a ready means of computing the requisite area of cross-section, when the depth and figure have been fixed beforehand.

Ribs of uniform stiffness are not of common occurrence in practice, but the formulæ relating to them may be applied with little error to flat segmental ribs of uniform section.

PROBLEM THIRD. *Case in which the Abutments yield proportionally to the Horizontal Thrust.*—Let the enlargement of the span of the arch due to horizontal thrust be expressed by the equation

$$u_1 = a\,\mathrm{H}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9\ \text{A.})$$

then equation 8 takes the following form:—

$$\mathrm{H} \left\{ a + \int_0^l \frac{\left(1 + \frac{d\,y^2}{d\,x^2}\right)^{\frac{3}{2}}}{\mathrm{E\,A}}\, d\,x \right\} + \int_0^l i\,\frac{d\,y}{d\,x}\,d\,x = 0; (8\ \text{C.})$$

and for a rib of uniform stiffness,

$$\mathrm{H} \left\{ a + \frac{1}{\mathrm{E\,A}_1} \int_0^l \left(1 + \frac{d\,y^2}{d\,x^2}\right) d\,x \right\} + \int_0^l i\,\frac{d\,y}{d\,x}\,d\,x = 0.\ (8\ \text{D.})$$

The co-efficient a may be determined by experiment. For example, in the course of some recent experiments, a stone pier, 24 feet broad and 11 feet thick at the base, was found to yield to the extent of ·27 of an inch to a thrust of 240,000 lbs. applied at a height of 25 feet above its base. In this case, the value of a was

$$\frac{\cdot 27}{240{,}000} = \cdot 000{,}001{,}125.$$

Further experiments are wanting to establish general principles as to the yielding of piers and abutments.

PROBLEM FOURTH. *Parabolic Rib with Rolling Load; the Ends fixed in direction; the Abutments immoveable.*—The following is the most useful case in practice:—Let the neutral curve B A B′ be a parabola, and the rib of uniform depth and uniform stiffness; and let the ends be broad and flat, and accurately bedded on the skewbacks from which they spring, so that their directions may be regarded as fixed; that is to say,

$$i_0 = i_1 = 0. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19.)$$

Take the origin of co-ordinates on a level with the summit of the neutral curve; then the equation of that curve is as follows, k being its rise:—

$$y = \frac{4\,k}{l^2}\left(\frac{l}{2} - x\right)^2; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20.)$$

Whence we have—

$$\left.\begin{array}{c} \dfrac{d\,y}{d\,x} = -\dfrac{8\,k}{l^2}\left(\dfrac{l}{2} - x\right);\ \dfrac{d\,y_0}{d\,x_0} = \dfrac{-4\,k}{l};\ 1 + \dfrac{d\,y^2}{d\,x^2} = 1 + \\ \dfrac{64\,k^2}{l^4}\left(\dfrac{l}{2} - x\right)^2;\ \displaystyle\int_0^l \left(1 + \dfrac{d\,y^2}{d\,x^2}\right) d\,x = l + \dfrac{16\,k^2}{3\,l}; \\ \dfrac{d^2\,y}{d\,x^2} = \dfrac{8\,k}{l^2}. \end{array}\right\} (21.)$$

We further find—

$$\int_0^l i\,\frac{d\,y}{d\,x}\,d\,x = \int_0^l \frac{d\,y}{d\,x}\,d\,v = (\text{because } v_0 = v_1 = 0)$$

$$-\int_0^l \frac{d^2\,y}{d\,x^2}\,v\,d\,x = -\frac{8\,k}{l^2}\int_0^l v\,d\,x; \quad \ldots\ldots\ldots\ldots (22.)$$

being in this case simply proportional to the *area of deflection*, $\int_0^l v\,d\,x$.

Let the rib be under an uniform fixed load, w_0 lbs. on the horizontal lineal inch, and a rolling load of w lbs. on the horizontal lineal inch; the rolling load, covering the horizontal length $r\,l$ of the rib at the end furthest from the origin of co-ordinates, leaves $(1 - r)\,l$ unloaded.

Then equations 2, 3, 4, and 5, become as follows:—formulæ relating to the unloaded division being denoted by A, and those relating to the loaded division by B,—

SHEARING FORCE,—

$$\left.\begin{array}{ll} \text{(A.)} & \mathrm{F} = \mathrm{F}_0 + \left(\dfrac{8\,k\,\mathrm{H}}{l^2} - w_0\right) x; \\ \text{(B.)} & \mathrm{F} = \mathrm{F}_0 + \left(\dfrac{8\,k\,\mathrm{H}}{l^2} - w_0\right) x - w\left\{x - (1 - r)\,l\right\}; \end{array}\right\} (23.)$$

BENDING MOMENT,—

$$\left.\begin{array}{ll} \text{(A.)} & \mathrm{M} = \mathrm{M}_0 + \mathrm{F}_0\,x + \left(\dfrac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\dfrac{x^2}{2}; \\ \text{(B.)} & \mathrm{M} = \mathrm{M}_0 + \mathrm{F}_0\,x + \left(\dfrac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\dfrac{x^2}{2} - \\ & \qquad \dfrac{w\left\{x - (1 - r)\,l\right\}^2}{2}; \end{array}\right\} \ldots\ldots (24.)$$

[illegible]

ALTERATION OF SLOPE,— $i = i_0 -$ [illegible]

$$\left.\begin{aligned}&\text{(A.)}\quad i = \frac{1}{q\,m'h^2\,\mathrm{E\,A_1}}\left\{-\mathrm{M_0}\,x - \mathrm{F_0}\frac{x^2}{2} - \left(\frac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\frac{x^3}{6}\right\};\\&\text{(B.)}\quad \left(\text{to the factor in brackets add} + \frac{w}{6}\left\{x - (1-r)\,l\right\}^3\right);\end{aligned}\right\}\text{(25.)}$$

DEFLECTION,— [illegible]

$$\left.\begin{aligned}&\text{(A.)}\quad v = \frac{1}{q\,m'h^2\,\mathrm{E\,A_1}}\left\{-\mathrm{M_0}\frac{x^2}{2} - \mathrm{F_0}\frac{x^3}{6} - \left(\frac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\frac{x^4}{24}\right\};\\&\text{(B.)}\quad \left(\text{to the factor in brackets add} + \frac{w}{24}\left\{x - (1-r)\,l\right\}^4\right);\end{aligned}\right\}\text{(26.)}$$

The equations of condition are the following:—

$i_1 = 0$ gives

$$-\mathrm{M_0} - \mathrm{F_0}\frac{l}{2} - \left(\frac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\frac{l^2}{6} + \frac{w\,r^3\,l^2}{6} = 0;\text{......(27.)}$$

$v_1 = 0$ gives

$$-\frac{\mathrm{M_0}}{2} - \mathrm{F_0}\frac{l}{6} - \left(\frac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\frac{l^2}{24} + \frac{w\,r^4\,l^2}{24} = 0;\text{...(28.)}$$

The condition that the abutments are immoveable, or $u_1 = 0$, gives [illegible] 21 + 22 [illegible]

$$-\left(l + \frac{4\,k^2}{3\,l}\right)\frac{\mathrm{H}}{\mathrm{E\,A_1}} + \frac{8\,k}{l^2}\int_0^l v\,dx = 0;$$

[illegible]

and multiplying both sides by $\frac{q\,m'\,h^2\,\mathrm{E\,A_1}}{8\,k\,l}$, we have

$$\left.\begin{aligned}-\frac{\mathrm{M_0}}{6} - \frac{\mathrm{F_0}\,l}{24} + \frac{w_0\,l^2}{120} + \frac{w\,r^5\,l^2}{120} - \mathrm{H}\left\{\frac{k}{15} + \frac{q\,m'\,h^2}{8\,k}\right.\\\left.\left(1 + \frac{4\,k^2}{3\,l^2}\right)\right\} = 0.\end{aligned}\right\}\text{(29.)}$$

By elimination between the three equations of condition, the following results are obtained:—

$$\text{make } \frac{45\,q\,m'\,h^2}{4\,k^2}\cdot\left(1 + \frac{4\,k^2}{3\,l^2}\right) = \mathrm{B},\ \text{..........(30.)}$$

then the horizontal thrust is

$$H = \frac{l^2}{8(1+B)k}\left\{w_0 + w(10r^3 - 15r^4 + 6r^5)\right\} \quad (31.)$$

the bending moment at the unloaded end,

$$-M_0 = \frac{w_0 l^2}{12}\cdot\frac{B}{1+B} + \frac{w l^2}{12}\left\{4r^3 - 3r^4 - \frac{10r^3 - 15r^4 + 6r^5}{1+B}\right\} \ldots\ldots(32.)$$

and at the loaded end,

$$-M_1 = \frac{w_0 l^2}{12}\cdot\frac{B}{1+B} + \frac{w l^2}{12}\left\{6r^2 - 8r^3 + 3r^4 - \frac{10r^3 - 15r^4 + 6r^5}{1+B}\right\} \quad (33.)$$

The greatest intensity of stress occurs at the loaded end of the rib; and its value is, for thrust;

$$p_1 = \frac{1}{A_1}\left(H + \frac{M_1}{qh}\right) = \frac{l^2}{8A_1}\left\{\frac{w_0}{1+B}\left(\frac{1}{k} + \frac{2B}{3qh}\right) + w\left(\frac{2}{3qh}(6r^2 - 8r^3 + 3r^4) - \frac{10r^3 - 15r^4 + 6r^5}{1+B}\left(\frac{2}{3qh} - \frac{1}{k}\right)\right)\right\}; \quad (34.)$$

for tension, let p'_1 denote the stress, and q' the value of the factor q; then

$$p'_1 = \frac{1}{A_1}\left(\frac{M_1}{q'h} - H\right) = \frac{l^2}{8A_1}\left\{\frac{w_0}{1+B}\left(\frac{2B}{3q'h} - \frac{1}{k}\right) + w\left(\frac{2}{3q'h}(6r^2 - 8r^3 + 3r^4) - \frac{10r^3 - 15r^4 + 6r^5}{1+B}\left(\frac{2}{3qh} + \frac{1}{k}\right)\right)\right\}. \quad (35.)$$

Let r_1 denote the value of r which gives the absolute maximum of thrust; r'_1 that which gives the absolute maximum of tension (if any), then

$$r_1 = \frac{2}{5} \cdot \frac{1 + \mathrm{B}}{1 - \frac{3\,q\,h}{2\,k}};\ r'_1 = \frac{2}{5} \cdot \frac{1 + \mathrm{B}}{1 + \frac{3\,q'\,h}{2\,k}};\ldots\ldots..(36.)$$

and those absolute maxima are,

$$\left.\begin{array}{l}\text{thrust } p_1 = \dfrac{l^2}{8\,\mathrm{A}_1}\left\{\dfrac{w_0}{1+\mathrm{B}}\left(\dfrac{2\,\mathrm{B}}{3\,q\,h} + \dfrac{1}{k}\right) + \dfrac{2\,w}{3\,q\,h}\right. \\ \qquad\qquad \left.\left(2\,r_1^2 - 2\,r_1^3 + \dfrac{3}{5}\,r_1^4\right.\right\}\end{array}\right]\ldots(37.)$$

$$\left.\begin{array}{l}\text{tension } p'_1 = \dfrac{l^2}{8\mathrm{A}_1}\left\{\dfrac{w_0}{1+\mathrm{B}}\left(\dfrac{2\,\mathrm{B}}{3\,q'\,h} - \dfrac{1}{k}\right) + \dfrac{2\,w}{3\,q'\,h}\right. \\ \qquad\qquad \left.\left(2\,r'^2_1 - 2\,r'^3_1 + \dfrac{3}{5}\,r_1^4\right)\right\}\end{array}\right]..(38.)$$

Equation 37 serves to compute the proper sectional area for the rib, when its depth and form have been fixed. If equation 38 gives a negative result, there is no tension at any point of the rib.

The vertical component of the shearing force at the unloaded end is

$$\mathrm{F}_0 = \frac{l}{2}\left\{\frac{w_0\,\mathrm{B}}{1+\mathrm{B}} + w\left(2\,r^3 - r^4 - \frac{10\,r^3 - 15\,r^4 + 6\,r^5}{1+\mathrm{B}}\right)\right\};(39.)$$

and this, together with the proper values of — M_0 and of H, being substituted in equation 26, enables the deflection at any point to be computed.

When $q\,h \div k$, $q'\,h \div k$, and B, are all very small fractions (as is often the case), the following equations are *nearly* true:—

$$r_1 = r'_1 = \frac{2}{5};\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(36\ \text{A.})$$

$$p_1 = \frac{l^2}{8\,\mathrm{A}_1}\left\{w_0\left(\frac{2\,\mathrm{B}}{3\,q\,h} + \frac{1}{k}\right) + 0{\cdot}138\,\frac{w}{q\,h}\right\};\ldots(37\ \text{A.})$$

$$p'_1 = \frac{l^2}{8\,\mathrm{A}_1}\left\{w_0\left(\frac{2\,\mathrm{B}}{3\,q'\,h} - \frac{1}{k}\right) + 0{\cdot}138\,\frac{w}{q'\,h}\right\}\ldots.(38\ \text{A.})$$

When, on the contrary $(1 + \mathrm{B}) \div \left(1 - \dfrac{3\,q\,h}{2\,k}\right)$ is equal to or greater than $5 \div 2$, the greatest intensity of thrust takes place when the beam is loaded along its whole length; and when

$(1 + \mathrm{B}) \div \left(1 + \frac{3\,q'\,h}{2\,k}\right)$ is equal to or greater than $5 \div 2$, the greatest intensity of tension also takes place when the beam is loaded along its whole length; that is to say, $r_1 = r'_1 = 1$; and then we have the following equations:—

$$\mathrm{H} = \frac{l^2\,(w_0 + w)}{8\,(1 + \mathrm{B})\,k}; \ldots\ldots\ldots\ldots\ldots\ldots (31\text{ B.})$$

$$-\mathrm{M}_0 = -\mathrm{M}_1 = \frac{l^2\,(w_0 + w)\,\mathrm{B}}{12\,(1 + \mathrm{B})}; \ldots\ldots\ldots\ldots (33\text{ B.})$$

$$p_1 = \frac{l^2}{8\,\mathrm{A}_1}\left\{\frac{w_0}{1 + \mathrm{B}}\left(\frac{2\,\mathrm{B}}{3\,q\,h} + \frac{1}{k}\right) + 0{\cdot}192\,\frac{w}{q\,h}\right\}; (37\text{ B.})$$

$$p'_1 = \frac{l^2}{8\,\mathrm{A}_1}\left\{\frac{w_0}{1 + \mathrm{B}}\left(\frac{2\,\mathrm{B}}{3\,q'\,h} - \frac{1}{k}\right) + 0{\cdot}192\,\frac{w}{q'\,h}\right\}. (38\text{ B}$$

The effect of an *auxiliary horizontal girder*, made fast to the arched rib at its crown, will be considered further on (pp. 313, 314).

PROBLEM FIFTH. *In the same case, when the Abutments yield to the thrust so as to enlarge the span to the extent* $u_1 = a\,\mathrm{H}$; it is only necessary to make, throughout the formulæ of Problem Fourth,

$$\mathrm{B} = \frac{45\,q\,m'\,h^2}{4\,k^2}\left(1 + \frac{\overset{16}{4}\,k^2}{3\,l^2} + \frac{a\,\mathrm{E}\,\mathrm{A}_1}{l}\right). \ldots\ldots\ldots (40.)$$

PROBLEM SIXTH. *Parabolic Rib of equal stiffness, supported at the ends, but not fixed.*—The formulæ of Problem Fourth are applicable to this case, with the modifications, that M_0 and M_1 are each $= 0$, and that i_0 becomes an indeterminate constant. Hence the following results, in which the terms enclosed in square brackets, [ ], have reference to the *loaded* division of the rib only:—

$$\mathrm{F} = \mathrm{F}_0 + \left(\frac{8\,k\,\mathrm{H}}{l^2} - w_0\right)x - \left[w\left\{x - (1 - r)\,l\right\}\right]; (41.)$$

$$\left.\begin{aligned}\mathrm{M} = \mathrm{F}_0\,x + \left(\frac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\frac{x^2}{2} - \Bigg[w\,\cdot\\ \frac{\left\{x - (1 - r)\,l\right\}^2}{2}\Bigg];\end{aligned}\right\} \ldots\ldots (42.)$$

$$i = i_0 - \frac{1}{q\,m'\,h^2\,\mathrm{E}\,\mathrm{A}_1}\left\{\mathrm{F}_0\,\frac{x^2}{2} + \left(\frac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\frac{x^3}{6} - \left[\frac{w}{6}\left\{x - (1 - r)\,l\right\}^3\right]\right\}; \qquad (43.)$$

$$v = i_0\,x - \frac{1}{q\,m'\,h^2\,\mathrm{E}\,\mathrm{A}_1}\left\{\mathrm{F}_0\,\frac{x^3}{6} + \left(\frac{8\,k\,\mathrm{H}}{l^2} - w_0\right)\frac{x^4}{24} - \left[\frac{w}{24}\left\{x - (1 - r)\,l\right\}^4\right]\right\}; \qquad (44.)$$

and $u_1 = a\,\mathrm{H}$ denoting the enlargement of the span, as in Problem Fifth, we have,— p. 302

$$0 = -\left(1 + \frac{4\,k^2}{3\,l^2} + \frac{a\,\mathrm{E}\,\mathrm{A}_1}{l}\right)\frac{l\,\mathrm{H}}{\mathrm{E}\,\mathrm{A}_1} + \frac{8\,k}{l^2}\int_0^l v\,dx; \ldots(45.)$$

which, being multiplied by $q\,m'\,h^2\,\mathrm{E}\,\mathrm{A}_1 \div 8\,k$, and proper substitutions made, gives the following equation of condition:—

$$0 = \frac{q\,m'h^2\,\mathrm{E}\,\mathrm{A}_1\,i_0}{2} - \frac{\mathrm{F}_0\,l^2}{24} + \frac{w_0\,l^3}{120} + \frac{w\,r^5\,l^3}{120} - \mathrm{H}\,l\left\{\frac{k}{15} + \frac{q\,m'\,h^2}{8\,k}\left(1 + \frac{4\,k^2}{3\,l^2} + \frac{a\,\mathrm{E}\,\mathrm{A}_1}{l}\right)\right\}; \ldots(46.)$$

The other two equations of condition are as follows:—

,= 0

$$0 = \frac{q\,m'\,h^2\,\mathrm{E}\mathrm{A}_1 v_1}{l} = q\,m'h^2\,\mathrm{E}\,\mathrm{A}_1\,i_0 - \frac{\mathrm{F}_0\,l^2}{6} + \frac{w_0\,l^3}{24} + \frac{w\,r^4\,l^3}{24} - \frac{\mathrm{H}\,l\,k}{3}; \qquad (47.)$$

//,= 0

$$0 = \frac{\mathrm{M}_1}{l} = \mathrm{F}_0 - \frac{w_0\,l}{2} - \frac{w\,r^2\,l}{2} + \frac{4\,k\,\mathrm{H}}{l}. \ldots\ldots\ldots(48.)$$

Equations 46 and 47 give, by eliminating i_0, and dividing by l^2, the following;—

$$0 = -\frac{\mathrm{F}_0}{12} + \frac{w_0\,l}{40} + \frac{w\,l}{120}\left(5\,r^4 - 2\,r^5\right) - \frac{\mathrm{H}\,k}{l}\left\{\frac{1}{5} - \frac{q\,m'h^2}{4\,k^2}\left(1 + \frac{4\,k^2}{3\,l^2} + \frac{a\,\mathrm{E}\,\mathrm{A}_1}{l}\right)\right\}; \ldots(49.)$$

and eliminating F_0 between this equation and 48, we obtain the following:—

$$\left.\begin{array}{l} 0 = -\frac{w_0 l}{5} - \frac{w l}{10}\left(5 r^2 - 5 r^4 + 2 r^5\right) + \frac{H k}{l} \\ \quad \left\{\frac{8}{5} + \frac{3 q m' h^2}{k^2}\left(1 + \frac{4 k^2}{3 l^2} + \frac{a E A_1}{l}\right)\right\}; \end{array}\right\} \ldots(50.)$$

whence, using the following abbreviation,—

$$C = \frac{15 q m' h^2}{8 k^2}\left(1 + \frac{4 k^2}{3 l^2} + \frac{a E A_1}{l}\right)\Big\}, \ldots\ldots\ldots(51.)$$

we have the following values of the horizontal thrust, and of the other constants,—

$$H = \frac{l^2}{8 k (1 + C)}\left\{w_0 + \frac{w}{2}\left(5 r^2 - 5 r^4 + 2 r^5\right)\right\}; \ldots(52.)$$

$$F_0 = \frac{l}{2}\left\{\frac{w_0 C}{1 + C} + w\left(r^2 - \frac{5 r^2 - 5 r^4 + 2 r^5}{2 (1 + C)}\right)\right\}; \quad (53.)$$

$$\left.\begin{array}{l} i_0 = \frac{l^3}{24 q m' h^2 E A_1}\left\{\frac{w_0 C}{1 + C} + w\left(2 r^2 - r^4 - \right.\right. \\ \qquad\qquad \left.\left.\frac{5 r^2 - 5 r^4 + 2 r^5}{2 (1 + C)}\right)\right\}. \end{array}\right\} \quad (54.)$$

The *shearing force at the loaded end of the rib* is (with the sign reversed)—

$$\left.\begin{array}{l} P = -F_1 = -F_0 + w_0 l + w r l - \frac{8 k H}{l} \\ \qquad = \frac{w_0 l}{2} + \frac{w l}{2}(2 r - r^2) - \frac{4 k H}{l} \\ = \frac{l}{2}\left\{\frac{w_0 C}{1 + C} + w\left(2 r - r^2 - \frac{5 r^2 - 5 r^4 + 2 r^5}{2 (1 + C)}\right)\right\}. \end{array}\right\} \quad (55.)$$

To avoid negative signs in what follows, this is denoted as above by P.

The *greatest bending moment* occurs at a point whose horizontal distance from the loaded end of the rib is

$$l - x = \frac{P}{w_0 + w - \frac{8 k H}{l^2}}; \ldots\ldots\ldots\ldots\ldots(56.)$$

P

and the value of that greatest bending moment is

l·x

$$M' = \frac{P\,(l - x)}{2} = \frac{P^2}{2\left[(w_0 + w) - \frac{8\,k\,H}{l^2}\right]}; \ldots\ldots\ldots(57.)$$

giving, for the greatest stress, a thrust whose intensity is

$$p_1 = \frac{1}{A_1}\left(\frac{M'}{q\,h} + H\right). \ldots\ldots\ldots\ldots\ldots\ldots(58.)$$

To find how much of the span of the rib must be loaded, in order to make this stress an absolute maximum, and what that maximum is, the value of r is to be deduced from the equation

$$\frac{d\,p_1}{d\,r} = 0. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(59.)$$

This equation is of the fourteenth order. One of its roots is $r = 1$, which in most cases gives a *minimum* value of p_1. Dividing the equation, therefore, by $1 - r = 0$, it is reduced to the thirteenth order; but it is still too complex to be employed as a formula for practical use.

It appears, however, by trial, that with those proportions which are common in practice, a *close approximation* to the absolute maximum value of the stress p_1 is formed by assuming *one half of the rib to be loaded;* that is—

$$r = \frac{1}{2}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(60.)$$

By introducing this value of r into the preceding formulæ, we obtain the following results:—

$$H = \frac{l^2}{8\,k\,(1 + C)}\left(w_0 + \frac{w}{2}\right); \ldots\ldots\ldots\ldots(52\text{ A.})$$

$$F_0 = \frac{l}{2}\left(w_0 + \frac{w}{2}\right)\frac{C}{1 + C}; \ldots\ldots\ldots\ldots(53\text{ A.})$$

$$i_0 = \frac{l^3}{24\,q\,m'\,h^2\,E\,A_1}\cdot\left\{\left(w_0 + \frac{w}{2}\right)\frac{C}{1 + C} - \frac{w}{16}\right\}; \ldots(54\text{ A.})$$

$$P = \frac{l}{2}\left\{\left(w_0 + \frac{w}{2}\right)\frac{C}{1 + C} + \frac{w}{4}\right\}; \ldots\ldots\ldots(55\text{ A.})$$

$$l - x = \frac{l}{4} \cdot \frac{w + 4\left(w_0 + \frac{w}{2}\right)\frac{C}{1+C}}{w + 2\left(w_0 + \frac{w}{2}\right)\frac{C}{1+C}}; \ldots\ldots\ldots (56\text{ A.})$$

$$M' = \frac{P\,(l - x)}{2} = \frac{l^2}{64} \cdot \frac{\left\{w + 4\left(w_0 + \frac{w}{2}\right)\frac{C}{1+C}\right\}^2}{w + 2\left(w_0 + \frac{w}{2}\right)\frac{C}{1+C}}. (57\text{ A.})$$

To illustrate this by a numerical example, let the following data be assumed:—

$$k = \frac{1}{8}\,l;\ h = \frac{1}{5}\,k = \frac{1}{40}\,l;\ m' = \frac{1}{2};$$

$q = \frac{1}{3}$ (this value requires an I-shaped section to realize it.

$a = 0$; (that is, let the abutments be immoveable).

Then,

$$C = \frac{1}{80} \times \frac{49}{48} = \cdot 013 \text{ nearly.}$$

Also, let the intensity of the rolling load be equal to that of the dead load, or $w = w_0$. Then

$$H = 1{\cdot}48\, l\, w;$$
$$P = 0{\cdot}13\, l\, w;$$
$$l - x = 0{\cdot}26\, l\, w;$$
$$M' = 0{\cdot}0169\, l^2\, w;$$

(being less than the bending moment due to a load of the intensity w over the whole span, in the ratio of 0·135 to 1).

$$p_1 = \frac{1}{A_1}\left(\frac{M'}{q\,h} + H\right) = \frac{l\,w}{A_1}\left(2{\cdot}03 + 1{\cdot}48\right) = 3{\cdot}51\,\frac{l\,w}{A_1}.$$

PROBLEM SEVENTH. *To find the greatest Deflection of an Arched Rib*, the greatest value of v is to be taken which corresponds to $i = 0$. It can be deduced from equations 25 and 26 of Problem Fifth, and 43 and 44 of Problem Sixth, that in all ordinary cases to which those problems relate, the absolute maximum deflection occurs in the middle of the rib, when it is loaded over its whole length; that is, when

$$r = 1;\ x = \frac{l}{2}.$$

Then in a rib of uniform stiffness, *fixed in direction at the ends,* we have, 31 p 306

32 p 106

$$\left.\begin{aligned} \mathrm{H} &= \frac{l^2 (w + w_0)}{8\, k (1 + \mathrm{B})};\ \mathrm{F}_0 = \frac{l (w + w_0)\, \mathrm{B}}{2 (1 + \mathrm{B})}; \\ -\mathrm{M}_0 &= -\mathrm{M}_1 = \frac{l^2 (w + w_0)\, \mathrm{B}}{12 (1 + \mathrm{B})};\ \text{and} \\ v &= \frac{l^4 (w + w_0)\, \mathrm{B}}{384\, q\, m'\, h^2\, \mathrm{E}\, \mathrm{A}_1 (1 + \mathrm{B})}. \end{aligned}\right\} \ldots. (61.)$$

In a rib of uniform stiffness, *not fixed* in *direction at the ends,* we have,

(32)

(34)

$$\left.\begin{aligned} \mathrm{H} &= \frac{l^2 (w + w_0)}{8\, k (1 + \mathrm{C})};\ \mathrm{F}_0 = \frac{l (w + w_0)\, \mathrm{C}}{2 (1 + \mathrm{C})}; \\ i_0 &= \frac{l^3 (w + w_0)\, \mathrm{C}}{24\, q\, m'\, h^2\, \mathrm{E}\, \mathrm{A}_1 (1 + \mathrm{C})};\ \text{and} \\ v &= \frac{5\, l^4 (w + w_0)\, \mathrm{C}}{384\, q\, m'\, h^2\, \mathrm{E}\, \mathrm{A}_1 (1 + \mathrm{C})}. \end{aligned}\right\} \ldots.. (62.)$$

In comparing these formulæ with equation 12, of Article 169, p. 273, for the deflection of straight beams under any load, it is to be observed that the total load in the present problem is $l\,(w + w_0)$, that $l^3 \div 384 = c^3 \div 48$, and that $q\ m'\ h^2\ \mathrm{A}_1 = \mathrm{I}$. Hence it appears that the deflection of an arched rib of uniform stiffness under an uniformly distributed load, is less than that of a straight beam whose section has the same moment of inertia with that of the arched rib at its crown, in the ratio of

B : 1 + B if the ends are fixed in direction (see pp. 305, 308).

C : 1 + C if the ends are merely supported (see p. 310).

PROBLEM EIGHTH. *Arched Rib of uniform stiffness fixed in direction at the ends, and fixed at the crown to a horizontal beam.*—In fig. 153, let B B′ as before be the arched rib, and E A E′ the horizontal beam. In the spandrils of the arch are vertical struts which transmit the vertical load to the curved rib, and cause the vertical components of the deflection

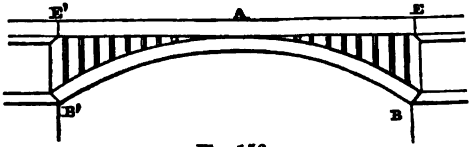

Fig. 153.

of the straight and arched beams to be the same at corresponding points.

The effect of these struts is taken into account by making the total moment of inertia of the cross-section, in the formulæ of Problem Fourth, viz. :—

$$I_1 = q\, m'\, h^2\, A_1,$$

include the moment of inertia of the straight beam; but the area A_1 is still to be that of *the arched beam only.*

Let the curved and straight beams be so firmly connected at the crown (A, fig. 153), that their *horizontal displacement u* is the same at that point; and let the horizontal beam *abut* at its ends, E, E′, either against the piers, or against some other part of the superstructure, so as to be capable of resisting a thrust. Then the horizontal thrust is no longer necessarily the same in the two divisions of the arched rib, A B, A B′; but when one of those divisions (as A B′) is more heavily loaded than the other, the horizontal thrust in the more loaded division is greater than in the less loaded division, the excess being resisted by that part of the horizontal beam (A E) which is above the less loaded division.

It is unnecessary to give here the complete detailed investigation of this case, or to do more than to state the most important result of that investigation, viz. :—that with the dimensions and under the circumstances that usually occur in practice, the effect of the resistance of the horizontal beam to a longitudinal thrust is to make the greatest intensity of stress in the arched rib under every partial load either less than, or not appreciably greater than, the greatest intensity of stress under a complete load, which thus becomes the absolute maximum of stress in the arched rib, and is given by equation 37 B for thrust, and by equation 38 B for tension, p. 308.

The greatest intensity of stress in the horizontal beam may be found approximately as follows, when its cross-section is nearly similar above and below :—Let h' denote its depth, A' its sectional area, — M_1 the greatest moment of flexure as computed by equation 33 B, p. 308, H the horizontal thrust as computed by equation 31 B, p. 308.

Then for thrust,—

$$p''_1 = \frac{M_1 h'}{2 E I} + \frac{H}{A_1 + A'}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (63.)$$

and for tension—

$$p'''_1 = \frac{M_1 h'}{2 E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (64.)$$

CHAPTER II.

OF EARTHWORK.

SECTION I.—*Strength and Stability of Earthwork in General.*

181. **General Principles—Adhesion—Friction—Natural Slope—Heaviness.**—Earthwork is of two kinds—excavation, or cutting, and filling, or embankment. The term "*earthwork,*" in its widest sense, comprehends excavation in rock, as well as in the looser materials of the earth's crust.

Earthwork gives way by the *slipping* or sliding of its parts on each other; and its stability arises from resistance to the tendency so to slip.

In solid rock, that resistance arises from the elastic stress of the material, when subjected to a shearing force; but in a mass of *earth*, as commonly understood, it arises partly from the friction between the grains, and partly from their mutual adhesion; which latter force is considerable in some kinds of earth, such as clay, especially when moist.

But the adhesion of earth is gradually destroyed by the action of air and moisture, and of the changes of the weather, and especially by alternate frost and thaw; so that its friction is the only force which can be relied upon to produce permanent stability.

The temporary additional stability, however, which is produced by adhesion, is useful in the execution of earthwork, by enabling the side of a cutting to stand for a time with a vertical face for a certain depth below its upper edge. That depth is greater the greater the adhesion of the earth as compared with its heaviness; it is increased by a moderate degree of moisture, but diminished by excessive wetness.

The following are some of its values:—

EARTH.	Greatest depth of temporary vertical face.
Clean dry sand and gravel,........................	0
Moist sand, and ordinary surface mould, from	3 to 6 feet.
Clay (ordinary),..................................from	10 to 16 feet.

One of the effects of the temporary stability due to adhesion is seen in the figure of the surface left after a "slip" has taken place in earthwork. That surface is not an uniform slope, inclined at the angle of repose, but is concave in its vertical section, being vertical at its upper edge, and becoming less and less steep downwards. It is not capable, however, of preserving that figure; for the action of the weather, by gradually destroying the adhesion of the earth, causes the steep upper part of the concave face to crumble down, so that the whole tends to assume an uniform slope in the end.

The *permanent stability* of earth, which is due to friction alone, is sufficient to maintain the side either of an embankment or of a cutting at an uniform slope, whose inclination to the horizon is the *angle of repose*, or angle whose tangent is the *co-efficient of friction*. This is called the *natural slope* of the earth. The customary mode of describing the slope of earthwork is to state the ratio of its horizontal breadth to its vertical height, which is the *reciprocal* of the tangent of the inclination.

Values of the angle of repose (φ) and co-efficient of friction (f), and its reciprocal ($1 \div f$), for various substances, have already been given in Article 110, p. 172; but for the sake of convenience, those which refer to the frictional stability of earth are here repeated, with a few additions:—

EARTH.		Angle of Repose. φ	Co-efficient of Friction. f	Customary designation of Natural Slope: $1 \div f$ to 1.
Dry sand, clay, and mixed earth, ...	from	37°	0·75	1·33 to 1
	to	21°	0·38	2·63 to 1
Damp clay, ...		45°	1·00	1 to 1
Wet clay, ...	from	17°	0·31	3·23 to 1
	to	14°	0·25	4 to 1
Shingle and gravel, ...	from	48°	1·11	0·9 to 1
	to	35°	0·70	1·43 to 1
Peat, ...	from	45°	1·0	1 to 1
	to	14°	0·25	4 to 1

The most frequent slopes of earthwork are those called 1½ to 1, and 2 to 1; corresponding respectively to the co-efficients of friction 0·67 and 0·5, and to the angles of repose 33½° and 26½°, nearly.

The presence of moisture in earth to an extent just sufficient to expel the air from its crevices, seems to increase its co-efficient of friction slightly; but any additional moisture acts like an unguent in diminishing friction, and tends to reduce the earth to a semifluid condition, or to the state of *mud*. In this state, although it has some cohesion, or viscidity, which resists rapid alteration of form, it has no frictional stability; and its co-efficient of friction, and angle of repose, are each of them null.

Hence it is obvious that the frictional stability of earth depends to a great extent on the ease with which the water that it occasionally absorbs can be drained away. The safest materials for earthwork are shivers of rock, shingle, gravel, and clean sharp sand, whether consisting wholly of small hard crystals, or containing a mixture of fragments of shells; for those materials allow water to pass through, without retaining more of it than is beneficial. The cleanest sand, however, may be made completely unstable, and reduced to the state of "quicksand," if it is contained in a basin of water-holding materials, so that water mixed amongst its particles cannot be drained off.

The property of retaining water, and forming a paste with it, belongs specially to clay, and to earths of which clay is an ingredient. Such earths, how hard and firm soever they may be, when first excavated, are gradually softened, and have both their frictional stability and their adhesion diminished by exposure to the air. In this respect, mixtures of sand and clay are the worst; for the sand favours the access of water, and the clay prevents its escape.

The properties of earth with respect to adhesion and friction are so variable, that the engineer should never trust to tables or to information obtained from books to guide him in designing earthworks, when he has it in his power to obtain the necessary data either by observation of existing earthworks in the same stratum, or by experiment.

The following are the weights of a cubic foot and of a cubic yard of the ordinary materials of earthwork:—

		Cubic Foot.			Cubic Yard.	
Chalk,.....................	from	117 to 174	lbs.	from	3160 to 4730	lbs.
Clay,......................	,,	120 to 135	,,	,,	3240 to 3645	,,
Gravel and Shingle,....	,,	90 to 110	,,	,,	2430 to 2970	,,
Marl,......................	,,	100 to 119	,,	,,	2700 to 3210	,,
Mud,......................		102	,,	,,	2750	,,
Sand, dry,................		89	,,	,,	2400	,,
,, damp,............		118	,,	,,	2190	,,
Shale,.....................		162	,,	,,	4370	,,

182. **Sides of Rock-Cuttings.**—When rock is firm and sound, so that the permanence of its cohesion may be depended upon, the sides of excavations in it may be made vertical, or nearly so.

How far the cohesion of the rock is to be depended upon, is a question to be solved rather by observation of the rock in each particular case, than by any general principles having regard to its geological position, mineralogical character, or chemical composition; for the geological position is fixed by the organic remains imbedded in the rock; and these have no connection with its mechanical properties; and rocks composed of the same species of minerals, and the same chemical constituents in the same or nearly the same proportions, show great differences in strength and durability.

It may be observed, however, that the cohesion of igneous and metamorphic rocks, such as granite, syenite, trap, gneiss, mica-slate, marble, quartz-rock, &c., may in general be trusted, unless they are much fissured, or contain potash-felspar, in which cases a sufficient slope must be given, to prevent fragments from falling into the cutting so as to do damage. Of the sedimentary rocks, those which contain much clay, such as shale, are to be treated with caution, how hard soever they may be when first cut; for they are liable to soften by the action of the weather. Sandstone and limestone, whether compact or granular, if fit for building purposes, will stand with vertical or nearly vertical faces; but those materials exist of every degree of hardness, from that of rock, properly speaking, to that of earth. Sandstone is met with which crumbles in the hand, and requires slopes of from 1 to 1 to 1½ to 1; and chalk, according to its degree of hardness and soundness, stands at slopes varying from ½ to 1 to 1½ to 1.

The stability of sedimentary rocks in the side of a cutting is greater when the beds are horizontal, or dip away from the cutting, than when they dip towards it.

183. **Theory of the Stability and Pressure of Loose Earth.** (*A. M.*, 194 to 198.)—The stress exerted in different directions through a given particle in a mass of earth is subject to the general principles which govern the compound internal stress of solids, as already stated in Article 108, pp. 166 to 170.

It is also subject, when friction alone is the cause of stability, to the *limitation* expressed by the following principle:—

I. **General Principle of the Stability of Loose Earth.**—*It is necessary to the stability of a granular mass, that the direction of the pressure between the portions into which it is divided by any plane should not, at any point, make with the normal to that plane an angle greater than the angle of repose.*

The plane in any mass on which the obliquity of the pressure is greatest, is perpendicular to the plane which contains the axes of greatest and least pressure.

Referring to fig. 85, p. 168, and to the description of that figure in pp. 168, 169, it is evident that the above principle is equivalent to stating that the greatest value of the angle of obliquity $\angle$N O R or $\hat{nr}$ in that figure shall not exceed ϕ, the angle of repose of the earth in question.

The greatest value of $\hat{nr}$ obviously occurs when O R is perpendicular to P Q, and is given by the following equation:—

$$\max \hat{nr} = \text{arc sin.} \frac{\text{M R}}{\text{O M}} = \text{arc sin} \frac{p_1 - p_2}{p_1 + p_2};$$

and this angle must not exceed the angle of repose; whence the condition of stability of the earth is expressed as follows:—

$$\frac{\text{M R}}{\text{O M}} = \frac{p_1 - p_2}{p_1 + p_2} \leqq \sin \phi; \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

or otherwise as follows:—

$$\frac{p_2}{p_1} \overset{\geq}{\#} \frac{1 - \sin \phi}{1 + \sin \phi}, \ldots\ldots\ldots\ldots\ldots\ldots (1\text{ A.})$$

which last equation gives the *least* intensity of pressure p_2 in a given direction, that is consistent with the repose of earth through which a pressure of a given intensity p_1 acts at right angles to the first mentioned direction, and serves to determine the least intensity of horizontal pressure which will maintain the stability of a mass of earth through which a vertical pressure of a given intensity acts.

II. **Conjugate Pressures in Earth.**—But it is necessary in some cases to determine the limiting ratio of the intensities of a pair of *conjugate pressures* in a mass of earth, which may or may not be at right angles to each other; and that problem is solved by the following geometrical construction, easily deduced from Proposition IV. of Article 108, p. 168.

In fig. 154, let C represent a section of a prismatic particle of earth, made by the plane of greatest and least pressures. Let that particle be a rhombic prism, on whose faces the pressures are "*conjugate;*" that is to say, let the pressures on the faces which are parallel to D G, act parallel to E F; while the pressures on the faces which are parallel to E F act parallel to D G.

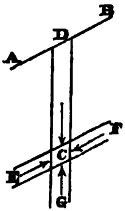

Fig. 154.

Let p be the intensity of the pressure parallel to D G, and p' that of the less pressure parallel to E F, each estimated *per unit of area of the plane to which it is conjugate.* Let θ be the angle of obliquity of the prism C;

$OP = OM \cos\theta + \sqrt{MP^2 - MN^2} = OM \cos\theta + OM \sqrt{\sin^2\varphi - \sin^2\theta}$

$= OM(\cos\theta + \sqrt{\cos^2\theta - \cos^2\varphi}$

that is, the difference between each of its angles and a right angle. This angle must not exceed φ, the angle of repose of the earth.

Then the intensities of the conjugate pressures, *per unit of area of planes perpendicular to their directions*, are respectively,—

$$\frac{p}{\cos\theta} \text{ and } \frac{p'}{\cos\theta}.$$

In fig. 155, from one point O, draw two straight lines, O M X and O R, making with each other the angle M O R = φ, the angle of repose. About any convenient point M in one of those straight lines, describe a semicircle Y R X, touching the other straight line in R. (This may be done by describing the dotted semicircle M R O, so as to find the point R.)

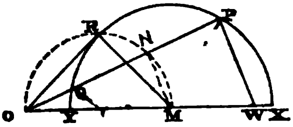

Fig. 155.

Through O draw the straight line O Q P, making the angle M O P = θ, the obliquity of the conjugate pressures, and cutting the semicircle Y R X in P and Q. Then the limits of the ratio of the intensities of the conjugate pressures are

$$\frac{\text{O Q}}{\text{O P}}, \text{ and } \frac{\text{O P}}{\text{O Q}};$$

that is to say, in algebraical symbols, AM 6/69

$$\frac{p'}{p} \text{ cannot be greater than } \frac{\text{O P}}{\text{O Q}} = \frac{\cos\theta + \sqrt{(\cos^2\theta - \cos^2\varphi)}}{\cos\theta - \sqrt{(\cos^2\theta - \cos^2\varphi)}}, \quad (2.)$$

$$\text{nor less than} \ldots\ldots\ldots\ldots \frac{\text{O Q}}{\text{O P}} = \frac{\cos\theta - \sqrt{(\cos^2\theta - \cos^2\varphi)}}{\cos\theta + \sqrt{(\cos^2\theta - \cos^2\varphi)}}; \quad (2\text{A}.)$$

being the solution of the problem.

The following are the extreme cases of the problem:—

When the prism C is rectangular, and the conjugate pressures perpendicular to each other, we have $\theta = 0$; O Q P coincides with O Y X, and consequently

$$\frac{p'}{p} \text{ cannot be greater than } \frac{\text{O X}}{\text{O Y}} = \frac{1 + \sin\varphi}{1 - \sin\varphi}, \ldots\ldots\ldots (3.)$$

$$\text{nor less than} \ldots\ldots\ldots\ldots \frac{\text{O Y}}{\text{O X}} = \frac{1 - \sin\varphi}{1 + \sin\varphi} \ldots\ldots (3\text{A}.)$$

When the obliquity of the prism C is the greatest possible, so that $\theta = \varphi$, the points P and Q coalesce in R, and the two limits of the ratio of the conjugate pressures become each equal to unity, giving the single equation,

$$p' = p \quad \text{.............................(4.)}$$

III. Pressure in a Mass of Earth with an unlimited plane upper Surface.—In fig. 154, p. 319, let A B represent part of the indefinitely extended plane upper surface of a mass of earth, either horizontal, or sloping at any given angle θ not exceeding the angle of repose φ. Conceive the whole mass to be divided into layers, such as E F, parallel to A B. The condition of all particles, such as C, into which one of those layers, as E F, can be divided by vertical planes, must be similar; whence it follows that the pressure exerted at any vertical plane is parallel to the surface A B, and the pressure at any surface parallel to A B is vertical. The particle C, formed by the intersection of the vertical column D G with the layer E F, is bounded by conjugate planes; and the conjugate pressures acting through it are respectively vertical, and parallel to the layer.

The vertical pressure p is due to the weight of the column of earth D C which rests on the particle. Let $x =$ D C be its depth, and w the weight of an unit of its volume; then

$$p = w\,x \cdot \cos\theta. \quad \text{.........................(5.)}$$

The pressure along the steepest slope of the layer E F, which is exerted through the vertical faces of the prism C, will, if the earth is laid down in layers, be of the *least* intensity sufficient to preserve the repose of the earth, as given by combining equation 2 A with equation 5; that is to say,

$$p' = w\,x \cdot \cos\theta \cdot \frac{\cos\theta - \sqrt{(\cos^2\theta - \cos^2\varphi)}}{\cos\theta + \sqrt{(\cos^2\theta - \cos^2\varphi)}}. \quad \text{........(6.)}$$

To represent these results graphically, construct fig. 155 as already described, with O M X horizontal, O R inclined at the "natural slope," and O Q P inclined at the actual slope,—that is, parallel to the steepest slope of the plane A B. From P draw the straight line P W perpendicular to O P, cutting O X in W.

Then

$$\mathrm{O\,W} : \mathrm{O\,P} : \mathrm{O\,Q} :: w\,x : p : p'. \quad \text{.............(7.)}$$

The extreme cases are as follows:—

When the upper surface of the earth is horizontal, W and P both coincide with X, and Q with Y; so that,

$$\mathrm{O\,X} : \mathrm{O\,Y} :: w\,x = p : p'; \text{ and } p' = w\,x \cdot \frac{1 - \sin\phi}{1 + \sin\phi}. \quad (8.)$$

When the upper surface of the earth slopes at the angle of repose, P and Q coincide with R, and W with M; so that

$$\mathrm{O\,M} : \mathrm{O\,R} :: w\,x : p' = p; \text{ and } p' = p = w\,x \cos\phi. \quad (9.)$$

There is a *third conjugate pressure*, exerted horizontally through the particle C, in a direction perpendicular to the vertical plane of steepest slope. Its intensity is represented in fig. 155 by O Y, and is given by the following equation:—

$$p'' = \frac{w\,x \cdot \cos\theta\,(1 - \sin\phi)}{\cos\theta + \sqrt{(\cos^2\theta - \cos^2\phi)}}; \ldots\ldots\ldots\ldots\ldots(10.)$$

and in the two extreme cases it takes the following values:—For a horizontal upper surface, or $\theta = 0$,

$$p'' = p' = w\,x\,\frac{1 - \sin\phi}{1 + \sin\phi}. \ldots\ldots\ldots\ldots\ldots\ldots(11.)$$

For the natural slope, or $\theta = \phi$,

$$p'' = w\,x\,(1 - \sin\phi). \ldots\ldots\ldots\ldots\ldots\ldots(12.)$$

The intensity of the *greatest pressure* exerted through a given particle of earth is represented by O X, and given by the following formula:—

$$p_1 = \frac{w\,x \cdot \cos\theta\,(1 + \sin\phi)}{\cos\theta + \sqrt{(\cos^2\theta - \cos^2\phi)}} \ldots\ldots\ldots\ldots\ldots(13.)$$

The direction of the *axis of greatest pressure* is at right angles to, and conjugate to, a plane bisecting the angle which a radius drawn from C to Q makes with the horizon; that is to say, the inclination of that axis to the horizon is given by the formula,—

$$\psi = \frac{1}{2}\left(\theta + 180^\circ - \text{arc} \cdot \sin\frac{\sin\theta}{\sin\phi}\right). \ldots\ldots\ldots (14.)$$

The extreme cases are,
When the upper surface is horizontal, or $\theta = 0$;

$$p_1 = w\,x; \ \psi = 90^\circ \text{ (or the axis is vertical)}. \ldots\ldots\ldots(15.)$$

When $\theta = \phi$;

$$p_1 = w\,x\,(1 + \sin\phi); \ \psi = \frac{\theta + 90^\circ}{2}; \ldots\ldots\ldots\ldots(16.)$$

or the axis of greatest pressure bisects the angle between the slope and the vertical.

The axis of *least pressure* in the plane of greatest slope is perpendicular to that of greatest pressure, and the intensity of the least pressure, being represented by O Y, has already been given in equation 10.

IV. **Pressure of Earth against a vertical Plane.**—In fig. 156, let O X represent a vertical plane in, or in contact with, a mass of earth, whose upper surface Y O Y is either horizontal or inclined at any angle θ, and is cut by the vertical plane in a direction perpendicular to that of steepest declivity. It is required to find the pressure exerted by the earth against that vertical plane *per unit of breadth*, from O down to X, at a depth O X $= x$ beneath the surface, and the direction and position of the resultant of that pressure.

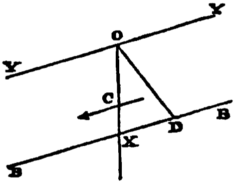

Fig. 156.

The *direction* of that resultant is already known to be parallel to the declivity Y O Y.

Let B B be a plane traversing X, parallel to Y O Y. In that plane take a point D, at such a distance X D from X, that the weight of a prism of earth of the length X D and having an *oblique* base of the area unity in the plane O X, shall represent the intensity of the conjugate pressure per unit of area of a vertical plane at the depth X; that is to say, construct fig. 155 as already described, and make

O P : O Q in fig. 155 : : O X : X D in fig. 156.

Draw the straight line O D; then will the ordinate, parallel to O Y, drawn from O X to O D at any depth, be the length of an oblique prism, whose weight, per unit of area of its oblique base, will be the intensity of the conjugate pressure at that depth. Let O D X be a triangular prism of earth of the thickness unity; the weight of that prism will be the *amount* of the conjugate pressure sought, and a line parallel to O Y, traversing its centre of gravity, and cutting O X in the *centre of pressure* C, will be the *position* of the resultant of that pressure. The depth O C of that centre of pressure beneath the surface is evidently two-thirds of the total depth O X.

To express this symbolically, make

$$\mathrm{X\,D} = x \cdot \frac{p'}{p} = x \cdot \frac{\cos\theta - \sqrt{(\cos^2\theta - \cos^2\phi)}}{\cos\theta + \sqrt{(\cos^2\theta - \cos^2\phi)}}; \;\ldots\ldots(17.)$$

then the amount of the conjugate pressure, represented by the weight of the prism O X D, is

$$P' = \frac{w x^2}{2} \cdot \cos \theta \cdot \frac{p'}{p} = \frac{w x^2}{2} \cdot \cos \theta \cdot \frac{\cos \theta - \sqrt{(\cos^2 \theta - \cos^2 \phi)}}{\cos \theta + \sqrt{(\cos^2 \theta - \cos^2 \phi)}}. \quad (18.)$$

In the extreme cases, equations 17 and 18 take the following forms:—For a horizontal surface;

$$\theta = 0; \; X\,D = x \cdot \frac{1 - \sin \phi}{1 + \sin \phi}; \; P' = \frac{w\,x^2}{2} \cdot \frac{1 - \sin \phi}{1 + \sin \phi} \ldots\ldots (19.)$$

For a surface sloping at the angle of repose;

$$\theta = \phi; \; X\,D = x; \; P' = \frac{w\,x^2}{2} \cdot \cos \phi. \ldots\ldots\ldots\ldots (20.)$$

Masses of earth with indefinitely extended plane upper surfaces do not occur in reality; but the formulæ which are applicable to them are applicable to real masses of earth with limited plane upper surfaces, with a degree of accuracy sufficient in most cases for practical purposes. (See *Phil. Trans.*, 1856-7).

TABLE OF EXAMPLES.

ϕ	0°	15°	30°	45°	60°
$(90° - \phi) \div 2$	45°	37°½	30°	22°½	15°
$f = \tan \phi$	0	0·268	0·577	1·000	1·732
$1 \div f = \text{cotan}\ \phi$	∞	3·732	1·732	1·000	0·577
$\sin \phi$	0	0·259	0·500	0·707	0·866
$\frac{1 - \sin \phi}{1 + \sin \phi}$	1	0·588	0·333	0·172	0·072
$\cos \phi$	1	0·966	0·866	0·707	0·500

There is a mathematical theory of the combined action of friction and adhesion in earth; but for want of precise experimental data, its practical utility is doubtful.

SECTION II.—*Mensuration of Earthwork.*

184. **Calculation of Half-breadths and Areas of Land.**—The boundaries of a piece of earthwork in general are as follows:—

I. The *base, forming*, or *formation*, being a surface nearly, and sometimes exactly, horizontal, which forms the bottom of a cutting, or the top of an embankment.

II. The original surface of the ground, which forms the top of a cutting and the bottom of an embankment.

III. The *sides*, or *slopes*, which connect the base with the natural surface, and whose inclination is the steepest consistent with the permanent stability of the material.

Figs. 157, 158, and 159, represent examples of *cross-sections* of pieces of earthwork, in each of which D E is the base, A B the natural surface, and D A and E B are the slopes. In fig. 157, the natural surface is horizontal; in figs. 158 and 159, it slopes sideways, being what is called "side-long ground."

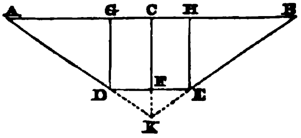

Fig. 157.

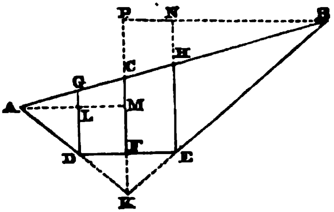

Fig. 158.

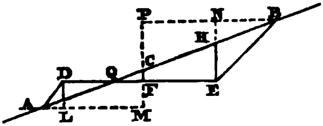

Fig. 159.

Figs. 157 and 158 represent cuttings; to represent embankments, it is only necessary to conceive them to be turned upside down.

Fig. 159 represents a piece of earthwork, of which one side, Q E B, is in cutting called "side cutting," and the other, Q D A, in embankment.

The *half-breadth* of a piece of earthwork has already been mentioned in Article 66, p. 112. It means the horizontal distance from a given point in the *centre line of the base* to one edge of the cutting or embankment; and although it is called "*half*-breadth," it is very generally different at opposite sides of that centre line.

Each half-breadth consists of two parts:—the real half-breadth of the base, which is fixed by the design of the work, and the horizontal breadth of one slope, which is to be found by calculation or by drawing.

In each of the figs. 157, 158, and 159, C represents a point in the centre line, as marked on the ground; F, the point vertically above or below it in the centre line of the base; D G and E H are vertical lines through the edges of the base; D F and F E are the half-breadths of the base.

In fig. 157, where the ground is level across, G A and H B are the breadths of the slopes, and C A and C B the half-breadths of the earthwork.

In figs. 158 and 159, where the ground slopes sideways, the vertical lines through D, F, and E are produced, if necessary, and are cut at right angles by horizontal lines, A L M, and B N R, drawn through the edges of the earthwork. A L and B N are the breadths of the slopes; and M A and P B are the half-breadths of the earthwork.

When the natural surface of the ground is rugged, the best method of determining the breadths of the slopes of earthwork is by measurement, upon a series of cross-sections of the proposed work, plotted to the same scale horizontally and vertically. (Article 11, p. 11; Article 60, pp. 97, 98.)

When the natural surface of the ground is level, or nearly level across, or has an uniform or nearly uniform sidelong slope, the breadths of slopes may be found by calculation, according to the rules now to be explained.

In each of the following problems, h denotes C F in figs. 157, 158, and 159, being the central depth of the earthwork at the given cross-section; b_0 the *half-breadth* of the base, F D or F E; s to I, the slope of the earthwork, meaning s horizontal to 1 vertical; b', the half-breadth of the slope.

Problem First. *To calculate the breadth of a slope, when the natural ground is level across.*—In fig. 157,

$$b' = \mathrm{HB} = \mathrm{GA} = s\,h. \quad\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

Problem Second. *To calculate the breadth of a slope when the natural ground has a given uniform sidelong inclination.*

Let the natural sidelong declivity be at the rate of r to 1; that is, let r be the cotangent of the angle which the line A B in figs. 158 and 159 makes with the horizon.

Case I.—When the ground, in proceeding from the centre to the edge of the earthwork, slopes away from the base, as in the right-hand side of figs. 158 and 159—

$$b' = \mathrm{BN} = \frac{r\,s}{r-s}\cdot\left(h + \frac{b_0}{r}\right)\ \ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

Here the factor $h + \frac{b_0}{r}$ represents H E, the depth of the earthwork at the edge of the base.

Case II.—When the ground, in proceeding from the centre to the edge of the earthwork, slopes towards the base, as in the left-hand side of fig. 158,—

$$b' = \text{A L} = \frac{r\,s}{r+s} \cdot \left(h - \frac{b_0}{r}\right) \text{.....................(3.)}$$

Here the factor $h - \frac{b_0}{r}$ represents G D, the depth of the earthwork at the edge of the base.

Case III.—When the ground intersects the base between the centre line and the edge of the earthwork, as at Q in the left-hand side of fig. 159—

$$b' = \text{A L} = \frac{r\,s}{r-s} \cdot \left(\frac{b_0}{r} - h\right) \text{.....................(4.)}$$

Here the factor $\frac{b_0}{r} - h$ represents G D, the depth of the earthwork at the edge of the base.

The horizontal distance of the point Q from the centre line is given by the formula

$$\text{F Q} = r\,h \text{ ....(5.)}$$

It is obvious that the formulæ of this article can be applied to cases in which the slope of the earthwork and the rate of declivity of the ground are different at the two sides of the centre line, as well as to those in which they are the same.

The half-breadths of the earthwork, $b_0 + b'$, to the right and left of the centre line at a given point, being each increased by the breadth required for fencing, give the *total half-breadths* at that point (as stated in Article 66, p. 113); and these being added together, give the *total breadth* of the land to be taken. From a series of those breadths, at different points in the centre line, the *area of land* to be taken may be calculated by the method of ordinates explained in Article 32, pp. 33, 34.

Or the total half-breadths may be plotted on a plan, the boundaries of the land to be taken drawn through them, and the area found by the Method of Triangles, p. 33, or by the use of the Planimeter, p. 34.

185. **Calculation of Sectional Areas of Earthwork.**—The computation of the areas of a series of cross-sections of a piece of earthwork is a step towards calculating its volume, or "quantity." If the ground is rugged, it may be necessary to find the area of each cross-section by measurements made upon a drawing; but if the ground is nearly or exactly level across, or has nearly or exactly an uniform sidelong slope, the area of a given cross-section can be computed from the same data which serve to compute the breadths of the slopes; that is to say,

The natural slope of the ground,...... r to 1;
The slope of the earthwork, ... s to 1;
The half-breadth of the base,.......... b_0;
The central depth, ... h.

In each case the area of cross-section required will be denoted by S.

Problem First. *To compute the area of cross-section of a piece of earthwork when the ground is level across*, as in fig. 157.

$$S = F\,C \cdot G\,B = h\,(2\,b_0 + b')$$
$$= 2\,b_0\,h + s\,h^2. \qquad \text{(1.)}$$

Problem Second. *To compute the area of cross-section of a piece of earthwork, when the ground has an uniform sidelong slope, not intersecting the base*, as in fig. 158.

The area of the trapezoid $G\,D\,E\,H = D\,E \cdot F\,C = 2\,b_0\,h$;

„ of the triangle $B\,H\,E = \frac{B\,N \cdot H\,E}{2}$ = (according to Article 184, equation 2) $\frac{r\,s}{2\,(r - s)} \cdot \left(h + \frac{b_0}{r}\right)^2$;

„ of the triangle $A\,G\,D = \frac{A\,L \cdot G\,D}{2}$ = (according to Article 184, equation 3) $\frac{r\,s}{2\,(r + s)} \left(h - \frac{b_0}{r}\right)^2$;

hence, adding those three parts together,

$$S = 2\,b_0\,h + \frac{r\,s}{2\,(r - s)} \cdot \left(h + \frac{b_0}{r}\right)^2 + \frac{r\,s}{2\,(r + s)} \cdot \left(h - \frac{b_0}{r}\right)^2. \quad \text{(2.)}$$

This formula may also be put in the following form:—

$$S = \frac{s\,b_0^2 + 2\,r^2\,b_0\,h + r^2\,s\,h^2}{r^2 - s^2}. \qquad \text{(3.)}$$

Another mode of expressing the same quantity is as follows,*—

---

* Suggested, so far as I know, by Mr. Thomas Roberts.

and is convenient for use in connection with a table of squares:—Produce B E and A D till they meet in K, in the vertical line C F produced. Then

$$S = \text{triangle A B K} - \text{triangle E D K}$$
$$= \frac{(\mathrm{A\,M} + \mathrm{B\,P}) \cdot \mathrm{C\,K} - \mathrm{D\,E} \cdot \mathrm{F\,K}}{2};$$

$$\text{but F K} = \frac{b_0}{s};\ \text{C K} = h + \frac{b_0}{s};\ \text{D E} = 2\,b_0 \text{ and}$$

$$\text{A M} + \text{B P} = \frac{2\,r^2}{r^2 - s^2}(s\,h + b_0);\ \text{consequently}$$

$$S = \frac{r^2\,s}{r^2 - s^2}\left(h + \frac{b_0}{s}\right)^2 - \frac{b_0^2}{s}. \ldots\ldots\ldots\ldots\ldots (4.)$$

PROBLEM THIRD. *To compute the areas of the two divisions of a cross-section of earthwork, when the ground intersects the base,* as in fig. 159.

The cross-section here consists of two similar triangles, Q B E and Q A D, one of which is in cutting and the other in embankment. In the figure, the larger triangle is in cutting; the same figure inverted will represent the case in which the larger triangle is in embankment. When Q, C, and F coincide, the triangles are equal.

Let S′ denote the larger and S″ the smaller triangle. Then

$$S' = \frac{(\mathrm{B\,P} + \mathrm{F\,Q}) \cdot \mathrm{E\,H}}{2} = \frac{(b_0 + r\,h)^2}{2\,(r - s)}; \ldots\ldots\ldots (5.)$$

$$S'' = \frac{(\mathrm{A\,M} - \mathrm{F\,Q}) \cdot \mathrm{D\,G}}{2} = \frac{(b_0 - r\,h)^2}{2\,(r - s)}. \ldots\ldots (6.)$$

186. **Calculation of Volumes or Quantities of Earthworks.**

CASE I. *When two cross-sections* S_0, S_1, *are given, with the longitudinal distance between them* x, the volume (V) of the earthwork between those cross-sections is given approximately, by the following formula, provided S_0 and S_1 are nearly equal, but not otherwise:—

$$V = x \cdot \frac{S_0 + S_1}{2}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

CASE II. *When three equidistant cross-sections* S_0, S_1, S_2, *are given, with the total length* x, *of the piece of earthwork between them,* the best approximation is,

$$V = x \cdot \frac{S_0 + 4\,S_1 + S_2}{6}. \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

CASE III. *Two cross-sections given, and one assumed.*—Equation 2 may also be used to give a closer approximation than equation 1, when the two endmost cross-sections only are given, S_0 and S_2, by putting for S_1 the area of an *assumed* cross-section midway between S_0 and S_2; its central depth being assumed to be a mean between the central depths of S_0 and S_2, and the sidelong slope of the ground (if any), at S_1 a harmonic mean between those at S_0 and S_2.

When the ground is level across, this last process gives the following result:—

Let h_0 be the central depth at S_0;
" h_2 " " " at S_2;

then the assumed central depth at S_1 is $\frac{h_0 + h_2}{2}$; and

$$V = x \left\{ b_0 \,(h_0 + h_2) + s \cdot \frac{h_0^2 + h_0\,h_2 + h_2^2}{3} \right\} \ldots\ldots (3.)$$

This formula is called the "Prismoidal Formula." Another form of the same formula, convenient for use in connection with a table of squares, is as follows:—

$$V = x \left\{ b_0 \,(h_0 + h_2) + s \left\{ \frac{(h_0 + h_2)^2}{4} + \frac{(h_0 - h_2)^2}{12} \right\} \right\} . (4.)$$

Formula 3 is the basis of Sir John Macneill's earthwork tables; formula 4 of Mr. Henderson's.

CASE IV. *An even number of equidistant cross-sections given,* S_0, S_1, S_2, &c. . . . S_m; the distance from section to section being $\Delta\, x$.

$$V = \Delta\, x \left\{ \frac{S_0}{2} + S_1 + S_2 + \&c. \ldots + \frac{S_m}{2} \right\} \ldots\ldots (5.)$$

CASE V. *An odd number of equidistant cross-sections given,* S_0, S_1, S_2, &c. . . . S_n; the distance from section to section being $\Delta\, x$.

$$\left. \begin{aligned} V = \frac{\Delta\, x}{3} \Big\{ & S_0 + 4\,S_1 + 2\,S_2 + 4\,S_3 + 2\,S_4 + \&c. \ldots \\ & + 2\,S_{n-2} + 4\,S_{n-1} + S_n \Big\}. \end{aligned} \right\} (6.)$$

Besides the earthwork tables already mentioned, many others have been published, such as Mr. Bidder's, Mr. Haskoll's, &c. Such tables generally give either the *mean sectional area* of a piece of earthwork of a given base and slope, and of given depths at the two ends, or a number proportional to it; which mean area or number being multiplied by the length, gives the volume.

Quantities of earthwork, in Britain, are usually stated in *cubic yards*, while their dimensions are given in feet. The expressions for volumes in this Article, being suited for the case in which the unit of volume is the cube described upon the linear unit, require to be divided by 27, when the dimensions are in feet, to reduce the volumes to cubic yards.

Sometimes, while the breadths and depths are given in feet, the lengths are stated in chains of 66 feet; and in that case, to give the volumes in cubic yards, the expressions in this Article should be multiplied by

$$\frac{66}{27} = \frac{22}{9} = 2{\cdot}44\dot{4}.$$

SECTION III.—*Of the Execution of Earthwork.*

187. **Borings and Trial Shafts.**—The ordinary method of ascertaining the nature of the material to be excavated, previous to the undertaking or execution of any piece of earthwork, is by boring a vertical hole of about 3½ or 4 inches in diameter in the ground, and bringing up specimens of the materials pierced through at different depths.

Inasmuch, however, as the specimens of materials so brought up are, in general, reduced to chips or to powder by the action of the boring-tool, and sometimes to paste or mud by the action of the water which is poured into the hole to keep the tool cool, and facilitate its working in hard strata, the information obtained by boring is not wholly satisfactory; for although it shows the mineralogical composition of the materials found at different depths, it leaves their probable stability in earthwork doubtful, except in so far as it can be inferred from the resistance met with by the boring tool; and this source of information is available to the engineer or contractor at second-hand only, through the statements of the borers. The smallness of the hole, too, makes the results of borings doubtful; for what seems to be a stratum of rock may sometimes prove to be only a solitary block or boulder.

To ascertain completely the nature and qualities of the materials of an intended cutting, trial-shafts or pits should be sunk down to the level of its bottom. The expense and time required for sinking shafts make it impracticable to use them exclusively. The best method is to combine shafts with borings, by sinking, in every important proposed cutting, one shaft at least, which should in general be at the point of greatest depth, and making, besides, a series of borings at points 200 or 300 yards apart. These borings will be sufficient to show whether any change in the strata occurs sufficient to make it advisable to sink one or more additional shafts in a given cutting.

Boring tools are made of wrought iron, steeled at the cutting edges and points. They are usually about 3 feet long, or a little more, about one-half of the length being the tool or boring instrument proper, and the remainder the shank, which is a bar of $1\frac{1}{8}$th inch square or thereabouts, having a screw at its upper end to connect it with the first of the *lengthening rods*. These are square bars, usually about 10 feet long, of the same diameter with the shank of the boring tool, with screws at their ends by which they can be united together to any length required by the depth of the bore. The uppermost rod is capable of being hung by a swivel and rope from a triangle or shears set up over the bore-hole, in order to haul up the rods when required. The working part of the tool is made of various figures, for penetrating various materials. The commonest forms are the *auger*, the *worm*, and the *jumper*. The *auger*, which is used for boring all ordinary earths, shale, and soft rock, is formed like a hollow cylinder, about $3\frac{1}{2}$-inches in diameter, with an open sharp-edged slit along one side, and slightly contracted at the lower end, which sometimes (for boring soft rock) has a small spiral point like that of a gimlet. It brings up specimens of the material bored in the inside of its hollow cylindrical body.

The *worm* is a sharp pointed spiral, used for boring rock too hard to be pierced by the auger. After the rock has been pierced by the worm, the auger is used to enlarge the hole and bring up the fragments.

Both the auger and the worm are worked by turning them continuously round towards the right (that is, in the direction of the motion of the hands of a watch), by means of a cross-head six feet long, or thereabouts, driven by two men.

To pierce rock that is too hard for the worm, a *jumper* is used. Jumpers are of various figures; some flat, like a chisel, with a sharp edge at the lower end; some square, with a four-sided pyramidal point, like a poker; some spear-pointed. The jumper is worked by raising it a short distance and letting it drop, turning it a little way round after each blow. It is sometimes simply hung by a rope, instead of being screwed to the lower end of the lengthening rods. The materials broken by the jumper are sometimes brought up by the auger, sometimes by a sort of bucket on the top of the jumper itself.

Bores in very soft materials sometimes require to be lined with a series of cast or wrought iron pipes, pushed down as the bore proceeds, to prevent its sides from falling in; the lowest pipe having a sharp serrated edge. These pipes may be made to screw together, so that they can be hauled up again.

The depth of a layer of moss, mud, or quicksand, at the surface of the ground, is sometimes probed or sounded with a long slender iron rod called a *pricker*.

The operations of sinking shafts will be described further on, under the head of TUNNELLING.

In marking the results of borings and trial shafts on a section (see Article 11, p. 10, and Article 17, p. 15), care is to be taken to show nothing on the paper except the facts actually observed, all conjectural sections of the strata lying between the borings and pits, whether marked by outlines, colour, shadings, or words, being rigidly excluded. The insertion of such conjectural sections, although it improves the appearance of the drawing, and makes it more readily intelligible, is done at the risk of misleading contractors, and involving the companies and engineers in heavy responsibility. The result of the pits and borings being shown exactly as observed, contractors and others are left to draw their own conclusions as to the intermediate strata.

188. **Equalizing Earthwork** is a term applied to the process of so adjusting the formation level of an intended work, that the earth from the cuttings shall be as nearly as possible sufficient to make the embankments, and no more. The art of making this adjustment by the eye upon a section of the ground with sufficient accuracy is soon acquired by practice. In most cases it is essential to economy in the cost of the work; for any surplus of embankment over cutting must be made up from "side cutting;" and the earth from any surplus of cutting over embankment must be formed into "spoil banks;" both of which works involve additional cost for labour and land. But cases sometimes occur, in which it is more economical to make an embankment from side-cutting close at hand, than to bring the necessary material from a far distant cutting on the line of works, or in which it is more economical to throw part of the material from a cutting into a spoil bank, than to send it to a far-distant embankment on the line of works; and these points must be decided by the engineer to the best of his judgment in each particular case.

189. The **Temporary Fencing,** erected before the earthwork is commenced, should enclose all the ground required for the undertaking; that is to say, it should run along the outer boundary of

the strip of land which is to be taken beyond the edge of the earthwork, and whose breadth is added to the half-breadths of the earthwork in calculating and setting out the total half-breadths (p. 113). In the open country, where the permanent fence is to be a hedge and ditch, the breadth of that strip of land is usually about *nine feet;* but where ground is valuable, as amongst gardens and pleasure-grounds, and in towns and suburbs, smaller breadths are used, as to which no general rule can be laid down.

The temporary fence usually consists of posts and rails of larch or oak; the posts being from 4 feet to 4 feet 6 inches apart, about 6 feet long, driven from 2 feet to 2 feet 6 inches into the ground, from 4 to 6 inches broad in a direction *across* the fence, and about 3 inches thick in a direction *along* the fence; the rails about 9 or 10 feet long, 3 inches deep, and 1½ inch thick, scarfed in mortises in every second post. Sometimes the posts in which the rails are scarfed are made stronger than the intermediate posts, and have diagonal stays to increase their stability, the foot of each stay being nailed to a small stake about 2 feet long.

The best site for permanent marks of the line and levels is near the fence (pp. 110, 111).

If the soil is wet, a *catchwater drain* may be made at the same time with the temporary fencing, at one or both sides of the earthwork, commencing at its outfall into an existing main drain or water course, and working upwards. When the ground has a sidelong slope the catchwater drain is indispensable at the up-hill side of the earthwork. Thus, in fig. 160, A B is part of the base and B C one of the slopes of an intended cutting; C G is part of the natural ground, sloping downwards towards C; D is a catchwater drain, to prevent surface water running from G towards C from injuring the slope of the cutting. In fig. 161, A B is part of the base and B C one of the slopes of an embankment; E F is part of the natural ground, sloping downwards from F towards E; D is a catchwater drain, to prevent surface water running from F towards C from collecting at C and injuring the embankment. The catchwater drain may be an open ditch, in ordinary cases from 3 to 4

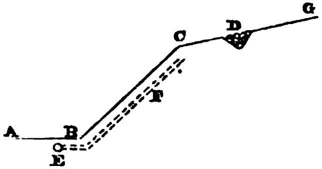

Fig. 160.

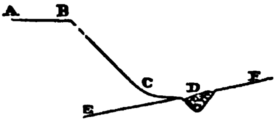

Fig. 161.

feet wide and from 2 to 3 feet deep; or it may be an underground drain, built of stone or brick, or made of earthenware tubes (as in the figures), with broken stone or clean gravel above it.

191. **Stripping the Soil.**—The soil or vegetable mould should be stripped from the site of an intended piece of earthwork, and laid down near the fence, in order that it may be afterwards used to re-soil the slopes of the earthwork. The usual depth of soil spread on these slopes varies from 3 to 6 inches.

192. **General Operations of Cutting.**—Where there is no reason to the contrary, it is desirable that the base of a cutting should have a declivity towards the point at which the work of excavation is commenced; for this renders more easy the removal of the earth in wagons, and the temporary and permanent drainage.

A cutting is usually commenced (if the earth will stand for a time with vertical sides) by making a "gullet," or vertical-sided excavation, wide enough to contain one or more lines of temporary rails for the passage of earth wagons. The widening of the cutting to its full width, and the formation of the slope, should be carried on so as never to be far behind the head or most advanced end of the gullet; for the strain thrown on a mass of earth by standing for a time with a vertical face has a tendency to produce cracks, which may extend beyond the position of the intended slopes, and so render the sides of the cutting liable to slip after they have been finished. The advanced end of a cutting of considerable depth, and the parts of its sides whose slopes have not been finished, consist, while the work is in progress, of a series of steps or stages called "lifts," rising one above another by six or eight feet, or thereabouts, the excavators working at the faces of these lifts so as to carry them on together.

From faces at the end or sides of the gullet, the earth is shovelled directly into the wagons. From the other faces of the cutting, the earth is wheeled in barrows along planks to points from which it can be tipped into the wagons.

193. **Draining the Base and Slopes.**—At the foot of each slope of a cutting it is almost always necessary to have a longitudinal drain called a "side-drain," of from 6 inches to 2 feet deep according to the circumstances of the case. It may be a small open ditch, or a channel pitched and faced with stone, or a covered stone or brick drain, or a line of tubes (as at E, fig. 160) with broken stone or gravel above. It may receive the waters of branch drains running across the base, should such be found necessary, and also of branch drains laid in the slopes, as F, fig. 160. When the latter are tubes, they may in general be laid about 2¼ feet below the surface. It is in general advisable so to place the side-drain E that its bottom shall not be below the prolongation of the plane

of the slope B C, unless there is a retaining wall; otherwise it may cause that slope to slip, and may itself be crushed or choked.

Springs rising in cuttings require special drains to carry away their waters.

194. The **Labour of Earthwork**, in ordinary cases, consists of *getting*, or excavating; *filling* into barrows or wagons; *wheeling* in barrows; *leading* in wagons; and *teeming* or *tipping*—that is, depositing the earth in the embankment where it is to rest. Other processes required in special cases will be considered farther on.

The labour of *getting* the earth depends mainly upon its adhesion. Loose sand and gravel, soft vegetable mould and peat, can be dug with the shovel or the spade alone; stiffer kinds of earth require to be loosened with the pick before being shovelled into barrows, and in some cases, with crowbars, wedges, or stakes; the softest kinds of rock can be broken up with the pick or crowbar; harder kinds require the action of wedges; harder still, especially if free from natural fissures, need blasting by gunpowder, which will be treated in a separate article.

Wheeling in barrows is performed upon planks, whose steepest inclination should not exceed 1 in 12, unless the men are assisted by means of ropes and winding machinery.

Leading is performed upon light temporary rails, in wagons called "earth-wagons," whose bodies can be tipped over by turning on a pair of horizontal trunnions, so as to empty the earth out: they are drawn by horses or by small locomotive engines.

The labour of shovelling *a given weight* of earth into barrows, and that of wheeling it from the face of the cutting to a given point, tipping it into the wagons, and leading it a given distance, are nearly the same for most ordinary kinds of earth. For *a given bulk* of earth, the labour of those operations varies nearly as the heaviness of the earth.

In order to execute an excavation with speed and economy, it is necessary to fix correctly both the absolute and the proportionate numbers of pickmen, shovellers, and wheelers, or barrowmen, so that all shall be constantly employed. The only method of doing this *exactly*, in any particular case, is by trial on the spot; but an approximation may be made beforehand by estimating from the data of experience.

The *absolute number of excavators* working at the face of a cutting is determined by the horizontal extent of face at which cutting is in progress at once; one excavator to five or six feet of breadth of face, is about as close as they can be placed without getting in each others' way.

The proportion of *wheelers to shovellers* may be estimated approximately by the fact, that a shoveller takes about as long to fill an ordinary barrow with earth as a wheeler takes to wheel a full barrow about 100 or 120 feet, on a horizontal plank, and return with an empty barrow.

If the full barrow has to be wheeled up an ascent, each foot of rise is to be considered equivalent to six additional feet of horizontal distance.

Hence the following approximate formula:—

Let l be the horizontal distance that the earth has to be wheeled, and h the height of ascent, if any; then

$$\text{number of wheelers to one shoveller} = \frac{l + 6\,h}{\text{from 100 to 120 feet}} \quad (1.)$$

The number of *barrows* required for each shoveller is one more than the number of wheelers.

A shoveller will throw each shovelful of earth from 6 to 10 feet horizontally, or from 4 to 5 feet vertically upwards. If the earth is to be thrown by the shovel to greater distances or heights, two or more ranks of shovellers must be employed.

The proportion of the pickmen to the shovellers (in a single rank) depends on the stiffness of the earth. The following are examples:—

	Pickmen to one Shoveller.
Loose sand and vegetable mould,...............	0
Compact earth,......................................	½
Ordinary clay, ...	from ½ to 1
Hard clay,..	„ 1½ to 2.

Earth is designated as "earth of one man," if one shoveller can keep one line of wheelers at work; "earth of a man and a-half," if two shovellers and a pickman are needed to keep two lines of wheelers at work; "earth of two men," if one shoveller and one pickman can keep one line of wheelers at work; and generally, "earth of so many men," according to the number of shovellers and pickmen together who are required to keep one line of wheelers at work. Let m denote that number; then the total number of shovellers, pickmen, and wheelers for each line of wheelers, will be approximately

$$\mathrm{M} = m + \frac{l + 6\,h}{\text{from 100 to 120 feet}}. \quad \ldots\ldots\ldots\ldots\ldots (2.)$$

The rate at which the cutting may be expected to advance, if no special difficulties occur, may be estimated for each line of wheelers (or for each shoveller in one rank), at about

$$\left.\begin{array}{l}\text{20 cubic yards of loose sand, or mould,}\\ \text{or 16 cubic yards of clay, or compact earth,}\end{array}\right\}\text{ per day.}$$

The labour of excavating is often considerably lessened, especially in widening a gullet at the sides, by undermining large masses of earth from below, and loosening them by driving stakes behind them from above. This is called "falling."

An earth-wagon holds about as much as 50 wheel-barrows, and if drawn at the walking pace of a horse, its speed may be taken as about one-fifth greater than that of the wheel-barrows; so that it is equivalent to about 60 wheel-barrows; and one earth-wagon going and returning a distance of about 6,000 feet horizontally, while another stands to be filled, will keep one shoveller at work. If loaded wagons have to be drawn up an ascent, and the temporary rails are in moderately good order, each foot of ascent may be considered as equivalent to about 150 feet of additional horizontal distance. Hence let L be the horizontal distance in feet to which the earth is to be led in earth-wagons drawn by horses, H the ascent, in feet, if any; then the *number of shovellers* (in single rank) *to each earth-wagon in motion at one time,* is about,

$$= \frac{6{,}000}{L + 150\,H} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

and the reciprocal of this expresses the *earth-wagons or fractions of an earth-wagon in motion at one time per shoveller;* but additional wagons, as to which no precise rule can be laid down, must be provided, in order to allow for those which are standing to be filled, and for those which are in the act of being tipped and reversed. With locomotive engines the speed can be increased, and the number of wagons proportionally diminished. The preceding calculations have reference to wagons which hold from 2 to $2\frac{1}{2}$ cubic yards of earth, or thereabouts, the weight of which is from $2\frac{1}{2}$ to 3 tons, the weight of the wagon itself being between a ton and a ton and a-half. The friction being taken at 15 lbs. per ton (or 1 — 150th of the gross load nearly), the force required to draw a wagon, or train of wagons, either on a level, or up or down a given declivity, can easily be calculated. In estimating the number of horses required, the force which a horse can exert when walking slowly may be estimated at about 120 lbs.

When the leading of earth is performed, not in wagons or temporary rails, but in two-wheeled one-horse carts on an ordinary roadway, the number of such carts required may be approximately computed from the data, that the net load of each such cart is about equal to that of twelve wheel-barrows, and its average speed going and returning about one-sixth part more than that of a wheel-barrow, so that each cart in motion is equivalent to about fourteen wheel-barrows in motion. In this as in the preceding case, in computing the total number of carts, allowance must be made for those which are standing to be filled, and those which are being turned and tipped.

195. **Benches** on the sides of cuttings are small platforms, level transversely, seldom exceeding about six feet in breadth. They are sometimes used in very deep cuttings, for the purpose either of intercepting the fall of boulders and pieces of rock from the higher slopes, or of facilitating the drainage. A bench ought to have a slight declivity lengthwise, and at the foot of the slope above it there should be a side-drain like that at the side of the base (E, fig. 160, p. 334), to catch and carry away all the surface-water from that slope.

196. **Prevention of Slips.**—As the slipping of the sides of cuttings is caused by the action of water, its prevention is promoted by efficient ordinary drainage, as described in Article 189, p. 334, and Article 193, p. 335. When ordinary methods of drainage are insufficient, other expedients must be adopted, such as the following:—To make the branch-drains of the slope very numerous and close; to make special drains for carrying down to the side-drain of the cuttings, the waters of such springs as may flow from the slope; to face the slope with a well-packed layer of stones laid dry, from 6 to 18 inches thick, according to the circumstances of the case; to cut away a portion of the lower part of the slope, and form in the space so left a bank of gravel or shivers of stone, against which the slope of earth may abut, with counterforts, made by digging trenches at right angles to the gravel bank, and filling them with gravel; this combination acts both as a retaining wall and as a system of drains; to build at the foot of the slope, so as at once to support and drain it, either a dry stone retaining wall, or a wall of brick or masonry laid in mortar, backed with a vertical layer of dry stones; to intercept underground waters on their way towards the slope, by means of a drift or mine.

Retaining walls will be further treated of under the head of MASONRY, and drifts under that of TUNNELLING.

In some instances, all remedies for slipping are found unavailing, and the material must be allowed to find its own angle of repose, care being taken to remove the earth which slides down from time to time, and to acquire the necessary additional land.

197. **Settlement of Embankments.**—Embankments subside or settle after their first formation, to an extent which varies considerably for different materials and under different circumstances, being seldom less than *one-twelfth*, and seldom more than *one-fifth*, of the original height. The best method of ascertaining the probable

proportionate settlement of a proposed embankment is by an experiment on a short length of it; allowance for the settlement so ascertained must then be made in constructing the remainder of the embankment.

198. The **Distribution of Earthwork** means the arrangement by which the materials obtained from different parts of the cuttings are distributed amongst different parts of the embankments, so as to insure the least possible expenditure of labour in the *leading* or conveyance of the earth. To attain this object, two rules are to be followed as closely as may be practicable;—to fill each portion of embankment from the *nearest accessible* portion of cutting; and to take care that the several routes by which earth is conveyed from cutting to embankment shall not cross each other.

The *mean distance* of lead, from a division of a cutting to that division of an embankment which is filled from it, is nearly equal to the distance between their centres of gravity.

199. **General Operations of Embanking.**—The best materials for embankments are those whose frictional stability is the greatest and the most permanent, such as shivers of rock, shingle, gravel, and clean sand. Clay also forms safe embankments, provided it is dry, or nearly dry, when laid down. Wet clay, vegetable mould, and mud, are unfit for use in embankments; so also is peat, except with certain precautions to be afterwards mentioned.

An embankment may be made in three ways:—I. In one layer. II. In two or more thick layers. III. In thin layers.

I. *In one layer.*—This being the cheapest and quickest method, consistent with stability, is that followed in all earthworks in which there is no special reason to the contrary. In fig. 162, B A C represents the natural surface of the ground; D A part of the base of a cutting; A E C an embankment, the construction of which is carried forward in the direction A E of its full width and height (including a sufficient allowance for subsidence), by running earth-wagons on temporary rails from the cutting along the top of the embankment, and tipping them at E, so that the earth runs down and spreads itself over the sloping end E C of the bank, which is called the "tip."

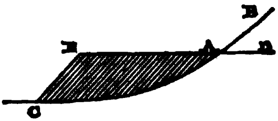

Fig. 162.

The sloping lines parallel to E C represent a series of successive previous positions of the tip, as the embankment advanced from A.

No tipping over the sides of embankments should be allowed; for the earth so tipped is liable afterwards to slip off.

II. *In thick layers.*—This process has been used in some embankments of great height. It consists in completing the construction of the embankment up to a certain height by the process of tipping over the end already described; leaving that layer for a time to settle, and then making a second layer in the same way, and so on. It involves much additional time and labour, and is seldom employed. It is, however, useful in making embankments of hard clay or shale, which, when first tipped, consists of angular lumps that lie with vacant spaces between them, and do not form a compact mass until partially softened and broken down by the action of air and moisture.

III. *In thin layers.*—This process consists in spreading the earth in horizontal layers of from 9 inches to 18 inches deep, and ramming each layer so as to make it compact and firm before laying down the next layer. Being a tedious and laborious process, it is used in special cases only, of which the principal are, the filling behind retaining walls, behind wings and abutments of bridges and culverts, and over their arches, and the embankments of reservoirs for water.

The *labour* of spreading earth in layers and ramming it may be estimated, in general, at from *once-and-a-sixth* to *once-and-a-third* that of shovelling it into a barrow. (See Article 194, p. 337.)

200. **Embanking on Sidelong Ground.**—When the natural ground has a steep sidelong slope, it is, in general, necessary to cut its surface into steps before making the embankment, in order that the latter may not slide down the slope. In the cross-section, fig. 163, the dotted line A B represents the natural surface of the ground, Q E B a side-cutting, and A D Q an embankment, resting on steps which have been cut between A and Q. The best position for those steps is *perpendicular to the axis of greatest pressure*, whose inclination to the vertical is given by equation 14 of Article 183, p. 322; so that, if A D is inclined at the angle of repose, the steps near A should be inclined to the horizon, in the opposite direction to A D, at the angle given by equation 16, p. 322; while the steps near Q may be level. It is better to make the steps steeper than the inclination given by this principle than to make them flatter.

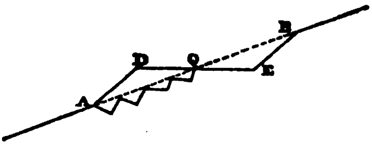

Fig. 163.

201. **Embanking over and near Masonry.**—In embanking over culverts (that is, covered drains of masonry or brickwork), near retaining walls, or near the abutment and wing walls of bridges, care must be taken not to injure the masonry by shocks from the fall of earth, or by ill-distributed or sudden pressures.

For the purpose of preventing shocks, the precaution is taken already mentioned in Article 199, above, of spreading the earth in immediate contact with the masonry in thin layers, and ramming each layer. For this purpose, dry materials should be chosen, that will let water drain off easily, such as shivers of stone, gravel, and clean coarse sand. This rammed earth should fill all the space between the wing walls of bridges, and extend back from retaining walls, and from the abutments of bridges and culverts, ten feet or thereabouts. Over the arches of culverts, the earth rammed in thin layers should rise to at least half the height of the proposed embankment; the remainder may be tipped in the common way.

For the purpose of preventing unequal lateral pressures against bridges and large culverts, care should be taken (by the aid, if necessary, of timber platforms to carry the temporary rails), that the embankment is carried up at both sides of the structure at once, and as nearly as possible to the same height at the same time.

202. **Drainage of Embankments.**—The position and use of the catchwater drain near the foot of the slope has already been explained in Article 189, p. 334. The construction of culverts, for carrying drainage-water below embankments, will be treated of under the head of MASONRY. Ground in which springs rise should be avoided altogether, if possible; but if it is absolutely necessary to embank over a spring, a culvert may be built to carry its water clear of the embankment.

203. **Embankment in a Great Plain.**—When a line of conveyance is carried across an extensive plain, it is almost always necessary, in order to keep its surface dry, that it should be raised above the general level of the ground; and where inundations occur, the requisite height may be considerable. In fig. 164, A represents a cross-section of an embankment for this purpose, the materials for which are obtained by digging a pair of trenches, B, C, alongside of it. These trenches, by collecting surface-water and discharging it into the nearest river or other main drainage channel, tend to shorten the duration of floods in the neighbourhood of the line.

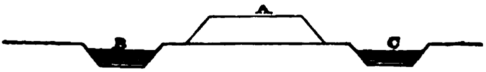

Fig. 164.

204. **Embankments on Soft Ground.**—When the ground is so soft that an embankment made in the ordinary way would sink in it, different expedients are to be employed, according to the kind and degree of difficulty to be overcome. The following list of expedients is arranged in the order of an increasing scale of difficulty:—

I. By digging side-drains parallel to the site of the intended embankment, the firmness of the natural ground may be increased.

II. If the material of the natural ground has a definite angle of repose, though much flatter than that of the material of the embankment, the slopes of the embankment may be formed to the same angle, thus giving it a broader foundation than it would have with its own natural slope.

III. A foundation may be made for the embankment by digging a trench and filling it with a stable material. In fig 165, A E F B represents the cross-section of an intended embankment, and A C D B that of the trench to be dug for its foundation, the edges of the base of the trench, C, D, being vertically below those of the top of the embankment, E, F. To design these cross-sections, proceed as follows:—

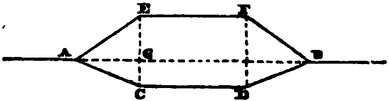

Fig. 165.

Let $h = \mathrm{G\,E}$ denote the height of the proposed embankment;

w, the weight of a cubic foot of its material;

w', the weight of a cubic foot of the material of the natural ground;

ϕ', its angle of repose;

$h' = \mathrm{G\,C}$, the required depth of the foundation;

also let $\frac{1 - \sin \phi'}{1 + \sin \phi'} = k'$;

then the depth of the foundation is given by the formula,

$$h' = \frac{h\,w\,k'^2}{w' - w\,k'^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

The slopes of the trench, C A, D B, should be inclined at the angle of repose of the soft material; so that the breadth of each will be

$$\mathrm{A\,G} = h' \cdot \mathrm{cotan}\ \phi'; \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

and this fixes also the inclination of the slopes of the embankment, A E, B F, without reference to the angle of repose of its material.

IV. The ground may be compressed and consolidated by means of short piles. This method will be further explained under the head of FOUNDATIONS.

V. The embankment may be made of materials light enough to form a sort of raft, floating on the soft ground, such as hurdles, fascines, or dry peat. The use of fascines will be further explained in a later chapter. Dry peat was the material used by George Stephenson to carry the Liverpool and Manchester Railway across Chat Moss. Its heaviness, when well dried in the air, is about

30 lbs. per cubic foot; and when saturated with water, 63 lbs. On the dry peat embankment was placed a platform of two layers of hurdles, to carry the ballast.

VI. Should all other expedients fail, a moss or bog may still be crossed by throwing in stones, gravel, and sand, until an embankment is formed, resting on the hard stratum below the moss, and with its top rising to the required level. It is found that the material of the embankment assumes the same natural slope that it would do in the air.

205. **Dressing, Soiling, and Pitching Slopes.**—The slopes, both of embankments and cuttings, are to be dressed to smooth and regular surfaces, and covered with a layer of soil, which varies from 3 inches to 6 inches in depth, according to the practice of different engineers, and is sown with grass-seed.

The *labour* of dressing slopes is nearly equivalent to that of digging about half-a-foot deep in loose mould over the same area of surface; and that of spreading the soil is about the same with that of shovelling it into a barrow. (See p. 337.)

Slopes of embankments which are exposed to still water may be faced or "pitched" with dry stone about a foot thick. The protection of slopes against waves and currents falls under the head of Hydraulic Engineering.

206. **Clay Puddle** is used to make embankments and channels water-tight, and to protect masonry against the penetration of water from behind. The proper material for it is clay, freed from all large stones, roots of plants, and the like, and containing as much sand and fine gravel as is consistent with its holding water; if there is too little sand, the puddle is liable to crack in dry weather. It is made by working the clay in layers about 9 inches thick, with enough of water to reduce it to a pasty condition, by means of a tool that has a sort of *poaching* action, until it becomes a perfectly uniform and compact mass. The labour is about five times that of shovelling the same quantity of material.

207. **Quarrying and Blasting Rock.**—Rock that is too hard to be split with the pick, the crowbar, or the quarryman's hammer, and not so hard as to require blasting with gunpowder, can be quarried in blocks, by cutting grooves, or boring holes, in the upper surface of a bed, inserting blunt steel wedges in them, and driving those wedges with a hammer until a block splits off from the layer. Gauthey's estimate of the labour of this operation, per cubic yard of rock, is about 0·4 of a day's work of a man; but it varies very much for different kinds of rock.

The processes of blasting with gunpowder may be divided into *small blasts* and *great blasts*.

I. A *small blast* is made by boring with a jumper (Article

187, p. 332), a hole in the rock, whose diameter varies from 1 inch to 6 inches, or thereabouts, and its depth from one foot to 30 feet. Part of the depth of the hole is filled with coarse-grained gunpowder, poured in through a tube reaching nearly to the bottom, and the remainder of the hole is rammed with what is called "tamping," consisting of chips of rock, sand, clay, and other such materials; the best material being dry clay. Care is to be taken never to use materials that may strike fire, and not to ram hard until there are some inches of material between the tamping-bar and the powder. The fuse may be protected by traversing a tube, or a groove in a piece of wood. It should burn at the rate of about 2 feet per minute. The best fuse for this purpose is known as "Bickford's."

The explosion of the powder splits and loosens a mass of rock whose volume is approximately proportional to the *cube of the line of least resistance,*—that is, in general, of the shortest distance from the charge to the surface of the rock—and may be roughly estimated at *twice* that cube; but this proportion varies very much in different cases.

The proportion of the *weight of rock loosened* to the *weight of powder* exploded ranges from about 7,000 : 1 to 14,000 : 1, and may be taken on an average at 10,000 : 1.

The ordinary rule for the weight of powder in small blasts is,

$$\text{powder in lbs.} = \frac{(\text{line of least resistance in feet})^3}{32} \ldots (1.)$$

A test of the strength of blasting powder is, that 2 ounces, or $\frac{1}{8}$th of an avoirdupois pound of it, being fired in an eight-inch mortar elevated at an angle of 45°, should throw a 68 lb. ball to a distance of 240 feet.

Another test is by firing 2 ounces of powder in the "eprouvette gun;" its bore is 27·6 inches long, and $1\frac{3}{4}$ inch in diameter; it weighs $86\frac{1}{2}$ lbs.; it is hung in a frame like a "ballistic pendulum," and its recoil is measured on a graduated arc. Good powder fit for blasting gives a recoil of about 20 degrees.

One lb. of powder in a loose state occupies about 30 cubic inches. By compression, it may be squeezed into $27\frac{1}{2}$, or thereabouts.

Thirty cubic inches are equal to 38·2 *cylindrical inches;* and this is the *length of hole, one inch in diameter, required to hold one lb. of powder.* The corresponding length for other diameters varies inversely as the square of the diameter.

A blast acts most efficiently when the line of least resistance (being, in sound rock of uniform strength, the shortest line from the charge to the surface), is perpendicular to the axis of the bore-hole. It acts least efficiently when the line of least resistance is the axis of the bore-hole itself. It is not always possible to jump a hole perpendicular to the intended line of least resistance; but the hole should always be made to form as great an angle with that line as possible.

If a charge fails to explode, the tamping may be bored out with the auger (p. 332), a new fuse put in, and the hole re-tamped. This process, however, is not wholly free from danger; and the safest method is to jump a new hole near the first, and put in a fresh charge of powder, the explosion of which will probably be communicated to the former charge.

The labour of jumping holes varies very much for different qualities of rock. It is performed either by two men striking the jumper with hammers, while a man or boy turns it, or by one or two men raising it and letting it drop; the latter being the more efficient method, but wearing out the jumper faster than the other. The jumper used for the latter process is called the "churn jumper."

The following are examples of the day's work per man performed in jumping holes:—

	Cylindrical Inches of Hole.
In granite, by hammering,	100 to 150
" by "churning,"	200 nearly
In limestone,	500 to 700

In granite, jumpers require to be sharpened about once for each foot bored, and steeled once for each 16 or 20 feet; and the length of iron wasted in using them is about one-tenth of the depth bored.

The lower ends of holes in limestone have sometimes been enlarged to form a chamber for the powder, by the aid of dilute nitric acid. A double tube, consisting of an outer tube of copper with a tube of lead within it, is passed down to the bottom of the hole; the inner tube has a funnel on the top into which the dilute acid is poured; it passes down, and dissolves the lime of the limestone; the carbonic acid gas disengaged forms, with the solution of nitrate of lime, a stream of froth, which rises through the space between the inner and outer tubes, and escapes through a lateral bent spout near the top of the latter.

II. A *great blast* is made by excavating a vertical shaft or a horizontal heading in the mass of rock, which should turn at right angles at least once on its way to the powder-chamber at its end, in order that the tamping may not be blown out. Such shafts and headings vary from 3½ feet square to 3½ feet by 5 feet or thereabouts, and the labour required to make them varies from 2 days' to 6 days' work of a miner per lineal foot. The mine being swept out and its floor covered with a matting of old sacks, the gunpowder is placed in the chamber in a deal box, whose size is regulated by the fact that 1 lb. of gunpowder fills about 30 cubic inches; a small quantity of finer powder in a bag or case forms the "bursting charge," and is traversed by a fine platinum wire, connecting a pair of copper conducting wires with each other. These are coated with Indian rubber or gutta-percha, or otherwise insulated, and protected by being placed in a groove in a wooden bar. The entrance of the chamber is closed with a wall of turf, and the rest of the mine "tamped" by being built up either with rubble masonry or with a mixture of stones and clay. When the workmen have removed to a safe distance, the conducting wires are connected with the opposite ends of a galvanic battery, when the electric current raises the platinum wire to a white heat, and fires the charge.

The chief use of the electrical apparatus is to fire several charges exactly at the same instant. When one charge only is to be fired, a safety fuse may be used.

According to Mr. Sim, the chamber of the mine should be so placed, that the line of least resistance may be about two-thirds of the height of the rock to be loosened.

In great blasts the proportion of the weight of the rock loosened to that of the powder exploded, ranges from 4,500 : 1 to nearly 13,000 : 1, and is on an average about 6,000 : 1 or 7,000 : 1.

The ratio of the number of lbs. of powder to the cube of the number of feet in the line of least resistance ranges from 1 : 32 to 1 : 10; but the best mode of fixing the quantity of powder is to estimate roughly the weight of the mass of rock which is likely to be loosened, and use from ¼ to ⅓ of a lb. of powder for each ton of rock.

In choosing the positions of bores and mines for blasting, regard should be had to the natural veins and fissures of the rock, as means of facilitating its detachment from its bed.

Blasting under water will be considered in a later part of this treatise.*

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* On the subject of blasting rock the following authorities may be consulted:—General Sir John Fox Burgoyne, "On Blasting and Quarrying;" a paper by Mr. William Sim, "On the Quarrying and Blasting of Rocks," read to the British Association in 1855; and a paper by Mr. George Robertson, "On the Large Blasts at Holyhead," read to the Royal Scottish Society of Arts, and published in the *Civil Engineer and Architect's Journal*, for February and March, 1861.

TABLE OF THE HEAVINESS OF ROCK.

	Lbs. in one Cubic Foot.		Lbs. in one Cubic Yard.		Cubic Feet to a Ton.
Basalt,..................	187	...	5060	...	12
Chalk,..................	117 to 174	...	3160 to 4730	...	19·1 to 12·9
Felspar,................	162	...	4370	...	13·8
Flint,....................	164	...	4430	...	13·6
Granite,................	164 to 172	...	4430 to 4640	...	13·6 to 13
Limestone, ...	169 to 175	...	4560 to 4720	...	13·2 to 12·8
„ magnesian,	178	...	4810	...	12·6
Quartz,.................	165	...	4450	...	13·6
Sandstone, average,...	144	...	3890	...	15·6
„ different kinds,............	130 to 157	...	3510 to 4240	...	17·2 to 14·3
Shale,..................	162	...	4370	...	13·8
Slate (Clay),...........	175 to 181	...	4720 to 4890	...	12·8 to 12·4
Trap,....................	170	...	4590	...	13·2

CHAPTER III.

OF MASONRY.

SECTION I.—*Of Natural Stones.*

208. **Structural Characters of Stones.**—In the last Article of the preceding chapter, rocks or natural stones were considered in the light of materials to be excavated. They have now to be considered in the light of materials for building.

The geological position of rocks has but little connection with their properties as building materials. As a general rule, the more ancient rocks are the stronger and the more durable; but to this there are many exceptions. The properties or characters of rocks which are of most importance in an engineering point of view are of two kinds; the structural and the chemical.

With respect to the structural character of their large masses, rocks may be divided into two great classes,—I. The unstratified, II. The stratified, according as they do not or do consist of flat layers.

I. The *Unstratified Rocks* are believed to have become solid more or less slowly, and under a greater or less pressure, from a melted state. They are, for the most part, hard, compact, strong, and durable.

It is in general obvious that the great masses of unstratified rocks are built, as it were, of blocks, which separate from each other when the rock decays. In granite, for example, those blocks are oblique hexaëdrons—in other words, rhomboidal prisms, sometimes of enormous size; in basalt, they are regular hexagonal or pentagonal prisms, built up into columns; in trap, they are irregular prisms, sometimes approximating imperfectly to the columnar form of basalt. In many cases the further progress of decay rounds off the corners and edges of the blocks, and converts them into boulders, which show a tendency to break up into concentric oval layers. In all cutting, quarrying, and blasting of unstratified rocks, the work is much facilitated by taking advantage of the natural joints between the blocks, at which the rock is more easily divided than elsewhere.

In their more minute structure the unstratified rocks present, for the most part, an aggregate of crystalline grains, firmly adhering together. In granite and syenite, these crystals are comparatively large and conspicuous; in trap, they are much smaller and less distinct; in basalt, they are almost invisible, and the structure is almost glassy; in lava, it is decidedly glassy. Amongst varieties of structure in unstratified rocks, are the porphyritic, where detached crystals of one substance are imbedded in a mass of another; and the cellular, where the mass contains a number of spherical or oval cavities, as if, in its former melted state, it had air-bubbles dispersed in it.

Masses of unstratified rock are often traversed by veins or cracks, sometimes empty, sometimes lined on the sides, and sometimes filled with crystalline masses of various minerals. Such veins facilitate the division of the rock where they traverse it.

II. *Stratified Rocks* consist of a series of parallel layers, evidently deposited from water, and originally horizontal, although in most cases they have become more or less inclined and curved by the action of disturbing forces. It is easier to divide them at the planes of division between those layers than elsewhere. They are traversed by veins or cracks, sometimes empty, sometimes containing crystals, sometimes filled with "dykes," or masses of unstratified rock. Those veins or dykes are often accompanied by a "fault," or abrupt alteration of the levels of the strata.

It is in the immediate neighbourhood of masses of unstratified rock that the stratified rocks show the greatest effects of the action of disturbing forces in the inclination, curvature, and distortion of their layers. In such positions, too, they often appear to have had their structure altered by heat and intense pressure, and to have been rendered harder and more compact.

Besides its principal layers or strata, a mass of stratified rock is in general capable of division into thinner layers; and although the surfaces of division of the thinner layers are often parallel to those of the strata, they are also often oblique, or even perpendicular to them. This constitutes a *laminated* structure. Laminated stones resist pressure more strongly in a direction perpendicular to their laminæ than parallel to them; they are more tenacious in a direction parallel to their laminæ than perpendicular to them; and they are more durable with the edges than with the sides of their laminæ exposed to the air; and, therefore, in building, they should be placed with their laminæ or "beds" perpendicular, or nearly so, to the direction of greatest pressure, and with the edges of these laminæ at the face of the wall.

In the more minute structure of stratified rocks the following varieties are distinguished:—

(1.) The *compact crystalline* structure, as in quartz rock and marble. This is accompanied by great strength and durability.

(2.) The *slaty* structure, when the rock, which is usually compact, can be split into innumerable thin layers, often highly inclined to the stratification. This structure is considered to have arisen from intense pressure, in a direction perpendicular to the layers. It facilitates quarrying. Some of the stones in which it occurs, as hard clay-slate and hornblende-slate, are amongst the strongest and most durable known. Others are soft and perishable.

(3.) The *granular crystalline* structure, in which crystalline grains either adhere firmly together, as in gneiss, or are cemented together into one mass by some other material, as in sandstone. This is accompanied by various degrees of compactness, porosity, strength, and durability, from the highest to the lowest, passing at the lowest extreme into sand.

(4.) The *compact granular* structure, where the grains are too small to be visible, and seem to form a continuous mass, as in blue limestone. This structure is usually accompanied with considerable strength and durability. It passes by gradations on the one hand into the compact crystalline structure (1), and in the other into,

(5.) The *porous granular* structure, in which the grains are not crystalline, and are often, if not always, minute shells cemented together, as in oolite. The porosity of rocks having this structure varies much; and so also do the strength and durability, which are seldom very high. In these respects the lowest example is soft chalk.

(6.) The *conglomerate* structure, where fragments of one material are imbedded in a mass of another, as in grauwacke.

The *fracture*, or appearance of the broken surface of a stone, is one of the means of showing its structural character. The following are examples:—

The *even* fracture, when the surfaces of division are planes in definite positions, is characteristic of a crystalline structure.

The *uneven* fracture, when the broken surface presents sharp projections, is characteristic of a granular structure.

The *slaty* fracture is even for planes of division parallel to the lamination, and uneven for other directions of division.

The *conchoidal* fracture presents smooth concave and convex surfaces, and is characteristic of a hard and compact structure.

The *earthy* fracture leaves a rough dull surface, and indicates softness and brittleness.

209. **Chemical Constituents of Stones.**—The numerous substances which have not yet been decomposed, and which are therefore provisionally called "elementary substances" in chemistry, are all found in the composition of stones. These elementary substances form, by their combinations, a vast variety of compounds called "simple minerals," or "mineral species." Each simple mineral is a definite chemical compound, and is a homogeneous substance; that

is to say, every particle of it perceptible to any means of observation is similarly composed to every other. Most simple minerals are distinguished also by definite primary forms of crystallization. Two minerals which have the same chemical composition may still be distinguished as distinct species, by having different primary crystalline forms. Thomson enumerates more than 500 mineral species; Jameson gives about 110 genera, each containing from one to 10 species.

The masses which form the earth's crust, whether stony or earthy, stratified or unstratified, are made up of simple minerals, either of one kind or of several kinds, *mixed*, not chemically combined.

There are a few simple minerals which are so much more abundant in the earth's crust than the others, that they fix the predominant characters, both chemical and mechanical, of the stones into whose composition they enter; and those minerals alone, with their principal chemical constituents, need be considered in such a treatise as the present.

The principal chemical constituents of those *predominant minerals* are four EARTHS, viz.:—

I. *Silica*, or pure flint. Its chemical composition is (according to the British scale),—

One equivalent of silicon, ...	7·5
One equivalent of oxygen, ...	8·0
One equivalent of silica, ...	15·5

Silica exists uncombined in great abundance, in the form of quartz, sand, and flint. With other earths and alkalies it combines, acting as an acid. It is not soluble in any acid except the fluoric, nor when crystallized is it soluble in water; but by an indirect process it can be made to form a gelatinous compound with water.

II. *Alumina*, the base of clay. Its chemical composition is

Two equivalents of aluminium,....................	27·4
Three equivalents of oxygen,......................	24·0
One equivalent of alumina,........................	51·4

Alumina exists uncombined in the ruby and sapphire alone. In combination with other earths, it exists in great abundance. It acts either as an acid or as a base. It forms a paste with water; and by indirect processes, can be made to form a gelatinous compound with water.

III. *Lime* is thus composed,—

One equivalent of calcium, ...	20·5
One equivalent of oxygen,..........................	8·0
One equivalent of lime,..............................	28·5

Lime does not exist in nature uncombined; but in combination with carbonic acid and with other earths it is very abundant. It is strongly alkaline, and soluble to a small extent in water.

IV. *Magnesia* is thus composed—

One equivalent of magnesium,......................	12·7
One equivalent of oxygen,..........................	8·0
	20·7

Magnesia is not found in nature uncombined; in combination with carbonic acid and with other earths it is abundant, though not so much so as the three earths before-mentioned. It is alkaline, but not so highly so as lime, and is very sparingly soluble in water.

In some of the predominant minerals the two following ALKALIES are found, combined with earths. Their presence in stone promotes its decomposition when exposed to the weather:—

Names.	Composition.	Equivalent.
V. *Potash*,............	Potassium 39·15 + oxygen 8	= 47·15
VI. *Soda*, ...	Sodium 23·3 + oxygen 8	= 31·3

The following ACID exists abundantly in combination with lime and magnesia.

VII. *Carbonic Acid*,... Carbon 6 + oxygen 16 = 22.

The presence of carbonic acid in stones is made known by their effervescing when acted upon by stronger acids.

The metals iron and manganese also enter into the composition of the predominant minerals, in quantities comparatively small. Their chemical equivalents are, Iron, 28; Manganese, 27·7.

210. The **Predominant Minerals in Stones** are the following:—

I. QUARTZ is pure silica. Its heaviness is from 2·5 to 2·7 times that of water. Its primary crystalline form is a rhombohedron. Its most common external crystalline form is a regular six-sided prism, with a six-sided pyramidal summit.

When it occurs in transparent crystals, colourless or coloured, it is called *rock-crystal*. In a compact, translucent mass, it is called *hornstone*. In dark-coloured, translucent lumps, which are scattered through the chalk, it is called *flint*. In grains, or small crystals, more or less rounded at the edges and corners, it forms *sand*. There are various other forms of quartz, which it is unnecessary to mention. It is the most hard and durable of all the predominant minerals.

II. **Felspar** is a mineral genus of compounds of earths and alkalies, of which the three species whose composition is given below are the most abundant, especially the first. Their heaviness is from 2·5 to 2·8 times that of water.

1. *Common Felspar*, or *Potash Felspar*, is composed of silica, alumina, and potash, in proportions which nearly agree with the following constitution:—

12 equivalents of silica;
1 equivalent of alumina;
1 equivalent of potash.

2. *Soda Felspar* has the whole or part of the potash replaced by an equivalent quantity of soda.

3. *Lime Felspar* has the whole or part of the potash replaced by an equivalent quantity of lime.

Felspar, with a crystalline or compact granular structure, forms the white or flesh-coloured grains and crystals which are seen in granite, porphyry, and some other rocks to be afterwards mentioned. With a slaty structure, it forms *clinkstone*. With a soft granular structure and earthy fracture, it forms *claystone*. It presents all degrees of hardness and durability.

III. **Hornblende** presents great varieties in appearance and composition. Its heaviness is from 2·7 to 3·2 times that of water. The composition of the white variety agrees nearly with the following constitution:—

9 equivalents of silica;
3 equivalents of magnesia;
1 equivalent of lime;

and there is also a small quantity of fluorine, which may be combined with part of the calcium. The most common varieties are the dark-green and the black, in which part of the silica appears to be replaced by alumina, in the proportion of one equivalent of alumina for three of silica, and part of the magnesia by an equivalent quantity of protoxide of iron.

Dark-green or black hornblende forms a great part of the mass of greenstone or trap. It occurs in crystals, fibres, and grains, and has a glassy lustre, and a fracture sometimes conchoidal, sometimes uneven, sometimes slaty. It is one of the toughest and most durable of minerals.

IV. **Augite** much resembles hornblende in all its properties. The composition of its white varieties agrees nearly with the following:—

4 equivalents of silica;
1 equivalent of magnesia;
1 equivalent of lime;

while in the green and black varieties, part of the magnesia appears to be replaced by an equivalent quantity of protoxide of iron.

V. MICA is distinguished by having a laminated structure, so that it either consists of or can easily be split into transparent or semi-transparent layers or scales. It is flexible, and so soft that it can be cut with a knife. Its heaviness is from 2·8 to 3 times that of water. The composition of one variety is nearly as follows:—

30 equivalents of silica;
4 equivalents of alumina;
3 equivalents of potash;
5 equivalents of oxides of iron and of manganese.

In other varieties part of the potash would seem to be replaced by lithia, and by an additional quantity of oxides of iron and of manganese. Some kinds contain fluorine.

VI. CHLORITE, or green earth, is a compound of the silicates ot magnesia, alumina, potash, and oxide of iron, with some water. It resembles mica in its laminated structure, and in its softness and flexibility. Its heaviness is from 2·7 to 2·8 times that of water. It occurs in small scales, in large sheets, and in slaty masses.

VII. CARBONATE OF LIME consists of one equivalent of carbonic acid and one of lime. It forms all the varieties of marble and limestone. These stones will be further described afterwards.

VIII. DOLOMITE is a compound of carbonate of lime and carbonate of magnesia, in the proportion of about two equivalents of the former to one of the latter. It forms various magnesian limestones, to be described further on.

211. **Stones Classed.**—The stones used in building are divided into three classes, each distinguished by the *earth* which forms its chief constituent. These are—

I. *Siliceous Stones.*
II. *Argillaceous Stones.*
III. *Calcareous Stones.*

212. **Siliceous Stones** are those in which silica is the characteristic earthy constituent. With a few exceptions their structure is *crystalline-granular*, and the crystalline grains contained in them are hard and durable; so that weakness and decay in them generally arise from the decomposition or disintegration of some softer and more perishable material, by which the grains are cemented together, or by the freezing of water in their pores, when they are porous.

The following are the principal siliceous stones used in building:—

I. GRANITE and SYENITE are unstratified rocks, consisting of quartz, felspar, mica, and hornblende. The name *granite* is specially applied to those specimens in which there is little or no hornblende;

the name *syenite* to those in which there is little or no mica; but both are popularly known as *granite*.

The quartz is in the form of clear, colourless or gray crystals; the hornblende (when present) in dark-green or black crystals; the mica in glistening scales, or grains composed of such scales; the felspar in compact opaque crystals, of a white, yellowish, or flesh colour.

Granite is found underlying the lowest or "primary" stratified rocks, and often rising through and over them in dykes, veins, and mountain masses, which naturally break up into large rhomboidal blocks, as stated in Article 208, p. 349.

The durability and hardness of granite are the greater the more quartz and hornblende predominate, and the less the quantity of felspar and mica, which are the more weak and perishable ingredients. Smallness and lustre in the crystals of felspar indicate durability; largeness and dullness, the reverse.

The best kinds of granite are the strongest and most lasting of building stones. The difficulty of working them, caused by their great hardness, is only overcome by long practice on the part of the stone-cutters. Minute ornaments cannot be carved in granite, and a simple and massive style of architecture is the best suited for it. It is used chiefly in works of great magnitude and importance, such as lighthouses, piers, breakwaters, and bridges over large rivers; and for such purposes it is brought from great distances at considerable cost, the stones being often cut to the required forms before leaving the quarry, with a view to save expense in carriage, and to obtain the benefit of the skill of stone-cutters accustomed to the material. It is only in districts where granite abounds that it is used for ordinary building purposes.

II. GNEISS and MICA SLATE consist of the same materials with granite, in a stratified form. They are found in the neighbourhood of granite, in strata much inclined, bent, and distorted, and often form great mountain masses. Gneiss resembles granite in its appearance and properties, but is less strong and durable. Mica slate is distinguished by containing little or no felspar, so that it consists chiefly of quartz and mica; it has a laminated or slaty structure, and the silky lustre of mica; it is a tough material, in directions parallel to its layers, but is more perishable than gneiss. Both these stones are used for ordinary masonry in the districts where they are found. Gneiss, from its stratified structure, is a good material for flag-stones. Mica slate, split into thin layers, may be used for covering roofs; but it is inferior for that purpose to clay slate.

III. GREENSTONE, WHINSTONE, or TRAP, and BASALT. These rocks are unstratified, and consist of granular crystals of hornblende or of augite, with felspar. In greenstone the grains are considerably finer than in granite; in basalt they are scarcely distinguishable. Greenstone breaks up into small blocks; basalt into regular prismatic columns. (Article 208, p. 349.) They are found in veins, dykes, and tabular masses, amongst stratified rocks of various ages. Greenstone is usually dark-green, rarely white or red; basalt nearly black. These varieties of colour are due to the hornblende or the augite, the felspar being white. Both these rocks are very compact, durable, hard, and tough. The smallness of the blocks in which they can be obtained, and the difficulty of working them, prevent their being used in large works of masonry; but they are well adapted for ordinary building, and especially well suited for paving and metalling roads.

IV. TALC, CHLORITE SLATE, SOAPSTONE. In these stones, silicate of magnesia predominates. *Talc* is in transparent or translucent sheets of a laminated structure; it is soft and easily cut. *Chlorite Slate* is also laminated, soft, and easily cut, but more opaque than talc; it is sometimes used for roofing, but is inferior to clay slate. It has a green or greenish-gray colour, and silky lustre.

Soapstone is translucent and soft, and greasy to the touch. It is valued for its power of resisting the action of fire.

V. QUARTZ ROCK, HORNSTONE, FLINT. These stones consist of quartz, pure, or nearly pure. *Quartz rock* and *Hornstone* are stratified, and appear to have been produced by the action of intense heat on standstone; they are both compact. Quartz rock is crystalline: hornstone is glassy. They are the strongest and most durable of all stones; but their hardness is so great as to make their use in masonry almost impracticable.

Flint is found in nodules or pebbles scattered through the chalk strata, and in beds of gravel, apparently left after the washing away of the chalk. It is hard and durable, but very brittle. Flints are used for building purposes by being made into a concrete with lime.

VI. HORNBLENDE SLATE is hard, tough, durable, and impervious to water, and is used for flag-stones.

VII. SANDSTONE is a stratified rock, consisting of grains of sand, that is, small crystals of quartz, cemented together by a material which is usually a compound of silica, alumina, and lime. In the strongest and most durable sandstone the cementing material is nearly pure silica; the weakest and least durable is that in which the cement contains much alumina, and resembles soft felspar or claystone. When there is much lime in the cementing matter of sandstone it decays rapidly in the atmosphere of the sea coast, and in that of towns where much coal is burned; in the former case the lime is dissolved by muriatic acid, in the latter by sulphuric acid. *Calciferous sandstones*, as those containing much lime are called, pass by insensible degrees into sandy limestones. The appearance of strong and durable sandstone is characterized by sharpness of the grains, smallness of the quantity of cementing material, and a clear, shining, and translucent appearance on a newly broken surface. Rounded grains, and a dull, mealy surface, characterize soft and perishable sandstone. The best sandstone lies in thick strata, from which it can be cut in blocks that show very faint traces of stratification; that which is easily split into thin layers is weaker. Sandstone is found in every geological formation above the primary rocks, amongst which its place is supplied by hornstone and quartz rock. The best kinds on the whole are those which belong to the coal formation; but they sometimes have their strength impaired by being divided into layers by extremely thin laminæ of coal.

The colours of sandstone are white, yellowish-red, and red, the latter colours being produced by the presence of peroxide of iron in the cementing material. Crystals of sulphuret of iron are sometimes imbedded in it; when exposed to air and moisture, they decompose, and cause disintegration of the stone. They are easily recognized by their yellow or yellowish-gray colour and metallic lustre. Sandstone is in general porous, and capable of absorbing much water; but it is comparatively little injured by moisture, unless when built with its layers set on edge, in which case the expansion of water in freezing between the layers makes them split or "scale" off from the face of the stone. When it is built "on its natural bed," any water which may penetrate between the edges of the layers has room readily to expand or escape.

The better kinds of sandstone are the most generally useful of building stones, being strong and lasting, and at the same time easily cut, sawn, and dressed in every way, and fit alike for every purpose of masonry.

213. **Argillaceous or Clayey Stones** are those in which alumina, although it may not always be the most abundant constituent, exists in sufficient quantity to give the stone its characteristic properties.

I. PORPHYRY consists of a mass of felspar, with crystals of felspar, and sometimes of quartz, hornblende, and other minerals, scattered through it. It occurs of all degrees of hardness. The variety in which the felspar matrix is soft and earthy is called *claystone porphyry;* it is of little or no value for building purposes. The hardest kind, in which the matrix is compact and crystalline, and the whole material beautifully coloured and capable of taking a high polish, is sometimes stronger than granite. It is rare, and is valued in building for ornamental purposes.

II. Clay Slate is a primary stratified rock of great hardness and density, with a laminated structure making in general a great angle with its planes of its stratification. (See Article 208, p.350.) Its colours are bluish-gray, blue, and purple, the darkest colours indicating in general the greatest strength and durability. It can be split into slabs and plates of small thickness and great area, and is nearly impervious to water; qualities which make it the best stony material for covering roofs, lining water-tanks, and similar purposes. The stronger kinds of clay slate have more tenacity along their laminæ than any other stone whose tenacity has been ascertained. The signs of good quality in slate are, compactness, smoothness, and uniformity of texture, clear dark colour, lustre, and the emission of a ringing sound when struck.

III. Grauwacke Slate is a laminated claystone, containing sand, and sometimes fragments of mica and other minerals. It is used for roofing and for flag-stones, but is inferior to clay slate.

214. **Calcareous Stones** are those in which carbonate of lime predominates. They effervesce with the dilute mineral acids, which combine with the lime, and set free carbonic acid gas. Sulphuric acid forms an insoluble compound with the lime. Nitric and muriatic acid form compounds with it, which are soluble in water. By the action of intense heat the carbonic acid is expelled in the gaseous form, and the lime left in its caustic or alkaline state, when it is called *quicklime*. Some calcareous stones consist of pure carbonate of lime; in others it is mixed with sand, clay, and oxide of iron, or combined with carbonate of magnesia. The durability of calcareous stones depends on their compactness: those which are porous being disintegrated by the freezing of water, and by the chemical action of an acid atmosphere. They are, for the most part, easily wrought.

I. Marble is compact crystalline carbonate of lime. It is found chiefly amongst the primary strata, and generally in the neighbourhood of igneous rocks. It is translucent, capable of a fine polish, sometimes white, and sometimes variously coloured. It is one of the most durable of all stones. Its scarcity and value prevent its being used except for ornamental buildings.

II. Compact Limestone consists of carbonate of lime, either pure, or mixed with sand and clay. It varies in hardness and compactness, sometimes approaching to the condition of marble, sometimes to that of granular limestone. Its most frequent colours are white, grayish-blue, and whitish-brown. It is found amongst primary and secondary strata, and abounds specially in the coal formation, and in the lias formation. It is very useful as a building stone, and is durable in proportion to its compactness.

III. GRANULAR LIMESTONE consists of carbonate of lime in grains, which are in general shells or fragments of shells, cemented together by some compound of lime, silica and alumina, and often mixed with a greater or less quantity of sand. It is always more or less porous, and the less porous the more durable. It is found of various colours, especially white, and light yellowish-brown. In many cases it is so soft when first quarried that it can be cut with a knife, and hardens by exposure to the air. It is found in various strata, especially the oolitic formation. It there appears in the form of *Oolite*, or *Roestone*, so called because its grains are round, and resemble the roe of a fish. The pleasing colour and texture of oolite, and the ease with which it is wrought, have caused it to be much used in building, especially where delicate carving is required. The durability of oolites varies extremely. The Portland stone, the Bath stone, and the Aubigny stone (from Normandy) are examples of durable oolites. The perishable kinds of oolite decay more rapidly than almost any other stone, especially in an acid atmosphere.

IV. MAGNESIAN LIMESTONE, or DOLOMITE, is found in various conditions, from the compact crystalline to the porous granular. In Britain it is found in the new red sandstone formation immediately above the coal. It is like limestone in appearance. Its durability depends mainly on its texture; when that is compact it is nearly as lasting as marble, which it resembles in appearance; when porous it is very perishable.

215. **Strength of Stones.**—The external appearances from which the probable comparative strength or weakness of stones may be inferred have been stated in the course of the preceding articles.

Amongst stones of the same kind, that which has the greatest heaviness is almost invariably the strongest.

The results of past experiments on the strength of stones of the same kind differ very much from each other, probably owing to variations in the strength of the specimens of stone experimented on. The results given in the tables of the strength of materials at the end of the volume are averages from discordant data.

The following table contains some additional information on the resistance of stones to crushing, extracted from a paper which was read by Mr. Fairbairn to the Manchester Philosophical Society, and published in his *Useful Information for Engineers*, second series:—

	Crushing Stress, in lbs. on the Square Inch.
Grauwacke from Penmaenmaur,	16,893
Basalt, Whinstone,	11,970
Granite (Mount Sorrel),	12,861
„ (Argyllshire),	10,917
Syenite, (Mount Sorrel),	11,820
Sandstone (Strong Yorkshire, mean of 9 experiments),	9,824
„ (weak specimens, locality not stated),	3,000 to 3,500
Limestone, compact (strong),	8,528
„ magnesian (strong),	7,098
„ „ (weak),	3,050

Mr. Fairbairn's experiments further show, that the resistance of strong sandstone to crushing in a direction parallel to the layers, is only *six-sevenths* of the resistance to crushing in a direction perpendicular to the layers.

The hardest stones alone give way to crushing at once, without previous warning. All others begin to crack or split under a load less than that which finally crushes them, in a proportion which ranges from a fraction little less than unity in the harder stones, down to about *one-half* in the softest.

The mode in which stone gives way to a crushing load is in general by *shearing*. (Article 157, pp. 235, 236.)

Experiments on the strength of stones have hitherto been made almost universally on cubical specimens. It is desirable that they should be made on prismatic specimens, whose heights are at least once and a-half their diameters; for an experiment made by crushing a cube indicates somewhat more than the real strength of the material.

When any building of importance is projected, the best course is not to trust to books for information as to the strength of the stone to be used, but to test it by special experiments, which can easily be made by the aid of a hydraulic press. As to the method to be followed in making those experiments, and calculating their results, in order to insure accuracy, see Article 144, pp. 223, 224.

The factor of safety in structures of stone should not be less than *eight*, in order to provide for variations in the strength of the material, as well as for other contingencies. In some structures which have stood it is less; but there can be no doubt that these err on the side of boldness.

216. **Testing Durability of Stones.**—The appearances which indicate probable durability have already been mentioned in describing particular kinds of stone; but they are often deceptive.

One test of the probable comparative durability of *stones of the same kind* is the smallness of the weight of water which a given weight of stone is capable of absorbing.

The following are examples:—

Granite absorbs one part of water in from 80 to 700 of stone.
Gneiss, „ „ „ „ about 40 „
Clay Slate, „ „ „ „ 80 to 700 „
Sandstone (Strong, Yorkshire), „ 30 to 60 „

Another test of probable comparative durability (invented by M. Brard) is to imitate the disintegrating action of frost by means of the crystallization of sulphate of soda, and weigh the fragments so detached from a block of a given size and surface in a given time. (*Annales de Chimie et de Physique*, vol. 38.)

The only sure test, however, of the durability of any kind of stone, is experience; and the engineer who proposes to use stone from a particular stratum in a particular locality in any important structure should carefully examine buildings in which that stone has been already used, especially those of old date.

The great difference which may exist in the durability of stones of the same kind, and presenting little difference in appearance, is strikingly exemplified at Oxford, where Christ Church Cathedral, built in the twelfth or thirteenth century, of oolite from a quarry about fifteen miles away, is in good preservation, while many colleges only two or three centuries old, built also of oolite, from a quarry in the neighbourhood of Oxford, are rapidly crumbling to pieces.

217. **Preservation of Stone.**—The various processes which have been tried or proposed for the preservation of naturally perishable stone all consist in filling the pores of the stone at and near its exposed surface with some substance which shall exclude air and moisture. In every case the surface of the stone should be prepared to receive the preserving material by expelling the existing moisture as completely as possible; and this is easily done by the aid of a portable furnace containing burning coke or charcoal.

The principal preserving materials are the following:—

Bituminous matter, such as coal tar, is very efficient; but unsightly from its colour. It is possible, however, that a colourless or light-coloured bituminous substance, suited for the preservation of stone, might be prepared by dissolving "paraffine" (so-called) in pitch-oil, or by some such process.

Drying Oil, such as linseed oil, either unmixed, or as an ingredient of paint, protects the stone for a time; but it is gradually destroyed by the oxygen of the air, so that it requires renewal from time to time; and it injures the appearance of the stone.

Silicate of Potash, or soluble glass, is applied in a state of solution in water, either alone or mixed with silica in fine powder. It gradually hardens, partly through the evaporation of its water, and partly through the removal of the potash by the carbonic acid of the air.

Silicate of Lime is produced by filling the pores of the stone with a solution of silicate of potash, and then introducing a solution of chloride of calcium, or of nitrate of lime. The chemical action of the two solutions produces silicate of lime, which forms an artificial stone, filling the pores of the natural stone, together with chloride of potassium or nitrate of potash, as the case may be, which salts, being soluble in water, are washed out.

The efficiency of the last two processes, and of various modifications of them, has of late been much contested. Time and experience only can show their real merits.

218. **Expansion of Stone by Heat.**—The following are the expansions in linear dimensions, according to the experiments of Mr. Adie, of some kinds of stone, when raised from the temperature of melting ice (32° Fahr.) to that of water boiling under the mean atmospheric pressure (212° Fahr.); that is, through 180° Fahr :—

Granite,	·0008 to ·0009
Marble,	·00065 to ·0011
Sandstone,	·0009 to ·0012
Slate,	·00104.

SECTION II.—*Of Bricks, and other Artificial Stones.*

219. **Clay for Bricks.**—The various sorts of clay, which are very numerous, are chemical compounds consisting of silicates of alumina, either alone, or combined with silicates of potash, soda, lime, magnesia, iron, and manganese. The complex clays approximate in their composition to felspar; and many of them may in fact be considered as soft varieties of felspar. (Article 210, p. 354.)

Clay and sand mechanically mixed constitute *loam;* clay and carbonate of lime mechanically mixed, *marl.* Amongst other substances which are found mixed with clay are peroxide of iron, sulphuret of iron, bitumen, &c.

Every kind of clay has the property, in its natural condition, of swelling and forming a paste when mixed with water. The expulsion of the water by heat is a slow process, and requires a high temperature, and is accompanied by shrinking and hardening of the mass of clay. It is doubtful at what temperature the expulsion of the water is complete; for so far as experiment has yet been carried, it appears that how high soever the temperature at which a mass of clay has been "burnt," as it is called, it will continue to shrink and to lose weight if raised to a higher temperature. A mass of burnt clay, at temperatures lower than that at which it has been burnt, expands with heat and contracts with cold like other solid substances.

By the operation of "burning," at a sufficiently high temperature, clay becomes hard and gritty, and loses either wholly or almost wholly the property of combining with water. Whether the clay afterwards slowly softens, and recovers that property or not, depends on its composition, and on the chemical agents to which it is exposed. The presence of alkaline constituents in the clay, and the action of acids upon it, tend to promote softening; and this goes on the more rapidly if it has been burned at too low a temperature.

Single earthy silicates, or compounds of silica with one other earth, are difficult of fusion, and resist the most intense heat of a furnace. This has been already exemplified in tersilicate of magnesia, or soapstone. (Article 212, p. 357.) Double, or more complex silicates, are more easily fusible, especially if one of the two or more silicates that are combined has for its base potash, soda, or lime. In conformity with this general law, the *refractory clays*, or those which resist fusion by the greatest heat of an ordinary furnace, are those which consist of silicates of alumina alone; and such clays only are fit to make fire-bricks and crucibles, and to cement together the parts of furnaces.

The following are examples:—

Porcelain Clay, or *Kaolin*, consists of

1 equivalent of alumina,
3 equivalents of silica,

which compound, in the natural state, is combined with two equivalents of water, nearly all of which can be expelled by a white heat. It is found in the neighbourhood of granitic rocks, having been formed by the slow decomposition of potash-felspar, under the action of the carbonic acid and moisture of the atmosphere, which have abstracted the potash and nine equivalents of silica. Its colour is white or cream-colour.

Stourbridge Fire Clay consists of

1 equivalent of alumina,
6 equivalents of silica,

with two equivalents of water, or thereabouts, which can be nearly all expelled by a white heat, and a small quantity of oxide of iron.

This and other fire-clays are found chiefly in the coal formation. Their colours are white, light-gray, and yellowish-gray, the colouring matter being in general a small quantity of oxide of iron.

Common Clays are rendered less difficult to fuse than porcelain clay and fire-clay, by the presence of silicates of lime, magnesia, and protoxide of iron; and the bricks made of them, when thoroughly burned, are partially vitrified. Of these constituents, protoxide of iron is the most favourable to the quality of the clay as regards the purpose of brickmaking, as it promotes the strength and hardness of the bricks. Its presence is shown by a dark greyish-blue colour, which is changed to red at and near the surface of the bricks by burning.

Silicate of lime in the clay in any considerable quantity makes it too fusible, so that the bricks soften in the kilns and become distorted.

Carbonate of lime, mixed with the clay in considerable quantity (indicated by effervescence with acids), loses its carbonic acid during the burning; and the quicklime which remains tends afterwards to absorb moisture, and cause disintegration of the brick. Clay containing this impurity should be avoided in making bricks.

Sand mixed with the clay in moderate quantity is beneficial, as tending to prevent excessive shrinking in the fire. Excess of sand makes the bricks too brittle. One part by volume of sand to four or five of pure clay is about the best proportion.

220. **Manufacture and Qualities of Bricks.**—In making bricks, the clay having been cleared of stones, is "tempered;" that is, mixed with about half its volume of water, and worked by stirring and kneading until it forms a perfectly uniform and homogeneous paste. The quality of the bricks depends mainly on the efficiency with which this is done. It may be performed by a machine called a "*pug-mill*," in which the clay, contained in a vertical cylinder or barrel, is stirred and mixed by flat arms projecting from a rotating vertical axis, and at the same time forced downwards by the obliquity of the surfaces of those arms, so as to be made to stream slowly from a hole near the lower end of the barrel.

The wet clay, having been properly tempered and worked, is formed into bricks in moulds, which are larger than the bricks are intended to be when burned, by about 1-10th or 1-12th of each dimension, that being the ordinary proportion in which the dimensions of the brick shrink in burning.

Ordinary moulds for bricks measure about 10 inches in length, 5 inches in breadth, and 3 inches or thereabouts in depth; but bricks for special purposes are moulded of a great variety of shapes. Various machines have been invented for moulding them.

The bricks, having been dried in the open air, or in a drying house with a temperature of from 50° to 70°, are burned in kilns whose temperature is gradually raised in the course of twenty-four hours to a white heat, maintained nearly at that temperature until the bricks are sufficiently burnt, and then allowed to cool by slow degrees. The duration of the process of burning and cooling varies; but fifteen days, or thereabouts, is not unusual.

From data contained in a paper by Mr. F. W. Simms, it appears that the labour of making bricks by hand is as follows:—

$$\text{Three men, viz.,....}\left\{\begin{array}{l}\text{one temperer,}\\ \text{one moulder,}\\ \text{one wheeler,}\end{array}\right.$$

$$\text{and two boys, viz.,}\left\{\begin{array}{l}\text{one carrier boy,}\\ \text{one picker-up boy,}\end{array}\right.$$

make 16,100 bricks per week. This is at the rate of

$$\left.\begin{array}{l}\text{1·12 days' work of a man, and}\\ \text{0·75 days' work of a boy,}\end{array}\right\}\text{per 1,000 bricks.}$$

The fuel consumed in burning bricks ranges from 5 to 10 cwt. per 1,000 bricks.

The following are characteristics of good bricks.

To be regular in shape, with plane parallel surfaces and sharp right-angled edges.

To give a clear ringing sound when struck.

When broken, to show a compact uniform structure, hard and somewhat glassy, and free from air-bubbles and cracks.

Not to absorb more than about one-fifteenth of their weight of water.

Bricks which answer the preceding description, when *set on end* in a hydraulic press, should require at least 1,100 lbs. on the square inch to crush them, agreeably to the strength of "strong red bricks," as stated in the table at the end of the volume; and they will sometimes bear considerably more. A *small pillar of brick-work,* made of bricks of this quality laid in cement, should require from 800 to 1,000 lbs. on the square inch to crush it.

Bricks in general begin to show signs of giving way, by splintering and cracking, when under about one-half or two-thirds of their crushing load.

The weaker qualities of bricks may be estimated as having from one-half to two-thirds of the strength stated above.

The bricks supplied for every building of importance should be carefully inspected, and the defective ones thrown away.

The expansion of bricks by heat, in rising from 32° to 212° Fahr. is as follows, according to Mr. Adie:—

Common brick,	·00355
Fire brick,	·0005.

221. **Compressed Bricks** are made by drying the clay, grinding it to a fine powder, putting it into moulds of proper shapes, subjecting it to a pressure of about 5 tons on the square inch, and baking the bricks in a pottery-oven. The bricks so made have about once and a-half the heaviness of ordinary bricks, and considerably greater strength. They shrink very little in baking.

222. **Other Artificial Stones.**—Artificial sandstone, closely resembling natural sandstone in appearance, strength, and durability, is made by cementing clean sharp sand together with silicate of potash, or silicate of lime.

A sort of artificial limestone has been made by compressing the chalky mud of the Thames in iron moulds by the blows of a steam-hammer.

The composition and use of *Concrete* will be explained in the next section.

Section III.—*Of Cementing Materials.*

223. **Analysis of Limestones and Cement Stones.**—Stones containing carbonate of lime in combination and mixture with other minerals are the most abundant and useful source of the cementing materials used in masonry. The following are their principal constituents, with their chemical equivalents:—

Carbonic Acid (see p. 353),	22	
Lime (see p. 352),	28·5	
Carbonate of Lime, 22 + 28·5 =		50·5
Magnesia (see p. 353),	20·7	
Carbonate of Magnesia, 22 + 20·7 =		42·7
Silica (see p. 352),	15·5	
Alumina (see p. 352),	51·4	
Protoxide of iron (see p. 353),	36	
Peroxide of iron (iron 56 + oxygen 24),	80·	
Water (hydrogen 1 + oxygen 8),	9·	

It would be out of place in this work to enter into details of chemical processes; nevertheless, it may be useful to give the following directions for determining roughly the proportions of these constituents of limestone which are of the greatest practical importance.

I. Weigh a specimen carefully; calcine it in a crucible, and weigh it again; the loss of weight shows the quantity of *carbonic acid* and *water* together in the specimen; but if it has been well dried previously, at a temperature not sufficient to expel carbonic acid (which requirès a bright red heat), the water remaining may be neglected, and the whole loss considered as *carbonic acid*.

II. Weigh another specimen of from 30 to 80 grains; reduce it to impalpable powder in a mortar, mix it with three times its weight of caustic potash or soda, and heat it to redness in a silver crucible; dissolve the whole in slightly diluted muriatic acid; the rapidity of solution may be increased by heating the diluted acid to near the boiling point of water. Evaporate the solution, taking care to stir it continually towards the end of the process, until it becomes thick and pasty: this shows that the silica has coagulated; mix the paste with eight or ten times its volume of boiling water:—this will dissolve every constituent except the silica:—filter the solution, washing the precipitate well with water, taking care to preserve all the water so used along with the original liquor; dry and calcine the precipitate left on the filter; weigh it:—this will give the quantity of *silica* in the specimen.

III. To the liquor add water of ammonia in excess; to precipitate the *alumina*, the *oxide of iron*, and *part of the magnesia*.

Then add lime-water by degrees as long as a precipitate falls. That precipitate is the *remainder of the magnesia*. Wash the whole precipitate; dry it; calcine it; weigh it. To the weight thus found add the weight of the silica found by operation II, and that of the carbonic acid as calculated from the result of operation I; subtract the sum from the whole weight of the specimen; the remainder will be the *lime*.

IV. From the total carbonic acid found by process I, (and reduced to the weight of the second specimen) subtract the weight of lime found by process III. $\times 22 \div 28{\cdot}5$; the remainder will be the quantity of carbonic acid in combination with magnesia; and that remainder $\times 20{\cdot}7 \div 22$ will give the quantity of *magnesia in combination with carbonic acid*.

V. Subtract the result of process IV. from that of process III.; the result will be the quantity of *alumina and oxide of iron*, and of *magnesia combined with silica*, if any; but in fact, so far as limestones are known, the whole of the magnesia is in the state of carbonate. For the present purpose it is unnecessary to separate the alumina from the oxide of iron (although it might be done by means of caustic potash, which dissolves the alumina and leaves the iron).

The most important result of the analysis is the proportion of *carbonates* to *silicates* in the stone. The quantity of carbonates may be approximated to in a rough way by multiplying the total quantity of carbonic acid, as found by the first process, by the following multipliers:—

If the limestone is not magnesian,................ 2·3;
if there is one equivalent of carbonate of magnesia for each equivalent of carbonate of lime, 2·12;

and the truth will almost always be between those limits. The remainder of the stone may be held to consist wholly or almost wholly of silicates.

The substances obtained by calcining different limestones and cement stones may be divided into the following four classes:—

I. *Pure, Rich,* or *Fat Lime,* produced from stones containing little or no silicate, which "slakes" by absorbing moisture, and having been made into a paste with water, hardens slowly in air, and not at all under water.

II. *Hydraulic Limes,* produced from stones containing moderate quantities of silicates (from 10 to 30 per cent.), which slake, but less rapidly than pure lime, and harden under water slowly. These pass by insensible gradations into

III. *Cements,* produced from stones containing from 40 to 60 per cent. of silicates, which do not slake, and which harden quickly under water.

IV. *Puzzolanas,* which contain silicates in excess, and are used to make cement by mixing them with pure lime.

224. **Pure, Rich,** or **Fat Lime** is made by calcining, at a bright red heat or somewhat higher, limestone that consists wholly or almost wholly of carbonate of lime, such as marble, or chalk. Such limestone loses 44 per cent. of its weight by burning, and leaves 56 per cent. of its weight of lime. Of this, about one-eighth is usually wasted.

The operation of lime-burning is performed in kilns of two sorts. In the more common kind, the kiln is circular in plan, and oval in vertical section, the diameter at the bottom being about 6-10ths of the greatest diameter; it seldom exceeds 10 or 12 feet in height, but may be less; it is filled with alternate layers of limestone and fuel, and when the burning is completed the whole charge is removed. The whole operation takes from 30 to 50 hours. Another sort of kiln is cylindrical, or nearly so, with its axis vertical. It is continuously fed with limestone at the top, which descends, and is calcined by the flame coming from a furnace at one side of the kiln, and reaches the bottom completely burnt, whence it is gradually removed.

The weight of the coal consumed is from 1-5th to 1-6th of that of the lime burned.

One cubic foot of chalk in block, weighs 90 lbs. nearly.
" of broken chalk,............ 63 "
" of the quicklime from the same chalk, in pieces, 35 "

When rich quicklime is moistened with water, it *slakes;* that is to say, it combines chemically with one equivalent of water (9 parts by weight) to one equivalent of lime (28·5 parts by weight), and forms *slaked lime*, or in chemical language, *hydrate of lime.* During this process, the lime swells to from twice-and-a-half to thrice-and-a-half its original bulk, becomes very hot, and falls to powder. The same process takes place slowly through absorption of moisture from the atmosphere; but lime for building purposes ought never to be "air-slaked," as this slow operation is called; for the lime thus exposed to the air absorbs not only water, but carbonic acid; and part of it returns to the state of carbonate of lime. To guard against this sort of deterioration, quicklime should be kept in barrels, or in a dry store, until it is required for use, and then rapidly slaked with water. The hardening of slaked lime is produced by gradual absorption of carbonic acid from the atmosphere, and crystallization of the carbonate of lime so formed. It is a very slow process, but produces, after the lapse of years, a very hard material.

225. **Hydraulic Limes** are obtained by burning limestones, which contain silicates of alumina, and sometimes carbonate of magnesia, and which in general are compact, and of a gray, blue, or brownish-yellow colour. Besides the test of chemical analysis already mentioned in Article 223, the following direct test may be applied to limestones supposed to be hydraulic.

Calcine two or three cubic inches of the stone in a crucible:—pound the calcined lime: make it into a stiff paste with water, and form it into a ball, which immerse in a glass of water. If it is *hydraulic lime*, it will harden under water so as to resist the pressure of the finger in a time varying from 24 hours to a fortnight, according to its composition; and if its quality is good, in a month it will be about as hard as weak limestone.

If it is *cement*, it will harden so as to resist the pressure of the finger in a few minutes.

The best kinds of hydraulic lime slake so imperfectly, that they must be pulverized by grinding them in the dry state in a mill consisting of a circular trough, in which two stone rollers shaped like millstones, at opposite ends of a bar rotating with a vertical shaft, roll round, and so crush and grind the lime to a fine powder. Hydraulic lime should be kept in sacks or barrels in a dry store, and exposed as little as possible to air and moisture until it is about to be used.

It is a mixture of quicklime with silicates of alumina and iron, and sometimes with magnesia. Its hardening under water arises from the formation of an artificial stone, consisting of compound silicates of lime, alumina, and the other bases. In hydraulic lime, as distinguished from cement, there would seem to be a greater or less surplus of lime beyond that which is capable of combining with the silica and alumina.

226. **Natural Cement** is obtained by burning stones in which carbonate of lime and silicates exist in such proportions that, when the carbonic acid is expelled, the lime is exactly in the proportion required to make a hard compound with the silica and alumina. From the experiments of M. Vicat and of General Sir Charles Pasley, on making artificial cements, it would appear that the best mixture for making cement consists, before burning, of

two equivalents of carbonate of lime,...	50·5 × 2 =	101·0
one equivalent of clay, of which the probable composition is		
one equivalent of alumina, ...	51·4	
six equivalents of silica,............	93·0	144·4
		245·4

so that the composition in one hundred parts is,

carbonate of lime,....................	41
clay,..................................	59
	100

and the rapidity with which the cement hardens under water depends on the nearness with which the composition of the stone approximates to these proportions.

Cement stones are usually found in thin strata amongst those of hydraulic limestone. Their most frequent colours are brown and fawn-coloured, their texture compact, and fracture earthy. After having been burned, they are ground to powder, which is packed in barrels, and carefully kept dry till required for use. In this state, it consists of a *mixture* of quicklime with silicate of alumina. So soon as it is made into a paste with water, chemical action takes place, and a double silicate of alumina and lime is formed, whose composition, in the best cement, would seem to be,

two equivalents of lime	28·5 × 2 =	57·0
one equivalent of alumina,..........		51·4
six equivalents of silica,............	15·5 × 6 =	93·0
		201·4

and this double silicate forms a compact artificial stone.

227. **Artificial Cement** is made by taking either ground chalk or slaked pure lime, and blue clay, in the proportions that will give the chemical composition stated in the preceding article, thoroughly mixing those ingredients into a paste with water in a pug-mill, making the paste into balls of 2 or 3 inches in diameter, drying those balls, calcining them, and grinding them to powder. It is equal, if not superior, to any natural cement.

228. **Puzzolanas** are mixtures analogous to cements, but containing an excess of silicates and a deficiency of lime, so that they must be mixed with pure lime in order to make cement, or hydraulic lime, according to the proportions.

One of the best puzzolanas is "Mine-dust," which is probably a silicate of protoxide of iron. If mixed with lime, so as to give the mortar a bluish-gray colour, it produces a cement of extraordinary hardness and tenacity, which is probably a double silicate of lime and protoxide of iron.

An ordinary proportion of mine-dust is about one part (by volume) to two parts of hydraulic lime, measured in the dry state; but the best way to fix the proportions is by trial.

Artificial puzzolana may be made by grinding bricks, or by burning good brick-clay and grinding it; in short, by any process which yields a dry powder of silicate of alumina, or compound silicate of alumina and iron.

229. **Mortar, Common and Hydraulic.**—Mortar is made by mixing lime and sand with enough of water to form them into a semifluid paste, in which state it is used as a binding material in masonry and brickwork.

Common Mortar, being made with pure lime, hardens in the air by the evaporation of the water, and by the absorption of carbonate of lime from the atmosphere, which crystallizes in the course of years. If the water evaporates too fast, the mortar falls to powder; if it does not evaporate, the mortar remains always soft. Very slow evaporation of the water is therefore favourable to the ultimate hardness of the mortar.

Hydraulic Mortar, being made with hydraulic lime, hardens partly by the formation and crystallization of carbonate of lime, as above stated, but principally by the formation and crystallization of a complex silicate of lime, alumina, and other bases. (See Article 225, p. 371.)

Common mortar may be made hydraulic by a mixture of puzzolana. (Article 228, above.)

The sand employed should be clean, sharp, and rather coarse than fine. In order to render sand which is naturally mixed with clay fit for use in making mortar, it should be washed by stirring it amongst water, a slight current of which will carry away the clay in suspension, and leave the sand. Good sand for mortar is obtained by crushing soft sandstone. Sea-sand should be washed with fresh water; otherwise the salts contained in it will keep the mortar always moist.

In hydraulic as well as in common mortar the sand remains in a state merely of mechanical mixture, so that the mortar, when hardened, becomes a sort of artificial sandstone, consisting of grains of sand imbedded in a matrix of carbonate of lime, or of silicate of lime and other bases, as the case may be. The uses of the mixture of sand with the lime are as follows:—

To save expense, by diminishing the bulk of the lime, which is the more costly material, required to fill a given joint in the masonry.

To increase the resistance of the mortar to crushing.

To lessen the amount of shrinking, and the consequent tendency to crack, during the drying of the mortar.

But, at the same time, the mixture of sand diminishes the tenacity of the mortar; and if too much be used, the mortar will become brittle, and fall to powder as it dries.

The proportion of sand which lime will "bear," as it is called, without making the mortar brittle, is the greater the purer the lime, and the less the more strongly hydraulic the lime is. The best proportions, according to Vicat, are—

2·4 measures of sand to 1 of pure slaked lime in paste;
1·8 measures of sand to 1 of good hydraulic lime in paste;

and lime of intermediate qualities bears intermediate proportions of sand.

When sand and puzzolana are mixed with pure lime to make hydraulic mortar, the sand and puzzolana together may measure from double to 2½ times the volume of the lime before slaking.

In mixing mortar, however, the best method is, to ascertain the proper proportions in each case by trial.

The *labour of mixing mortar by the shovel* may be estimated at about

¾ of a day's work of a man per cubic yard.

A *two horse pug-mill* mixes mortar at the rate of from 20 to 25 cubic yards per day.

As hydraulic mortar tends to *set*, or harden, even in the wet state, it should not be mixed until immediately before it is required for use.

230. **Concrete and Beton.**—Common concrete is a mixture of mortar with gravel, in proportions such that the gravel and sand together are about six times the volume of the lime. It may be mixed either by hand or by the pug-mill.

Strong Hydraulic Concrete, to which the name *beton*, borrowed from the French, has been applied, is made by mixing angular fragments of stone of from $1\frac{1}{4}$ to 2 inches in diameter, with hydraulic mortar in such proportions, that the mortar is a little more than enough to fill the spaces between the stones:—a proportion which is easily found by trial in each case. It may, however, be estimated as varying from

1 volume of stones and 1 volume of mortar, to
2 volumes of stones and 1 volume of mortar.

Concrete and beton, when mixed, occupy at first from two-thirds to three-fourths of the total volume of their materials before mixing; and when laid and rammed, they undergo a further settlement of about one-sixth; so that the final volume of concrete and beton varies from

$\frac{5}{9}$ to $\frac{5}{8}$ of the volume of the materials when unmixed.

When rounded stones only can be obtained, such as flints from the chalk strata, they may be made fit for the composition of beton by breaking them with the hammer into angular pieces.

231. **Mixed Cement.**—Cement which is to dry and set when fully exposed to the air, as at the outer edges of joints, or on the face of a wall, should be mixed with sand, to prevent unequal drying, and consequent shrinking and cracking. The proportions vary from

1 measure of sand and 2 of cement, to
1 measure of sand and 1 of cement.

Every mixture of sand diminishes the tenacity of cement; so much so that a mixture of equal parts of sand and cement has only one-fourth part of the tenacity of pure cement. Where the surface of the cement, therefore, is not exposed to the air, the only advantage of a mixture of sand is the saving of expense; and where great tenacity is required, pure cement should be used.

232. **Strength of Mortar, Cement, Concrete, and Beton.**—To the data given in the tables in the appendix, the following results of experiment may be added:—

A YEAR AND A-HALF AFTER MIXTURE.	CRUSHING FORCE in lbs. on the Square Inch.
Mortar of Lime and River-Sand, ...	440
" " " beaten,.........	600
Mortar of Lime and Pit-Sand, ...	580
" " " beaten, ...	800
Hydraulic Mortar, of lime and pounded tiles,......	680
" " " beaten,	930
Beton, or concrete, of mortar and broken flints,	420

SIXTEEN YEARS AFTER MIXTURE, the increase of strength is in the following proportions:—

For common mortar,	1-8th
For hydraulic mortar,	1-4th.

(The above results are given on the authority of Rondelet.)

ONE YEAR AFTER MIXTURE.		Tenacity in lbs. on the Square Inch.
Good hydraulic lime,		170
Ordinary hydraulic lime	from	140
	to	100
Rich lime,		40
Good hydraulic mortar,		140
Ordinary hydraulic mortar,		85
Good common mortar,		50
Bad common mortar,		20

(The above are from Vicat.)

SIX MONTHS AFTER MIXTURE.	
Adhesion of common mortar to compact limestone,	15
Adhesion of common mortar to brick,	33

(The above are from Rondelet.)

Cement from Chalk Lime and Blue Clay, a few days after mixture (Sir C. W. Pasley),	125
Portland Cement (from compact limestone and clay) 30 to 50 days after mixture,	1200 to 1550.

233. **Gypsum—Plaster of Paris.**—Gypsum is a compound of sulphate of lime with water, in the following proportions:—

one equivalent of sulphuric acid (sulphur 16 + oxygen 24) =	40·0
one equivalent of lime,	28·5
two equivalents of water,	18·0
	86·5

It is found stratified, and in various conditions, crystalline, laminated, granular, and earthy. It is translucent, usually white or gray, has a pearly lustre, and can be easily scratched with a knife, being intermediate in hardness between rock-salt and calcareous spar.

By calcining gypsum the water is expelled, and it becomes a dry white powder of sulphate of lime, known as "*Plaster of Paris.*" When this powder is rapidly mixed with water, so as to form a paste, it immediately begins to combine with part of the water, so as to reproduce gypsum in a compact granular state; heat is at the same time developed, which hastens the evaporation of the superfluous water. The mixture should be made by putting the powder into the water, not the water amongst the powder. The proportion of water used varies according to the purpose to which the plaster is to be applied; on an average it is about equal in bulk to the powder.

The tenacity of plaster, after it has "set," or hardened, according to Rondelet, is about 70 lbs. per square inch.

234. **Bituminous Cement and Concrete.**—A bituminous cement is a mixture of a pitchy or bituminous substance with an earthy substance.

For example, *Asphaltic Mastic* is made by mixing *Bitumen*, or mineral tar, obtained from bituminous shale or sandstone, with the powder of bituminous limestone or *asphalt*:—a mineral which consists of carbonate of lime, containing in its pores from 3 to 15 per cent. of bitumen.

The asphalt may either be broken into small fragments or ground to powder. It is then combined with the bitumen by heating the latter in an iron boiler, and adding the asphalt by degrees, taking care to mix the ingredients well. The proportions vary with the composition of the asphalt, less bitumen being required for that asphalt which contains much bitumen. The average proportion may be estimated at about 1 part by measure of bitumen to 7 or 8 of asphalt.

Artificial asphaltic mastic may be made by substituting coal-tar, or a solution of pitch in pitch-oil, for bitumen, and adding to it finely ground limestone till a proper consistency is attained.

A mastic composed of coal-tar and finely ground fireclay, in proportions which have never been exactly determined, but which are adjusted by trial until the mixture when cool is just soft enough to yield visibly to the pressure of the nail, and no more, has been found exceedingly useful to make tight joints in pipes, especially those traversed by strong acids, which would act upon a mastic containing limestone.

A bituminous or asphaltic mortar, as it may be called, is made by adding to the before-mentioned mixture of bitumen and powdered asphalt, about a thirty-fifth of its bulk of resin oil, and three-fifths of its bulk of sand.

11 Measures of this mixture, with 9 of broken stone, make a sort of *bituminous concrete*, suited for covering the surfaces of roads. Its use will be further described under the head of ROADS.

SECTION IV.—*Of Ordinary Foundations.*

235. **Ordinary Foundations Defined and Classed.**—The *foundation* of a work of masonry on land consists, in the first place, of an excavation in the ground, and secondly, if required, of a structure at the bottom of that excavation, suited to form a firm base for the masonry. The foundations to which this section relates are those in which either an excavation alone is required, or an excavation partially filled with sand, stones, concrete, or beton. Foundations of a more difficult character, and requiring more complex works to render them secure, will be treated of in a later chapter.

Ordinary foundations are ranged under three classes, viz.:—

I. *Foundations in Rock*, or material whose stability is not impaired by saturation with water.

II. *Foundations in firm earth*, such as sand, gravel, and hard clay.

III. *Foundations in soft earth.*

The base of every foundation should be as nearly as possible perpendicular to the direction of the pressure which it is to sustain, and of sufficient area to bear that pressure with safety. The area is increased to any required extent by making the lowest courses of masonry or brickwork in the building spread out by a series of steps; by supporting them on a sufficiently broad layer of concrete or beton; by making inverted arches under openings; and by other contrivances. The *centre of resistance* of the foundation of a piece of masonry (or point traversed by the resultant of the pressure), should not deviate from the centre of gravity of its figure beyond certain limits, which will be afterwards specified in particular cases.

236. **Rock Foundations.**—To prepare a rock foundation for being built upon, the following are in general all the operations that are required:—

I. To cut away all loose and decayed parts of the rock.

II. To cut and dress the rock to a plane surface, or to a set of plane surfaces like those of steps, perpendicular, or nearly perpendicular, to the pressure to be sustained.

III. To fill, if necessary, hollows in rock with beton, or with rubble masonry.

IV. In some cases it is advisable, in order to distribute the pressure, that the rock should be covered with a layer of beton, whose thickness, in different examples, ranges from a few inches to six feet and upwards.

The intensity of the pressure on a rock foundation should at no point exceed one-eighth of the pressure which would crush the rock.

(See Article 215, p. 36.) The following are examples of the actual intensity of the pressure on some existing rock foundations:—

	Pressure in lbs. on the Square Foot.		Pressure in lbs. on the Square Inch.
Average of ordinary cases, the rock being at least as strong as the strongest red bricks, ...	20,000	...	140
Pressures at the base of St. Rollox Chimney (450 feet below the summit):—			
On a layer of strong concrete or beton, 6 feet deep, ...	6,670	...	46
On sandstone below the beton, so soft that it crumbles in the hand, ...	4,000	...	28

The last example shows the pressure which is safely borne in practice by one of the weakest substances to which the name of *rock* can be applied.

The proper rule for limiting the deviation of the centre of resistance of a rock foundation from the centre of gravity of its figure is, that *there should be no tension at any point of the base.* The following is the formula for calculating the greatest value of the deviation in question which is consistent with that limitation:—*

Let A denote the area of the base;

y the distance from the centre of gravity of the figure of the base to the edge furthest from the centre of resistance;

h the total breadth of the base in the same direction;

I the moment of inertia of that figure, computed as for the cross-section of a beam relatively to a neutral axis traversing the centre of gravity at right angles to the direction of the deviation to be found. (See Article 162, pp. 252-254, and Article 179, pp. 294, 295.)

δ the deviation to be found;

then

$$\delta = \frac{\mathrm{I}}{\mathrm{A}\,y} = q\,h; \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1.)}$$

in which expression, q has the same meaning as in Article 179, pp. 294, 295, where a table of its values for various figures is given.

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* See *Applied Mechanics*, Article 94, pp. 76, 77; Article 205, pp. 228, 229.

The only assumption involved in this equation is, that the pressure on the foundation is an *uniformly varying stress.*

237. **Theory of Earth Foundations.** (*A. M.*, 199.)—In earth whose friction is alone to be relied on for resistance to displacement by the pressure of a building, the weight of earth displaced by the foundation should not bear a less ratio to the weight of the building than that given by the following equations, in each of which

x represents the depth of the foundation;
w the weight of a cubic foot of the earth;
φ its angle of repose.

CASE I. Let the weight of the building be uniformly distributed over its base, and let p_0 be the intensity of the pressure produced by it. Then

$$\frac{w\,x}{p_0} \geqq \left(\frac{1 - \sin\varphi}{1 + \sin\varphi}\right)^2. \quad \text{(1.)}$$

CASE II. When the weight of the building is so distributed that there is an uniformly varying pressure on the foundation, as assumed in Article 236, let p_1 be the greatest, p_2 the least, and p_0 the mean intensity of that pressure; then the two following conditions must be fulfilled:—

$$\frac{w\,x}{p_1} \geqq \left(\frac{1 - \sin\varphi}{1 + \sin\varphi}\right)^2; \quad \text{(2.)}$$

$$\frac{w\,x}{p_2} \leqq 1; \quad \text{(3.)}$$

Whence are deduced the following restrictions as to the extent of variation of the intensity of the pressure on the base, and the deviation of its centre of resistance from the centre of gravity of its figure.

$$\frac{p_1}{p_2} \leqq \left(\frac{1 + \sin\varphi}{1 - \sin\varphi}\right)^2; \quad \text{(4.)}$$

$$\delta = q\,h \cdot \frac{p_0 - p_2}{p_0}. \quad \text{(5.)}$$

When the figure of the foundation, as is usually the case, is symmetrical about its neutral axis, we have

$$p_0 = \frac{p_1 + p_2}{2};$$

and consequently,

$$\frac{w\,x}{p_0} \geqq \frac{(1 - \sin\varphi)^2}{1 + \sin^2\varphi} = 1 - \frac{2\sin\varphi}{1 + \sin^2\varphi}; \quad \text{(6.)}$$

$$\delta = q\,h \cdot \frac{p_1 - p_2}{p_1 + p_2} = q\,h \cdot \frac{2 \sin \varphi}{1 + \sin^2 \varphi} \text{............} (7.)$$

The following table gives some examples of the values of the functions of the angle of repose which occur in the preceding formulæ:—

φ	15°	20°	25°	30°	35°	40°	45°
$\dfrac{1+\sin\varphi}{1-\sin\varphi}$	1·700	2·145	2·464	3·000	3·690	4·599	5·826
$\dfrac{1-\sin\varphi}{1+\sin\varphi}$	0·588	0·466	0·406	0·333	0·271	0·217	0·172
$\left(\dfrac{1+\sin\varphi}{1-\sin\varphi}\right)^2$	2·890	4·472	6·070	9·000	13·619	21·152	33·94
$\left(\dfrac{1-\sin\varphi}{1+\sin\varphi}\right)^2$	0·346	0·224	0·165	0·111	0·073	0·047	0·0295
$\dfrac{1+\sin^2\varphi}{(1-\sin\varphi)^2}$	1·945	2·736	3·535	5·000	7·310	11·076	17·47
$\dfrac{(1-\sin\varphi)^2}{1+\sin^2\varphi}$	0·514	0·421	0·283	0·200	0·137	0·090	0·057
$\dfrac{2\sin\varphi}{1+\sin^2\varphi}$	0·486	0·579	0·717	0·800	0·863	0·910	0·943

238. **Foundations in Firm Earth.**—When a foundation is to be made in such earth as hard clay, clean dry gravel, or clean sharp sand, —that is to say, in earth which has considerable frictional stability, and is not liable to have that stability diminished by becoming saturated with water,—it is rarely necessary to apply the principles of the preceding article; because the depth to which the foundation must be sunk, in order that the building may rest on earth below the reach of the disintegrating effects of frost and drought, is almost always greater than those principles require. In Britain that depth should be at least 3 feet for sand and 4 feet for clay. In continental regions, where the climate has greater extremes of heat and cold, a greater depth is necessary. For example, in Germany, it appears that the depths of ordinary foundations are from 4 to 5 feet, and in North America from 4 to 6 feet.

Care should be taken to divert surface-water, which may tend to run into the foundation, by means of catchwater drains, just as in other cuttings (Article 189, p. 334); and, if necessary, drains ought also to be made at the bottom of the foundation.

The greatest intensity of pressure on foundations in firm earth is usually from 2,500 to 3,500 lbs. per square foot, or from 17 to 23 lbs. per square inch.

In fixing the "*spread*," or additional breadth given to the "footings" or foundation courses of the masonry or brickwork of ordinary walls, the usual rule is to make the breadth of the base once-and-a-half the thickness of the body of the wall in compact gravel, and twice that thickness in sand and stiff clay.

239. In **Foundations on Soft Earth** care must be taken that the depth of the foundation is not less, as compared with the pressure of the buildings, and the deviation of the centre of resistance not greater, as compared with the breadth of base, than the limits given by the formulæ of Article 237. Those objects are promoted by making the breadth of the base of the masonry as great as is practicable, so as at once to distribute its weight over a large surface, and to increase the breadth as compared with the deviation of the centre of resistance from the centre of gravity of the base.

If practicable, the ground should be well drained before the digging of the foundation is commenced, in order to increase its firmness as far as possible.

Precisely as in the case of an embankment on soft ground (Article 204, p. 343), a trench may be dug and filled with a stable material, such as sand or concrete, in order to distribute the pressure, and convey it to a sufficiently low stratum of the softer material. To find the proper depth for the trench—

Let p_1 be the greatest intensity of pressure of the intended building on its base, in lbs. per square foot. In calculating this quantity, if the trench is to be filled with sand, the area over which the weight of the building is distributed should be taken as simply equal to the area of the lowest course of foundation stones. But if the trench is to be filled with beton, the weight may be considered (as in an example given in p. 378) to be distributed over the whole area of the layer of beton, provided the edges of that layer do not project beyond the edges of the foundation stones to a distance greater than the depth of the layer of beton.

Let w be the weight in lbs. of a cubic foot of the material with which the trench is to be filled; being about 90 lbs. for sand and 130 for strong concrete or beton—

w', the weight of a cubic foot of the soft earth;

ϕ', its angle of repose;

also let $\frac{1 - \sin \phi'}{1 + \sin \phi'} = k'$; (for values of k' and of k'^2, see p. 380);

and the required depth of the trench $= x'$; then

$\frac{w'x'}{k'} =$ [illegible] k'

$$x' = \frac{p_1 k'^2}{w' - w k'^2} \text{ ....(1.)}$$

The material for filling the trench should be laid and rammed in layers of about a foot deep. If concrete is used, the effect of ramming may be produced by throwing it down from a scaffolding at least ten feet high.

If the trench is filled with sand, the building may be founded on the layer of sand as on a natural sand foundation.

If the trench is filled with concrete, the building is to be built on the upper surface of that layer as soon as it is *set*, care being taken that the intensity of the pressure on the concrete does not anywhere exceed one-eighth part of its resistance to crushing; that is to say, about

$$\frac{53{,}300}{8} = 6{,}660 \text{ lbs. on the square foot, or}$$

$$\frac{555}{8} = 46 \text{ lbs. on the square inch.}$$

In buildings which contain a number of openings, such as arches, windows, doorways, &c., the distribution of the load on the foundation over an increased area may be effected by means of inverted arches under the openings, provided those arches are very accurately built.

The more difficult class of foundations in soft ground, which require the use of timber or iron to make them safe, will be treated of in a later chapter.

When, by trial-pits and borings, it is shown that *a soft stratum underlies a firm one*, equation 1 should be applied in order to determine whether the depth (x') of the firm stratum is sufficient to make the foundation safe. When the firm stratum consists of sound rock, the intensity p_1 of the pressure on the soft stratum due to the weight of the building may be computed according to the same rule as for a layer of concrete. The results of this method will err on the safe side.

SECTION V.—*Construction of Stone-Masonry.*

240. **General Principles.**—The following principles are to be observed in the building of all classes of stone-masonry.

I. To build the masonry, as far as possible, in a series of courses, perpendicular, or as nearly perpendicular as possible, to the direction of the pressure which they have to bear; and to avoid all long continuous joints parallel to that pressure by "breaking joint."

II. To use the largest stones for the foundation course.

III. To lay all stones which consist of layers or "beds" in such a manner that the principal pressure which they have to bear shall act in a direction perpendicular, or as nearly perpendicular as possible, to the direction of the layers. This is called "*laying the stone on its natural bed*," and is of primary importance to strength and durability, as has been already explained in various Articles.

IV. To moisten the surface of dry and porous stones before bedding them, in order that the mortar may not be dried too fast, and reduced to powder by the stone absorbing its moisture. (Article 229, p. 372.)

V. To fill every part of every joint, and all spaces between the stones, with mortar; taking care at the same time that such spaces shall be as small as possible.

241. **Masonry Classed.**—The *face* of a stone is its outer surface which is exposed to view. Its *beds* are the surfaces parallel to the layers. Its *sides* are the surfaces which bound it in a direction transverse both to the face and to the beds. The term *bed* is also applied to the joints between or parallel to the courses, through which the principal pressures act; these joints are also called *bed-joints;* the *side-joints*, or joints transverse both to the beds and the face, are often called simply *joints*.

The classification of masonry for engineering purposes is based almost entirely on the size and figure of the stones, and on the manner in which the joints, whether bed-joints or side-joints, are formed and executed, the appearance of the face being a matter of secondary importance.

The principal tools employed in the dressing of stone are, the scabbling hammer, whose head is pointed at one end like a pick, and axe-formed at the other, and various chisels, of which one is pointed at the end, and the others flat, and of breadths ranging from one to three inches, or thereabouts.

The scabbling hammer produces a rough approximation to a plane surface; the point gives a closer approximation, producing a surface covered with a number of small parallel ridges and furrows; the "inch-tool" and other flat-ended chisels cut away the ridges left by the point, producing still greater smoothness. Stone thus dressed is said to be "droved."

There are an indefinite number of different qualities of masonry, from "perpend ashlar," in which every stone is hewn to a regular figure and exactly fitted to the adjoining stones, to common rubble, in which the stones are built nearly as they come from the quarry, great irregularities of figure alone being reduced by means of the hammer.

For engineering purposes, masonry may be classed generally under four principal kinds, viz.:—Ashlar — Block-in-course — Coursed Rubble—and Common Rubble—and the combinations of those four kinds.

242. **Ashlar Masonry**, or hewn stone, consists of blocks cut to regular figures, generally rectangular, and built in courses of an uniform depth, which is seldom less than a foot.

In order that the stones may not be liable to be broken across, no stone of a soft material, such as the weaker kinds of sandstone and granular limestone, should have a length greater than 3 times its depth; in harder materials, the length may be 4 or 5 times the depth. The breadth, in soft materials, may range from 1½ times to double the depth; in hard materials, it may be 3 times the depth.

The bed-joints and side-joints are dressed to plane surfaces (and in some exceptional cases to be afterwards specified, to curved surfaces). In the case of plane joints this is done by making an accurately plane chisel-draught all round the edges of the surface to be shaped, and if the stone is large, some additional transverse chisel-draughts in the same plane, and dressing the remainder of the surface by the point down to the plane of the chisel-draught, which serves as a guide. The accuracy with which this is done is of special importance in the case of bed-joints; for if any part of the surface projects beyond the plane of the chisel-draught, that projecting part will have to bear an undue share of the pressure, which will be concentrated upon it; and the joint, which will gape at the edges, constituting what is called an *open joint*, will be wanting in stability. On the other hand, if the surface of the bed is concave, having been dressed down below the plane of the chisel-draughts, the pressure is concentrated on the edges of the stone, to the risk of splintering them off. Such joints are said to be *flushed*. They are more difficult of detection after the masonry has been built than open joints, and are often executed by design, in order to give a neat appearance to the face of the building; and therefore their occurrence must be guarded against by careful inspection of the progress of the stone-cutting.

When the stone has been dressed so that all the small ridges on its surface are in one plane with the chisel-draughts, the pressure is distributed with a near approach to uniformity; for the mortar serves to transmit it to the furrows between the ridges.

Great smoothness is not desirable in the joints of ashlar masonry intended for strength and stability; for a moderate degree of roughness adds at once to the resistance to displacement by sliding, and to the adhesion of the mortar.

Each stone should first be fitted into its place dry, in order that any inaccuracy of figure may be discovered and corrected by the stone-cutter, before it is finally laid in mortar, and settled in its bed. No side-joint in any course should be directly above a side-joint in the course below; but the stones should overlap or *break joint* to an extent of from once to once-and-a-half the depth of a course. This is called the *bond* of the masonry: its effect is to make each stone be supported by at least two stones of the course below, and assist in supporting at least two stones of the course above; and its objects are twofold: *first*, to distribute the pressure; so that inequalities of load on the upper part of the structure, or of resistance at the foundation, may be transmitted to and spread over an increasing area of bed in proceeding downwards or upwards, as the case may be; and *secondly*, to tie the building together, or give it a sort of tenacity, both lengthwise and from face to back, by means of the friction of the stones where they overlap.

A stone which lies with its greatest length parallel to the face of the building is called a *stretcher*. A stone which lies with its greatest length perpendicular to the face of the building is called a *header*. Stretchers tie the building together lengthwise, headers crosswise. The strongest bond in ashlar masonry is that in which each course at the face of the building contains a header and a stretcher alternately, the outer end of each header resting on the middle of a stretcher of the course below; so that rather more than *one-third* of the area of the face consists of ends of headers. This proportion may be deviated from when circumstances require it; but in every case it is advisable that the ends of headers should not form less than *one-fourth* of the whole area of the face of the building.

Quoins, or corner-stones, which should be of large size and chosen with special care, are at once headers and stretchers; each quoin being a header relatively to one of the two faces of the building which it connects, and a stretcher relatively to the other.

The *thickness of mortar* in the joints of well-executed ashlar masonry should be about an eighth of an inch. The *volume of mortar* required in all is about one-eighth part of the volume of the stone.

Ashlar masonry is used in engineering chiefly for the piers, abutments, arches, and parapets of bridges, for hydraulic works to be afterwards specified, for facing, quoins, string courses, and coping to inferior kinds of masonry, and to brickwork, and in general, for works in which great strength and stability are required.

A rougher kind of ashlar masonry is built with stones of the sizes and figures already mentioned, but scabbled or dressed with the hammer. It may be considered as intermediate between ashlar and block-in-course.

It what manner soever the faces of ashlar stones are dressed, or even should they be "quarry-faced," there ought to be a chisel-draught round the edges of the face, forming sharp and straight edges with the chisel-draught of the beds and joints, in order that the stone may be accurately set.

243. **Block-in-Course Masonry** differs from hammer-dressed ashlar chiefly in being built of smaller stones. The usual depth of the courses is from 7 to 9 inches. The same rules apply to breaking joint, and to the proportions which the lengths and breadths of the stones should bear to their depths, as in ashlar; and as in ashlar also, *at least one-fourth* of the face of the building should consist of headers, whose length should be from 3 to 5 times the depth of a course.

Block-in-course masonry is used for spandrils and wing-walls of bridges, the facing of retaining walls, and similar purposes.

244. In **Coursed Rubble Masonry** the building consists of a series of horizontal courses, seldom exceeding one foot in depth, each of which is correctly levelled before another is built upon it; but the side-joints are not necessarily vertical. *One-fourth part* at least of the face in each course should consist of bond-stones or headers; each header to be of the entire depth of the course, of a breadth ranging from 1½ times to double that depth, and of a length extending into the building to from 3 to 5 times that depth, as in ashlar. Those headers should be roughly squared with the hammer, and their beds hammer-dressed to approximate planes; and care should be taken not to place the headers of successive courses above each other; for that arrangement would cause a deficiency of bond in the intermediate parts of the course. Between the headers, each course is to be built of smaller stones, of which there may be one, two, or more, in the depth of the course. These are sometimes roughly squared, so as to have vertical side-joints; sometimes the stones are taken as they come, so that the side-joints are irregular; but no side-joint should form an angle with a bed-joint sharper than 60°. Care should be taken, not only that each stone shall rest on its natural bed, but that the sides parallel to that natural bed shall be the largest, so that the stone may lie flat, and not be set on edge or on end. Howsoever small and irregular the stones may be, care should be taken to make the courses break joint. Hollows between the larger stones should be carefully filled with smaller stones, completely imbedded in mortar.

Coursed rubble masonry requires great care in the inspection of its progress, to see that the preceding rules are observed; and especially, that the interior of the wall contains neither empty hollows, nor spaces filled wholly with mortar or with rubbish where pieces of stone ought to be inserted, and that each stone is laid flat, and on its natural bed. Care must be taken that the headers or bond-stones are really what they profess to be, and not thin stones set on edge at the face of the wall.

A cubic yard of rubble masonry requires, in order to allow for waste, about 1⅓ cubic yard of stones, and ⅓ cubic yard of mortar.

The resistance of good coursed rubble masonry to crushing is about four-tenths of that of single blocks of the stone that it is built with.

Coursed rubble is used for retaining walls and wing-walls that require less strength than those built of block-in-course or ashlar, for the backing of pieces of masonry that are faced with ashlar or block-in-course, for fence-walls, and for various other purposes.

Rubble is often built in "random courses;" that is to say, each course rests on a plane bed, but is not necessarily of the same depth or at the same level throughout, so that the beds occasionally rise or fall by steps.

245. **Common Rubble Masonry** differs from coursed rubble in not being built in courses; but in other respects the same rules are to be observed. The resistance of common rubble to crushing is not much greater than that of the mortar which it contains; it is therefore not to be used where strength is required, unless built with strong hydraulic mortar. Its chief use in engineering is for fence walls.

246. **Ashlar and Block-in-Course backed with Rubble.**—In this sort of masonry the stones of the ashlar or block-in-course face should have their beds and joints accurately squared and dressed with the hammer, or the point, as the case may be (see Articles 242, 243, pp. 384 to 386), for a breadth of from once to twice (or on an average, once and a-half) the depth of the course, inwards from the face; but the backs of these stones may be rough. The proportion and length of the headers should be the same as in ashlar, and the "tails" of those headers, or parts which extend into the rubble backing, may be left rough at the back and sides; but their upper and lower beds should be hammer-dressed to the general planes of the beds of the course. These tails may taper slightly in breadth, but should not taper in depth.

The rubble backing, built as described in Article 244, p. 386, should be carried up at the same time with the face-work, and in courses of the same depth, the bed of each course being carefully formed to the same plane with that of the ashlar or block-in-course facing.

In estimating the labour or cost of building such masonry as is here described, the area of the face, multiplied by the distance inwards to which the dressing of the joints is carried, may be taken as ashlar or block-in-course, as the case may be, and the remainder as rubble.

These combinations of masonry are the most generally useful in engineering works; and they are especially suitable in a mechanical point of view where the pressure is concentrated towards the face of the building, as in retaining walls.

For the abutments of bridges they are *not* mechanically suitable, because the pressure is concentrated towards the back; but if in any bridge coursed rubble is strong enough to resist the pressure at the back of the abutments, it may be used for that purpose, and faced with block-in-course, or ashlar, for the sake of appearance, and of protection from the weather.

Coursed rubble masonry is often used in combination with ashlar quoins, to which the remarks in Article 242, p. 385, are applicable.

247. **String Courses and Copes.**—A string course is a course of large stones slightly projecting beyond the face of a building, and dressed and built like ashlar or block-in-course, as the case may be. Setting aside its architectural appearance, its mechanical use is to support some load and distribute it upon the masonry below it. For example, when a coursed rubble or block-in-course wing-wall or spandril of a bridge has to support an ashlar parapet, a string course must first be placed on the wall, to give a steady base for the parapet, and to distribute its weight over the smaller stones below.

The *Cope* of a wall consists of large and heavy stones, slightly projecting over it at both sides, accurately bedded on the wall, and jointed to each other with hydraulic mortar, or with cement. Its use is to shelter the mortar in the interior of the wall from the weather, and to protect, by its weight, the smaller stones below it from being knocked off or picked out. Cope-stones should be so shaped that water may rapidly run off from them.

Rough rubble coping forms an exception to the general rule that laminated stones should be laid with their layers parallel to the beds of the courses. In this case the stones are very often set on edge, with their layers vertical, and perpendicular to the length of the wall, so that the edges of the layers alone are exposed to the air, at the top, as well as at the sides of the cope.

Additional stability is given to a cope by so connecting the cope-stones together that it is impossible to lift one of them, without, at the same time, lifting the ends of the two next it. This is done either by means of iron cramps inserted into holes in the stones, and fixed there with lead, or better still, by means of *dowels* of some very hard and strong stone, such as greenstone or granite. These are small prismatic or cylindrical blocks, each of which fits into a pair of opposite holes in the contiguous ends of a pair of cope-stones, where it is fixed with cement or hydraulic mortar.

Cast iron and wrought iron dowels are also used, but they are inferior in durability to those of hard stone, though superior in strength. Copper dowels are strong and durable, but expensive.

Cramps or dowels may be used in string courses, or in any part of a piece of masonry.

Fence-walls are sometimes coped with sods, or with clay-puddle. (Article 206, p. 344.)

248. **Pointing** a piece of masonry consists in scraping the mortar from the outer edges of the joints, at the face of the building, as far as the point of the trowel will reach, and filling the groove so made with mixed cement, or with hydraulic mortar, to keep out moisture. As to mixed cement, see Article 231, p. 374.

In sea-walls exposed to hard blows from the waves, cement put into the joints by ordinary pointing is apt to jump out in pieces; and it is best to *lay* the stones in cement for two or three inches inwards from the face of the wall.

249. **Dry Stone Walls** should be built according to the principles already laid down for rubble masonry in Articles 244, 245, pp. 386, 387, with the single exception that the mortar is to be omitted. It is often advisable to make the cope of a dry stone wall waterproof, in order that water may not lodge in the joints of the wall and force the stones from their places by its expansion in freezing. In such cases the cope may be made of stones set on edge, and jointed with mortar; or of bituminous concrete (Article 234, p. 376); or if great cheapness be desired, of clay puddle. (Article 206, p. 344.)

If a dry stone wall is intended to be permanent, rounded boulders should not be used in their natural condition to build it, but should first be broken into flat and angular pieces.

Dry stone building is employed for fence-walls, and sometimes for a backing to retaining walls, in order at once to diminish the pressure of earth against them, and to drain away water by letting it escape between the crevices of the stones.

It is also used in retaining walls of small height, and in facing earthen slopes exposed to the action of water (Article 196, p. 339; and Article 205, p. 344); and in the latter case the beds of the courses are laid perpendicular to the direction of the steepest slope.

250. **Labour of Stone-Masonry.**—The following information as to the labour required to execute different kinds of work connected with stone-masonry is given chiefly on the authority of Gauthey:—

RUBBLE STONE, one cubic yard.	Day's Work of a Man.
Loading barrows with stone, ...	0·06
Wheeling one relay = about 100 feet on a level, (As in earthwork, each foot of ascent is equivalent to *six feet* of additional distance.)	0·045
Unloading barrows, ...	0·03

As to mixing mortar, see Article 229, p. 373; and as to the proportions of mortar and stone, see Articles 242 to 245, pp. 385 to 387.

Kinds of Building.		Day's Work of a Man per Cubic Yard.			
		Breaking Stone.	Stone-Cutting.	Building.	Labourers' Work.
Dry stone,		0·64	—	1·00	0·50
Coursed rubble,		0·64	—	0·90	0·90
Block-in-course,		0·90	1·5	0·90	0·90
Block-in-course arching,		0·90	2·25	0·90	0·90
Ashlar (soft sandstone),	from	1·80	2·50	1·00	1·00
	to ...	2·50	6·00	2·00	2·00

Facing ashlar, per square foot (soft sandstone)—

Broached with the point, 0·05; Droved, 0·07; Polished, 0·10.

Labour of breaking and stone-cutting for harder stones—

Hard sandstone = soft sandstone × 2;
Hard limestone, marble, granite = soft sandstone × from 3 to 4.

$$\text{Curved facing} = \text{flat} \times \left(1 + \frac{2\frac{1}{2}}{\text{radius in feet}}\right).$$

Taking down old masonry, 0·5 day's work of a man per cubic yard.

251. **Mechanism for moving large Stones.**—There are various ways of laying hold of stones that are too heavy to be moved by hand, the most usual being the following:—

I. By nippers or tongs, the claws of which enter a pair of holes in the sides of the stone. Those holes should be situated in a horizontal line passing through or a little above the centre of gravity of the stone.

II. By a single iron plug, very slightly tapered, and driven tightly with the hammer into a vertical cylindrical hole in the top of the stone, directly above its centre of gravity. At the upper end of the plug is an eye, to which the chain for lifting the stone is hooked. After the stone has been laid in its place, a few sharp taps given sideways with the hammer loosen the plug. This method answers best with the hardest stones, such as granite.

III. By a pair of iron plugs, inserted into two holes in the top of the stone, which converge towards each other at a right angle, being inclined in opposite directions at angles of 45°. The eyes at the upper ends of the plugs are attached to a pair of chains, which, when the stone hangs by them, are at right angles to their respective plugs, and meet each other at a right angle, where they are attached to the lower end of one main chain. The two plug-holes should be in a vertical plane traversing the centre of gravity of the stone, and equally distant from it. The tension on each of the branch chains is

$$= \text{weight of the stone} \times \cdot 707.$$

IV. By the *Lewis*, a truncated iron wedge or dovetail with the larger end downwards, made of three pieces, which can be put into or taken out of a similarly shaped hole in the top of the stone one by one, but not together. The lewis-hole is made from 2 inches to 10 inches deep, according to the weight of the stone.

The most generally useful machine for lifting and shifting large stones in ordinary buildings is the moveable jib-crane. In large buildings a travelling crab or winch is used, running on a travelling platform; that is to say, a framework of timber and iron is erected, consisting of two parallel lines of posts with sufficient diagonal bracing, supporting a pair of parallel beams, which extend along the whole length of the intended building, and include its greatest breadth between them; each of those longitudinal beams carries an iron rail; upon the pair of longitudinal rails so carried run the wheels supporting the travelling platform, which spans over the whole breadth of the building, and is made sufficiently strong and stiff by tubular iron beams or otherwise; it carries a pair of transverse rails, upon which runs a four-wheeled truck, carrying the crab or winding machine, which can thus be moved to any part of the building. The whole apparatus may be worked by a steam-engine.

252. **Instruments used in Building.**—In Article 65, p. 111, and Article 68, p. 113, it has been already explained how the situations and levels of those leading points upon which the situations and levels of all other points in a piece of masonry depend, are to be set out by the engineer.

The principal instruments used during the progress of the building are the cord, for setting out long straight lines, such as the edges of the bed-joints; the straight-edge, for shorter straight lines and for plane surfaces; the square and the bevel, for right and oblique angles; the plumb-rule, for vertical or nearly vertical lines; the level, for horizontal lines and planes, which may be like an inverted ⊥, with a plummet to set the stem vertical, or what is better, a spirit-level.

When the face of a wall is to be vertical, it can be set out, and its accuracy tested, by a plumb-rule, being a flat, straight-edged piece of board, with a line marked on it parallel to one of its edges, which line is set truly vertical by a plummet.

When the face of the wall is to have "*a straight batter*,"—that is, to be inclined at an uniform angle to the vertical, the rule to be used is still straight-edged, but the edge is inclined to the plumb-line at the proper angle of batter. The batter of a wall is usually described by stating the extent of deviation from the vertical in a given height; for example, "one in twelve," or "one inch in a foot."

When the vertical section of the face of a wall is to be curved, it is said to have a "curved batter," and it must be set out by means of a "face-mould,"—that is to say, a narrow, flat board, having one of its edges of the intended figure of the face of the wall, and having a straight line marked upon it, which is set truly vertical by means of a plummet. Great care should be bestowed on preparing the face-moulds of important pieces of masonry; in some cases, which will be exemplified farther on, every course of stones ought to be marked on the edge of the mould.

Large face-moulds are sometimes made of several pieces of timber framed together.

When the beds of the courses are to be plane and level, they can be set correctly by the level and common straight-edge. When they are to be planes having a given slope, a rule must be employed having two straight edges inclined to each other at such an angle that, when one edge is set horizontal by the spirit-level, the other has the proper inclination. If the beds of the courses are to be perpendicular to a straight or curved battering face, their position can be set out and tested by the square.

Curved beds, such as are employed for some special purposes, require the use of suitably curved "*bed-moulds*."

In all cases in which economy of time and money has to be studied, the engineer should, as far as practicable, avoid curved figures in masonry; for not only are they more tedious and expensive to set out and to build than straight and plane figures, but it is more difficult to test the accuracy with which they have been executed. A single glance will detect the smallest appreciable inaccuracy in a wall with a straight batter, while the same process in the case of a wall with a curved batter would require either a long series of measurements, or the application of a cumbrous face-mould to various parts of the wall; and this becomes a matter of serious importance in large structures, where errors in form may affect the strength and stability.

253. **Mensuration of Masonry.**—For engineering purposes, quantities of the rougher kinds of masonry are stated in cubic yards, and of the finer, in cubic feet.

But there are also special units of measure for masonry, such as the following:—

A *rood* of masonry means, when applied to surface, 36 square feet, and when applied to volume, 36 square feet of a wall of a specified thickness, such as 2 feet. In estimating a building according to this system, the superficial measure of the face is taken in roods of 36 square feet, in order to estimate the cost of the face-work; and then the area in superficial roods of the face of each portion of the building is multiplied by the ratio which its thickness bears to 2 feet, so as to compute the cubic contents in solid roods of 36 square feet in area and 2 feet thick, in order to estimate the cost of the masonry exclusive of the face. This method is better suited to architectural than to engineering purposes.

SECTION VI.—*Construction of Brickwork.*

254. **General Principles.**—The following principles are to be observed in building with bricks:—

I. To reject all misshapen and unsound bricks. (See Article 220, p. 366.)

II. To place the beds of the courses perpendicular, or as nearly perpendicular as possible, to the direction of the pressure which they have to bear; and to make the bricks in each course break joint with those of the courses above and below by over-lapping to the extent of from one quarter to one half of the length of a brick.

III. To cleanse the surface of each brick, and to wet it thoroughly before laying it, in order that it may not absorb the moisture of the mortar too rapidly.

IV. To fill every joint thoroughly with mortar, taking care at the same time that the thickness of mortar shall not exceed about a quarter of an inch.

In order to prevent the use of too great a thickness of mortar, it is usual in specifications to prescribe a certain depth which a certain number of courses of brickwork shall not exceed. For example, if the bricks are 2¾ inches deep, it may be specified that four courses of bricks, when built, shall not measure more than one foot in depth; a condition which implies that the average thickness of mortar in the joints shall be ¼ inch.

V. To use no "bats," or pieces of bricks, except when absolutely necessary, in order to make a "closure,"—that is, to finish the end or corner of a wall, or the side of an opening; and even then, to use no piece less than half a brick.

In stating the length and breadth of masses of brickwork, it is usual to employ the length of a brick as an unit of measure. For example, if bricks are used which build to 9 inches in length,

½ brick means 4½ inches.
1 ,, ,, 9 inches.
1½ ,, ,, 1 foot 1½ inch.
2 ,, ,, 1 foot 6 inches.

And so on.

The *volume of mortar* required for good brickwork is about *one-fifth* of the volume of the bricks.

255. **Bond in Brickwork.**—The bricks used in a given building being of uniform or nearly uniform size and figure, are to be built according to an uniform system, which is called the *bond* of the brickwork.

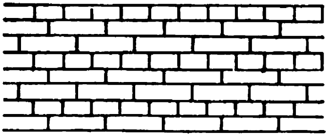


Fig. 166.

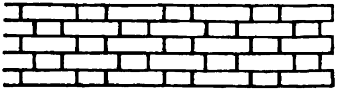


Fig. 167.

As in ashlar masonry, so in brickwork, a *header* is a brick whose length lies perpendicular to the face of the wall; a *stretcher*, one whose length lies parallel to the face of the wall. As the length of a brick is almost exactly double of its breadth, one stretcher occupies the same area on the face of the wall with two headers.

I. *English Bond*, which is considered the strongest and most stable arrangement, consists in laying entire courses of headers and of stretchers periodically, as in fig. 166. Sometimes the courses of headers and stretchers occur alternately; sometimes there is only one course of headers for every two, three, or four courses of stretchers. The stretchers tie the wall together lengthwise, the headers crosswise. The proportionate numbers of the courses of headers and stretchers should depend on the relative importance of transverse and longitudinal tenacity. (*A. M.*, 202.) The proportion shown in fig. 166, of one course of headers to two of stretchers, is that which gives equal tenacity to the wall lengthwise and crosswise, and which therefore may be considered the best in ordinary cases.

In a factory chimney, the longitudinal tenacity, which resists any force tending to split the chimney, is of more importance than the transverse tenacity; therefore, in these buildings, it is advisable to have a greater proportion of stretchers, such as three or four courses of stretchers to one course of headers.

In building brickwork in English Bond, it is to be borne in mind that there are twice as many vertical or side-joints in a course of headers as there are in a course of stretchers; and, therefore, that unless great care is taken in laying the headers to make these joints very thin, two headers will occupy a little more length than one stretcher, and the correct breaking of the joints, to the extent of exactly a quarter of a brick, will be lost. This is often the case in carelessly built brickwork, in which at intervals vertical joints are seen nearly or exactly above each other in successive courses.

II. In *Flemish Bond* (fig. 167) a header and a stretcher are laid alternately in each course, and so placed that the outer end of each header lies on the middle of a stretcher in the course below. The number of vertical joints in each course is the same, so that there is no risk of the correct breaking of the joints by a quarter of a brick being lost; and the wall presents a neater appearance than one built in English Bond. English Bond, however, when correctly built, is considered to be stronger and more stable than Flemish Bond.

256. **Hoop Iron Bond.**—Pieces of hoop iron are sometimes laid flat in the bed-joints of brickwork, to increase its longitudinal tenacity. They should break joint with each other; and the ends of each piece of hoop iron should be bent down at right angles for the length of two inches or thereabouts, and inserted into vertical joints of the course of bricks on which the hoop iron lies.

The total sectional area of the hoop iron needs not exceed about 1-300th of that of the brickwork.

257. **Pointing Joints.**—(See Article 248, p. 388.)

258. The **Foundation Courses** of a piece of brickwork usually spread downwards by steps of a quarter of a brick at the face and back, until a sufficient breadth is gained to support the weight of the building, according to the principles already explained in Section IV. of this Chapter, pp. 377 to 381.

259. **String Courses and Copes.** Brick string courses ought to consist entirely of headers, and so also ought copes built with ordinary bricks. Coping for brick walls is sometimes made with large bricks moulded expressly for that purpose. Stone string courses and coping are frequently used along with brick building, especially where strength and stability are required.

260. In **Brickwork with Stone Quoins** special care must be taken that the layer of mortar in each bed-joint of the brickwork is as thin as possible; for as the bed-joints of the brickwork are three or four times as numerous as those of the stone quoins, any superfluous thickness of the former will cause the brickwork to settle more than the stone quoins, the effect of which will be to disfigure, crack, and perhaps destroy the building.

261. **Labour of Brickwork.**—The following data are given on the authority of Gauthey:—

	Day's Work of a Man per Cubic Yard.		
	Bricklayer.	Labourer.	Erecting Scaffolding.
Ordinary Bricklaying,......	0·6	0·6	0·2
Brick Arching,..............	0·9	0·9	various.

In the case of arching, the labour of erecting scaffolding includes that of putting up "centres," or timber frames for the arches to rest on while in progress. These structures will be considered further on.

262. **Mensuration of Brickwork.**—For engineering purposes in Britain, quantities of brickwork are usually computed and stated in cubic yards; but there are also special modes of stating them, such as the following:—

Each piece of brickwork has its thickness stated in bricks and half-bricks; the area of its face is calculated: from that area is computed the area of a wall of the standard thickness of *one brick and a-half*, and of the same cubic contents, and the reduced area so obtained is stated in *rods* of $30\frac{1}{4}$ square yards, and in square yards.

A *rod* of brickwork, of a brick and a-half thick, if each brick be 9 inches long, is equal to $11\frac{1}{3}$ cubic yards very nearly.

SECTION VII.—*Of Buttresses and Retaining Walls.*

263. The **Stability of Blocks of Masonry and Brickwork in General** (*A.M.*, 211) depends on the conditions already stated in Article 139, p. 220—viz., that of *stability of position*, which requires that the structure shall not give way by overturning; and that of *stability of friction*, which requires that it shall not give way by the sliding of one course upon another; and those two conditions ought to be fulfilled *at the bed-joint of each course.*

The following are the most convenient ways of expressing these conditions by means of formulæ suitable for calculation:—

I. *Stability of Position* is insured when the moment of the force tending to overturn the mass above a given bed-joint does not exceed the moment of stability of the mass of masonry above that bed-joint.

To express the *moment of stability at a given bed-joint* symbolically, it is necessary, in the first place, to determine the greatest distance to which the "*centre of pressure*" or "*of resistance*" at that bed-joint may deviate from the middle of the bed, without endangering the stability of the structure.

Let q denote the greatest safe ratio of the deviation to the thickness of the masonry at the given bed-joint.

In *flying buttresses*, and *piers and abutments* of arches and of frames, it is in general advisable to limit q according to the rule already given for rock foundations, Article 236, p. 378—viz., *that there shall be no tension at any point of the bed*, the pressure being supposed to be an uniformly varying stress. For various values of q, see Article 179, pp. 294, 295. The value of most common occurrence is that for solid rectangular structures—viz., $q = \frac{1}{6}$.

In *retaining walls* for sustaining the pressure of earth or of water, the following are average values of q deduced from the dimensions of actual retaining walls:—

According to the practice of British engineers, $q = \cdot 375$ nearly.

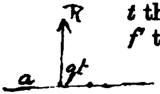
According to the practice of French engineers, $q =$ from ·3 to ·25.

The following is a method of determining the greatest value of q for a rectangular structure, consistent with safety from crushing of the material, based on the supposition that the intensity of the pressure diminishes at an uniform rate from the compressed edge of the bed-joint inwards, that the mortar exerts no appreciable tension, and that consequently the distance of the centre of resistance from the compressed edge is one-third of the thickness throughout which the pressure is distributed:—

Let R be the total pressure at the given bed-joint;

$\left.\begin{array}{l} b \text{ the breadth,} \\ t \text{ the thickness} \end{array}\right\}$ of the mass of masonry at that joint, in feet;

f the greatest safe pressure in lbs. on the square foot (being about one-eighth of the crushing pressure); then



$a = \left(\frac{1}{2} - q\right) t$

$t = 3a = \left(\frac{3}{2} - 3q\right) t$

$\frac{f\,bt}{2} = R$

$$f = 2\,R \div \left(\frac{3}{2} - 3\,q\right) b\,t; \text{ and therefore,}$$

$$q = \frac{1}{2} - \frac{2\,R}{3\,f\,b\,t} \quad \text{.......................(1.)}$$

The value of q having been fixed, let

$r\,t$ denote the distance from the middle point of the bed to the point where the bed is cut by a vertical line let fall from the centre of gravity of the mass of masonry above it;

W, the weight of that mass; and

j, the inclination to the horizon of a line in the plane of the bed, connecting the limiting position of the centre of resistance with the point directly below the centre of gravity before mentioned.

Then the moment of stability is,

$$M = W\,(q \pm r)\,t \cos j; \quad \text{.......................(2.)}$$

the sign $\left\{\begin{array}{c} + \\ - \end{array}\right\}$ being used according as the centre of resistance, and the vertical line through the centre of gravity, lie towards $\left\{\begin{array}{l} \text{opposite sides} \\ \text{the same side} \end{array}\right\}$ of the middle of the diameter.

The following modification of this expression is convenient in comparing structures of similar figures and different dimensions:—

Let h denote the height of the structure above the middle of the given bed-joint, b the breadth of that bed in a direction perpendicular or conjugate to the thickness t, and w the weight of an unit of volume of the material. Then

$$W = n \cdot w\, h\, b\, t, \text{ ....(3.)}$$

where n is a *numerical factor* depending on the *figure* of the structure, and on the angles which the dimensions, h, b, t, make with each other; that is, the angles of obliquity of the co-ordinates to which the figure of the structure is referred. Introducing this value of the weight of the structure into the formula 2, we find the following value for the moment of stability:—

$$M = n\,(q \pm r) \cos j \cdot w \cdot h\, b\, t^2 \text{ ....(4.)}$$

This quantity is divided by points into three factors, viz.:—

(1.) $n\,(q \pm r) \cos j$, a *numerical factor*, depending on the *figure* of the structure, the *obliquities* of its co-ordinates, and the *direction* in which the applied force tends to overturn it.

(2.) w, the heaviness of the material.

(3.) $h\, b\, t^2$, a geometrical factor, depending on the dimensions of the structure.

Now the first factor is the same in all structures having figures of the same class, with co-ordinates of equal obliquity, and exposed to similarly applied external forces; that is to say, to all structures whose figures, together with the lines of action of the applied forces, are *parallel projections of each other, with co-ordinates of equal obliquity.* (See Articles 101, 140, pp. 150, 220.) Hence for any set of structures which fulfil that condition, the moments of stability are proportional to

The heaviness of the material;
The height;
The breadth;
The *square* of the thickness; that is, of the dimension of the base which is parallel to the vertical plane of the applied force.

The following is the general expression for the moment, relatively to the limiting position of the centre of resistance, of an externally applied force, tending to overturn the mass of masonry above the given bed-joint.

Let P denote the magnitude of that force;

θ the angle which its direction makes with the horizon in a direction *contrary* to that of the slope j of the bed;

x', the vertical height, and y', the horizontal distance $\Big\}$ of its point of application from the centre of resistance of the bed;

then the perpendicular distance of P from the centre of resistance is

$x' \cos \theta - y' \sin \theta$; and the required moment is given by the following formula, which also expresses the condition, that that moment shall not exceed the moment of stability of the masonry:—

$$\mathrm{P}\,(x' \cos \theta - y' \sin \theta) \leqq \mathrm{M}. \quad \ldots\ldots\ldots\ldots\ldots (5.)$$

II. *Stability of Friction* is insured when the resultant pressure makes with a normal or line perpendicular to the bed, an angle not exceeding the angle of repose of the materials.

Let φ denote that angle. (See Article 110, p. 172.)

The angle made by the resultant pressure with the vertical is

$$\operatorname{arc} \cdot \tan \cdot \frac{\mathrm{P} \cos \theta}{\mathrm{W} + \mathrm{P} \sin \theta};$$

and the condition of stability of friction is given by the equation,

$$\operatorname{arc} \cdot \tan \cdot \frac{\mathrm{P} \cos \theta}{\mathrm{W} + \mathrm{P} \sin \theta} - j \leqq \varphi. \quad \ldots\ldots\ldots\ldots (6.)$$

This condition can always be fulfilled by properly adjusting the declivity of the bed-joint, j.

264. **Stability of a Vertical-faced Buttress with Horizontal Beds.** (*A. M.*, 213.)—Let fig. 168 represent a vertical section of a buttress, with a vertical face C D, against which a strut, rib, or piece of framework abuts at C, exerting a given force P in a given direction C A. In order that the buttress may be stable, it must fulfil the conditions of stability at each of its horizontal bed-joints. Let D E be one of those joints.

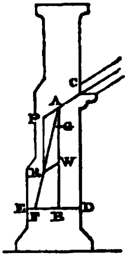


Fig. 168.

Should several pressures abut against the buttress, the force P acting in the line C A may be held to represent the resultant of all the forces which are applied above the particular joint D E under consideration.

Let G be the centre of gravity of that part of the buttress which is above the joint D E, and let W denote the weight of the same part. Through G draw the vertical line A G B, cutting the direction of the lateral thrust in A, and the joint D E in B; make $\overline{\mathrm{A\,W}} = \mathrm{W}$, $\overline{\mathrm{A\,P}} = \mathrm{P}$; complete the parallelogram A P R W; then $\overline{\mathrm{A\,R}}$ will represent the resultant of all the forces which act on the part of the buttress above the joint D E, to which the resultant of the resistance at that joint must be equal and directly opposed. A R being produced, cuts D E in F, the centre of resistance of that joint, which must not fall beyond a certain prescribed limit, that the condition of stability of position may be fulfilled. In order that the condition of stability of friction may be fulfilled, the angle A F B must not be less than the complement of the angle of repose.

In expressing this algebraically, it is to be observed that

$$\mathrm{C\,D} = x'; \ \mathrm{D\,F} = y'; \ j = 0;$$

and consequently that equation 5 of the preceding Article, p. 399, becomes,

$$\mathrm{P}\,(x' \cos \theta - y' \sin \theta) \leqq n\,(q \pm r)\,w\,h\,b\,t^2; \ \ldots\ldots\ldots (1.)$$

and equation 6,

$$\frac{\mathrm{P} \cos \theta}{n\,w\,h\,b\,t + \mathrm{P} \sin \theta} \leqq \tan \varphi. \ \ldots\ldots\ldots\ldots\ldots (2.)$$

By means of these fundamental equations the following problems are solved:—

I. The relation between the weight and the dimensions of the part of the buttress under consideration being given (in other words, the factor n being given), it is required to find the least thickness t at the joint D E consistent with stability of position.

In equation 1, make $y' = \left(q + \frac{1}{2}\right) t$, and put $=$ instead of $\leqq$; then

$$n\,(q + r)\,w\,h\,b\,t^2 = \mathrm{P}\,\left(x' \cos \theta - \left(q + \frac{1}{2}\right) t \sin \theta.\right)$$

To simplify the form of this quadratic equation, make,

$$\frac{\mathrm{P}\,x' \cos \theta}{n\,(q + r)\,w\,h\,b} = \mathrm{A}, \ \frac{\left(q + \frac{1}{2}\right) \mathrm{P} \sin \theta}{2\,n\,(q + r)\,w\,h\,b} = \mathrm{B};$$

then it becomes

$$t^2 = \mathrm{A} - 2\,\mathrm{B}\,t,$$

the solution of which is

$$t = \sqrt{(\mathrm{A} + \mathrm{B}^2)} - \mathrm{B} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

II. To find the least weight of material above the point C, consistent with stability of friction.

The greatest obliquity of pressure occurs at that joint which is immediately below the point of abutment C. Let h_0 denote the height of material above that joint, b_0 the breadth, and t_0 the required thickness; then,

$$n\,w\,h_0\,b_0\,t_0 = \mathrm{P}\,\left(\frac{\cos \theta}{\tan \varphi} - \sin \theta\right). \ \ldots\ldots\ldots\ldots (4.)$$

III. *Particular Case—Rectangular Buttress.* (*A. M.*, 214.)—In a rectangular buttress, the breadth b and thickness t are constant; and if h_0 be taken to denote the height of the top of the buttress above the point C,

$$h = h_0 + x$$

will be its height above a given joint. Also, because the centre of gravity of the portion above any bed-joint is vertically above the centre of the joint, $y' = 0$; and because
r

$$\mathrm{W} = w\,h\,b\,t,$$

$n = 1$.

These values being substituted in equation 3, give the following results, in which x denotes the depth of the base of the wall below C.

$$\mathrm{A} = \frac{\mathrm{P}\,x\cos\theta}{q\,w\,(h_0 + x)\,b};\ \mathrm{B} = \frac{\left(q + \frac{1}{2}\right)\mathrm{P}\sin\theta}{2\,q\,w\,(h_0 + x)\,b};\ t = \surd\,(\mathrm{A} + \mathrm{B}^2) - \mathrm{B} \ldots (5.)$$

As the depth x increases without limit, the thickness required for the wall approaches the following limit:—

$$t = \sqrt{\left(\frac{\mathrm{P}\cos\theta}{q\,w\,b}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

which depends on the horizontal component of the applied force alone.

Supposing this value to be adopted for the thickness of the buttress, in order that it may be stable, how deep soever the base may be below the point C, then to insure stability of friction, the height of the top above C must have the following value:—

$$h_0 = q\,t \cdot \frac{\cos(\varphi + \theta)}{\sin\varphi\cos\theta} = \frac{\cos(\varphi + \theta)}{\sin\varphi} \cdot \sqrt{\left(\frac{q\,\mathrm{P}}{w\,b\cos\theta}\right)} \ldots\ldots (7.)$$

Instead of the rectangular mass $h_0\,b\,t$, there may be substituted a *pinnacle* of the same weight, and of any figure.

265. **Stability of Retaining or Revêtement Walls in General.** (*A.M.*, 217.)—Figs. 169 and 170 represent vertical sections of retaining walls against which banks of earth abut. In each figure a vertical layer of the masonry and of the earth is supposed to be considered, whose length is unity. D E is the base of the layer of masonry, F the centre of resistance of that base, B a point vertically below G, the centre of gravity of the weight which rests on that base, $\overline{\mathrm{AW}}$ a line representing that weight, $\overline{\mathrm{A\,P}}$ a line representing the thrust of the earth; $\overline{\mathrm{A\,R}}$, the diagonal of the parallelogram A P R W, is a

line representing the resultant pressure at the base DE, and cutting that base in the centre of resistance F.

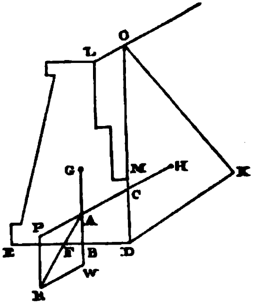


Fig. 169.

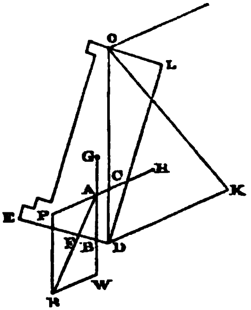


Fig. 170.

In each figure, D O is a vertical plane traversing the inner edge D of the base of the wall, and cutting the plane of the surface of the bank in O. In fig. 169, the whole of the wall lies in front of that vertical plane; so that the weight, represented by $\overline{\mathrm{A\,W}}$ (or by W simply), which rests on the base D E, consists of the weight of the masonry *together with the weight of the mass of earth, if any* (represented by O L M), *which is vertically above that base;* and G is the common centre of gravity of the compound mass of masonry and earth, which is situated in front of the plane O D.

In fig. 170, on the other hand, a part of the masonry, represented by D L O, lies *behind* the plane O D. If the prism D L O consisted of earth, its weight would be supported by the earth beneath it; therefore the earth beneath that prism exerts a pressure vertically upwards sufficient to sustain the weight of a prism of earth of a volume equal to that of the prism of masonry; therefore the weight represented by A W (or by W simply), which rests on the base D E, consists of the weight of the masonry in the vertical layer of the wall, *less* the weight of earth which would fill D L O; and G is the common centre of gravity of the masonry E D O which lies before the plane O D, and of the prism D L O, considered as having a heaviness equal to the *excess of the heaviness of masonry above that of earth.*

It has already been shown in Article 183, Division IV., p. 323, that the pressure of the earth against the vertical plane O D (which pressure is parallel to the surface of the bank, and represented by A P, or by P simply), is equal to the weight of the prism of earth O D K, in which D K, parallel to the surface of the bank, is equal to the vertical depth O D multiplied by the ratio of the conjugate pressures at a point,

$$\frac{p'}{p} = \frac{\cos\theta - \sqrt{(\cos^2\theta - \cos^2\varphi)}}{\cos\theta + \sqrt{(\cos^2\theta - \cos^2\varphi)}},$$

which ratio depends on the slope θ of the bank, and angle of repose φ; and that the resultant of that pressure traverses C, at the height

$$\overline{\mathrm{C\,D}} = \frac{\overline{\mathrm{O\,D}}}{3} = \frac{x}{3}$$

above D. For the sake of brevity (w' being the weight of unity of volume of the earth), let

$$w' \cos\theta \frac{p'}{p} = w_1;$$

then equation 18 of Article 183, p. 324, becomes

$$\mathrm{P} = \frac{w_1 x^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

This force has to be multiplied, as in Article 263, by the perpendicular distance of F from C P, to give the moment of the couple which tends to overturn the wall. Let t be the thickness $\overline{\mathrm{D\,E}}$, and j the angle of inclination of D E to the horizon; then the arm of the couple in question is

$$\frac{x\cos\theta}{3} - \left(q + \frac{1}{2}\right) t \cdot \sin(\theta + j);$$

which being multiplied by the force P, and equated to the moment of stability of the weight which rests on the base D E, gives the following condition of stability of position:—

$$\mathrm{W}\,(q \pm q')\, t \cdot \cos j = \frac{w_1 x^3 \cos\theta}{6} - \frac{w_1 x^2 t}{2}\left(q + \frac{1}{2}\right) \sin(\theta + j) .. (2.)$$

Now suppose (as in Article 263, p. 398) that W bears a definite ratio n to the weight $w\,x\,t \cdot \cos j$ of a rectangle of masonry whose height is $\overline{\mathrm{O\,D}} = x$, and its breadth the horizontal distance of E from O D, $t \cos j$; then the first side of equation 2, being the moment of stability, becomes as follows:—

$$n\,(q \pm q')\, w\, x\, t^2 \cos^2 j.$$

Divide both sides of the equation by

$$n\,(q \pm q')\, w\, x^3 \cos^2 j,$$

and for brevity's sake, let

$$\frac{w_1 \cdot \cos \theta}{6\, n\, (q \pm q')\, w \cos^2 j} = a;$$

$$\frac{w_1 \left(q + \frac{1}{2}\right) \sin (\theta + j)}{4\, n\, (q \pm q')\, w \cos^2 j} = b;$$

then

$$\frac{t^2}{x^2} = a - 2\, b\, \frac{t}{x} \text{.........................} (3.)$$

and consequently

$$\frac{t}{x} = \sqrt{a + b^2} - b \text{.......................} (4.)$$

The inclination of the resultant A R to the vertical is given by the equation

$$\tan \angle \text{W A R} = \frac{\text{P} \cos \theta}{\text{W} + \text{P} \sin \theta} \text{..................} (5.)$$

When the base D E is horizontal, this should not exceed the tangent of the angle of repose. When that base is inclined at the angle j, the condition of frictional stability is thus expressed:—

$$\angle \text{W A R} - j \leqq \varphi'; \text{.....................} (6.)$$

φ' being the angle of repose of the foundation of the wall.

The object of giving the base of the wall an inclined position is to diminish the obliquity of the pressure on it, and so to enable the condition of frictional stability to be fulfilled.

As to the values of q in practice, see Article 263, pp. 396, 397.

266. **Stability of Upright Rectangular Retaining Walls.** (*A. M.*, 218.)—In a vertical rectangular wall, $n = 1$, $q' = 0$, $j = 0$; so that, in equations 3 and 4 of Article 265,

$$a = \frac{w_1 \cos \theta}{6\, q\, w};\ b = w_1 \left(q + \frac{1}{2}\right) \sin \theta \div 4\, q\, w \text{.........} (1.)$$

Case I. When the surface of the bank is horizontal, so that $\theta = 0$, then

$$w_1 = w' \frac{1 - \sin \varphi}{1 + \sin \varphi};\ b = 0;$$

and the proportion of the thickness of the wall to its height is

$$\left.\begin{aligned} \frac{t}{x} = \sqrt{a} &= \sqrt{\left\{\frac{w'(1-\sin\phi)}{6\,q\,w\,(1+\sin\phi)}\right\}} \\ &= \tan\left(\frac{90^\circ-\phi}{2}\right)\sqrt{\frac{w'}{6\,q\,w}}. \end{aligned}\right\}\dots\dots\dots\dots(2.)$$

Equation 5 of Article ~~217~~ 265 becomes

$$\left.\begin{aligned} \tan \angle \mathrm{W\,A\,R} &= \frac{\mathrm{P}}{\mathrm{W}} = \frac{w_1 x}{2\,w\,t} \\ &= \sqrt{\left\{\frac{3\,q\,w'(1-\sin\phi)}{2\,w\,(1+\sin\phi)}\right\}} \leqq \tan\phi' \end{aligned}\right\}\dots\dots\dots(3.)$$

If the material on which the wall rests is the same with that of the bank, we may assume $\phi' = \phi$; in which case, by squaring equation 3, and attending to the fact that

$$\tan^2\phi = \frac{\sin^2\phi}{1-\sin^2\phi} = \left(\frac{\sin\phi}{1-\sin\phi}\right)^2 \cdot \frac{1-\sin\phi}{1+\sin\phi},$$

we obtain the equation

$$\frac{3\,q\,w'}{2\,w} \leqq \left(\frac{\sin\phi}{1-\sin\phi}\right)^2 \dots\dots\dots\dots\dots\dots\dots\dots(4.)$$

Assuming that the specific gravity of the earth is four-fifths of that of the masonry, or $w \div w' = 5 \div 4$, we find that this equation is fulfilled for the ordinary value of q, $3 \div 8$, so long as ϕ exceeds 27°. Should equation 4 not be fulfilled for $q = 3 \div 8$, a smaller value of q must be determined by the following equation:—

$$q = \frac{2\,w}{3\,w'} \cdot \left(\frac{\sin\phi}{1-\sin\phi}\right)^2, \dots\dots\dots\dots\dots\dots\dots\dots(5.)$$

and introduced into equation 2 to find the ratio $t \div x$.

CASE II. When the surface of the bank slopes at the angle of repose ϕ, then $w_1 = w' \cos\phi$, and

$$a = \frac{w' \cos^2\phi}{6\,q\,w};\; b = \frac{\left(q+\frac{1}{2}\right) w' \cos\phi \sin\phi}{4\,q\,w}; \dots\dots\dots(6).$$

which values, being introduced into equation 4 of Article 265, p. 404, give the ratio $t \div x$.

267. **Stability of Battering-faced Retaining Walls.** (*A. M.*, 219.)—In fig. 171, let E Q represent the vertical face of a rect-

angular wall, suited to sustain the thrust of a given bank, and let F be the centre of resistance of the base. Make $\overline{QN} = 3\,\overline{EF} = 3\left(\frac{1}{2} - q\right) t$; then the centre of gravity g of the triangular prism of masonry E Q N will be vertically above the centre of resistance F; therefore if that prism be removed, so as to reduce the cross-section of the wall to a trapezoid with a battering face E N, the position of the centre of resistance F will not be altered, and the wall will still fulfil the condition of stability of position, the thickness t being determined as for a rectangular wall. The thickness of the wall at the summit is

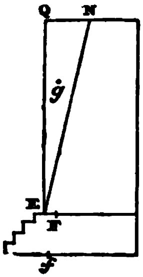


Fig. 171.

$$\left(3\,q - \frac{1}{2}\right) t.$$

The tangent of $\angle$ W A R (the inclination of the resultant pressure to the vertical) is increased in the ratio $\frac{1}{4} + \frac{3\,q}{2} : 1$; so that it may in some cases be necessary to make the base slope backwards, as in fig. 170.

268. **Stability of Battering Walls of Uniform Thickness.** (*A. M.*, 220.)—When a wall for supporting a *horizontal-topped* bank is of uniform thickness, and has a sloping or curved face, as in figs. 172 and 173, its moment of stability may be determined with a degree of accuracy sufficient for practical purposes, in the following manner:—

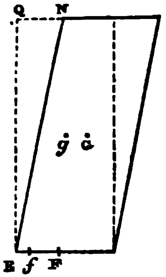


Fig. 172.

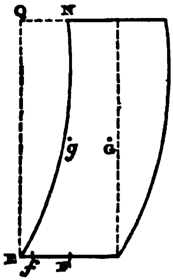


Fig. 173.

Let E Q in each figure represent the vertical face of a rectangular wall of the same height x and thickness t with the proposed wall, and let g be the centre of gravity of that rectangular wall. Then

$$\mathrm{W} \cdot q\, t = q\, w\, x\, t^2$$

will be its moment of stability per unit of length.

Divide the area E Q N included between the vertical face E Q and the face of the proposed wall, E N, by the height x. Then

$$\therefore\ q'\, t = \overline{g\,G} = \frac{\mathrm{E\,Q\,N}}{x}, \text{ .... (1.)}$$

will be the distance of the centre of gravity G of the sloping or curved wall from that of the rectangular wall; and the change of figure will increase the stability in the ratio $q + q' : q$; that is to say, the moment of stability will now be

$$\mathrm{W}\,(q + q')\,t = (q + q')\,w\,x\,t^2. \dots\dots\dots\dots\dots (2.)$$

If E N is a straight line (fig. 103),

$$q'\,t = \frac{\overline{\mathrm{Q\,N}}}{2} \dots\dots\dots\dots\dots\dots\dots\dots (3.)$$

If E N is a parabolic arc,

$$q'\,t = \frac{2\,\overline{\mathrm{Q\,N}}}{3}; \dots\dots\dots\dots\dots\dots\dots (4.)$$

a formula which is also sensibly correct when E N is an arc of a circle.

Walls with a "curved batter" are usually built as shown in fig. 174, with the bed-joints perpendicular to the face of the wall. This diminishes the obliquity of the pressure on the base.

269. **Counterforts** (*A. M.*, 222) are projections from the inner face of a retaining wall. A wall and its counterforts, if the bond of the masonry is well preserved by means of long bond-stones connecting the counterforts with the wall, are equivalent to a wall having successive divisions of its length alternately of greater and of less thickness. The moment of stability of such a wall, per unit of length, when the wall is well bonded, may be found, with sufficient accuracy for practical purposes, by adding together the moments of stability of one of the parts between two counterforts, and of one of the parts whose thickness is augmented

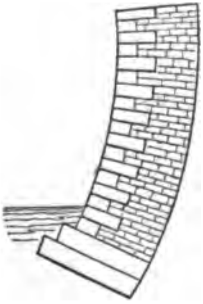


Fig. 174.

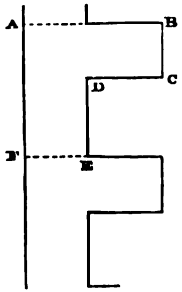


Fig. 175.

by the addition of a counterfort, and dividing the sum by the joint length of those two parts.

For example, let fig. 175 represent a portion of the *plan*, or horizontal section, of a vertical rectangular retaining wall whose height is h, with a row of rectangular counterforts of the same height with the wall. Let $t = \overline{\mathrm{F\,E}}$ be the thickness of a part of the wall between two counterforts, and $b = \overline{\mathrm{E\,D}}$ its length; let $\mathrm{T} = \overline{\mathrm{A\,B}}$ be the thickness of a counterforted part of the wall, including the counterfort, and $c = \overline{\mathrm{B\,C}}$ its length.

The moment of stability of the first part is

$$q\,w\,h\,b\,t^2;$$

and that of the second part,

$$q\,w\,h\,c\,\mathrm{T}^2.$$

Adding together those moments, and dividing their sum by the total length $b + c = \overline{\mathrm{A\,F}}$, the mean moment of stability per unit of length is found to be

$$q\,w\,h \cdot \frac{b\,t^2 + c\,\mathrm{T}^2}{b + c} \text{..........................(1.)}$$

This is the same with the moment of stability per unit of length of a wall of the uniform thickness,

$$t_1 = \sqrt{\left\{\frac{b\,t^2 + c\,\mathrm{T}^2}{b + c}\right\}} \text{.....................(2.)}$$

which may be called the *equivalent uniform wall.*

The quantity of masonry in the counterforted wall is to the quantity in the equivalent uniform wall in the ratio

$$b\,t + c\,\mathrm{T} : (b + c)\,t_1,$$

which is always less than unity; so that there is a saving of masonry (though in general but a small one) by the use of counterforts.

270. **Surcharged Retaining Wall.**—This term is applied to a wall for supporting a bank of earth which rises from the top of the wall at the natural slope for a certain height, called the *height of the surcharge*, beyond which it is horizontal. The thrust of such a bank is intermediate in amount and in direction between that of a horizontal-topped bank and that of a bank with an indefinitely long natural slope.

The following formula serves to compute approximately the thickness of an upright rectangular retaining wall for supporting such a bank:—

Let x, as before, denote the height of the wall,

c the height of the surcharge,

t the thickness required to sustain a horizontal bank, as computed by equation 2 of Article 266, p. 405,

t' the thickness required to sustain a bank with an indefinitely long natural slope, as computed by equations 6 of Article 266, p. 405, and 4 of Article 265, p. 404,

t'' the thickness required for the surcharged wall; then

$$t'' = \frac{x\,t + 2\,c\,t'}{x + 2\,c}, \text{ nearly} \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

When the foot of the slope of the bank rests on the top of the wall, nearly above the centre of resistance of the base, the following formula may be used:—

$$\frac{t''}{x} = \cos\varphi . \sqrt{\left(\frac{w'}{6\,q\,w}\right) \cdot \left(\frac{x}{1 + \sin\varphi} + 2\,c\right)} \div (x + 2\,c) \ldots . (2.)$$

271. **Construction of Retaining Walls.**—The foundation courses of retaining walls have their width increased beyond the thickness of the wall by a series of steps in front, as shown in figs. 171 and 174. The objects of this are, at once to distribute the pressure over a greater area than that of any bed-joint in the body of the wall, and to diffuse that pressure more equally, by bringing the centre of resistance nearer to the middle of the base than it is in the body of the wall, according to the principles already explained in Section IV. of this Chapter, pages 377 to 382.

The body of the wall may be either entirely of brick, or of ashlar backed with brick or with rubble, or of block-in-course backed with rubble, or of coursed rubble, built with mortar or built dry. As the pressure at each bed-joint is concentrated towards the face of the wall, those combinations of masonry in which the larger and more regular stones form the face, and sustain the greater part of the pressure, and are backed with an inferior kind of masonry, whose use is chiefly to give stability by its weight, are well suited for retaining walls (see Article 246, p. 387), special care being taken that the back and face are well tied together by long headers, and that the beds of the facing stones extend into the wall to a distance of about as far inwards from the centre of pressure at the base of the wall as that centre of pressure lies inwards from the face.

Along the base and in front of a retaining wall there should run a drain, like that at the foot of the slope of a cutting. (See Article 193, p. 335.) In order to let water escape from behind the wall, it has small upright oblong openings through it, called "weeping-holes," which are usually two or three inches broad, and of the depth of a course of masonry, and are distributed at regular distances, an ordinary proportion being one weeping-hole to every 4 square yards of face of wall.

The back of a retaining wall should be rough, in order to resist any tendency of the earth to slide upon it. This object is promoted by building the back in steps, as exemplified in fig. 169.

In retaining walls of great size, both back and face may be built of block-in-course, or the face of ashlar and the back of block-in-course, the "heart" of the wall being of coursed rubble, or of beton or strong concrete laid in regular courses of the same depth with those of the face and back.

When the material at the back of the wall is clean sand or gravel, so that water can pass through it readily, and escape by the weeping-holes, it is only necessary to ram it in layers, as already described in Articles 198 and 200, pp. 341, 342. But if the material is retentive of water, like clay, a *vertical layer* of stones or coarse gravel, at least a foot thick, or a dry stone rubble wall, must be placed at the back of the retaining wall, between the earth and the masonry, to act as a drain.

A catchwater drain behind a retaining wall is often useful. It may either have an independent outfall, or may discharge its water through pipes into the drain in front of the base of the wall.

When the material at the back of the wall is of a loamy description, and liable to be reduced to quicksand or mud by saturation with water, and there are no means of preventing such saturation by efficient drainage, one way of making provision to resist the additional pressure which may arise from such saturation is to calculate the requisite thickness of wall, as if the earth were a fluid, making $\phi = 0$ in the formulæ.

Another way of providing against such a contingency is to construct, sloping against the back of the wall, a bank of shivers of stone or of coarse gravel, whose angle of repose is not affected by the presence of water, and then to fill in the softer material. The pressure against the wall in this case will not at any time greatly exceed that of a bank of the firm material employed, sloping at its own angle of repose.

The subject of relieving retaining walls from pressure by the aid of arches, and that of securing their foundations by special contrivances in swampy ground, will be considered further on.

The cope of a retaining wall should consist of large flat stones laid as headers.

272. **Land Ties for Retaining Walls.**—Retaining walls having to bear a great pressure, while they rest on a yielding foundation, may have their stability increased by being tied or anchored by iron rods to vertical or nearly vertical plates of iron imbedded in a firm stratum of earth at a distance behind the wall sufficient

to prevent its being disturbed by the operations of excavation, building, and embanking, connected with the erection of the wall.

The holding power, per foot of breadth, of a rectangular vertical anchoring plate, the depths of whose upper and lower edges below the surface are respectively x_1 and x_2, may be approximately calculated from the following formula:—

Let w' be the weight of a cubic foot of the earth;
φ' its angle of repose;
H, the holding power per foot of breadth; then

$$H = w' \cdot \frac{x_2^2 - x_1^2}{2} \cdot \frac{4 \sin \varphi}{\cos 2 \varphi} \text{.....................} (1.)$$

The depth of the centre of pressure of the plate below the surface of the ground is given by the following expression:—

$$\frac{2}{3} \cdot \frac{x_2^3 - x_1^3}{x_2^2 - x_1^2}; \text{............................} (2.)$$

and to that centre the tie-rod should be attached.

If the retaining wall depends on the tie-rods alone for its security against sliding forward, they should be fastened to plates on the face of the wall in the line of the resultant pressure of the earth behind the wall; that is, at one-third of the height of the wall above its base. But if the resistance to sliding forward is to be distributed between the foundation and the tie-rods, they are to be placed at a higher level; for example, if half the horizontal thrust is to be borne by the foundation, and half by the tie-rods, the latter should be fixed to the wall at two-thirds of its height above the base.

273. **Struts for Retaining Walls.**—The base of a retaining wall may be prevented from sliding forward by a series of horizontal struts of masonry or brickwork, abutting against rectangular masses whose resistance to displacement depends on the same principles with the holding power of anchoring plates, stated in the last article.

When a cutting in soft ground has a retaining wall at each side of it, the foundations of the walls may be kept asunder, and thus prevented from sliding forward, by means of a series of inverted arches extending between them, across and below the base of the cutting, so as to act as transverse struts.

The upper parts of such walls may also be held asunder by slightly arched ribs of cast iron or of brick. These ribs abut against the walls at about two-thirds of their height above their base.

274. **Relieving Arches.**—A row of arches having their axes and the faces of their piers at right angles to the face of a bank of earth are called "relieving arches." There may be either one or several tiers of them, and their front ends may be closed by a vertical wall, which thus presents the appearance of a retaining wall, although the length of the archways is such as to prevent the earth from abutting against it. Fig. 175 represents a vertical transverse section of such a wall, with two tiers of relieving arches behind it. To compute the length of a relieving arch from its clear height, or its clear height from its length, the following approximate formulæ may be used, in which

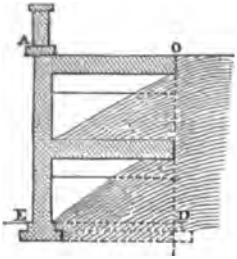


Fig. 175.

x denotes the depth of the crown of an arch below the surface,
h, its clear height,
l, its length, and
ϕ, the angle of repose of the earth.

$$l = \text{cotan}\,\varphi\left(h + \frac{x}{(1+\sin\varphi)^2}\right);\ \ldots\ldots\ldots\ldots\ldots(1.)$$

$$h = l\cdot\tan\varphi - \frac{x}{(1+\sin\varphi)^2}.\ \ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

To determine the conditions of stability of such a structure as a whole, the horizontal pressure against the vertical plane O D may be determined, and compounded with the weight of the combined mass of masonry and earth O A E D in front of that plane, to find the resultant pressure on the base.

In soft ground the bases of the piers of the lowest tier of relieving arches should be connected by means of inverted arches, so as to distribute the pressure over the whole area covered by the structure.

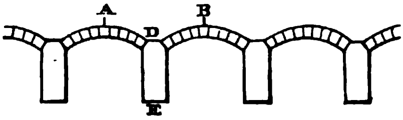


Fig. 176.

275. **Buttressed Horizontal Arches.**—Fig. 176 represents a plan, or horizontal section, of part of a row of buttresses, connected by horizontally arched walls.

To find the thickness, T = D E, required for such buttresses, let

B denote A B, the breadth of the mass of earth which one buttress has to sustain;
b, the breadth of the buttress;

t, the thickness which would be required for an uniform wall, to sustain the same bank of earth, computed as in Article 266, equation 2, p. 405; then

$$T = t \cdot \sqrt{\frac{B}{b}}; \; b = B \cdot \frac{t^2}{T^2} \text{ ....(1.)}$$

In soft ground the bases of the buttresses may be connected by means of inverted arches, to distribute the pressure; and their tops may, if necessary, be connected by means of arches, in order to support a platform, or a surcharged bank of earth. In the last-mentioned case, t is to be computed as in Article 270, p. 409.

SECTION VIII.—*Of Stone and Brick Arches.*

276. **General Structure of Arches of Stone.** (*A. M.*, 223.)—An arch of masonry consists of a sector of a ring, composed of courses of wedge-formed stones, called *arch-stones* or *voussoirs*, pressing against each other at surfaces called *bed-joints*, which are, or ought to be, perpendicular, or nearly perpendicular, to the *soffit*, or internal concave surface of the arch. The soffit is also called, in mathematical language, the *intrados*. The word *extrados* is applied sometimes to the upper surface of the ring of arch-stones; sometimes to that of the solid masonry or *backing* above them; sometimes to that of the entire mass of permanent loading material. (See also p. 203.) The outer or convex surface of the ring of arch-stones, which may be either a curved surface or a series of steps, sustains the vertical pressure of that part of the load which arises from the weight of materials other than the arch-stones themselves; and that outer surface also exerts in many cases a horizontal or inclined thrust against the *spandrils* and *abutments*. The abutments sustain also the thrust of the lowest voussoirs, vertical or inclined, as the case may be. The course of stones from which an arch springs is called the *springing-course* or *skew-back*, the latter term being used when its upper and lower beds are oblique to each other. Sometimes an arch springs at once from the ground, so that its abutments are its foundations.

A wall standing on an arch, in the plane of the arch-ring, is called a *spandril wall*. The arch of a bridge requires a pair of *external spandril walls*, one over each *face* of the arch; the space between them is filled up to a certain level with solid masonry, and above that level it is sometimes filled with earth or rubbish, and sometimes occupied by a series of *internal spandril walls* parallel to the external spandril walls, and having vacant spaces between them—a mode of construction favourable both to stability and to lightness. In order to form a continuous platform for the roadway, the spaces between the internal spandril walls are sometimes covered with flags of some strong stone (such as slate), and sometimes arched over with small transverse arches. The external spandril walls are the abutments of those arches, and must have stability sufficient to sustain their thrust: when the spandrils are filled with earth or rubbish, the external spandril walls must have stability sufficient to withstand the pressure of the filling material.

277. **Masonry of Arches—Backing.**—The description of masonry used for arches is either ashlar or block-in-course; the beds being perpendicular or nearly perpendicular to the direction of the thrust through the arch-ring, and the side-joints perpendicular to the beds and to the soffit. In common, or *square* arches, in which the axis of the archway is perpendicular to each face of the arch, the bed-joints are plane; in oblique, or *skew* arches, they are curved surfaces, shaped according to principles which will be explained in a later article.

The principles according to which the masonry of arches is to be built are in other respects the same with those already explained in Articles 242 and 243, pp. 384 to 386. Special care is to be taken to cut and lay the beds of the stones accurately, and to make the bed-joints thin and close, in order that the arch may be strained as little as possible by settling. To insure this, some engineers have caused arches to be built dry, *grout* or liquid mortar being afterwards run into the joints; but the advantage of this method is doubtful. Others have placed sheets of lead in the bed-joints, to distribute the pressure between the stones.

The *backing* of an arch consists of block-in-course, coursed rubble, or random rubble, and sometimes of concrete. When the backs of the arch-stones are cut into steps, the backing is built in courses of the same depth with those steps, and thus bonded with them. Sometimes the backing is built in radiating courses, whose beds are prolongations of the bed-joints of the arch-stones. Both these methods are favourable to strength and stability.

The height to which the solid backing should be built is regulated by principles which will be explained in subsequent articles.

The upper surface of the backing, and of that part of the arch, if any, near the crown, which is without backing, is *coated* with a layer of waterproof material, such as clay puddle (Article 206, p. 344), mixed cement (Article 231, p. 374), or bituminous concrete (Article 234, p. 376); the last being the best. Any rain-water which penetrates the structure above the arch flows to the valleys or lowest parts of this coating, whence it is carried away by tubes or other convenient outlets.

278. **Brick Arches** may be built either of gauged or wedge-formed bricks, moulded or rubbed so as to suit to the radius of the soffit, or of bricks of the common shape. In the former case, the methods of bonding the bricks are the same with those employed in walls (Article 255, p. 394); in the latter, the bricks are accommodated to the curved figure of the arch by making the bed-joints thinner towards the intrados than towards the extrados, or, if the curvature is sharp, by driving thin pieces of slate into the outer edges of those joints; and different methods are followed for bonding them. The most common way is to build the arch in concentric rings, each half-a-brick thick: this is, in fact, to lay the bricks all stretchers, and to depend upon the tenacity of the mortar or cement for the connection of the several rings. It is deficient in strength, unless the bricks are laid in cement at least as tenacious as themselves. Another way is to introduce courses of headers at intervals, so as to connect pairs of half-brick rings together. This may be done either by thickening the joints of the outer of a pair of half-brick rings with pieces of slate, so that there shall be the same number of courses of stretchers in each ring between two courses of headers; or by placing the courses of headers at such distances apart that between each pair of them there shall be one course of stretchers more in the outer than in the inner ring. The former method is the best suited to arches of long radius, the latter to those of short radius.

Hoop-iron bond (Article 256, p. 395), laid *round the arch*, between half-brick rings, as well as longitudinally and radially, is very useful for strengthening brick arches. The bands of hoop-iron which traverse the arch radially may be bent, and prolonged in the bed-joints of the backing and spandrils. By the aid of hoop-iron bond Sir Marc-Isambard Brunel built a half-arch of bricks laid in strong cement, which stood projecting from its abutment like a bracket, to the distance of 60 feet, until it was destroyed by its foundation being undermined.

279. **Use of Centres.**—A centre is a temporary structure of timber or iron (but most commonly of timber), by which the voussoirs of an arch are supported until the arch is completed, and capable of supporting itself. The principles of the strength, stability, and construction of centres will be explained under the head of CARPENTRY.

A centre consists of parallel frames or *ribs* about 5 or 6 feet apart, curved on the outside to a figure parallel to that of the soffit of the arch, and supporting a series of transverse planks called *laggings*, upon which the archstones directly rest.

The oldest and most common kind of centre is one which can be lowered or "struck" all in one piece, by driving out wedges from below it, so as to remove the support from below every part of the arch at once. An improved kind was first introduced by Hartley, in which *each lagging* is supported upon the ribs by wedges or screws of its own, so that the support can be removed from the arch-stones course by course, and replaced in the event of the settlement proving too rapid.

The centre of an arch should not be struck until the solid part of the backing has been built, and the mortar has had time to set; and when an arch forms one of a series of arches, with piers between them, no centre should be struck so as to leave a pier with an arch abutting against one side of it only, unless the pier is what is termed an "abutment-pier;" that is, a pier which has sufficient stability to act as an abutment.

280. **Line of Pressures in an Arch—Condition of Stability.** (*A. M.*, 224.)—If a straight line be drawn through each bed-joint of the arch-ring, representing the position and direction of the resultant of the pressure at that joint, the straight lines so drawn form a polygon, and each of the angles of that polygon is situated in the line of action of the resultant external force acting on the arch-stone which lies between the pair of joints to which the contiguous sides of the polygon correspond; so that the polygon is similar to a polygonal frame, loaded at its angles with the forces which act on the arch-stones (their own weight included). A curve inscribed in that polygon, so as to touch all its sides, is the *line of pressures* of the arch. The smaller and the more numerous the arch-stones into which the arch-ring is subdivided, the more nearly does the polygon coincide with the curve; and the curve or line of pressures represents an ideal *linear arch*, which would be balanced under the continuously-distributed forces which act on the real arch under consideration. From the near approach of this linear arch to the polygon whose sides traverse the centres of resistance of the bed-joints, the points where the linear arch cuts those joints may be taken without sensible error for the centres of resistance.

Now in order that the stability of the arch may be secure, it is necessary that no joint should tend to open either at its outer or at its inner edge; and in order that this may be the case, the centre of resistance of each joint should not deviate from the centre of the joint by more than one-sixth of the depth of the joint; that is to say, the centre of resistance should lie within the *middle third* of the depth of the joint; whence follows this

THEOREM. *The stability of an arch is secure, if a linear arch, balanced under the forces which act on the real arch, can be drawn within the middle third of the depth of the arch-ring.* It is through this theorem that the principles of the stability of ideal linear arches or ribs, already explained in Article 132, p. 202, and the previous articles referred to in that article, and also in Articles 133 to 139, pp. 203 to 218, become applicable to real arches of masonry and brickwork.

It may be held that in most practical examples the tenacity of fresh mortar is not sufficiently great to be taken into account in determining the stability of masonry; and hence, where cement is not used, all horizontal or oblique conjugate forces which maintain the equilibrium of the arch-ring must be pressures, acting on the arch from without inwards. The linear arch, therefore, is limited in such cases to those forms which are balanced under *pressures from without alone;* that is to say, that the intensity of the horizontal or conjugate pressure, denoted by p_y in Article 138, equation 4, p. 214, *must not at any point be negative.*

It is true that arches have stood, and still stand, in which the centres of resistance of joints fall beyond the middle third of the depth of the arch-ring; but the stability of such arches is either now precarious, or must have been precarious while the mortar was fresh.

When tenacity to resist horizontal or oblique tension is given to the spandrils of an arch, and to the joints between them and the arch-stones, by means of cement, hoop-iron bond, iron cramps, or otherwise, the conjugate tension denoted by $-p_y$ must not at any point exceed a safe proportion of that tenacity; that is to say, about one-eighth. By this means stability may be given to arches of seemingly anomalous figures; but such structures are safe on a small scale only.

281. **Relation between Linear Rib and Intrados of Real Arch.**—There are numerous cases in which the form of a linear rib, suited to sustain a given load, may at once be adopted for the intrados of a real arch for sustaining the same load, with sufficient exactness for practical purposes. The following is the test whether this method is applicable in any given case. Let A C B in fig. 177 be one half of the ideal rib which it is proposed to adopt as the intrados of a real arch. Draw A a normal to the rib at the crown, so as to represent a length not exceeding two-thirds of the intended depth of the keystone, and conceive a couple applied to the keystone, consisting of tension at A equal and opposite to the thrust along the rib there, and of an equal thrust at a. Draw a normal B b at the springing, and make

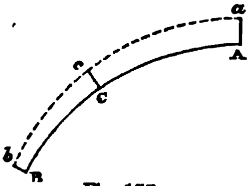


Fig. 177.

$$\frac{\mathrm{B}\,b}{\mathrm{A}\,a}=\frac{\text{thrust along rib at A}}{\text{thrust along rib at B}},$$

and conceive a couple of equal moment to the first, consisting of tension at B and thrust at b, to be applied at the springing. The pair of couples thus introduced, being equal and opposite, do not alter the equilibrium, their only effect being to transfer the line of pressures from the intrados or ideal rib A C B to a line $a\,c\,b$, whose perpendicular distance C c from the intrados at any point is inversely as the thrust along the rib at that point. Then if $a\,c\,b$ lies within the middle third of the arch-ring, the ideal rib A C B is of a suitable form for the intrados.

The process may, if required, be commenced by making B b not less than one-third of the depth of the arch at the springing, that depth having first been fixed with due regard to the conditions of resistance to crushing, which will be considered further on.

282. **Use of Equilibrated or Transformed Catenary Arches.**—The transformed catenary has already been fully described in Article 131, pp. 200 to 202. When used for the intrados of an arch, it is commonly called the "curve of equilibrium." It is suited for the support of any load whose pressure is wholly vertical, and whose extrados is either a horizontal plane coinciding with the *directrix* O X of the transformed catenary (fig. 114), or another transformed catenary having the same directrix.

When the method of Article 281 is applied to an intrados of this figure, the resulting curve, $a\,c\,b$ of fig. 177, is simply the same curve shifted vertically upwards through the height A a.

In making use of the transformed catenary in practice, there are usually given, the directrix, the crown of the arch, and the points from which it is to spring. From these data the *modulus* m is to be computed by means of equation 4 of Article 131, p. 202; and then the vertical ordinate y from the directrix at any given horizontal distance x from the crown of the arch is given by the formula,—

$$y = \frac{y_0}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right); \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

in which y_0 is the depth of the curve of the arch below the directrix. (See table, Article 298, p. 436.)

In applying the formulæ (3) of Article 131, p. 202, to this arch, *thrust* is to be read instead of *tension;* and the symbol w is to be understood to stand for *the load per square foot of the vertical area between the intrados and the directrix.*

For example, let B denote the breadth of the arch, or length of the archway, in feet; let w_1 be the weight of a cubic foot of the masonry; let the extrados be a transformed catenary, each of whose ordinates, measured from the directrix, is equal to the corresponding ordinate of the intrados multiplied by a fraction n; and let a fraction k be the ratio of the volume of solid building to the whole volume, the remainder $1 - k$ consisting of spandril-voids. Then,

$$w = n\,k\,w_1. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

In order that the arch may be equilibrated under its own weight and that of the solid backing alone, as well as when the whole structure is finished, the figure of the upper surface of the solid backing should itself either be a transformed catenary, or approximate to that curve.

283. **Use of the Hydrostatic Arch.**—The mathematical and mechanical properties of this arch, considered as a linear rib, have been explained in Article 136, pp. 208 to 212.

Inasmuch as the thrust through this arch is uniform, the application of the method of Article 281 to it produces simply a curve parallel to it; so that if it be used for the intrados of an arch-ring of uniform thickness, and the centre of resistance at the keystone be at the middle of the thickness, the line of pressures will be at the middle of the thickness of the arch-ring throughout, or approximately so. The word "approximately" is used, because the thrust along the real arch is not exactly uniform, like that in the ideal rib; for at the springing it is greater than at the crown, by an amount equal to the weight of the prism of masonry which stands vertically above the springing-course; but that difference is practically unimportant.

The application of the hydrostatic arch to practice is founded on the fact, that every arch, after having been built, subsides at the crown, and spreads, or tends to spread, at the haunches, which therefore press horizontally against the filling of the spandrils; from which it is inferred as probable, that if an arch be built of a figure suited to equilibrium under fluid pressure—that is, pressure of equal intensity in all directions—it will spread horizontally, and compress the masonry of the spandrils, until the horizontal pressure at each point becomes of equal intensity to the vertical pressure, and therefore sufficient to keep the arch in equilibrio.

In addition to the methods already explained in Article 136 for drawing the figure of the arch, the following method may be given for describing an approximation to it about five centres. It is very simple, and has been found by trial to answer well.

In fig. 178, let F B be the half-span and F A the rise of the proposed arch. Make A C = ϱ_0, and B D = ϱ_1, the radii of curvature at the crown and springing, as calculated by the formulæ (11 and 12) of Article 136, p. 211.* Then C will be one of the

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* **The first of the formulæ (11), p. 211, *ought to be* as follows:—**

$$b = y_1 + \frac{y_1^2}{30\,a}.$$

In the second of the formulæ (12), for 2 x_2 read 2 x_1.

centres, and D another. About D, with the radius D E = F A − B D, describe a circular arc, and about C with the radius C E = C F, describe another circular arc; let E be the point of intersection of those arcs; this will be a third centre; and two more centres will be similarly situated to D and E with respect to the other half-arch.

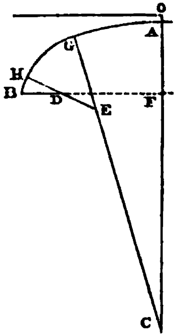

Fig. 178.

Then about C with the radius C A, draw the circular arc A G till it cuts C E produced in G; about E, with the radius E G = F A, draw the circular arc G H till it cuts E D produced in H; about D, with the radius D B, draw the circular arc H B. This completes one half-arch, and the other is drawn in the same manner.

The curve thus described falls a little beyond the true hydrostatic arch at G, and a little within it at H.

To give the greatest possible security to a hydrostatic arch, especially if the span is great compared with the rise, the backing ought to be built of solid rubble masonry up to the level of the crown of the extrados, before the centre is struck.

Many *semi-elliptic arches* may be treated as approximate hydrostatic arches. In fact, many of the arches called semi-elliptic approximate more nearly to the figure of the hydrostatic arch than to that of the true semi-ellipse. The true semi-ellipse of a given span and rise differs from the hydrostatic arch by being of somewhat sharper curvature at the crown and springing, and somewhat flatter curvature at the haunches, and by enclosing a somewhat less area.

284. **Use of the Geostatic Arch.**—The derivation of the figure of this arch, by transformation from that of the hydrostatic arch, and most of its properties, have been explained in Article 137, pp. 212, 213. The following problem only remains to be solved. Given in a geostatic arch, the rise a, the half-span s, and the depth of load at the crown x_0; it is required to find the proportion c, which the half-span and other horizontal dimensions bear to the corresponding dimensions of a hydrostatic arch whose vertical dimensions are the same:—

Make

$$b = a \cdot \left(\frac{x_0 + a}{x_0}\right)^{\frac{1}{3}}; \text{ and } y_1 = b - \frac{b^2}{30\,a}; \text{ then}$$

$$c = \frac{s}{y_1} \text{ nearly. } \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

This method may be applied to those *semi-elliptic arches* which are not approximate hydrostatic arches.

285. **Stability of any proposed Arch.** (*A. M.*, 225.)—The first step towards determining whether a proposed arch will be stable, is to *assume* a linear arch parallel to the intrados or soffit of the proposed arch, and loaded vertically with the same weight, distributed in the same manner. The *size* of this assumed linear arch is a matter of indifference, provided each point in it is considered as subjected to the same forces which act at the *corresponding joint* in the real arch; that is, *the joint at which the inclination of the real arch to the horizon is the same with that of the assumed arch at the given point.*

The assumed linear arch is next to be treated according to the method of Article 138, pp. 213 to 216; and by equation 4 of that Article, p. 214, is to be determined, either a general expression, or a series of values, of the intensity p_y of the conjugate pressure, horizontal or oblique, as the case may be, required to keep the arch in equilibrio under the given vertical load. If that pressure is nowhere negative, a curve similar to the assumed arch, drawn through the middle of the arch-ring, will be either exactly or very nearly the line of pressures of the proposed arch; p_y will represent, either exactly or very nearly, the intensity of the lateral pressure which the real arch, tending to spread outwards under its load, will exert at each point against its spandril and abutments; and the thrust along the linear arch at each point will be the thrust of the real arch at the corresponding joint.

On the other hand, if p_y has some negative values for the assumed linear arch, there must be a pair of points in that arch where that quantity changes from positive to negative, and is equal to nothing. The angle of inclination i_0 at that point, called the *angle of rupture*, is to be determined by solving Problem IV. of Article 138, pp. 215, 216. The corresponding joints in the real arch are called the *joints of rupture;* and it is below those joints that conjugate pressure from without is required to sustain the arch, and that consequently the backing must be built with squared side joints.

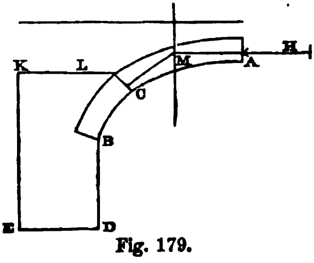

Fig. 179.

In fig. 179, let B C A represent one-half of a symmetrical arch, K L D E an abutment, and C the joint of rupture, found by the method already described. The *point* of rupture, which is the centre of resistance of the joint of rupture, is somewhere within the middle third of the depth of that joint; and from that point down to the springing joint B, the line of pressures is a curve similar to the assumed linear arch, and parallel to the intrados, being kept in equilibrio by the lateral pressure between the arch and its spandril and abutment.

From the joint of rupture C to the crown A, the figure of the true line of pressures is determined by the condition, that it shall be a linear arch balanced under vertical forces only; that is to say, the horizontal component of the thrust along it at each point is a constant quantity, and equal to the horizontal component of the thrust along the arch at the joint of rupture.

That horizontal thrust, denoted by H_0, is found in solving Problem IV., Article 138, p. 213, and is the horizontal thrust of the entire arch.

[If the arch is distorted, *conjugate thrust* is to be read instead of "*horizontal thrust*," wherever that phrase occurs.]

The only point in the line of pressures above the joints of rupture which it is important to determine, is that which is at the crown of the arch, A; and it is found in the following manner:—

Find the centre of gravity of the load between the joint of rupture C and the crown A; and draw through that centre of gravity a vertical line.

Then if it be possible, from one point, such as M, in that vertical line, to draw a pair of lines, one parallel to a tangent to the soffit at the joint of rupture, and the other parallel to a tangent to the soffit at the crown, so that the former of those lines shall cut the joint of rupture, and the latter the keystone, in a pair of points which are both within the middle third of the depth of the arch-ring, the stability of the arch will be secure; and if the first point be the point of rupture, the second will be the centre of resistance at the crown of the arch, and the crown of the true line of pressures.

When the pair of points related to each other as above do not fall at opposite limits of the middle third of the arch-ring, their exact positions are to a small extent uncertain; but that uncertainty is of no consequence in practice. Their most probable positions are equi-distant from the middle line of the arch-ring.

Should the pair of points fall beyond the middle third of the arch-ring, the depth of the arch-stones must be increased.

286. **Circular Arch, not less than a Quadrant.**—In applying the principles of the preceding article to an arch whose intrados is an arc of a circle, it is necessary to modify the equations 9 and 11 of Article 138, Example IV., p. 217, in order to take into account the weight of a given portion of the ring of arch-stones, as distinguished from that of the material which rests upon it. The results are as follows:—

In a circular arch with a horizontal platform above it, let

r denote the radius of the intrados;

r' that of the extrados of the *arch-ring*, which is supposed uniformly thick;

c the depth of loading material above the crown of the arch;

w the weight of a cubic foot of the arch-stones;

w' the *mean weight* of a cubic foot of the superstructure, including voids ($= \frac{1}{2} w$ nearly);

Then the solution of the following equation gives the *angle of rupture* i_0;

$$0 = \left\{ \frac{w'}{w} (1 - \cos i_0) + \left(1 - \frac{w'}{w}\right) \frac{i_0 - \cos i_0 \sin i_0}{2 \sin^3 i_0} \right\} r'^2$$

$$+ \frac{w'}{w} c\, r' - \frac{i_0 - \cos i_0 \sin i_0}{2 \sin^3 i_0} r^2; \text{ ....} (1.)$$

which having been solved, the following equation gives the *horizontal thrust for each unit of breadth of the arch*:—

$$\mathrm{H}_0 = w' r^2 \left\{ \left(1 + \frac{c}{r}\right) \cos i_0 - \frac{\cos^2 i_0}{2} - \frac{i_0 \cot\!an\, i_0}{2} \right\}$$

$$+ w (r'^2 - r^2) \frac{i_0 \cot\!an\, i_0}{2}. \text{ .... } (2.)$$

Equation 1 is here given in the form best suited for practical use, being that of a quadratic equation between r', r, and c, with co-efficients depending on the angle i_0, and on the comparative heaviness of the arch and of the superstructure. The value of i_0 can be calculated from a given set of values of r', r, c, and $\frac{w'}{w}$, by a series of trials only; but if a value of i_0 be assumed, then any one of those four quantities can be computed exactly by solving the quadratic equation.

Problem.—*The radius of the intrados* r, *and the height* c *of the horizontal platform above the crown of the arch being given, to find the outer radius* r' *of the arch-ring corresponding to an assumed angle of rupture* i_0. In this case r' is the unknown quantity; and if equation 1 be denoted for brevity's sake by

$$\mathrm{A}\, r'^2 + \frac{w'}{w} c\, r' - \mathrm{B}\, r^2 = 0,$$

its solution is

$$r' = \sqrt{\left(\frac{\mathrm{B}}{\mathrm{A}} r^2 + \frac{w'^2 c^2}{4 w^2 \mathrm{A}^2}\right)} - \frac{w'}{2 w \mathrm{A}}. \text{ ....} (3.)$$

In most of the examples of circular arches which occur in practice, the angle of rupture lies between 45° and 55°; so that if the squared backing is carried up to that part of the arch which is inclined at an angle of 45° to the horizon, its height will be sufficient, at least.

It further appears, by trial, that the following approximate rule seldom errs by so much as one-twentieth part in giving the horizontal thrust of the arch.

The horizontal thrust is nearly equal to the weight supported between the crown and that part of the soffit whose inclination is 45°; or in symbols,

$$H_0 \text{ nearly} = w' r' (\cdot 0644\, r' + \cdot 7071\, c) + \cdot 3927\, w\, (r'^2 - r^2) \ldots (4.)$$

Thus, in fig. 180, let A C B represent one-half of a circular arch, O being the centre of the intrados and O A its radius, $= r$; let O P $= r'$, P U $= c$; U V being the horizontal platform. Draw O C F, making the angle A O C $= 45°$ with the vertical; then the horizontal thrust of the arch will be nearly equal to the weight of the mass A C F V U, which lies between the joint C F and the crown. The point F is that up to whose level it is advisable to build the backing solid, or, at all events, to bond and joint it in such a manner that it shall be capable of transmitting a horizontal thrust. Draw F T horizontal; then P T = ·7071 O P.

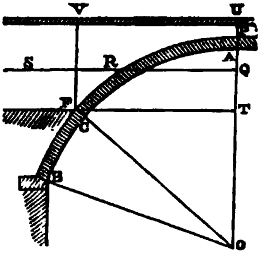

Fig. 180.

287. **Stability of an Unloaded Arch-Ring.**—Before the centre of an arch is struck, the spandril walls (or spandril filling, as the case may be) should be built to such a height that the part of the arch-ring near the crown, which is left for the time unloaded, shall fulfil the condition of stability which consists in having the line of pressures within the middle third of its thickness, when under its own weight alone. The exact investigation of this question is very complex, and it is unnecessary to give it here in detail. An approximation sufficiently accurate for practical purposes is as follows:—

Make the depth of the lowest point of the extrados of the unloaded arc below its highest point a mean proportional between the thickness of the arch and the radius of the intrados.

That is to say, in fig. 180 (last Article) make

$$P\,Q = \sqrt{P\,A \cdot A\,O} = \sqrt{r'\,r - r^2}; \ldots\ldots\ldots\ldots (1.)$$

then the horizontal line Q R S will show the level up to which the spandril walls or spandril filling are to be built before the centre is struck.

288. **Circular Arch less than a Quadrant.**—In this case the rule of the preceding article is to be applied exactly as in the case of an arch not less than a quadrant; but in computing the horizontal thrust, it is sufficient to take the weight of a half-arch with its load, and multiply by the co-tangent of the inclination of the intrados to the horizon at the springing.

289. **Tie-Walls**, in the hollow spandrils of arches, are transverse walls at right angles to the spandril walls. The distance from centre to centre of the tie-walls may be from three to five times the distance from centre to centre of the spandril walls.

290. **Depth of Keystone.**—To determine with precision the depth required for the keystone of an arch by direct deduction from the principles of stability and strength, would be an almost impracticable problem from its complexity. That depth is always many times greater than the depth necessary to resist the direct crushing action of the thrust. The proportion in which it is so in some of the best existing examples has been calculated, and found to range from 3 to 70. The smaller of these factors may be held to err on the side of boldness, and the latter on the side of caution; good medium values are those ranging from 20 to 40. The best course in practice is to assume a depth for the keystone according to an empirical rule, founded on dimensions of good existing examples of bridges.

The following is such a rule:—

For the depth of the keystone, take a mean proportional between the radius of curvature of the intrados at the crown, and a constant, whose values are,

for a single arch,............................... ·12 foot;
for an arch forming one of a series,........... ·17 „

That is to say, in symbols,

Depth of keystone for a single arch,

$$\text{in feet} = \sqrt{(\cdot 12 \times \text{radius at crown})}. \ldots\ldots\ldots\ldots(1.)$$

Depth of keystone for an arch of a series,

$$\text{in feet} = \sqrt{(\cdot 17 \times \text{radius at crown})}. \ldots\ldots\ldots\ldots(2.)$$

The following are examples:—

Single Arches.	Radius at Crown. Feet.	Depth of Keystone.	
		Calculated Feet.	Actual Feet.
Bridge over the Severn, at Gloucester (by Telford); elliptic arch; span 150 feet; rise 35 feet,......	160·7	4·39	4·5
Bridge at Turin, over the Dora Riparia (by Mosca); arch segmental; span 147·6 feet; rise 18 feet,............................	160	4·38	4·9
Grosvenor Bridge, over the Dee, at Chester (by Hartley and Harrison); segmental arch; span 200 feet; rise 42 feet,............	140	4·1	4·0
Ordinary bridge over a double line of railway; elliptic arch; span 30 feet; rise 7 feet 6 inches,......	30	1·9	1·83 to 2
Arches in Series.			
Bridge over the Thames, near Maidenhead (by I. K. Brunel); arch (of brick in cement) nearly elliptic; span 128 feet; rise 24·25, ...	169	5·36	5·25
London Bridge (by Sir John Rennie); elliptic arch; span 152 feet, ...	162	5·25	5·00
Bridge of Neuilly (by Perronet); basket-handle arch; span 39 mètres = 128 feet nearly; rise 9·75 mètres = 32 feet nearly,...	159	5·20	5·13
Bridge of St. Maxence (by Perronet); segmental arch; span about 76·7 feet; rise about 6·4 feet, ...	119	4·49	4·79
Waterloo Bridge (by Rennie); elliptic arch; span 120 feet; rise 32 feet,............................	112·5	4·37	5·00
Ballochmyle Bridge, over the Ayr, (by Miller); semicircular arch; span 181 feet; rise 90·5 feet, ...	90·5	3·92	4·5
Dean Bridge, near Edinburgh; segmental arch; span 90 feet; rise 30 feet,............................	48·75	2·88	3·00

It is evident from the law approximately followed by the examples in the preceding table, that the depth required for the arch-ring is regulated chiefly by the necessity for providing against deviations of the line of pressures, produced by temporary partial loads; and because such loads on a large arch are less as compared with the weight of the arch itself than in a small arch, the depth of the arch-stones increases more slowly than the general dimensions of the arch—viz., proportionally to the square root of the radius at the crown.

The *probability* of such a rule being found to answer in practice might have been inferred from equation 38 A of Article 180, p. 7 ~~370~~, by assuming that, owing to the plasticity of the mortar, the dead load of the arch, there denoted by w_0, produces no bending action (which is equivalent to omitting the term in B); and then determining the depth h of the arched rib so that the tension p'_1 shall be $= 0$, the arch being considered as sensibly flexible *between the joints of rupture only.* This last condition makes the rise k of the sensibly flexible part of the arch equal to a certain fraction of the radius at the crown; say $n\,r$, so that the equation referred to is reduced to the following:—

$$0{\cdot}138 \frac{w}{w_0\, q'\, h} - \frac{1}{n\, r} = 0.$$

But $q' = \frac{1}{6}$; and $w \div w_0$, which expresses the ratio of the intensity of the external load to the weight of the arch itself, may be replaced by $h' \div h$; h being the required depth of keystone, and h' the depth of the same material which is equivalent to the external load. The equation thus becomes,

$$0{\cdot}828 \frac{h'}{h^2} - \frac{1}{n\, r} = 0; \text{ or}$$

$$h^2 = {\cdot}828\, n\, h'\, r; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

that is to say, *for equally intense external loads, and equal angles of rupture, the square of the thickness of the keystone should vary as the radius of the intrados;* being very nearly the rule deduced empirically from practical examples. The co-efficients ·12 and ·17 in equations 1 and 2 correspond to the factor ·828 $n\, h'$ in equation 3. It is probable that the necessity for a larger co-efficient in the case of an arch which forms one of a series arises from the fact, that when one arch of a series is loaded externally, and the adjoining arches unloaded, the piers yield slightly, so as to lower the position of the joints of rupture.

291. An **Abutment in Radiating Courses** forms in truth a continuation of the arch, and is the strongest and most stable kind of

abutment where the foundation is firm, and the height from which the arch springs is moderate. One of the best examples is the Grosvenor Bridge at Chester. The real face of such an abutment is a continuation of the intrados of the arch, and its back is a continuation of the extrados of the solid spandril backing, which ought to be built in radiating courses also; but the custom is, in violation of good taste, to disguise the real structure by means of a casing of masonry with vertical and horizontal joints.

292. **Vertical Abutments** depend for their stability on the same principles which regulate that of buttresses, and which have been fully explained in Articles 263 and 264, pp. 396 to 401. The points to be chiefly attended to are, that if there is any horizontal thrust through the spandril, the part of the abutment above the springing of the arch shall have sufficient weight to resist by friction the tendency to sliding produced by that thrust; that above the bed-joint next below the springing of the arch, the weight of material, including that of the half-arch itself with its load, shall produce friction enough to resist the whole thrust of the arch, whether exerted through the spandrils or at the springing; and that the centre of resistance at the base of the abutment shall not deviate from the centre of the base by more than the proper fraction, 1 : q, of the thickness of that base. (See Article 263, p. 396, and Article 179, pp. 294, 295.)

It is highly advantageous, in point both of stability and economy, to build abutments with hollows in them, or with narrow archways passing through them, perpendicular to the main archways which the abutments support. These archways should have inverted arches at the bottom, to distribute the load over as large a base as possible. The hollows or archways may occupy about one-third of the whole volume of the abutment.

When an arch, as in fig. 179, p. 421, has a joint of rupture such as C, the part of the arch below that joint, together with its spandril backing and the load directly resting on it, may be considered as forming part of the abutment.

In some of the best examples of bridges, the thickness of the abutments ranges from *one-third* to *one-fifth* of the radius of curvature of the arch at its crown.

293. **Piers of Arches.**—Each pier of a series of arches ought to have sufficient stability to resist the thrust which acts upon it when one only of the arches which spring from it is loaded with a travelling load. That thrust may be roughly computed by multiplying the travelling load per lineal foot by the radius of curvature of the intrados at its crown in feet.

Each pier should always give sufficient space on its top for the two arches to spring from.

Either of these rules gives in general a less thickness than those adopted for piers in practice, which range from *one-tenth* to *one-fourth* of the span of the arches; the latter thickness, and those approaching to it, being suitable for "*abutment-piers.*" The most common thickness, for ordinary piers, is from one-sixth to one-seventh of the span of the arches.

Piers, like abutments, are advantageously lightened, especially when very lofty, as in viaducts, by being built hollow, or by having archways traversing them, with inverted arches at the base.

294. **Ribbed Arches, Abutments, and Piers.**—Arches and their abutments and piers may be made at once light and stiff, by building them in parallel deep ribs, with thinner portions of masonry between them; but this of course involves additional workmanship.

295. **Skew Arches** are of figures derived from those of symmetrical arches by distortion in a horizontal plane. The elevation of the face of a skew arch, and every vertical section parallel to its face, being similar to the corresponding elevation and vertical section of a symmetrical arch, the forces which act in a vertical layer or rib of a skew arch with its abutments, are the same with those which act in an equally thick vertical layer of a symmetrical arch with its abutments, of the same dimensions and figure, and similarly and equally loaded.

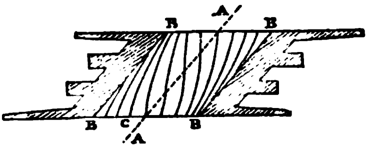

Fig. 181.

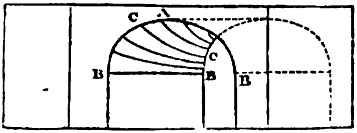

Fig. 182.

Fig. 181 represents a plan of a skew arch, with counterforted abutments. The *angle of skew*, or *obliquity*, is the angle which the axis of the archway, A A, makes with a perpendicular to the face of the arch, B C A B. The span of the archway, "*on the square,*" as it is called (that is, the perpendicular distance between the abutments), is less than the span *on the skew*, or parallel to the face of the arch, in the ratio of the cosine of the obliquity to unity. It is the span *on the skew* which is equal to that of the corresponding symmetrical arch.

The best position for the bed-joints of the arch-stones is perpendicular to the thrust along the arch. If, therefore, there be drawn on the soffit of a skew arch, a series of parallel curves, made by the intersections of the soffit with vertical planes parallel to the face of the arch, the best forms for the bed-joints will be a series of curves drawn on the soffit of the arch so as to cut the whole of the former series of curves at right angles, such as C C in figs. 181 and 182. Joints of the best form being difficult to execute, spiral joints are used in practice as an approximation.

Preparatory to the execution of a skew arch, a large drawing of the soffit must be prepared, showing the exact figure and position of every arch-stone. That drawing represents the curved surface of the soffit as if it were *spread out flat*, and is called the "*development*" of that curved surface. In general it is sufficient to draw one-half of the soffit, the other half being similar. The following are the processes by which that drawing is prepared:—

I. *To draw the development of the soffit, and of its vertical sections on the skew.* Fig. 183, No. 2, represents a plan of one half of the arch, H A K being the crown of the soffit, and I B L the face of one of the abutments. The line A C B represents the position of a vertical section *on the skew*, and A E D, perpendicular to H K, that of a vertical section *on the square:* $\angle$ B A D being the angle of obliquity.

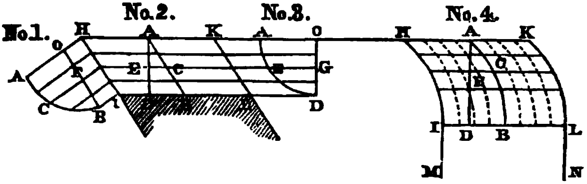

Fig. 183.

Assume any convenient number of points in H I, through which draw a set of lines (such as E C E G) $\parallel$ H K, and also a set of lines $\perp$ H I. Draw O B $\parallel$ H I, cutting these lines; and on O B as half-span, construct the vertical section of the arch *on the skew*, represented by No 1, in which A C B is the line on the soffit corresponding to A C B in No. 2.

Construct the vertical section *on the square*, No. 3, by drawing O D $\parallel$ A D to represent the half-span on the square, and transferring the ordinates of No. 1 to the corresponding points in No. 3; for example, F C to G E.

Then construct the development No. 4 in the following manner: —Produce the centre line of the soffit, H A K A O H A K. From any convenient point A, No. 4, draw A E D $\perp$ H K, in which take distances A E, A D, &c., equal in length to the arcs A E, A D, &c., which are cut off on the curve A E D, No. 3, by its several ordinates. Then will the straight line A E D, No. 4, be the development of the *section on the square*, A E D, Nos. 2 and

3. Through the points of division of A E D, No. 4, draw lines parallel to H K, such as E C, I D B L, &c., on which lay off ordinates, such as E C, D B, &c., equal respectively to the corresponding ordinates, E C, D B, &c., in the plan, No. 2, and through the ends of those ordinates draw a curve A C B, No. 4; this will be the development of the vertical section *on the skew*, A C B, Nos. 1 and 2.

Draw also the curves H I, K L parallel, similar, and equal to A C B, and at distances from it on either side, H A = A K, of half the length of the archway. Then I H K L will be the development of half the soffit. Draw I M and L N perpendicular to I L; then M I L N will be the development of part of the face of an abutment. Draw also any convenient number of intermediate curves, such as those shown by dots, parallel, similar, and equal to A C B, to represent the development of several parallel skew vertical sections of the soffit, and to indicate, at the same time, the direction of the thrust at each point which they traverse. These may be called "*curves of pressure.*"

II. *To draw on the development of the soffit, the bed-joints and side-joints of true courses.* The bed-joints are drawn by sketching with the free hand a series of curves, cutting all the curves of pressure at right angles, and called the *orthogonal trajectories of the curves of pressure.* The side-joints, being perpendicular to the bed-joints, are parts of curves of pressure themselves. (See fig. 184.) The courses become thinner towards the acute angle of the abutment, and thicker towards the obtuse angle, so that it may be sometimes advisable to introduce intermediate bed-joints near the obtuse angle, as shown near L in fig. 184. In the illustrations, the arch springs vertically from the abutments, so that none of the bed-joints intersect the line of springing, I L, to which they are all asymptotes. Had the arch been segmental, some of the bed-joints would have intersected that line obliquely, making necessary the use of skew-backs of the kind shown in the next figure, but not so oblique.

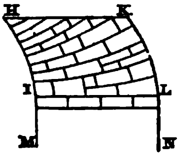

Fig. 184.

III. *To draw, on the development of the soffit, the bed-joints and side-joints of spiral courses.* (See fig. 185.) On the development of the soffit, draw a series of parallel equidistant straight lines, perpendicular to the direction of the thrust at the crown of the arch; these will represent the bed-joints, and the side-joints will be perpendicular to them. Between I and L are shown the *skew-backs*, or stones which connect the slanting courses of the arch with the horizontal courses of the abutment.

Spiral courses are perpendicular to the thrust at the crown of the arch only, and become more and more oblique to it the nearer they are to the springing.

296. **Ribbed Skew Arch.**—A substitute for an ordinary skew arch is sometimes composed of a series of ribs placed side by side, as in fig. 186. This mode of construction contracts the span *on the square* as compared with that of an ordinary skew arch having the same span *on the skew*, by the following quantity:—

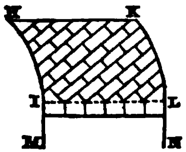

Fig. 185.

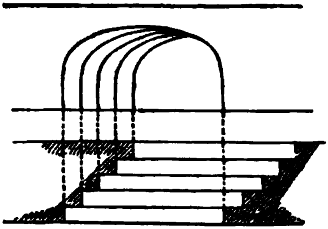
Fig. 186.

Let a denote the projection of each rib of the abutment beyond the preceding rib;

b, the breadth of a rib; then

$$\text{contraction of span on the square} = \frac{2\,a\,b}{\sqrt{a^2 - b^2}} \ldots\ldots (1.)$$

If the span *on the square* has already been fixed, the *span on the skew* for a ribbed arch is to be made greater than that for a common skew arch, simply by the projection a of a rib of the abutment.

297. **Strength of Stone and Brick Arches.**—A well-designed stone or brick arch of sufficient stability has usually a great surplus of strength. In the case, however, of a proposed arch of unusual dimensions or figure, especially if the material is comparatively weak, it is advisable, after the figure and dimensions have been planned with a view to stability, to test whether the strength is likely to be sufficient. This may be done with a sufficiently close approximation, by the aid of equations 30, 36, and 37 of
307 Article 180, p. 370, by making the following substitutions:—

Case I. *In a transformed Catenary Arch, or a circular segment of less than a quadrant;* make

h' = depth of arch-stones at springing × cos · inclination of arch at that point;

$$q = \frac{1}{6};\ m' = \frac{1}{2};\ l = \text{span};\ k = \text{rise};$$

$$\text{and consequently, } \mathrm{B} = \frac{15\,h'^2}{16\,k^2}\left(1 + \frac{16\,k^2}{3\,l^2}\right); \ldots\ldots (1.)$$

$$r_1 = \frac{2}{5} \cdot (1 + \mathrm{B}) \left(1 - \frac{h'}{4\,k}\right); \ldots\ldots\ldots\ldots\ldots(2.)$$

also, considering a rib of a foot in breadth, make $\mathrm{A}_1 = h'$; for w_0 put the dead load in lbs. per square foot of platform *at the crown of the arch*, and for w the rolling load in lbs. per square foot of platform.

CASE II. *In hydrostatic, elliptic, and semicircular arches, and circular segments greater than a quadrant*, make $l =$ the span, and $k =$ the rise, of the part of the arch lying between the two joints which make angles of 45° with the horizon. In hydrostatic and elliptic arches, make $h' =$ thickness of arch-ring; in circular arches, make $h' =$ thickness at the joints first mentioned $\times \cdot 707$. Then proceed in other respects as in Case I.

In all cases omit the term $\frac{l^2\,w_0}{8\,\mathrm{A}_1\,(1 + \mathrm{B})\,k}$, and substitute for it $\mathrm{H} \div h'$, H being the horizontal thrust as found by the methods of Articles 132 to 138, pp. 202 to 218, Article 286, pp. 423, 424, and Article 288, p. 425, for a rib one foot in breadth.

Equation 37, giving the greatest intensity of thrust, p_1, in *lbs. on the square foot*, thus becomes

$$p_1 = \frac{\mathrm{H}}{h'} + \frac{l^2}{2\,h'^2} \left\{ \frac{w_0\,\mathrm{B}}{1 + \mathrm{B}} + w \left(2\,r_1^2 - 2\,r_1^3 + \frac{3}{5}\,r_1^4\right) \right\}. \,(3.)$$

When $h' \div k$, and consequently B, are small fractions, so that $r_1 = \frac{2}{5}$ nearly, the following formula may be used:—

$$p_1 = \frac{\mathrm{H}}{h'} + \frac{l^2}{2\,h'^2} \left\{ \mathrm{B}\,w_0 + 0{\cdot}207\,w \right\}. \,\ldots\ldots\ldots(3\,\mathrm{A.})$$

When, on the other hand, $(1 + \mathrm{B}) \div \left(1 - \frac{h'}{4\,k}\right)$ is equal to or greater than $\frac{5}{2}$, so that $r_1 = 1$, the following formula is to be used:—

$$p_1 = \frac{\mathrm{H}}{h'} + \frac{l^2}{2\,h'^2} \left\{ \frac{\mathrm{B}\,w_0}{1 + \mathrm{B}} + 0{\cdot}288\,w \right\}. \,\ldots\ldots\ldots(3\,\mathrm{B.})$$

In transformed catenarian and circular segmental arches, an approximation to uniformity of strength may be obtained by making the depth of each voussoir proportional to the secant of the inclination of its bed to the vertical.

297 A. **Underground Archways—Tunnels—Culverts.**—If the depth of a buried archway, such as a tunnel or culvert, beneath the surface of the ground, is great compared with the height of the archway, the proper form for the line of pressures, which must lie within the

middle third of the thickness of the arch, is the elliptic linear arch 201 of Article 134, p. ~~202~~, in which the ratio of the horizontal to the vertical semi-axis is the square root of the ratio of the horizontal to the vertical pressure of the earth; that is to say,

$$\frac{\text{horizontal semi-axis}}{\text{vertical semi-axis}} = c = \sqrt{\frac{p_y}{p_x}} = \sqrt{\left(\frac{1 - \sin \varphi}{1 + \sin \varphi}\right)}; \quad (1.)$$

φ being the angle of repose.

If the earth is firm, and little liable to be disturbed, the proportion of the half-span, or horizontal semi-axis, to the rise, or vertical semi-axis, may be made *greater* than is given by the preceding equation, and the earth will still resist the additional horizontal thrust; but that proportion should never be made *less* than the value given by the equation, or the sides of the archway will be in danger of being forced inwards.

In a drainage tunnel or culvert the entire ellipse may be used as the figure of the arch; but in a railway tunnel, where it is necessary to have a flat floor, the sides and roof of the tunnel comprise in height the upper two-thirds, or three-fourths, of the ellipse, which is closed below by a circular segmental inverted arch of slight curvature, its depression being one-eighth of its span, or thereabouts. By this mode of construction the vertical pressure of the sides of the tunnel is concentrated upon foundation courses directly below them, from which they spring. The ratio which the entire width of the tunnel, measured *outside* the masonry or brickwork, bears to the joint width of that pair of foundations, must not exceed the limit of the ratio of the weight of a building to the weight of earth displaced by it, as given by Article 237, equation 1, p. 379. The inverted arch serves to prevent the foundations of the sides of the tunnel from being forced inwards by the horizontal pressure of the earth.

The *exact* form for the line of pressures in the sides and roof of a tunnel is the *geostatic arch* of Article 137, pp. 212, 213. This principle requires attention when the roof of the tunnel is near the surface. Let x_0 be the depth of the crown of the tunnel, and x_1 that of its greatest horizontal diameter, beneath the surface. From those ordinates as data, design a *hydrostatic arch*, by the aid of the formulæ (12) of Article 136, p. 211; contract the horizontal ordinates of that arch in the ratio $c : 1$ (see equation 1, above); and the result will be the figure of the geostatic arch required.

The greatest intensity of pressure in a buried archway occurs usually in its sides, at the ends of the shorter diameter of the oval intrados; and that intensity is given approximately by the following equation. Let x_1 be the depth of the shorter diameter below the surface of the ground, b' the half-span of the archway, a' its rise, t the thickness of its side, w the weight of a cubic foot of the earth; then the greatest pressure, in *lbs. on the square foot*, is

$$q = \frac{w\{x_1(b'+t) - 0\cdot 8\, a'\, b'\}}{t}; \ldots\ldots\ldots\ldots\ldots (2.)$$

and this should not exceed the resistance of the material to crushing, divided by a proper factor of safety.

It appears that in the brickwork of various existing tunnels, the factor of safety is as low as *four*. This is sufficient, because of the steadiness of the load; but in buried archways exposed to shocks, like those of culverts under high embankments, the factor of safety should be greater; say from *eight* to *ten*.

How small soever the load may be, there is a certain minimum thickness for an underground archway, for determining which the following empirical rule, exactly similar to that for finding the depth of the keystone of an arch, has been deduced from practical examples. The rise and half-span being denoted as before by a' and b', compute approximately the longest radius of curvature of the intrados by the formula

$$r = \frac{a'^2}{b'}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

then

$$\text{least thickness } t \text{ in feet} = \sqrt{0\cdot 12\, r}. \ldots\ldots\ldots\ldots (4.)$$

This is applicable where the ground is of the firmest and safest kind. In soft and slippery materials, the thickness ranges from *once and a-half* to *double* that given by equation 4; that is to say,

$$\text{from } \sqrt{0\cdot 27\, r} \text{ to } \sqrt{0\cdot 48\, r}. \ldots\ldots\ldots\ldots\ldots (4\text{ A.})$$

The thickness of an underground arch at the crown may be made less than at the sides in the ratio $b' : a'$; but the more common practice is to make it uniform.

As to the precautions to be observed in embanking over and near archways, see Article 201, p. 341.

298 **Table of Co-ordinates and Slopes of Catenarian Curves.**—(See Article 131, pp. 200, 201, and Article 282, p. 418.)

Let m denote the modulus, or parameter;

y_0, the ordinate from the directrix to the crown;

x, any abscissa, measured horizontally from the crown;

y, the corresponding ordinate from the directrix;

$\frac{dy}{dx}$, the tangent of the slope of the curve at the end of that ordinate.

$\frac{x}{m}$	$\frac{y}{y_0}$	$\frac{m\,dy}{y_0\,dx}$	$\frac{x}{m}$	$\frac{y}{y_0}$	$\frac{m\,dy}{y_0\,dx}$
0	1·0000	·0000	1·6	2·5774	2·3755
0·2	1·0200	·2013	1·8	3·1074	2·9421
0·4	1·0810	·4107	2·0	3·7622	3·6269
0·6	1·1854	·6366	2·2	4·5679	4·4571
0·8	1·3373	·8880	2·4	5·5569	5·4662
1·0	1·5431	1·1752	2·6	6·7690	6·6947
1·2	1·8106	1·5094	2·8	8·2526	8·1918
1·4	2·1509	1·9043	3·0	10·0676	10·0178

To interpolate the ordinate $y \pm v$ corresponding to an intermediate abscissa $x \pm u$, when $\frac{y}{y_0}$ corresponds to $\frac{x}{m}$ in the table; make

$$\frac{y \pm v}{y_0} = \frac{y}{y_0}\left(1 + \frac{u^2}{2\,m^2} + \frac{u^4}{24\,m^4}\right) \pm \frac{m\,d\,y}{y_0\,d\,x}\left(\frac{u}{m} + \frac{u^3}{6\,m^3}\right). \quad (1.)$$

This computation is to be performed by addition to the number next below in the table, or by subtraction from the number next above, according as the intermediate abscissa lies nearer to the one next below it or to that next above it.

When very great precision is not required, the terms in u^3 and u^4 may be neglected; but those in u and u^2 should *always* be computed. The greatest possible error within the limits of the table, by using equation 1 as it stands, is about ·00005; by neglecting u^3 and u^4, that limit of error is increased, for the greatest intermediate ordinate in the table, to about ·0015.

298 A. **List of Authorities on Masonry.**—(Stones, Limes, and Cements)—Berthier, *Traité des Essais par la Voie sèche;* Vicat, *Traité des Mortiers;* Pasley on *Cements and Mortars.* (Masonry in general)—Rondelet, *Traité de l'Art de Bâtir;* Gauthey, *Traité de la Construction des Ponts;* Tredgold on *Masonry* (*Encyc. Brit.*)

CHAPTER IV.

OF CARPENTRY.

SECTION I.—*Of Timber.*

299. **Structure of Timber.**—Timber is the material of trees belonging almost exclusively to that class of the vegetable kingdom in which the stem grows by the formation of successive layers of wood all over its external surface, and is therefore said by botanists to be *exogenous.*

The exceptions are, trees of the palm family, and tree-like grasses, such as the bamboo, which belong to the *endogenous* class; so called because, although the stem grows partly by the formation of layers of new wood on its outer surface, the fibres of that new wood do nevertheless cross and penetrate amongst those previously formed in such a manner as to be mixed with them in one part of their course, and internal to them at another.

The stems of endogenous trees, though light and tough, are too flexible and slender to furnish materials suitable for important works of carpentry. They will therefore not be further mentioned in this section except to refer to the tables at the end of the volume for the tenacity and heaviness of bamboo.

The stem of an exogenous tree is covered with bark, which grows by the formation of successive layers on its inner surface, at the same time that the wood grows by the formation of successive layers on its outer surface. This double operation takes place in the narrow space between the previously-formed wood and bark, during the circulation of the sap. The sap ascends from the roots to the leaves through vessels contained in the outer layers of the wood; at the surface of the leaves it acquires carbon from the atmosphere, and becomes denser, thicker, and more complex in its composition; it then descends from the leaves to the roots through vessels contained chiefly in the innermost layers of the bark. It is believed that the formation of new wood and bark takes place either wholly or principally from the descending sap.

The circulation of the sap is either wholly or partially suspended during a portion of each year (in tropical climates during the dry season, and in temperate and polar climates during the winter); and hence the wood and bark are usually formed in distinct layers, at the rate of one layer in each year; but this rule is not universal. Each such layer consists of parts differing in density and colour to an extent which varies in different kinds of trees.

The tissues of which both wood and bark consist are distinguished into two kinds—*cellular tissue*, consisting of clusters of minute cells; and *vascular tissue*, or *woody fibre*, consisting of bundles of slender tubes; the latter being distinguished from the former by its fibrous appearance. The difference, however, between those two kinds of tissue, although very distinct both to the eye and to the touch, is really one of degree rather than of kind; for the fibres or tubes of vascular tissue are simply very much elongated cells, tapering to points at the ends, and "breaking joint" with each other.

The tenacity of wood when strained "along the grain" depends on the tenacity of the walls of those tubes or fibres; the tenacity of wood when strained "across the grain" depends on the adhesion of the sides of the tubes and cells to each other. Examples of the difference of strength in those different directions will be given afterwards.

When a woody stem is cut across, the cellular and vascular tissue are seen to be arranged in the following manner:—

In the centre of the stem is the *pith*, composed of cellular tissue, enclosed in the medullary sheath, which consists of vascular tissue of a particular kind. From the pith there extend, radiating outwards to the bark, thin partitions of cellular tissue, called *medullary rays;* between these, additional medullary rays extend inwards from the bark to a greater or less distance, but without penetrating to the pith.

When the medullary rays are large and distinct, as in oak, they are called "*silver grain.*"

Between the medullary rays lie bundles of vascular tissue, forming the woody fibre, arranged in nearly concentric rings or layers round the pith. These rings are traversed radially by the medullary rays. The boundary between two successive rings is marked more or less distinctly by a greater degree of porosity, and by a difference of hardness and colour.

The annual rings are usually thicker at that side of the tree which has had most air and sunshine, so that the pith is not exactly in the centre.

The wood of the entire stem may be distinguished into two parts—the outer and younger portion, called "*sap-wood*," being softer, weaker, and less compact, and sometimes lighter in colour, than the inner and older portion, called "*heart-wood.*" The heart-wood is alone to be employed in those works of carpentry in which strength and durability are required. The boundary between the sap-wood and the heart-wood is in general distinctly marked, as if the change from the former to the latter occurred in the course of a single year. The following examples of the proportion of sap-wood to the entire volume are given on the authority of Tredgold. (*Principles of Carpentry*, Section X.)

Tree.	Age. Years.	Diameter. Inches.	Rings of Sap-wood.	Thickness of Sap-wood. Inches.	Proportion of Sap-wood to whole Trunk.
Chestnut,.........	58	15½	7	⅜	0·1
Oak, ...	65	17	17	1¼	0·294
Scotch Fir,........	?	24	?	2½	0·416

The following data are given on the authority of Mr. Robert Murray, C.E. (*Encyc. Brit.*, Article "Timber.")

Tree.	Rings of Sap-wood.
English Oak (*Quercus pedunculata*),...........	12 to 15
Durmast Oak (*Quercus sessiliflora*), ...	20 to 30
Chestnut (*Castanea Vesca*),.......................	5 or 6
Elm (*Ulmus campestris*),..........................	about 10
Larch (*Larix Europæa*),..........................	„ 15
Scotch Fir (*Pinus sylvestris*), ...	„ 30
Memel Fir (*Pinus sylvestris*), ...	„ 44
Canadian Yellow Pine (*Pinus variabilis*),.....	„ 42

The structure of a *branch* is similar to that of the trunk from which it springs, except as regards the difference in the number of annual rings, corresponding to the difference of age. A branch becomes partially imbedded in those layers of the trunk which are formed after the time of its first sprouting; it causes a perforation in those layers, accompanied by distortion of their fibres, and constitutes what is called a *knot*. (On various matters mentioned in this Article, see Balfour's *Manual of Botany*, Part I., chaps. i. and ii.)

300. **Timber Trees Classed—Fir-wood—Hardwood.**—For purposes of carpentry trees may be classed according to the mechanical structure of the wood. It has already been stated that the botanical classes of Endogens and Exogens correspond to essential differences of mechanical structure.

In further dividing the class of Exogenous trees, or timber-trees proper, according to the structure of the wood, a division into two classes at once suggests itself, which exactly corresponds with a botanical division, viz.:—

Fir-wood, comprising all timber-trees belonging to the coniferous order; and

Hardwood, comprising all other timber-trees.

Beyond this primary division, the place of a tree in the botanical system has little or no connection with the structure of its timber.

A classification of timber according to its mechanical structure was proposed by Tredgold, founded, in the first place, on the greater or less distinctness of the medullary rays; and secondly, on the greater or less distinctness of the annual rings. According to that classification, fir-wood, or coniferous timber, is placed in the same class with hardwood that has the medullary rays indistinct; and this is certainly a fault in the system. If, however, fir-wood be placed in a class apart, Tredgold's system may very well be applied to divide and subdivide the class of hardwood; but it is to be observed, that the characters on which that system is founded, being mere differences in degree, and not in kind, are not of that definite sort which a thoroughly satisfactory system of classification requires; and if they are adopted, it is because no better set of distinguishing characters has yet been proposed.

The following is a condensed view of the classification of exogenous timber, as above described:—

CLASS I. Fir-wood. (Natural order *Coniferæ*).—Examples:—Pine, Fir, Larch, Cowrie, Yew, Cedar, Juniper, Cypress, &c.

CLASS II. Hardwood. (Non-coniferous trees.)

- Division I. With distinct large medullary rays. (The trees in this division form part of the natural order *Amentaceæ*.)
  - *Subdivision* I. Annual rings distinct.—Example:—Oak.
  - *Subdivision* II. Annual rings indistinct.—Examples:—Beech, Alder, Plane, Sycamore, &c.
- Division II. No distinct large medullary rays.
  - *Subdivision* I. Annual rings distinct.—Examples:—Chestnut, Ash, Elm, &c.
  - *Subdivision* II. Annual rings indistinct.—Examples:—Mahogany, Walnut, Teak, Poplar, Box, &c.

The chief practical bearings of this classification are as follows:—

Fir-wood, or coniferous timber, in most cases, contains turpentine. It is distinguished by straightness in the fibre and regularity in the figure of the trees; qualities favourable to its use in carpentry, especially where long pieces are required to bear either a direct pull, or a transverse load, or for purposes of planking. At the same time, the lateral adhesion of the fibres is small; so that it is much more easily shorn and split along the grain, or torn asunder across the grain, than hardwood; and is therefore less fitted to resist thrust or shearing stress, or any kind of stress that does not act along the fibres. Even the toughest kinds of fir-wood are easily wrought. A peculiar characteristic of fir-wood (but one which requires the microscope to make it visible) is that of having the vascular tissue "*punctated;*" that is to say, there are small lenticular hollows in the sides of the tubular fibres. This structure is probably connected with the smallness of the lateral adhesion of those fibres to each other.

In Hardwood, or non-coniferous timber, there is no turpentine. The degree of distinctness with which the structure is seen, whether as regards medullary rays or annual rings, depends on the degree of difference of texture of different parts of the wood. Such difference tends to produce unequal shrinking in drying; and consequently those kinds of timber in which the medullary rays, and the annual rings, are distinctly marked, are more liable to warp than those in which the texture is more uniform. At the same time, the former kinds of timber are, on the whole, the more flexible, and in many cases are very tough and strong, which qualities make them suitable for structures that have to bear shocks.

301. **Appearance of good Timber.**—There are certain appearances which are characteristic of strong and durable timber, to what class soever it belongs.

In the same species of timber, that specimen will in general be the strongest and the most durable which has grown the slowest, as shown by the narrowness of the annual rings.

The cellular tissue as seen in the medullary rays (when visible) should be hard and compact.

The vascular or fibrous tissue should adhere firmly together, and should show no wooliness at a freshly-cut surface, nor should it clog the teeth of the saw with loose fibres.

If the wood is coloured, darkness of colour is in general a sign of strength and durability.

The freshly-cut surface of the wood should be firm and shining, and should have somewhat of a translucent appearance. A dull, chalky appearance is a sign of bad timber.

In wood of a given species, the heavier specimens are in general the stronger and the more lasting.

Amongst resinous woods, those which have least resin in their pores, and amongst non-resinous woods, those which have least sap or gum in them, are in general the strongest and most lasting.

It is stated by some authors that in fir-wood, that which has most sap-wood, and in hardwood, that which has least, is the most durable; but the universality of this law is doubtful.

Timber should be free from such blemishes as clefts, or cracks radiating from the centre, "cup-shakes," or cracks which partially separate one annual layer from another; "upsets," where the fibres have been crippled by compression; "rind-galls," or wounds in a layer of the wood, which have been covered and concealed by the growth of subsequent layers over them; and hollows or spongy places, in the centre or elsewhere, indicating the commencement of decay.

302. **Examples of Fir-wood,—Pine, Fir, Larch, Cowrie, Cedar, &c.** —The following are examples of timber of this class:—

I. Pine timber of the best sort is the produce of the Red Pine, or Scotch Fir (*Pinus sylvestris*), grown in Norway, Sweden, Russia, and Poland. The best is exported from Riga, the next from Memel and from Dantzic. The same species of tree grows also in Britain, but is inferior in strength. The annual rings, when this timber is of the best kind, consist of a hard part, of a clear dark-red colour, and a less hard part, of a lighter colour, but still clear and compact. The thickness of the rings should not exceed one-tenth of an inch. The most common size of the logs to be met with in the market is about 13 inches square. This is the best of all timber for straight beams, straight ties, and straight pieces in framework generally, and for the spars of ships.

Pine timber for the same purposes is also obtained from various other species, chiefly North American, of which the best are the Yellow Pine (*Pinus variabilis*), and White Pine (*Pinus Strobus*). It is softer and less durable than the Red Pine of the North of Europe, but lighter, and can be had in larger logs.

II. White Fir, or Deal timber of the best kind, is the produce of the Spruce Fir (*Abies excelsa*), grown in Norway, Sweden, and Russia. The best is that known as Christiania Deal. Much of this timber is sawn up for sale into pieces of various thicknesses suited for planking, which,

when 7 inches broad are called "*battens.*"
when 9 " " "*deals.*"
when 11 " " "*planks.*"

They are to be had of various lengths; but the most usual length is about 12 feet.

This is an excellent kind of timber for planking, light framing, and joiners' work, and for the lighter spars of ships.

Amongst other kinds of spruce fir, applied to the same purposes, are the North American White Spruce (*Abies alba*), and Black Spruce (*Abies nigra*).

III. The Larch (*Larix Europæa*), grown in various parts of Europe, furnishes timber of great strength, and remarkable for durability when exposed to the weather; but harder to work and more subject to warp than red pine. The best sort has the harder part of the rings of a dark-red, and the softer part of a honey-yellow; and its rings are somewhat thicker than those of red pine.

Two North American species, the Black Larch, or Hackmatack (*Larix pendula*), and the Red Larch (*Larix microcarpa*), produce timber similar to that of the European Larch.

IV. The Cowrie or Kawrie (*Dammara Australis*), a coniferous tree, grown in New Zealand, produces timber similar in its properties to the best kinds of pine, except that it is said to be more liable to warp, and more variable in quality. It is of a brownish-yellow colour, and more uniform in its texture than red pine and larch.

V. The term Cedar is applied, not only to the timber of the true Cedar (*Cedrus Libani*), but also to that of various large species of Juniper (such as *Juniperus Virginiana*) and of Cypress. All these kinds of wood are remarkable for durability, in which they excel all other timber; but they are deficient in strength.

303. **Examples of Hardwood—Oak, Beech, Alder, Plane, Sycamore.** —The kinds of timber which head this article belong to the first division of Tredgold's system, being that in which there are distinct large medullary rays. Of the examples cited, the Oak alone belongs to the first subdivision, in which the divisions between the annual rings are distinctly marked by circles of pores. The other examples belong to the second subdivision, in which the rings are less distinctly marked.

I. Oak timber, the strongest, toughest, and most lasting of those grown in temperate climates, is the produce of various species or varieties of the botanical genus *Quercus*. In Europe there are two kinds of oak trees; and it is doubtful whether they are distinct species or varieties of one species. They are—

The old English Oak, or Stalk-fruited Oak (*Quercus Robur*, or *Quercus pedunculata*), in which the acorns grow on stalks, and the leaves close to the twig, and

The Bay Oak, or Cluster-fruited Oak (*Quercus sessiliflora*), in which the acorns grow in close clusters, and the leaves have short stalks.

Both those kinds of oak come to their greatest perfection in Britain.

The wood of the stalk-fruited oak is lighter in colour, and has more numerous and distinct medullary rays than that of the cluster-fruited oak, in which they are sometimes so few and indistinct as to have caused it in some old buildings to be mistaken for chestnut. The stalk-fruited oak is the stiffer and the straighter-grained of the two, the easier to work, and the less liable to warp; it is therefore preferable where stiffness and accuracy of form are desired; the cluster-fruited oak is the more flexible, which gives it an advantage where shocks have to be borne.

The best oak timber when new is of a pale brownish-yellow, with a perceptible shade of green in its composition, a firm and glossy surface, very small and regular annual rings, and hard and compact medullary rays. Thick rings, many large pores, a dull surface, and a reddish, or "foxy" hue, are signs of weak and perishable wood.

It is considered that oak timber comes to maturity at the age of 100 years, at which period each tree produces on an average about 75 cubic feet of timber; and that it should not be felled before the sixtieth year of its age, nor later than the 200th.

The species of oak in North America are very numerous. The best of them are, the Red Oak (*Quercus rubra*), and White Oak (*Quercus alba*), which are little inferior to the best European kinds, and the Live Oak (*Quercus virens*) which is said to be superior in strength, toughness, and durability, to all other species, but is so rare as to be reserved exclusively for ship-building.

The wood of the oak contains gallic acid, which probably contributes to the durability of the timber, but tends to corrode iron fastenings.

The following are examples of trees belonging to the second subdivision:—

II. Beech (*Fagus sylvatica*), common in Europe;

III. Alder (*Alnus glutinosa*), also common in Europe;

IV. American Plane (*Platanus occidentalis*), common in North America.

V. Sycamore (*Acer Pseudo-platanus*), also called Great Maple, and in Scotland and the North of England, Plane; common in western Europe.

All these afford compact timber of uniform texture. They are not used for great works of carpentry; but are valuable where blocks of wood are required to resist a crushing force. They last well when constantly wet, and are therefore suited for piles that are to be always under water; but when alternately wet and dry they decay rapidly.

304. **Hardwood continued.—Chestnut, Ash, Elm.**—The examples of timber in this article belong to the first subdivision of the second division, according to Tredgold's system, having no large distinct medullary rays, and having the divisions between the

annual rings distinctly marked by a more porous structure. They are in general strong, but flexible.

I. The CHESTNUT (*Castanea vesca*) yields timber resembling that of the cluster-fruited oak, except that it is without large medullary rays, and has less sap-wood. Its properties resemble those of oak timber, except that the chestnut timber is less durable, especially when obtained from old trees.

II. The ASH (*Fraxinus excelsior*) furnishes timber whose toughness and flexibility render it superior to that of all other European trees for making handles of tools, shafts of carriages, and the like; but which is not sufficiently stiff and durable to be used in great works of carpentry. The colour of the wood is like that of oak, but darker, and with more of a greenish hue; the annual rings are broader than those of oak, and the difference between their compact and porous parts more marked.

III. The common ELM (*Ulmus campestris*) and Smooth-leaved Elm (*Ulmus glabra*) yield timber which is valued for its durability when constantly wet, and is specially suited for piles and for planking in foundations under water. Its strength across the grain, and its resistance to crushing, are comparatively great; and these properties render it useful for some parts of mechanism, such as naves of cart wheels, shells of ships' blocks, and the like. It is not suited for great works of carpentry. There are other European species of elm, such as the Wych Elm (*Ulmus montana*), but their timber is inferior to that of the two species named.

A North American species, the Rock Elm, is said to be not only durable under water, but straight-grained and tough, so as to be well suited for long beams and ties.

305. **Hardwood continued.—Mahogany, Teak, Greenheart, Mora.** —These kinds of timber are examples of the second subdivision of Tredgold's second division, having no large distinct medullary rays, and no distinct difference of compactness in the rings. This uniformity of structure is accompanied by comparative freedom from warping.

I. MAHOGANY (*Swietenia mahogani*) is produced in Central America and the West Indian Islands, that of the former region being commonly known as "Bay Mahogany;" that of the latter as "Spanish Mahogany." When of good quality, it is very strong in all directions, very durable, and preserves its shape under varying circumstances as to heat and moisture better than any other kind of timber which can be procured in equal abundance. Mahogany varies much in quality; bay mahogany being in general superior to Spanish mahogany in strength, stiffness, and durability, and in the size of the logs. Spanish mahogany is the more highly valued for ornamental purposes.

Spanish mahogany is distinguished by having a white chalky substance in its pores, those of bay mahogany being empty.

II. TEAK (*Tectona grandis*), from its great strength, stiffness, toughness, and durability, is the most valuable of all woods for carpentry, especially for ship-building. It is produced in the mountainous districts of south-eastern Asia and the East India Islands. The best comes from Malabar, Ceylon, and Java.

Good teak resembles oak in colour and lustre, is very uniform and compact in texture, and has very narrow and regular annual rings. It contains a resinous, oily matter in its pores, in order to extract which, the tree is sometimes tapped; but this injures the strength and durability of the timber, and ought to be avoided. Insects do not attack teak, and iron is not corroded by contact with it, unless it has been grown in a marshy soil.

III. GREENHEART (*Nectandra Rodiæi*), a tree of British Guiana, yields a very strong and durable timber, considered of the first quality for ship-building and all kinds of carpentry, and also for piled foundations and other structures under water.

IV. MORA (*Mora excelsa*), also a tree of British Guiana, yields a first-class timber for ship-building.

306. **Hardwood continued.—Iron-bark, Blue-gum, Jarrah.**—These are three of the numerous species of the genus *Eucalyptus*, peculiar to Australia. They yield timber of great size, strength, and durability; and that of the iron-bark, in particular, is held to be of the first-class for ship-building. The wood of iron-bark is white or yellowish; that of blue-gum, straw-coloured; that of jarrah resembles mahogany, and is sometimes called "Australian Mahogany." The *Eucalypti*, in common with some other Australian trees, are distinguished from the trees of other quarters of the globe by being more easily split in concentric layers, than in planes radiating from the pith; and the most frequent blemish in their timber is the occurrence of cylindrical clefts of that kind, filled with gum.

307. **Influence of Soil and Climate on Trees.**—Most timber trees are capable of flourishing in a great variety of soils. The best soil for all of them is one which, without being too dry and porous, allows water to escape freely, such as gravel mixed with sandy loam.

The most injurious soil to trees is that of swampy ground containing stagnant water: it never fails to make the timber weak and perishable.

As to the influence of climate, two general laws seem to prevail: that the strongest timber is yielded, amongst *different species* of trees, by those produced in tropical climates; and amongst trees of *the same species*, by those grown in cold climates. The first law is exemplified in such woods as teak, iron-wood, ebony, and lignum-vitæ, surpassing in strength all those of temperate climates: the second, in the red pine of Norway, as compared with that of Scotland, in the oak of Britain as compared with that of Italy, and even in the oak of Scotland and the North of England, as compared with that of the South of England.

308. **Age and Season for felling Timber.**—There is a certain age of maturity at which each tree attains its greatest strength and durability. If cut down before that age, the tree, besides being smaller, contains a greater proportion of sap-wood, and even the heart-wood is less strong and lasting; if allowed to grow much beyond that age, the centre of the tree begins either to become brittle, or to soften, and a decay commences by slow degrees, which finally renders the heart hollow. The age of maturity is therefore the best age for felling the tree to produce timber. The following data respecting it are given on the authority of Tredgold:—

	Age of Maturity. Years.
Oak,	60 to 200 average 100
Ash, Elm, Larch,	50 to 100
Fir,	70 to 100

The best season for felling timber is that during which the sap is not circulating—that is to say, the winter, or in tropical climates, the dry season; for the sap tends to decompose, and so to cause decay of the timber. The best authorities recommend, also, as a means of hardening the sap-wood, that the bark of trees which are to be felled should be stripped off in the preceding spring.

Immediately after timber has been felled, it should be *squared*, by sawing off four "slabs" from the log, in order to give the air access to the wood and hasten its drying. If the log is large enough, it may be sawn into quarters.

309. **Seasoning, Natural and Artificial.**—Seasoning timber consists in expelling, as far as possible, the moisture which is contained in its pores.

Natural Seasoning is performed simply by exposing the timber freely to the air in a dry place, sheltered, if possible, from sunshine and high winds. The seasoning yard should be paved and well drained, and the timber supported on cast iron bearers, and piled so as to admit of the free circulation of air over all the surfaces of the pieces.

Natural seasoning to fit timber for carpenters' work usually occupies about two years; for joiners' work, about four years; but much longer periods are sometimes employed.

To steep timber in water for a fortnight after felling it extracts part of the sap, and makes the drying process more rapid.

The best method of *Artificial Seasoning* consists in exposing the timber in a chamber or oven to a current of hot air. In Mr. Davison's process, the current of air is impelled by a fan at the rate of about 100 feet per second; and the fan, air-passages, and chamber are so proportioned, that one-third of the volume of air in the chamber is blown through it per minute. The best temperature for the hot air varies with the kind and dimensions of the timber; thus, for

Oak, of any dimensions, the temperature should not exceed	105° Fahr.
Hardwoods in general, in logs or large pieces,	90° to 100°
Fir-woods, in thick pieces,	120°
„ in thin boards,	180° to 200°
Bay mahogany, in boards one inch thick,...	280° to 300°

The time required for drying is stated to be as follows:—

Thickness in inches,	1,	2,	3,	4,	6,	8;
Time in weeks,	1,	2,	3,	4,	7,	10,

the current of hot air being kept up for *twelve hours per day* only.

The drying of timber by hot air from a furnace has also been practised successfully by Mr. James Robert Napier, in a brick chamber, through which a current is produced by the draught of a chimney. The equable distribution of the hot air amongst the pieces of timber is insured by introducing the hot air close to the roof of the chamber, and drawing it off through holes in the floor into an underground flue. The hot air on entering, being more rare than that already in the chamber, which is partially cooled, spreads into a thin stratum close under the roof, and gradually descends amongst the pieces of wood to the floor. The air is introduced at the temperature of 240° Fahr. The expenditure of fuel is at the rate of 1 lb. of coke for every 3 lbs. of moisture evaporated.

Many experiments have been made on the loss of weight and shrinkage of dimensions undergone by timber in seasoning; of which the details may be found in the works of Fincham on *Ship-building*, Tredgold on *Carpentry*, Mr. Murray on *Ship-building*,

&c. The results of these experiments vary so much that it is almost impossible to condense them into any general statement. The following shows the limits within which they generally lie:—

Timber.		Loss of Weight per Cent.	Transverse Shrinking per Cent.
Red Pine,	from	12 to 25	2½ to 3
American Yellow Pine,	„	18 to 27	2 to 3
Larch,	„	6 to 25	2 to 3
Oak (British),	„	16 to 30	about 8
Elm „	„	about 40	about 4
Mahogany,	„	16 to 25	

310. **Durability and Decay of Timber.**—All kinds of timber are most lasting when kept constantly dry, and at the same time freely ventilated.

Timber kept constantly wet is softened and weakened; but it does not necessarily decay. Various kinds of timber, some of which have been already mentioned, such as elm and beech, possess great durability in this condition.

The situation which is least favourable to the duration of timber is that of alternate wetness and dryness, or of a slight degree of moisture, especially if accompanied by heat and confined air. For pieces of carpentry, therefore, which are to be exposed to these causes of decay, the most durable kinds of timber only are to be employed, and proper precautions are to be taken for their preservation.

Slaked lime hastens the decay of timber, which should therefore, in buildings, be protected against contact with the mortar.

Timber exposed to confined air alone, without the presence of any considerable quantity of moisture, decays by "*dry rot*," which is accompanied by the growth of a fungus, and finally converts the wood into a fine powder.

The following table shows the comparative durability of some kinds of timber for ship-building, as estimated by the committee of Lloyd's.

12 years.	Teak, British Oak, Mora, Greenheart, Iron-bark, Saul.
10 „	Bay Mahogany, Cedar (*Juniperus Virginiana.*)
9 „	European Continental Oak, Chestnut, Blue-gum, Stringy-bark (*Eucalyptus gigantea.*)
8 „	North American White Oak, North American Chestnut.
7 „	Larch, Hackmatack, Pitch Pine, English Oak.
6 „	Cowrie, American Rock Elm.

5 years. Red Pine, Grey Elm, Black Birch, Spruce Fir, English Beech.
4 " Hemlock Pine (North American.)

311. **Preservation of Timber.**—Amongst the most efficient means of preserving timber, are good seasoning and the free circulation of air.

Protection against moisture is afforded by oil-paint, provided that the timber is perfectly dry when first painted, and that the paint is renewed from time to time. A coating of pitch or tar may be used for the same purpose.

Protection against the dry rot may be obtained by saturating the timber with solutions of particular metallic salts. For this purpose Chapman employed copperas (*sulphate of iron*); Mr. Kyan, corrosive sublimate (*bichloride of mercury*); Sir William Burnett, *chloride of zinc.* All these salts preserve the timber so long as they remain in its pores; but it would seem that they are gradually removed by the long-continued action of water.

Dr. Boucherie employs a solution of *sulphate of copper* in about one hundred times its weight of water. The solution, being contained in a tank about 30 or 40 feet above the level of the log, descends through a flexible tube to a cap fixed on one end of the log, whence it is forced by the pressure of the column of fluid above it through the tubes of the vascular tissue, driving out the sap before it at the other end of the log, until the tubes are cleared of sap and filled with the solution instead.

Timber is protected not only against wet rot and dry rot, but against white ants and sea-worms, by Mr. Bethell's process of saturation with the liquid called commercially "*creosote*," which is a kind of pitch oil. This is effected by first exhausting the air and moisture from the pores of the timber in an air-tight vessel, in which a partial vacuum is kept up for a few hours, and then forcing the creosote into these pores by a pressure of about 150 lbs. on the square inch, which is kept up for some days. The timber absorbs from a *ninth* to a *twelfth* of its weight of the oil.

312. **Strength of Timber.**—Amongst different specimens of timber of the same species, those which are most dense in the dry state are in general also the strongest.

Tables of the average results of the most trustworthy experiments on the strength of different kinds of timber strained in various ways are given at the end of the volume; and a supplementary table containing some additional results, at the end of this section, p. 452. As to the strength of timber posts, see Article 158, p. 238.

The following are some general remarks as to the different ways in which the strength of timber is exerted:—

I. The TENACITY *along the grain*, depending, as it does, on the tenacity of the fibres of the vascular tissue, is on the whole greatest in those kinds and pieces of wood in which those fibres are straightest and most distinctly marked. It is not materially affected by temporary wetness of the timber, but is diminished by long-continued saturation with water, and by steaming and boiling.

The *tenacity across the grain*, depending chiefly on the lateral adhesion of the fibres, is always considerably less than the tenacity along the grain, and is diminished by wetness and increased by dryness. Very few exact experiments have been made upon it. Its smallness in fir-wood as compared with hardwood forms a marked distinction between those two classes of timber, the proportion which it bears to the tenacity along the grain having been found to be, by some experiments,

In fir-wood, from 1-20th to 1-10th.
In hardwood, from 1-6th to 1-4th, and upwards.

II. The RESISTANCE TO SHEARING, by sliding of the fibres on each other, is the same, or nearly the same, with the tenacity across the grain. As to *shearing across the grain*, see Article 322, p. 460.

III. The RESISTANCE TO CRUSHING along the grain, depending, as it does, on the resistance of the fibres to being crippled or "upset," and split asunder, is greatest when their lateral adhesion is greatest, and has been found by Mr. Hodgkinson to be nearly twice as great for dry timber as for the same timber in the green state. In most kinds of timber, when dry, it ranges from one-half to two-thirds of the tenacity (p. 236.)

Experiments have been made on the crushing of timber across the grain, which takes place by a sort of shearing; but they have not led to any precise result, except that the timber is both more compressible and weaker against a transverse than against a longitudinal pressure; and consequently, that intense transverse compression of pieces of timber ought to be avoided.

IV. The MODULUS OF RUPTURE of timber, which expresses its resistance to cross-breaking, is usually somewhat less than its tenacity, but seldom much less. (See Article 162, p. 252.)

V. The FACTOR OF SAFETY, in various actual structures of carpentry, ranges from 4 to 14, and is on an average about 10.

SUPPLEMENTARY TABLE OF PROPERTIES OF TIMBER GROWN IN CEYLON; SELECTED AND COMPUTED FROM A TABLE OF THE PROPERTIES OF NINETY-SIX KINDS OF TIMBER BY MODLIAR ADRIAN MENDIS.

TIMBER.	Modulus of Elasticity in lbs. on the Square Inch.	Modulus of Rupture in lbs. on the Square Inch.	Weight of a Cubic Foot in lbs.
Aludel (*Artocarpus pubescens*),...	1,850,000	12,800	51
Burute (*Chloroxylon Swietenia*),	2,700,000	18,800	55
Caha Milile (*Vitex altissima* ?),...	2,000,000	13,900	56
Caluvere. See "Ebony."			
Cos (*Artocarpus integrifolia*),......	1,810,000	11,000	42
Ebony or Caluvere (*Diospyros Ebenus*),........................	1,360,000	13,000	71
Gal or Hal Mendora (*Vateria sp.* — ?)........................	1,530,000	13,300	57
Ilal Milile (*Berrya Ammonilla*),	970,000	15,200	48
Ironwood. See "Naw."			
Jack. See "Cos."			
Mee (*Bassia longifolia*),.	1,880,000	13,000	61
Meean Milile (*Vitex altissima*),...	2,040,000	14,200	56
Naw (*Mesua Nagaha*), ...	2,580,000	17,900	72
Palmira. See "Tal."			
Paloo (*Mimusops hexandra*),......	2,430,000	18,900	68
Satinwood. See "Burute."			
Sooriya (*Thespesia populea*),.......	2,610,000	12,700	42
Tal (*Borassus flabelliformis*),......	2,810,000	14,700	65
Teak (*Tectona grandis*),............	2,800,000	14,600	55

ADDITIONAL DATA FROM THE EXPERIMENTS OF CAPTAIN FOWKE, R.E. AND MODLIAR MENDIS.

Teak from Cochin-China,..........	1,990,000	12,100	44
Teak from Moulmein,..............	1,900,000	11,520	42
Iron-bark (*Eucalyptus*—?) from Australia,......................	964,000	24,400	64
Iron-bark, rough-leaved,...........	1,157,000	22,500	64
Jarrah. See "Australian Mahogany," in Tables at end of volume.			
Stringy-bark (*Eucalyptus gigantea*) from Australia,......	1,709,000	13,000	54

312 A. **Comparative Value of Timber.**—The following table shows approximately the comparative values of different kinds of timber in Britain, that of the best Russian Red Pine being taken as the unit.*

Yellow Pine,	0·78 to 0·89
Red Pine,	0·92 to 1·00
Beech,	1·00
Elm,	1·00
Ash,	1·33
Oak—from old ships,	0·83
„ Canadian,	1·00
„ British—according to size,	1·33 to 2·00
Teak—African,	1·50
„ Indian,	2·17
Bay Mahogany,	2·50 and upwards.
Spanish Mahogany,	4·67 and upwards.

SECTION II.—*Of Joints and Fastenings in Carpentry.*

313. **Classification and General Principles.**—The *joints*, or surfaces at which the pieces of timber in a frame of carpentry touch each other, and the *fastenings* which connect those pieces together, are of various kinds, according to the relative positions of the pieces, and the forces which they exert on each other. *Joints* have been classed by Robison and Tredgold; and those authors are very nearly followed in the following classification, which will be the better understood by referring to the previous portion of this work which relates to framework in general, viz.:—Part II., Chapter I., Section IV., Articles 111 to 122, pp. 173 to 185:—

I. Joints for lengthening ties.
II. Joints for lengthening struts.
III. Joints for lengthening beams.
IV. Joints for supporting beams on beams.
V. Joints for supporting beams on posts.
VI. Joints for connecting struts with ties.

Fastenings may be classed as follows:—

I. Pins, including treenails, nails, spikes, screws, and bolts; being fastenings which are exposed principally to shearing and bending stress.

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* These comparative values are given according to Laxton's *Builder's Price Book* for 1861, the price of the best red pine in scantlings being 3s. per cubic foot.

II. Straps and tie-bars, including iron stirrups and suspending-rods, being fastenings which are exposed principally to tension.

III. Sockets.

In designing and executing all kinds of joints and fastenings, the following general principles are to be adhered to as closely as may be practicable:—

I. To cut the joints and arrange the fastenings so as to weaken the pieces of timber that they connect as little as possible.

II. To place each abutting surface in a joint as nearly as possible perpendicular to the pressure which it has to transmit.

III. To proportion the area of each such surface to the pressure which it has to bear, so that the timber may be safe against injury under the heaviest load which occurs in practice; and to form and fit every pair of such surfaces accurately, in order to distribute the stress uniformly.

IV. To proportion the fastenings, so that they may be of equal strength with the pieces which they connect.

V. To place the fastenings in each piece of timber so that there shall be sufficient resistance to the giving way of the joint by the fastenings shearing or crushing their way through the timber.

314. **Lengthening Ties** is performed by *fishing* or by *scarfing*. In a fished joint the two pieces of the tie abut end to end, and are connected together by means of "fish-pieces" of wood or iron which are bolted to them; in a scarfed joint the ends of the two pieces of the tie overlap each other. Fig. 187 is a fished joint; figs. 188, 189, and 190 are called scarfs; though in figs. 188 and 190 the ties are in fact fished with iron as well as scarfed.

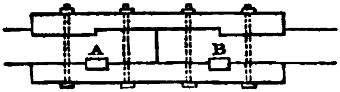

Fig. 187.

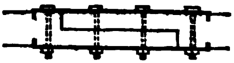

Fig. 188.

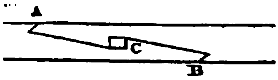

Fig. 189.

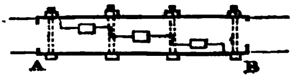

Fig. 190.

In a *plain fished joint* the fish-pieces have plane surfaces next the tie, so that the connection between them and the tie for the transmission of tension depends wholly on the strength of the bolts, together with the friction which they may cause by pressing the fish-pieces against the sides of the tie. The tie is only weakened so far as its effective sectional area is diminished by the bolt-holes. The joint sectional area of the fish-pieces should be equal to that of the ties. The joint sectional area of the bolts should be at least *one-*

fifth of that of the timber left after cutting the bolt-holes; and the bolts should be square rather than round. The bolt-holes should be so distributed, and placed at such distances from the ends of the two parts of the tie, that the joint area of both sides of the layer of fibres, which must be sheared out of one piece of the tie before the bolts can be torn out of its end, shall be as much greater than the effective area of the tie as the tenacity of the wood is greater than its resistance to shearing; as to which proportion, see Article 312, p. 450. The same rule regulates the places of the bolt-holes in the fish-pieces.

The fish-pieces and the parts of the tie may also be connected by *indents*, as at the upper side of fig. 187, or by *joggles* or *keys*, as at the lower side of the same figure. In either case the effective area of the tie is reduced by the cutting of the indents or of the key-seats, at A and B. The area of abutting surface of the indents, or of the key-seats, should be sufficient to resist safely the greatest force to be exerted along the tie; and their distances from the ends of the fish-pieces and of the parts of the tie should be sufficient to resist safely the tendency of the same force to shear off two layers of fibres.

A timber tie may be fished with plates of iron, due regard being paid to the greater tenacity of the iron in fixing the proportions of the parts, and the iron fish-plates may be indented into the wood. Fig. 188 represents a joint in which the parts of the timber tie are scarfed together, and at the same time fished with iron plates, which are indented into the wood at the ends.

Fig. 189 represents a scarfed joint for a tie, which will hold without the aid of bolts or straps. At C is a key or joggle of some hard kind of wood, which is wedged in so as to tighten the joint moderately. The depth of the key is one-third of the depth of the beam. It is evident that this joint, as shown in the figure, has only one-third of the strength of the solid timber tie; but its strength may be considerably increased by bolting on iron fish-plates at A and B.

Fig. 190 shows a scarfed joint with several keys, which should all be driven equally tight. It is also fished with iron plates, indented into the wood at the ends.

The following practical rules are given by Tredgold for the proportion which the length of a scarf (between A and B in each of the figures) should bear to the depth of the tie:—

	Without Bolts.	With Bolts.	With Bolts and Indents.
Hardwood (as Oak, Ash, or Elm), ...	6	3	2
Fir-wood, ...	12	6	4

315. **Lengthening Struts.**—At each joint in a post, pillar, or other strut, the two pieces should abut against each other at a plane surface, perpendicular to the direction of the thrust; and to keep them steady they may either be fished on all four sides, or have their abutting ends enclosed in an iron socket made to fit them. Joints in struts ought if possible to be stayed laterally. (As to the strength of timber struts, see Article 158, p. 238).

316. **Lengthening Beams** may be performed either by fishing or by scarfing; and in either case the joints should as far as practicable be placed where the bending moment is small. The construction of the joints should be the same with that of joints for lengthening ties, with the following qualifications:—

I. At the compressed side of the beam, its two pieces should have a square abutment against each other; hence oblique surfaces, such as those in fig. 189, are to be avoided.

II. The surfaces of the scarf ought to be parallel to the direction of the load; (that is to say, in general, vertical: so that in figs. 188 and 190, the plane of the paper shall represent a horizontal plane); for it was found, in experiments by Colonel Beaufoy, that a scarfed beam was stronger with the scarf "up and down" than "flatwise." (See Barlow *On the Strength of Timber*, Article 71.)

317. **Notching Beams.**—When a joist or cross-beam has to be supported on a girder or main beam, the method which least impairs the strength of the main beam is simply to place the cross-beam above it; a shallow notch being cut on the lower side of the cross-beam, so as to fit the main beam.

318. **Mortising Beams—Shouldered Tenon.**—When the space is not sufficient to admit of placing the cross-beam above the main beam, the connection may be made by means of a *mortise and tenon joint;* the *tenon* being a projection from the end of the cross-beam, and the *mortise*, a cavity in the side of the main beam, cut so as exactly to fit the tenon. The tenon may be fixed in its place by means of a pin, or of a screw. It is evident that in order to weaken the main beam as little as possible, the mortise should be cut at the middle of its depth, so that the centre of the mortise may be at the neutral axis of the beam.

To find in what proportion a beam is weakened by a plain rectangular mortise cut in the position above prescribed, let h be the depth and b the breadth of the beam, h' the depth of the mortise, and b' the distance to which it penetrates into the beam; then the beam is weakened in the following ratio:—

$$b\,h^3 - b'\,h'^3 : b\,h^3. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

(See Article 162, pp. 249 to 253.)

To keep a cross-beam steady in its proper position, a tenon requires length; to bear its share of the load, it requires depth; but a tenon at once long and deep would too much weaken the main beam. To avoid this difficulty the *shouldered tenon* is used, as shown in fig. 191. A is a cross-section of a main beam; B is one end of a cross-beam. C is the shoulder, which bears the load of that end of the cross-beam, and penetrates into the side of the main beam for a distance of one-sixth of the depth of the cross-beam or thereabouts; the depth of the shoulder below the upper side of the cross-beam is about two-thirds or three-fourths of the total depth of that beam. D is the tenon proper, whose depth is only one-sixth of that of the cross-beam, while its length is about double of its own depth. Its use is to give the joint sufficient hold, so that there shall be no risk of the shoulder being dislodged from its place in the mortise.

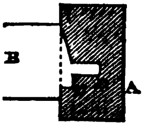

Fig. 191.

Mortises cut by hand are always rectangular. Those cut by machinery are made by a boring tool, so that although their longest sides are plane, their ends are semicylindrical; and tenons to fit them must be cut of the same shape.

319. **Post and Beam Joints.**—To support the end of a horizontal beam at one side of a post, a shouldered mortise-and-tenon joint is to be used. The shoulder should be like that on the end of the cross-beam in fig. 191; but the long tenon should be *on edge*, or have its narrowest dimension horizontal, in order that the mortise for it may weaken the post as little as possible.

When the beam is to rest on the top of the post, the joint may be secured simply by means of a small tenon in the centre of the top of the post fitting into a mortise in the under side of the beam; but there are other methods, two of which are shown in fig. 192. B B is the beam. A is a post, the top of which is fitted into a shallow rectangular notch in the under side of the beam. That notch does not extend completely across the beam, but is divided into two parts by a *bridle*, of about one-fifth of the breadth of the beam, which is left uncut in the middle of the notch. To receive the bridle, a groove of the same breadth is cut in the middle of the top of the post, as indicated by the dotted line C D. The post E is also fitted into a notch-and-bridle joint F G, the only difference being that the figure of the notch in the under side of the beam is an obtuse angled triangle instead of a rectangle. This last form is recommended by Tredgold. He also recommends a joint of the same class, in which the notch in the under side of the beam has the figure of a circular arc; but from

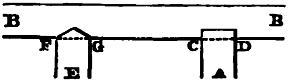

Fig. 192.

the experiments of Mr. Hodgkinson on the strength of flat-ended and round-ended pillars, it must be inferred that this construction would weaken the post. (Article 158, pp. 236 to 238.)

The same joints are applicable to the case in which a post is supported on a beam.

320. **Strut-and-Tie Joints.**—A strut and a tie meeting at an oblique angle are to be connected by means of a shoulder on the end of the strut, fitting into a notch in the side of the tie, to transmit the pressure, and of a tenon on the strut fitting into a mortise in the tie, or a bridle on the tie fitting into a groove in the shoulder of the strut, to keep the joint steady. Such joints are exemplified in figs. 193 and 194, in each of which B represents a tie-beam and A the foot of a strut or rafter. C D is the *shoulder* of the rafter, fitting into a notch in the tie-beam, and having a plane surface, which in fig. 193 has a depth equal to half of the depth of the rafter, and bisects the obtuse angle between the directions of the tie-beam and rafter; while in fig. 194 it is perpendicular to the length of the rafter, and of somewhat more than half its depth. In fig. 193 the dotted lines at F represent a *tenon and mortise*, whose breadth is one-fifth of that of the rafter. In fig. 194, the dotted line C F shows the upper surface of a *bridle*, left uncut in the middle of the breadth of the notch C D F in the tie-beam, and fitting into a groove in the shoulder of the rafter. The breadth of the bridle is one-fifth of the breadth of the tie-beam.

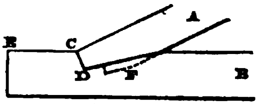

Fig. 193.

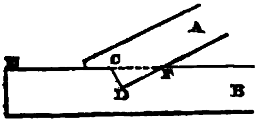

Fig. 194.

In making each of those joints, care must be taken that the length of the fibres left between the notch C D and the end E of the tie-beam is sufficient to resist safely the tendency of the longitudinal component of the thrust against the notch to shear them off; that is to say, let H be that component of the thrust of the rafter, b the breadth of the tie-beam in inches, l the distance in inches from the notch to the end of the tie-beam, f' the resistance of the wood to shearing, s a factor of safety; then

$$l = \frac{s\,H}{f'\,b}. \quad \text{.............................(1.)}$$

According to Tredgold, 4 is a sufficient value for s in this case; and hence, taking f' at 600 lbs. per square inch for fir, and 2,300 lbs. per square inch for oak, we have

$$\text{For oak, } l = \frac{H}{575\,b}; \text{ for fir, } l = \frac{H}{150\,b}. \text{ ....(1 A.)}$$

These joints may be made more secure by binding the rafter and tie together with a bolt or a strap, in a direction making as acute an angle with the tie as is practicable. The chief object of this is to hold the rafter in its place in case the end of the tie should give way. (See fig. 197, p. 462.)

321. **Suspending Pieces** in frames of carpentry are called by the very inappropriate names of *king-posts* and *queen-posts*, a king-post being a single suspending piece in the centre of a frame, and queen-posts, suspending pieces in other positions. A suspending piece hangs from the point of junction of two struts or rafters, and supports at its lower end either a beam or the ends of one or more struts.

A strut or rafter may be connected with a suspending piece by abutting against a notch cut in its side, or against a shoulder formed by an enlargement at the end of the suspending piece; and in either case the distance of the notch or shoulder from the end of the piece is to be determined by the formulæ of the preceding article. When a single suspending piece supports a beam at its lower end they are connected by means of an iron stirrup.

A better method is to make suspending pieces in pairs, so that the rafters from which they hang may abut between them directly against each other, as shown by the cross-section fig. 195, and the side view fig. 196. C and F are the ends of a pair of rafters abutting against each other; A and B the upper ends of a pair of suspending pieces, notched upon the rafters, and bolted to each other through the blocks or filling-pieces D and E. If these figures be turned upside down they will represent the lower ends of a pair of suspending-pieces, forming a wooden stirrup for the support of a beam, or of the ends of a pair of struts, as the case may be.

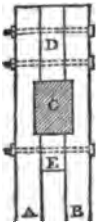

Fig. 195.

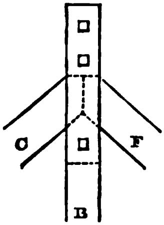

Fig. 196.

322. **Pins—Treenails.**—Wooden pins, as fastenings for joints, when of large diameter, are known as *treenails*. Experiments have been made on their resistance to a cross strain by Mr. Parsons, for the details of which, see Murray *On Ship-building;* the results may be summed up with sufficient exactness for practical purposes by saying—

I. That the ultimate resistance of English oak treenails to a shearing stress across the grain is about 4,000 lbs. per square inch of section.

II. That in order to realize that strength, the planks connected by the treenails should have a thickness equal to about three times the diameter of the treenails.

323. **Nails and Spikes.**—Where nails are exposed to any considerable strain those made by hand should be used, as they are stronger than those made by machinery.

The weight in lbs. of a thousand of the "flooring brads" commonly used in carpentry may be roughly computed by taking *twice the square of their length in inches.*

The nails or spikes used for fastening planks to beams are usually of a length equal to from twice to twice and a-half the thickness of the planks.

The following are the results, as stated by Tredgold, of experiments by Bevan on the force required to draw nails of different sizes out of *Dry Christiania Deal,* into which they had been driven to different depths *across the grain:*—

Kind of Nails.	Length. Inches.	No. to the Lb.	Inches driven.	Force to draw. Lbs.
Sprigs, ...	0·44	4,560	0·4	22
"	0·53	3,200	0·44	37
Threepenny brads,	1·25	618	0·50	58
Cast iron nails, ...	1·00	380	0·50	72
Fivepenny nails,	2·00	139	1·50	320
Sixpenny nails, ...	2·50	73	1·00	187
" ...	2·50	73	1·50	327
" ...	2·50	73	2·00	530

So far as these results can be expressed by a general law, they seem to indicate that the force required to draw a nail, driven across the grain of a given sort of wood, varies nearly as the *cube of the square root of the depth to which it is driven;* and that it increases with the diameter of the nail, but in a manner which has not yet been expressed by a mathematical law.

The following are the results of Bevan's experiments on the force required to draw a "sixpenny nail" of 73 to the lb., which had been driven one inch into different sorts of timber:—

Deal, across the grain, ...	187	lbs. (as above.)
Oak, "	507	"
Elm, "	327	"
Beech, "	667	"
Green Sycamore, "	312	"
Deal, endwise, ...	87	"
Elm, "	257	"

The following were the forces required to draw asunder a pair of planks joined by *two nails* of 73 to the lb.:—

Deal ⅞ inch thick, ...	712 lb.
Oak 1 inch thick, ...	1009 „
Ash 1 inch thick, ...	1420 „

324. **Screws.**—The holding power of screw-nails, or "wood-screws," is probably proportional nearly to the product of the diameter of the screw, and of the depth to which it is screwed into the wood. The following are the results of Bevan's experiments, quoted by Tredgold, on the force required to draw screws out of planks of *half-an-inch thick*, the screws being 0·22 inch in diameter over all, and 0·035 inch in depth of thread, with 12 threads to the inch.

Beech, ...	460 to 990 lbs.
Ash, ...	790 lbs.
Oak, ...	760 „
Mahogany, ...	770 „
Elm, ...	665 „
Sycamore, ...	830 „

325. **Bolts—Washers.**—The rules for proportioning bolts which have to withstand a shearing stress in carpentry have already been stated in Article 314, p. 455.

The sides of a piece of timber should always be protected against the crushing action of the head and nut of a bolt by means of flat rings called "*washers;*" the area of each washer being at least as many times greater than the sectional area of the bolt as the tenacity of the bolt is greater than the resistance of the timber to crushing; that is to say, for fir the diameter of the washer may be made about 3½ times the diameter of the bolt, and for oak about 2½ times.

When a bolt is oblique to the direction of the beam that it traverses, the timber may either have a notch cut in it with a surface perpendicular to the bolt, to bear the pressure of the washer, or it may be notched to receive a bevelled washer of cast iron, one of whose surfaces fits the notch in the wood, while another being perpendicular to the axis of the bolt, bears the pressure of the nut or head, as the case may be.

The screws of bolts are usually made of the following proportions, or nearly so: the depth of the thread one-tenth, and the pitch one-fifth of the internal diameter. A bolt which has to be often removed may be made fast by having a slot or oblong hole in one of its ends, through which a key or wedge is driven.

326. **Iron Straps** are used nearly in the same manner with bolts, to bind pieces of timber together. They have the advantage of not requiring so much of the timber to be cut away as bolts do.

According to the usual proportions of straps the breadth ranges from four times to eight times the thickness. When a strap has eyes in its ends, for bolting them to the sides of a beam, it ought to be either broadened or thickened round each eye, so that the sectional area of the iron may be at least as great at the sides of the eye as in other parts of the strap. When a strap is to embrace completely a piece or pieces of timber, it may, when practicable, be welded into a rectangular hoop, and driven on from one end of the timber; but when that is impracticable or inconvenient, it must be made with screws on its ends, of the same sectional area with its flat part, upon which screws a cross-piece is to be made fast with nuts.

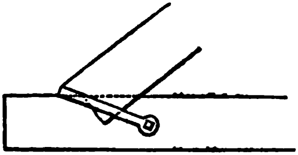

Fig. 197.

327. **A Stirrup** is a strap which supports a beam, or sustains the thrust of one end of a strut. If the tie or suspending piece is of wood, the ends of the stirrup are bolted through it; if of iron, the stirrup and tie, or suspending rod, are usually welded into one piece.

328. **Iron Tie-Rods** may be used instead of timber ties and suspending pieces in all these parts of a frame of carpentry in which tension alone is to be borne, and is not combined with a bending action, nor alternated with thrust. They may be connected with the timber pieces of the frame by means of screws and nuts, eyes and bolts, slots and wedges, stirrups or sockets; and they should be capable of being tightened when required, by means of screws or of wedges. Care must be taken that the points of attachment of the ends of a long iron tie-rod are free to change their distance from each other to an extent sufficient to allow of the changes of length of the rod which are produced by changes of temperature, at the rate of about

·0012 of the length of the rod, for 180° of change of temperature on Fahrenheit's scale.

329. **Iron Sockets,** made to fit the ends of pieces of timber furnish a convenient means of making various joints in framework, especially at points where struts meet each other, or have to be connected with tie-rods. If thrust alone is to be borne by the socket, cast iron is the most convenient material; if any considerable tension is to be borne, strong wrought iron plates are best.

330. **Protection of Iron Fastenings.**—The iron fastenings of timber, especially if in contact with oak, rust very rapidly unless properly protected. Amongst the most efficient means of protection are the following:—

I. Boiling in coal-tar, especially if the pieces of iron have first been heated to the temperature of melting lead.

II. Heating the pieces of iron to the temperature of melting lead, and smearing their surfaces, while hot, with cold linseed oil, which dries and forms a sort of varnish. This is recommended by Smeaton.

III. Painting with oil-paint, which must be renewed from time to time. The linseed oil process is a good preparation for painting.

IV. Coating with zinc, commonly called galvanizing. This is efficient, provided it is not exposed to acids capable of dissolving the zinc; but it is destroyed by sulphuric acid in the atmosphere of places where much coal is burned, and by muriatic acid in the neighbourhood of the sea.

SECTION III.—*Of Timber Built Beams and Ribs.*

331. **Joggled and Indented Built Beams.**—In fig. 198 two pieces of timber are built into one beam of double the depth of either, by the aid of hardwood *keys* or *joggles*, which resist the shearing stress at the surface of junction, and of vertical bolts in the spaces between the keys. It is obvious that no key nor bolt should be put at the middle of the span; because in general there is no shearing stress there; and also because the bending moment is in general a maximum there, and it is desirable to weaken the cross-section as little as possible. The grain of the keys should run vertically. According to Tredgold, the joint depth of all the keys should amount to *once and a-third* the total depth of the beam, and the breadth of each key should be twice its depth.

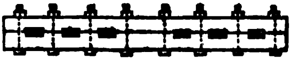

Fig. 198.

Considering that the stress at the neutral surface is equivalent to thrust in a direction sloping at 45°, combined with tension in a direction sloping at 45° the opposite way (see Article 162, p. 250), it would seem that the best position for the keys would be that shown in fig. 199, their fibres being made to slope in the direction of the thrust, and the bolts being made to slope in the direction of the tension. This, however, so far as I know, has never yet been tried.

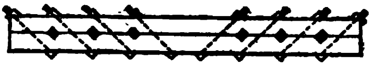

Fig. 199.

In fig. 200 the two pieces of which the beam is built are indented into each other, a sacrifice of depth being thus incurred equal to the depth of an indent. The abutting surfaces of the indents face outwards in the upper piece, and inwards in the lower, so as to resist the tendency to slide. According to experiments by Duhamel, the joint depth of the indents should amount to *two-thirds* of the total depth of the beam. The beam in the figure is slightly tapered from the middle towards the ends, in order that the hoops which are used to bind it may be put on at the ends and driven tight with a mallet.

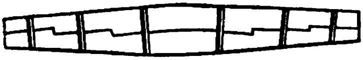
Fig. 200.

When a beam is built of several pieces in length as well as in depth, they should break joint with each other. The lower layer should be scarfed or fished like a tie (Article 314, p. 454), and the upper layer should have plain butt joints. The upper layer of a built beam is sometimes made of hardwood, and the lower layer of fir, in order to take advantage of the resistance of the former to crushing and the tenacity of the latter.

332. **Bent Ribs** are sometimes obtained from naturally bent pieces of timber, called "knees."

Naturally straight pieces of timber may be permanently bent by steaming them until the wood is softened, and while in that condition bending them by combinations of screws, and keeping them bent until they dry and stiffen. By this process there is a risk of injuring the tenacity of the fibres at the convex side of the piece, unless they are prevented from stretching by the following contrivance (see fig. 201):—A A is the piece of wood to be bent. Its ends abut against the bent parts of a strip of boiler-plate B B, which has two eyes C C, that are drawn together by a pair of tightening-screws at D till the required curvature is produced. The whole of the fibres of the timber are compressed, and none of them have their tenacity injured; and it is found by experiment that bent ribs made in this way are as strong as natural knees.

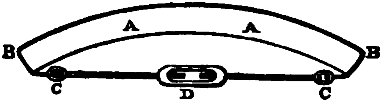

Fig. 201.

333. **Built Ribs** are best made by a method invented by Philibert de l'Orme, and represented in figs. 202, 203, and 204. Fig. 202 is a side view, and fig. 203 a plan of a rib made of several layers of planks *set on edge*, breaking joint with

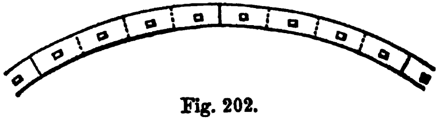
Fig. 202.

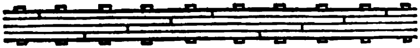
Fig. 203.

each other (as the plan shows), and connected together by square bolts or wedges.

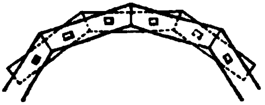

Fig. 204.

In fig. 202 the edges of the planks are supposed either to have been originally curved, or have had the corners smoothed off: in fig. 204 it is shown how they may be used with straight edges.

A built rib of this sort, properly constructed, is nearly as strong as a solid rib of the same depth, and of a breadth less by the thickness of *one layer*.

334. **Laminated Ribs** are composed, as in fig. 205, of layers of plank *laid flatwise*, breaking joint and bolted together. They are easily made, and very often used in bridges and roofs; but the experiments of Ardant have shown that they are weaker than solid ribs of the same dimensions, nearly in the ratio of unity to the number of layers into which they are divided.

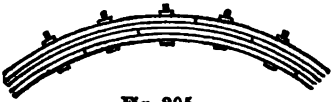

Fig. 205.

SECTION IV.—*Of Timber Frames and Trusses.*

335. **General Remarks on the Balance, Stability, and Strength of Timber Framing.**—The general principles of the balance and stability of frames and ribs of any material, already given in Part II., Chapter I., Section IV., pp. 173 to 203, and the general principles of the strength of materials, given in the same chapter, Section V., pp. 221 to 314, serve to solve all problems relating to the balance, stability, and strength of structures in carpentry. In the present section it will only be necessary to add some explanations of matters of detail in those particular cases which occur most frequently in practice. In fixing the transverse dimensions, or "*scantlings*," of the main pieces of timber which compose a structure of carpentry, made of good pine, fir, or oak, it is usual to limit the greatest intensity of the stress, whether compressive or tensile, to 1,000 *lbs. per square inch of section;* and when this is compared with the tenacity, resistance to crushing, and modulus of rupture, of those kinds of timber, it appears that the *factor of safety* ranges from 6 to 14, or thereabouts, and is on an average 10, as has been stated in Article 143, p. 222.

336. **Platforms** of timber consist of planks resting on beams. The beams upon which the planks rest may either be the main beams or *girders* of the structure, or they may be cross-beams or *joists*, supported by those girders. (Articles 317, 318, pp. 456,

457.) The former mode of construction is that which enables a given strength to be attained with the least expenditure of material and labour at the outset; but the latter, in most cases, is the more economical in the end; for although it causes a greater expenditure of material in joists than it saves by requiring thinner planking, the saving in the quantity of planking is productive of the greatest saving of expense; for the planking requires more frequent renewal than the joists.

It would be foreign to the purpose of this book to describe the various modes of constructing the floors of houses. The timber platforms with which the civil engineer is chiefly concerned are those of bridges and of foundations. The latter will be described further on.

The usual thickness of the planking for the platform of a bridge with joists is from 3 to 4 inches, the joists being placed at distances of from 2 feet to 4 feet from centre to centre. That thickness has been found by experience to be requisite in order to withstand the shocks, friction, and wear, to which the planking is subjected, and is in general much greater than is required for mere strength to support the greatest load with safety.

In bridges supporting railways where chairs are used, the joists are usually so arranged as to be directly under the chairs.

The breadth of the joists is from one-eighth to one-quarter of their distance apart; and their transverse dimensions are fixed with reference to the greatest load upon them and to the width which they span over between the girders. For timber bridges and platforms not carrying railways, that load, *in lbs. per square foot of platform*, is nearly as follows:—

Weight of a closely-packed crowd, estimated at	120	lbs. per sq. ft.
Add for the planking and joists, say	30	„ „
Gross load for a single wooden platform, ...	150	„ „
If there is a broken stone or gravel roadway, add	100	„ „
Making in all	250	„ „

When the platform carries a railway, the scantling of each joist must be regulated by the fact, that the load on a pair of driving wheels of the heaviest engine used on the line may rest above a certain pair of points in the joist. Should the rails be either directly above the girders, or so nearly above them that this rule gives a less scantling than the former, the rule for platforms not carrying railways is to be followed.

The best mode, in general, of designing the joists, is to fix the ratio of the depth to the span with a view to stiffness, as explained in Article 170, p. 275, and then compute the breadth with a view to strength.

The following formulæ express the results of these rules algebraically.

CASE I. For platforms not carrying railways, let

B be the distance from centre to centre of the joists.
b, the breadth of a joist.
h, its depth.
l, the span from centre to centre of the girders; then the greatest moment of flexure is to be as follows:—

$$\left.\begin{aligned} \frac{1{,}000\, b\, h^2}{6} &= \frac{150\, \mathrm{B}\, l^2}{8 \times 144} \text{ for plank roadways;} \\ \text{or } &\frac{250\, \mathrm{B}\, l^2}{8 \times 144} \text{ for broken stone roadways;} \end{aligned}\right\} \text{......(1.)}$$

and consequently,

$$\left.\begin{aligned} \frac{b}{\mathrm{B}} &= \frac{l^2}{1{,}280\, h^2} \text{ for plank roadways;} \\ \frac{b}{\mathrm{B}} &= \frac{l^2}{768 h^2} \text{ for broken stone roadways.} \end{aligned}\right\} \text{..........(2.)}$$

CASE II. For a platform carrying a railway, in which one line of rails lies midway between a pair of girders; let

W be the load on a pair of driving wheels of the heaviest engine, in lbs.
k, the gauge of the rails, *from centre to centre in inches;* then, l being also expressed in inches,

$$\frac{1{,}000\, b\, h^2}{6} = \frac{\mathrm{W}\,(l-k)}{4}; \text{....................(3.)}$$

and therefore

$$b = \frac{3}{2{,}000} \cdot \frac{\mathrm{W}\,(l-k)}{h^2}. \text{....................(4.)}$$

Example.—Let $l = 90$ inches; $k = 60$ inches; $h = 12$ inches; $\mathrm{W} = 30{,}000$ lb.; then $b = 9{\cdot}375$ inches.

As to the length and weight of the spikes to be used for nailing the planks to the joists, see Article 323, p. 460.

When a platform has both girders and joists, it may be stiffened against distortion by laying the planks diagonally. When separate diagonal braces are used for that purpose, their dimensions should be regulated by the horizontal shearing stress which the wind may produce when blowing against the side of the structure, as calculated by the formula for F in Case VI. of the table, p. 246. The greatest intensity of the pressure of the wind hitherto observed in Britain is 55 lbs. on the square foot; in tropical climates it is said sometimes to reach double that amount.

When there is no special reason for making a timber platform close-jointed, it is advisable to lay the planks with openings between them of from ¼ inch to ½ inch in width, in order to let rain-water escape and air circulate.

337. **Roofs—Covering and Load.**—The parts of a roof may be distinguished into the *covering* and the *framework*. The extent of the covering of a roof is usually expressed in *squares* and *feet*, a *square* of roofing being 100 square feet. The following table shows the structure and weight in lbs. per square foot of the most usual kinds of covering for timber roofs, and their flattest ordinary slopes:*

MATERIAL.	Flattest Ordinary Slope.	Weight per Square Foot—Lbs.
Sheet copper, about ·022 of an inch thick,	4°	1·00
Sheet lead,	4°	7·00
Sheet zinc,	4°	1·25 to 1·625
Sheet iron, plain, $\frac{1}{16}$ inch thick, ...	4°	3·00
„ corrugated,	4°	3·40
Cast iron plates, ⅜ inch thick,	4°	15·00
Slates,	30° to 22½	5·00 to 11·20
Tiles,	30° to 22½	6·50 to 17·80
Boarding, ¾ inch thick, (Weight of other thicknesses in proportion.)	22½°	2·50
Thatch,	45°	6·50
For the timbering of slated and tiled roofs, add per square foot,		from 5·50 to 6·50
For the pressure of the wind, according to Tredgold, there is to be taken into account an additional load per square foot of		40

Sheet copper is nailed on boards. Sheet lead, zinc, and iron, slates, and tiles, may be either nailed on laths or battens (which are slender pieces of timber of from 1 inch by 1½ inch to 1½ inch by 3 inches, or thereabouts, nailed across the rafters), or upon boarding of from $\frac{1}{2}$ inch to $\frac{3}{4}$ inch thick. Sheet iron may be nailed or screwed directly to the rafters, and cast iron plates screwed or bolted to the *principal rafters* to be afterwards mentioned. Roofs in which the framework as well as the covering is of iron will be treated of in another chapter.

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* The angles set down for the slopes of roofs in this table are all aliquot parts of a circumference; such angles being at once the most convenient in designing framework, and the most pleasing to the eye. (The latter fact appears to have been first pointed out by Mr. Hay in his *Theory of Beauty*.)

The *steepest* ordinary declivity in Gothic roofs is 60°; but by the Metropolitan Building Act, 1855, the declivity of the roofs of buildings used for purposes of trade is limited to 47°.

338. **Rafters and Purlins** are those parts of the framework of a roof which lie immediately below the covering, so as to form with it a more or less sloping platform. In fig. 206, A A is one of the *common rafters*, which are placed from 1 foot to 2 feet apart from centre to centre, and are supported by the *purlins*, to which they are spiked or screwed. B is a cross-section of one of the purlins, which lie from 6 feet to 8 feet apart from centre, and are slightly notched where they cross the *principal rafters*. The side of the purlin which faces down the slope is supported by means of the block C, which is screwed to the principal rafter D D. The principal rafters form parts of a series of frames or *trusses*, which are placed at from 5 to 10 feet apart. In order to prevent the action of transverse loads on the principal rafters, they are to be supported below each point where the purlins cross them by struts, such as that of which the upper end is shown at E.

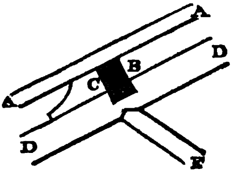

Fig. 206.

Diagonal Braces, to stiffen the roof and stay the trusses against upsetting sideways, may be framed either between the rafters or between the purlins. No precise rule can be given for their scantling; but they will in general be strong and stiff enough if each transverse dimension is made one-twentieth part of the unsupported length. When the roof is boarded, the same purpose may be answered by laying the boards diagonally.

339. **Roof-Trusses** are frames of the kinds already discussed in Articles 114 to 120, pp. 176 to 184, in which the principles that regulate the thrusts and tensions along the several pieces have been explained. In the present Article it is only necessary to state what particular cases of such frames are the most common in practice.

I. TRIANGULAR TRUSS.—Fig. 207 is a skeleton figure of the simplest form of truss, which is an isoceles triangle, B B being the tie-beam, and A and C equally inclined principal rafters. 2 and 3 are the points of support, 1 the ridge. D is a suspending-piece, which, when of wood, is called the *king-post*, and when of iron, the *king-bolt;* it supports the weight of the middle half of the tie-beam, and of any floor or other load with which that beam may be loaded.

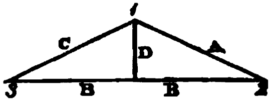

Fig. 207.

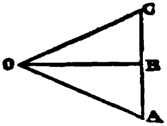

Fig. 208.

In the diagram, fig. 208, the vertical line C A represents the load on the point 1; that is, half the gross weight of the roof; O C and O A, parallel to the two rafters, represent the thrusts along them; and the horizontal line O B represents the tension along the tie-beam.

The algebraical expression of this is as follows:—

Let W be the gross weight of the truss, together with that of the division of the roof, of which it occupies the middle, and that of the floor, or other load supported by the tie-beam.

c, the half-span of the truss.
k, its rise.
H, the tension along the tie-beam.
T, the thrust along each of the rafters; then

$$H = \frac{W}{4}\frac{c}{k};\quad T = \sqrt{\left(H^2 + \frac{W^2}{16}\right)}. \quad \ldots\ldots\ldots\ldots(1.)$$

II. Trapezoidal Truss.—In fig. 209, B B B is the tie-beam, A and C two equally inclined principal rafters, F a horizontal rafter or *straining-piece.* D and E are suspending-pieces, to carry part of the weight of the tie-beam, and also that of the floor, which usually rests on the tie-beam between the points 5 and 6, together with its load.

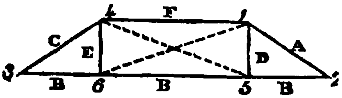

Fig. 209.

The same diagram of forces as in the former case, fig. 208, applies to this case; it being understood that C B = B A represent the loads on the points 1 and 4 respectively; that is, on each of those points, *one quarter of* the weight of the roof and truss, and *half* the weight of the floor between the points 5 and 6. The horizontal line O B represents at once the tension along the tie-beam, and the thrust along the straining-piece F.

The part of the roof above the straining-piece F may either be flat, or may be supported by a small triangular *secondary truss* (see Article 121, p. 184), similar to fig. 207, and resting on the points 1 and 4. The straining-piece F of the principal truss may be made to act also as the *tie-beam* of the secondary truss; in which case the thrust along it will be the *excess of the horizontal stress* H *in the principal truss above that in the secondary truss.*

III. SECONDARY TRUSSING UNDER PRINCIPAL RAFTERS.—The direct support of the points where the purlins cross the rafters, already mentioned in Article 338, p. 469, is effected by means of a system of secondary trussing, of which fig. 210 may be taken as an example. That figure represents a truss in which the main tie and the suspending-pieces are all iron rods; but it is applicable also to the case in which either some or all of those pieces are of timber. (*A. M.*, 159.)

Let W be the weight of the roof distributed over the points 3, 4, 6, 1, 8, 10, 2, so that *one-twelfth* rests directly on each of the points of support 2 and 3, and *one-sixth* on each of the five intermediate points; 2 3 is the great tie-rod; 1 7, 6 5, 8 9, suspending-rods; 7 6, 7 8, 5 4, 9 10, struts.

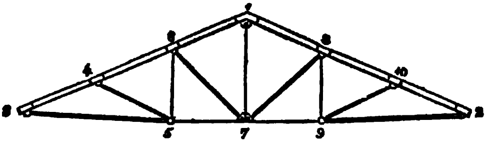

Fig. 210.

(1.) *Primary Truss* 1 2 3.—The load at 1, as before, is to be taken as $= \frac{1}{2}$ W, and the stresses found by equation 1 of this article.

(2.) *Secondary Trusses* 7 6 3, 7 8 2.—The load at 6 is to be held to consist of one-half of the load between 6 and 1, and one-half of the load between 6 and 3; that is, one-half of the load between 1 and 3, or $\frac{1}{4}$ W. The trusses are triangular, each consisting of two struts and a tie, and the stresses are to be found as in Article 115, p. 177; that is to say, let H′ denote the horizontal stress in each of these secondary trusses; T′ the thrust along the rafters between 6 and 3, and between 8 and 2, due to their places in those trusses; and S′ the thrust along the struts 67 and 87; then

$$H' = \frac{W c}{12 k};\ T = \sqrt{\left(H'^2 + \frac{W^2}{144}\right)};\ S' = \sqrt{\left(H'^2 + \frac{W^2}{36}\right)}. \quad (2.)$$

The suspension-rod 1 7 supports two-thirds of the load on 7 6 3, and two-thirds of the load on 7 8 2; that is, $\frac{2}{3} \cdot \frac{2}{4} \cdot W = \frac{1}{3}$ W; and this, together with $\frac{1}{6}$ W, which rests *directly* on 1, makes up the load of $\frac{1}{2}$ W, already mentioned.

(3.) *Smaller Secondary Trusses* 3 4 5, 9 10 2.—Each of the points 4 and 10 sustains a load of $\frac{1}{6}$ W, from which the stresses on the bars of those smaller trusses can be determined as follows:—

$$\mathrm{H}'' = \frac{\mathrm{W}\,c}{12\,k};\ \mathrm{T}'' = \mathrm{S}'' = \sqrt{\left(\mathrm{H}''^2 + \frac{\mathrm{W}^2}{144}\right)}. \quad \text{.......(3.)}$$

One-half of the load on 4, that is, $\frac{1}{12}$ W, hangs by the suspension-rod 6 5; and this, together with $\frac{1}{6}$ W, which rests directly on 6, makes up the load of $\frac{1}{4}$ W on that point, formerly mentioned. The same remarks apply to the suspension-rod 8 9.

(4.) *Resultant Stresses.*—The pull between 5 and 9 is the sum of those due to the primary and larger secondary trusses; that between 5 and 3, and between 9 and 2, is the sum of the pulls due to the primary, larger secondary, and smaller secondary trusses; that is to say,

$$\mathrm{H} + \mathrm{H}' = \frac{\mathrm{W}\,c}{3\,k};\ \mathrm{H} + \mathrm{H}' + \mathrm{H}'' = \frac{5\,\mathrm{W}\,c}{12\,k}; \quad \text{......(4.)}$$

The thrust on 1 6 is due to the primary truss alone; that on 6 4 to the primary and larger secondary truss; that on 4 3 to the primary, larger secondary, and smaller secondary trusses; and similarly for the divisions of the other rafter.

(5.) *General Case.*—Suppose that instead of only three divisions, there are n divisions in each of the rafters 1 3, 1 2, of fig. 78; so that besides the middle suspension-rod 1 7, there are $n - 2$ suspension-rods under each rafter, or $2\,n - 4$ in all; and $n - 1$ sloping-struts under each rafter, or $2\,n - 2$ in all. There will thus be $2\,n - 1$ centres of resistance; that is, the ridge-joint 1 and $n - 1$ on each rafter; and the load *directly supported* on each of these points will be $\frac{\mathrm{W}}{2n}$.

The total load on the ridge-joint 1, will be as before, $\frac{\mathrm{W}}{2}$; that is to say, $\frac{\mathrm{W}}{2n}$ directly supported, and $\frac{\mathrm{W}}{2}\left(1 - \frac{1}{n}\right)$ hung by the middle suspension-rod.

The total load on the upper joint of any secondary truss, distant from the ridge-joint by m divisions of the rafter, will be, $\frac{n - m + 1}{4\,n}$ W; that is to say, $\frac{\mathrm{W}}{2n}$ directly supported; and $\frac{n - m - 1}{4\,n}$ W hung by a suspension-rod.

The stresses on the struts and tie of each truss, primary and secondary, being determined as in Article 115, are to be combined as in the preceding examples.

The following formulæ give the horizontal stress H_m, the thrust along the rafter T_m, and the thrust along the strut S_m, in that secondary truss which has its highest point at m divisions of the rafter from the ridge-joint:—

$$H_m = \frac{W c}{4 n k}; \text{ (being the same for each secondary truss);...(5.)}$$

$$\left.\begin{array}{c} T_m = \sqrt{\left(H_m^2 + \frac{W^2}{16 n^2}\right)} = \frac{W}{4 n} \sqrt{\left(\frac{c^2}{k^2} + 1\right)}; \\ \text{(also the same for each secondary truss.)} \end{array}\right\} \text{.....(6.)}$$

$$S_m = \sqrt{\left\{ H_m^2 + \frac{W^2}{16 n^2} \cdot (n - m)^2 \right\}} = \frac{W}{4n} \left(\frac{c^2}{k^2} + (n - m)^2\right). \text{ (7.)}$$

It follows that the total tensions on the several divisions of the tie-rod and thrusts on the several divisions of the rafters, commencing at the divisions next the middle suspending-rod, are as follows $\left(\text{making } \frac{W c}{4 k} = H, \text{ and } \frac{W}{4} \sqrt{\left(\frac{c^2}{k^2} + 1\right)} = T, \text{ as in the equations 1}\right)$;

$$H\left(1 + \frac{1}{n}\right); H\left(1 + \frac{2}{n}\right); \text{ \&c., } \quad . \quad . \quad . \quad . \quad H \cdot \frac{2n - 1}{n}; \text{ (8.)}$$

$$T; T\left(1 + \frac{1}{n}\right); T\left(1 + \frac{2}{n}\right); \text{ \&c., } \quad . \quad . \quad . \quad T \cdot \frac{2n - 1}{n}. \text{ (9.)}$$

In timber roofs, instead of resisting the horizontal thrust of such struts as 4 5 and 9 10 by means of tie-rods, it is usual to make their lower ends abut against a horizontal strut or straining-piece laid on the top of the main tie-beam, and extending from 5 to 9; the object being to give transverse strength to the tie-beam. In that case the tension is uniform along the whole length of the tie-beam, being $H \cdot \frac{2n - 1}{n}$.

IV. GOTHIC ROOF-TRUSSES belong to the class of "Open Polygonal Frames," already mentioned in Article 117, p. 179; and they exert oblique thrust against the walls or buttresses which support them. The framing is so designed as to make the horizontal component of that thrust as small as possible.

Fig. 211 may be taken as an example. In this truss B C and B′ C′ are horizontal ties, each extending over one-quarter of the span, and E F is a suspending-piece; all the other pieces are struts. The struts A C and A′ C′ are curved for the sake of architectural effect; their straight lines of resistance, which are parallel to the rafters, are marked by dots. The curved pieces C F, C′ F, are mere stays, to provide against casual irregularities of the load.

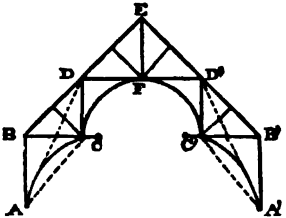

Fig. 211.

The lines of resistance of the *primary truss* are the horizontal line D D′, and the dotted lines A D, A′ D′. The diagram of forces is formed thus:—In fig. 212, draw O H horizontal, H G vertical, and O G || A D. Take H G to represent 3-8ths of the weight of the truss with its load; then will O H represent the horizontal stress, and O G the oblique thrust exerted along D A against the abutment.

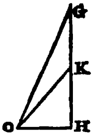

Fig. 212.

The dotted line D A is the line of resistance of a frame or compound strut, consisting of the four struts A B, A C, B D, and C D, and the tie B C. The stresses on these pieces are represented as follows:—

the tension on B C, by O H (fig. 212.)
the thrusts along B D and A C, by O K || B D || A C; (K bisects H G);
the thrust on D C by K H = $\frac{3}{16}$ths of gross load;
the thrust on B A by $1\frac{2}{3}$ K H = $\frac{5}{16}$ths of gross load.

D E D′ forms a secondary truss, loaded at E with one-quarter of the gross load; D D′ is the tie of this truss as well as the straining piece of the primary truss; and the tension arising from the action of the secondary truss is to be subtracted from the thrust due to the action of the primary truss, to find the resultant thrust along D D′, which is thus found to be represented by $\frac{2}{3}$ O H. The thrust along E D is represented by $\frac{1}{3}$ O K.

340. **Strength of Tie-Beams, Strut-Beams, and Bent Struts.**—Let H be the greatest direct working stress, whether tension or thrust, along the line of resistance of a given piece whose breadth is b and depth h; M the greatest working bending moment, whether arising from a transverse load, or from the neutral axis of the piece not coinciding with the line of resistance (in which latter case M = H × greatest distance of the neutral axis from the line of resistance); f the greatest safe working intensity of stress; then,

$$f' = \frac{\mathrm{H}}{b\,h} + \frac{6\,\mathrm{M}}{b\,h^2}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

and if h has been fixed beforehand, b is given by the formula

$$b = \left(\frac{\mathrm{H}}{h} + \frac{6\,\mathrm{M}}{h^2}\right) \div f'. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

As already stated, $f' = 1{,}000$ lbs. per square inch in ordinary carpentry.

341. **Bridge-Trusses.**—A bridge-truss is usually one of two or more parallel frames of carpentry, which act as girders, in supporting the cross-beams or joists of the platform of a bridge. (Article 336, p. 465.) The principal struts which it contains may spring either from a tie-beam, like the rafter of a roof, from iron sockets connected by means of a tie-rod, or from suitable piers and abutments of timber or stone. The most usual elementary figures of bridge-trusses are, like those of roof-trusses, the triangle (fig. 207), and the trapezoid (fig. 209); and the principles of their stability and equilibrium are the same, except that in a bridge-truss, special provision must be made for the unequal distribution of the load, both transversely and longitudinally.

I. *Load Unequal Transversely.*—This case occurs chiefly in bridges for double lines of railway, when one track is loaded and the other unloaded. The proportions in which the rolling load is distributed over the girders, when there are only two of them, is simply the inverse ratio of the horizontal distances of its centre of gravity from the two girders (Article 112, p. 174); but there are often more than two girders, most frequently four; and then, in order to determine the proportions in which the load is distributed over them, the assumption is made that the cross-beams remain sensibly straight; so that the difference between the deflections of any two of the girders, and consequently the difference between the shares of the load borne by them, is proportional simply to the distance between them.

To illustrate the application of this, let the girders, and the rolling load which by means of a cross-beam is made to rest on them, be arranged in cross-section as follows:—

$$\begin{array}{ccccc} & \mathrm{W'} & & & \\ \mathrm{A} & \mathrm{B} & \mathrm{O} & \mathrm{C} & \mathrm{D} \end{array}$$

W′ denotes the position of the centre of gravity of the rolling load; O the centre line of the platform; A, B, C, D, the four girders. Then, the *mean* share of the rolling load borne by each girder will be $W' \div 4$.

To find the deviations from that mean share, let

$$O B = z_1;\ O A = z_2;\ O C = -z_1;\ O D = -z_2;$$

and let the *horizontal* distance from O to W′ be z_0.

The deviation from the mean of the load on any girder whose distance from the centre line is z must be $a z$; a being a co-efficient to be determined by the condition that the moment of W′ relatively to O is equal and opposite to the sum of the moments of the resistances of the beams relatively to the same axis. This condition, expressed in symbols, gives $W' z_0 = 2 a (z_1^2 + z_2^2)$; whence $a = \frac{W' z_0}{2 (z_1^2 + z_2^2)}$; and the shares of the rolling load on the four girders are as follows:—

$$\left.\begin{array}{l}
\text{on A; } \frac{W'}{4} + a z_2 = W' \left(\frac{1}{4} + \frac{z_0 z_2}{2 (z_1^2 + z_2^2)}\right); \\
\text{on B; } \frac{W'}{4} + a z_1 = W' \left(\frac{1}{4} + \frac{z_0 z_1}{2 (z_1^2 + z_2^2)}\right); \\
\text{on C; } \frac{W'}{4} - a z_1 = W' \left(\frac{1}{4} - \frac{z_0 z_1}{2 (z_1^2 + z_2^2)}\right); \\
\text{on D; } \frac{W'}{4} - a z_2 = W' \left(\frac{1}{4} - \frac{z_0 z_2}{2 (z_1^2 + z_2^2)}\right).
\end{array}\right\} \ldots\ldots(1.)$$

When the share of the load on D, as often happens, proves to be *negative*, it shows that the girder furthest from the loaded track is *pulled upwards* by the platform.

As a numerical example, let the bridge be one under an ordinary narrow gauge railway, and let the four girders be exactly under the four rails respectively; so that we may make, with sufficient accuracy for the present purpose,

$$z_1 = 3 \text{ feet};\ z_2 = 8 \text{ feet};\ z_0 = 5\tfrac{1}{2} \text{ feet};$$

then,

$$\begin{array}{rl}
\text{load on A} = & W' \left(\frac{1}{4} + \frac{22}{73}\right) = + \cdot 551\, W' \\
\text{,, B} = & W' \left(\frac{1}{4} + \frac{33}{292}\right) = + \cdot 363\, W' \\
\text{,, C} = & W' \left(\frac{1}{4} - \frac{33}{292}\right) = + \cdot 137\, W' \\
\text{,, D} = & W' \left(\frac{1}{4} - \frac{22}{73}\right) = - \cdot 051\, W'
\end{array}$$

These results have been verified by careful experiments on a great scale.

The most important of them practically is the share of the load on A, being the greatest share. In order to arrange the girders so that this share shall not exceed *one-half*, the following equation should be fulfilled:—

$$z_1^2 = 2\,z_0\,z_2 - z_2^2 \text{......................} (2.)$$

For example, let $z_0 = 5\frac{1}{2}$ feet; $z_2 = 10$ feet; then $z_1 = \sqrt{10} = 3{\cdot}16$ feet.

II. *Load Unequal Longitudinally.*—This sort of inequality must be provided for in every case in which the figure of the truss has more sides than three.

The most important example in practice is that of the trapezoidal truss, whether springing from a tie-beam, as in fig. 209, p. 470, or from a pair of abutments, or from sockets connected by means of a tie-rod.

There are two means of enabling the truss to resist a partial load: by the stiffness of a longitudinal beam, and by diagonal bracing.

The longitudinal beam is either the tie-beam, or, in the absence of a tie-beam, a beam resting on the top of the truss, and bolted to the straining-piece F in the figure.

Let c denote the half-span of the truss; x, the distance of the points 4 and 1 from the middle of the truss.

Let a partial load W′ be applied at one of these points, the other being unloaded. Then the longitudinal beam has to resist a bending action, which is greatest at the loaded point and at the unloaded point, producing convexity downwards at the loaded point, and upwards at the unloaded point; the bending moment has the following value:—

$$\mathrm{M}' = \frac{\mathrm{W}'\,x\,(c - x)}{2\,c}; \text{......................} (3.)$$

?

and the stress produced by it must be taken into account in fixing the dimensions of the longitudinal beam. For example, if $x = c \div 3$, $\mathrm{M}' = \mathrm{W}'\,c \div 9$.

To provide resistance to a partial load by *diagonal bracing*, there should be two diagonal struts, in the positions shown by the dotted lines 4 5 and 6 1 in fig. 209; 4 5 to act when the partial load is on 4, and 6 1 when the partial load is on 1. The greatest thrust S along either of them is given by the following formula:—Let k be the depth of the truss, from the centre line of F to the centre line of B. Then

$$\mathrm{S} = \mathrm{W}'\,\frac{c - x}{2\,c\,k} \cdot \sqrt{4\,x^2 + k^2}. \text{...........} (4.)$$

342. **Compound Bridge-Truss.** (*A. M.*, 160.)—The general nature of a compound truss has been explained in Article 121, p. 184. Fig. 213 is a skeleton diagram of a compound timber bridge-truss, on the principle of those of the celebrated bridge of Schaffhäusen.

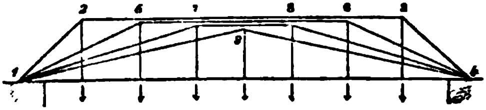

Fig. 213.

It consists of four elementary trusses, viz.:—

1 2 3 4 loaded at 2 and 3,
1 5 6 4 „ 5 „ 6,
1 7 8 4 „ 7 „ 8,
1 9 4 „ 9;

but all those trusses have the same tie-beam, 1 4; and the pull along that tie-beam is the sum of the pulls due to the four trusses.

The vertical lines represent suspending-pieces, from which the tie-beam is hung. The tie-beam supports the cross-beams of the platform.

An arrangement of struts similar to that in the figure, but without the tie-beam or suspending-pieces, and supporting the platform above, is often used for timber bridges with abutments. Stay-pieces, however, are required, nearly in the position of the upper parts of the suspending-pieces in the figure, to give sufficient stiffness to the struts.

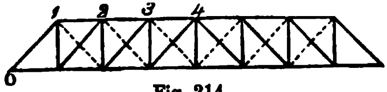

Fig. 214.

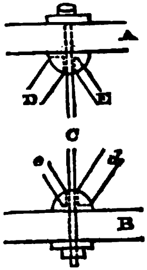

Fig. 215.

343. **Diagonally-braced Girder.**—This sort of girder, of which fig. 214 is a skeleton diagram, was first introduced in America by Mr. Howe. The two horizontal bars, or "booms," resist the bending moment of the load; they are made of layers of planks set on edge, and bolted together so as to break joint, as in the built ribs of Article 333, p. 464. The shearing action of the load is resisted by the vertical suspending-pieces (which are iron rods), and the diagonal timber struts, which abut into iron sockets, as shown on a larger scale in fig. 215. In the latter figure A is the upper or compressed boom, and B the lower or extended boom; C,

a suspending-rod; D, d, struts sloping up towards the middle of the span, and indicated by plain lines in fig. 214; E, e, struts sloping up towards the nearest point of support, and indicated by dotted lines in fig. 214.

The diagonals shown by full lines are all that would be required if the load were always uniformly distributed over the girder. Those shown by dots are necessary in order to resist travelling loads.

The actions of the load on this girder are computed by the method already explained in Article 160, pp. 230 to 243, as applied to a beam loaded at detached points. The formula for the bending moment at any cross-section has already been given in Article 161, Case VIII., p. 247. In computing the shearing force, regard must be had to the action of a travelling load, as explained in Article 161, Case IX., pp. 247, 248.

The following are the most convenient formulæ in practice. One of the points of support being numbered 0, the joints of the upper boom are to be numbered consecutively from that end of the girder towards the middle, as in fig. 214:—

Let n denote the number of any joint, and N the total number of divisions in the beam. (In the figure $\mathrm{N} = 8$; and for the middle joint, $n = 4$. When N is odd, there is no middle joint.)

Let k denote the height of the girder, measured from centre to centre of the horizontal booms;

l, its span; so that $l \div \mathrm{N}$ is the length of a division;

s, the length of a diagonal, measured along its line of resistance,

$$= \sqrt{k^2 + \frac{l^2}{\mathrm{N}^2}};$$

w, the uniform steady load upon each joint;

w', the greatest travelling load upon each joint.

The divisions of the horizontal booms are to be numbered 1, 2, 3, 4, from the ends towards the middle; so that in fig. 214, Division No. 1 of the upper boom lies between 1 and 2; Division No. 1 of the lower boom lies between 0 and the suspending-rod 1, &c.

Suspending-rods and diagonals are designated by the number of the joint where their *upper* ends meet; thus, in fig. 215, if n be the number of the rod C, it is also the number of the larger diagonal D, and the smaller diagonal E; while the number of d is $n+1$, and that of e, $n-1$.

Let H_n be the thrust and tension along the divisions n of the upper and lower booms;

V_n, the tension on the vertical rod n;

T_n, the thrust on the large diagonal n;

t_n, the thrust on the small diagonal n.

Then

$$H_n = \frac{(w+w')\,l}{k}\cdot\frac{n\,(N-n)}{2\,N};\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

V_n (when the platform is *hung* from the girder, for all except the middle rod) $= w\left(\frac{N+1}{2}-n\right)+w'\cdot\frac{(N-n)\,(N-n+1)}{2\,N};\ldots(2.)$

V (when the platform is hung from the girder, for the middle rod) $= w + \frac{w'}{4}\cdot\left(\frac{N}{2}+1\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.)$

(When the platform rests on the top of the girder, subtract $w+w'$ from each of the above values of V.)

$$T_n = \frac{w\,s}{k}\left(\frac{N+1}{2}-n\right)+\frac{w'\,s}{k}\cdot\frac{(N-n)\cdot(N-n+1)}{2\,N};\ldots\ldots(4.)$$

$$t_n = -\frac{w\,s}{k}\left(\frac{N-1}{2}-n\right)+\frac{w'\,s}{k}\cdot\frac{n\cdot(n+1)}{2\,N}.\ldots\ldots\ldots\ldots(5.)$$

When the last formula (5) gives a null or negative result, it shows that the smaller diagonal in the division in question is unnecessary.

The best angle of inclination for the diagonals is 45°; the corresponding value of $s \div k$ is 1·414, and that of $l \div k$ is N.

The following is a numerical example:—

Span 80 feet, in eight equal divisions; that is, $l = 80$; $N = 8$.
$k = 10$ feet; $s = 14{\cdot}14$ feet.
$w = 5000$ lbs.; $w' = 10{,}000$ lbs.
Platform hung below girder.

n	H lbs.	V lbs.	T lbs.	t lbs.
1	52,500	52,500	74,235	negative.
2	90,000	38,750	54,792	negative.
3	112,500	26,250	37,128	7,070
4	120,000	17,500	21,210	

The last column shows that, in the example chosen, the dotted diagonals are required in the two middle divisions only.

The value of H for $n = 4$ applies to the lower boom alone, as the upper boom has only three divisions on each side of the middle.

344. **Lattice-work Girders** of timber were first introduced by Mr. Ithiel Towne. The lattice-work consists of planks inclined at 45° to the horizon, crossing each other at right angles, and pinned together with treenails.

A lattice girder, even without horizontal booms (as in fig. 216), s capable of supporting a certain load, provided its ends are made fast to stable piers; and, under these circumstances, its moment of resistance at any cross-section is simply the sum of the moments of resistance of the planks intersected by that cross-section. But this mode of construction is unfavourable both to economy and to stiffness. When horizontal booms are bolted to the lattice-work at its upper and lower edges, they may be considered, without sensible error, as sustaining all the bending moment, like those in the example of the last article; while the lattice-work bears the shearing action of the load, distributed with approximate uniformity amongst the bars or planks.

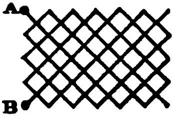

Fig. 216.

345. **Timber Arches.**—When a timber arch is exactly or nearly of the form of an equilibrated rib of uniform strength under the steady part of its load, and is subject besides to a rolling load, its strength is to be computed according to the methods of Article 180, pp. 296 to 314.

The usual form for timber arches is a segment of a circle; but the formulæ for a parabolic rib may be used in practice without material error. In almost every case the rib may be considered as *fixed in direction at the ends;* so that if the abutments are immoveable, the formulæ to be employed will be those of Problem IV., equations 30 to 38 B, pp. 305 to 308; and if the abutments are sensibly moveable, equation 40, p. 308, is to be used instead of equation 30.

In designing a timber arch, the greatest working deflection should be computed by the equation 61 of Article 180, p. 313; and the pieces of timber in the arch and superstructure should be proportioned as if the platform were to have an upward convexity or "camber," with a rise in the centre of the span equal to the calculated deflection. The result will be, that the platform will become horizontal, or nearly so, when fully loaded.

Semicircular timber ribs are now often employed to support roofs, for the sake of architectural appearance. In fig. 217, let A C B be a quadrant of such a rib, under a load uniformly distributed horizontally, O being its centre. Draw B D and A D tangents to the neutral layer at the springing and at the crown; bisect A D in E; then, if the arch be *jointed* or *hinged* at A and B, E B will be the direction of the thrust at B; and its horizontal component will be *half the load on the quadrant;* that is,

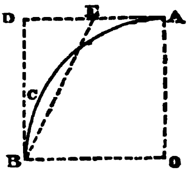

Fig. 217.

$$H = \frac{w\,r}{2}; \quad \text{..........................(1.)}$$

r being the radius O A, and w the load per lineal unit of span. The greatest bending moment, on the same supposition, occurs at C, 30° above the springing. That moment tends to make *curvature sharper* at that point; and its value is

$$M = \frac{w\,r^2}{8}. \quad \text{..........................(2.)}$$

The value of the direct thrust at C is H, as given by equation 1. By the use of these values in the formulæ of Article 340, p. 475, the proper scantling for the rib may be computed. The supposition of the rib being hinged at A and B is not perfectly realized in practice; but it will not lead to any error of importance.

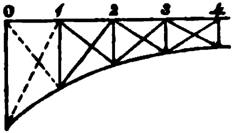

Fig. 218.

346. **Timber Spandrils.**—When timber arches support a level platform, each spandril in general contains a series of upright posts for transmitting the load from the platform to the arch. A horizontal beam on the top of each row of posts should have strength and stiffness sufficient to resist the load between each pair of posts.

To stiffen the frame transversely, the posts which stand side by side should have diagonal braces between them; the smallest transverse dimension of any brace not being less than about one-twentieth part of its length.

To stiffen the frame longitudinally, diagonal braces may be placed as in fig. 218. To find the stress which any one of those diagonal braces should be capable of resisting with safety, let the upright posts be numbered from one end of the arch to the middle, 0, 1, 2, 3, &c. (like the suspending-rods in Article 343). Let

N be the total number of longitudinal divisions in the platform.
n and $n+1$, the numbers of the posts between which a given diagonal brace is situated.
s, its length, and k the difference of level of its ends.
w', the greatest *travelling* load on one post.
T, the greatest amount of thrust along the diagonal; then

$$T = \frac{w'\,s}{k} \cdot \frac{n\,(n+1)}{2\,N}. \quad \text{.....................(1.)}$$

For the diagonals between 0 and 1, indicated by dots in the figure, this expression is $= 0$; but nevertheless a pair of diagonals may be placed there, of the same size with the smallest of those between 1 and 2, in order to give additional stiffness.

It is possible that when the arch is partially loaded with a travelling load, some of the upright pieces which, when the load is uniform, are posts, may have to act occasionally as suspending-pieces. To find whether this is the case for any given upright piece, let n be its number, and w'' the *dead* load resting upon it; then compute the value of the following expression:—

$$V = w' \cdot \frac{n(n+1)}{2N} - w''; \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

and if this is positive, it will give the greatest tension on the upright; if null or negative, it will show that the upright acts always as a strut or post, and never as a suspending-piece or tie.

Another mode of construction is to make all the diagonals iron tie-bolts. In this case equation 1 will give the greatest tension on any given bolt. The uprights will always act as posts, and the greatest load on each will be given by the following formulæ:—

$$V' = w'' + w' \left\{ 1 + \frac{n(n-1)}{2N} \right\}. \ldots\ldots\ldots\ldots (3.)$$

347. **Timber Bowstring Girder.** (Fig. 219.)—In a girder of this kind, a timber arch springs from a tie-beam, which supports the cross-beams of the platform, and is hung from the arch at intervals by vertical suspending-pieces or rods, with diagonal braces between them.

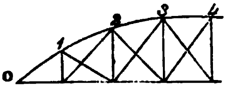

Fig. 219.

The tie-beam has to bear at once a tension equal to the horizontal thrust of the arch, and a bending action due to the load supported on it between a pair of suspending-pieces; and its strength depends on the principles explained in Article 340, p. 474.

The greatest tension on any suspending-piece is to be found by means of equation 3 of Article 346, above.

The greatest thrust along any diagonal is to be found by means of equation 1 of the same Article, p. 482.

The horizontal tie of a timber bowstring girder should never be made of iron, as its expansion and contraction would strain and at length destroy the timber arch.

348. **Timber Piers.**—A timber pier for supporting arches or girders may consist of any convenient number of posts, either vertical or slightly raking, and connected together by horizontal and diagonal braces.

Each post should be braced at every point where there is a joint in it, and at additional points if necessary, in order that the distance between the braced points may not be less than about 18 or 20 times the diameter of the post. (As to lengthening posts, see Article 315, p. 456.)

Should the pier have lateral thrust to bear, whether from the action of the wind or from that of the load upon the superstructure, the following principles are to be attended to:—

I. The posts at the base of the pier should, if possible, spread to such a distance from each other that the lateral thrust may cause *no tension* on any one of them. For example, conceive a pier of a timber viaduct to consist of two parallel rows of posts; let the greatest horizontal thrust in a direction perpendicular to the rows be H, acting at the height Y above the base of the pier, so that H Y is its moment; let W be the gross vertical load of the pier, and B the required distance from centre to centre between the two rows of posts at the base of the pier; then make

$$B = \frac{2 H Y}{W}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

and there will never be tension on any of the posts. If this arrangement be made, the *whole load* W will be concentrated on *one row of posts* when the greatest thrust acts. In other cases, the load on the row of posts furthest from the side of the pier on which the thrust acts will be,

$$\frac{W}{2} + \frac{H Y}{B}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

If the pier consists of more than two rows of posts, let n denote the number of rows, and let them be equidistant from each other, B being still the distance from centre to centre of the outside rows. Let P denote the share of the load which rests on the row of posts furthest from the side the thrust is applied to, and P′ the share which rests on the row nearest that side. Then

$$P = \frac{W}{n} + \frac{H Y (n - 1)^2}{B \{(n - 1)^2 + (n - 3)^2 + \&c.\}}; \quad \ldots\ldots (3.)$$

$$P' = \frac{W}{n} - \frac{H Y (n - 1)^2}{B \{(n - 1)^2 + (n - 3)^2 + \&c.\}}; \quad \ldots\ldots (4.)$$

the series in the denominator of the second term being carried on as long as the numbers in the brackets are positive.

The best value for B is found by making P′ = 0; that is to say,

$$B = \frac{H\,Y \cdot n\,(n - 1)^2}{W\left\{(n-1)^2 + (n-3)^2 + \&c.\right\}}; \ldots\ldots\ldots(5.)$$

in which case,

$$P = \frac{2\,W}{n}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.)$$

II. The horizontal and diagonal braces are to be calculated to resist the horizontal thrust, in the same manner that the suspending-pieces and diagonal struts of a diagonally-braced girder are calculated to resist the shearing stress, supposing that shearing stress to be the same at all points of the girder, and = H.

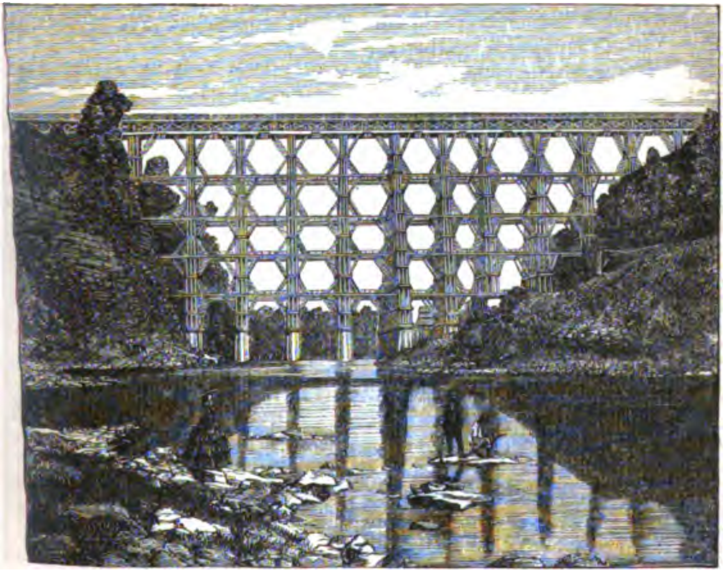

Fig. 220.—[Portage Bridge over the Genesee River, from a Photograph.]

349. **Centres for Arches.**—The use and general construction of centres for arches have already been explained in Article 279, p. 415. The present article relates to the figure and strength of the ribs or frames which support the laggings.

I. Action of Load on Centre.—The building of the arch should be carried up simultaneously at the two sides of the centre, so that the load on the centre may never be sensibly unsymmetrical. The loading of the centre will thus advance from both ends towards the middle; and its most severe action, whether compressive, shearing, or bending, will take place just before the key-stones are driven into their places.

If there were no friction between the arch-stones, the load upon the centre could be computed exactly. The friction between them renders all formulæ for that purpose uncertain.

It is usually stated that the arch-stones do not begin to press against the centre until courses are laid the slope of whose beds is steeper than the angle of repose; that is to say, from 25° to 35°, or on an average, about 30°; but in order that this may be true, the lower part of the arch must be so thick as to have no tendency to *upset inwards*. A thickness equal to about one-tenth of the radius of curvature of the intrados is in general sufficient for that purpose; but still any accidental disturbance of the arch-stones may make them press against the centre.

Each successive course of arch-stones that is laid causes the pressure exerted by the previous courses against the centre to diminish; and when a semicircular arch is completed all but the key-stone, the stones whose beds slope less steeply than 30° have ceased to press against the centre, and that *even although there should be no friction*. In fact, when the load on the centre reaches its greatest amount, its action is nearly the same whether friction operates sensibly or not; and considering this fact, and also the fact that any errors in calculation caused by neglecting the friction of the stones on each other must be on the side of safety, it appears that for practical purposes it is sufficient to calculate the load on a centre as if the friction between the stones were insensible.

The following are the results:—

(1.) *General Case.*—Let w denote the weight *per lineal foot of the intrados* of the arch resting on a given rib of a centre.

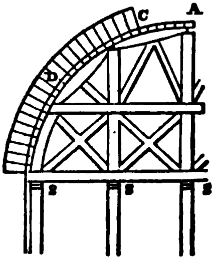

Fig. 221.

Let the co-ordinates of any point (such as D, fig. 221) in the intrados be measured from its highest point A; x being measured horizontally, and y vertically downwards.

Let x_0 and y_0 be the co-ordinates of the point C.

Let r be the radius of curvature of the intrados at the point D.

θ, its inclination to the horizon.

p, the *normal* pressure against the rib at the point D, *per lineal* foot of intrados; then, friction being insensible,

$$p = w \cdot \cos\theta - \frac{1}{r}\int_{y_0}^{y} w\,d\,y; \;\ldots\ldots\ldots\ldots\ldots (1.)$$

and the greatest value of this is

$$w \cdot \cos \theta. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1 \text{ A.})$$

Let P be the total *vertical* load arising from the pressure of the arch-stones on the rib between C and D; then

$$\mathrm{P} = \int_{x0}^{x} p\, d\, x, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

and the value of this, when the arch is complete all but the keystone, is

$$\mathrm{P}_1 = \int_{0}^{x_1} p\, d\, x \text{ (making } y_0 = 0\text{)}; \ldots\ldots\ldots\ldots(2 \text{ A.})$$

x_1 being the horizontal distance from the middle of the span to a point for which $p = 0$.

If the rib, instead of resting on a series of posts, as in fig. 221, is supported *as a girder* on the abutments or piers of the arch, or on timber piers of its own,

Let c be the half-span of that girder;

M, the moment of flexure in the middle of the span; then

$$\mathrm{M} = \mathrm{P}\, c - \int_{x0}^{x_1} p\, x\, d\, x; \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

and the greatest value of this is

$$\mathrm{M}_1 = \mathrm{P}_1\, c - \int_{0}^{x_1} p\, x\, d\, x \text{ (for } y_0 = 0\text{)}. \ldots\ldots\ldots(3 \text{ A.})$$

Formula 1 A. serves to compute the greatest load to be borne by the *laggings* or *bolsters*; equation 2 serves to compute the load on any vertical post, or the vertical component of the load on any given *back-piece*, or segment of the rib immediately under the laggings; and the total transverse load on such a piece is

$$\mathrm{P} \sec \theta'. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

θ' being its inclination to the horizon.

Equation 2 A. gives the greatest vertical load on each half of the rib, and serves to compute the total strength required for its vertical supports; and equation 3 A serves to compute the strength required if the rib acts as a girder.

(2.) *Circular Arch not exceeding* 120°.—In an arch with a circular intrados, we have—

$$x = r \sin \theta\,;\ y = r\,(1 - \cos \theta\,);$$
$$x_0 = r \sin \theta_0;\ y_0 = r\,(1 - \cos \theta_0);$$
$$x_1 = r \sin \theta_1;\ y_1 = r\,(1 - \cos \theta_1);$$

Let s, s_0, s_1, denote lengths of arcs measured from A in feet.

Let the weight per foot of intrados w be constant. Then the normal pressure per foot of intrados is

$$p = w\,(2\cos\theta - \cos\theta_0) = w\cdot\frac{r - 2y + y_0}{r}; \quad \text{.....(5.)}$$

and its greatest value for a given point,

$$w\cos\theta = w\cdot\frac{r - y}{r}. \quad \text{..................(5 A.)}$$

The vertical load between C and D is

$$\left.\begin{aligned} \mathrm{P} &= w\,r\left\{\theta - \theta_0 - \sin\theta\,(\cos\theta_0 - \cos\theta)\right\} \\ &= w\left\{s - s_0 - \frac{x}{r}\,(y - y_0)\right\}; \end{aligned}\right\} \text{.....(6.)}$$

which, when the load is complete up to A, and D is *at the springing*, becomes

$$\mathrm{P}_1 = w\,r\left\{\theta_1 - \sin\theta_1\,(1 - \cos\theta_1)\right\} = w\left\{s_1 - \frac{x_1\,y_1}{r}\right\}. \text{(6 A.)}$$

in which last expression, the load on the half-rib is given in terms of its length, s_1, its *half-span*, x_1, and its *rise*, y_1.

The greatest moment of flexure, M_1, on a girder-rib of the half-span c, is as follows:—

$$\left.\begin{aligned} \mathrm{M}_1 &= \mathrm{P}_1\,c - w\,r^2\left(\frac{1}{6} + \frac{\cos^2\theta_1}{2} - \frac{2\cos^3\theta_1}{3}\right) \\ &= w\left\{c\,s_1 - \frac{c\,x_1\,y_1}{r} - \frac{r^2}{6} - \frac{(r - y_1)^2}{2} + \frac{2(r - y_1)^3}{3\,r}\right\}. \end{aligned}\right\} \text{(7.)}$$

In employing these formulæ, it may often be convenient to use the following expressions for computing the radius r and length s of any given arc from its half-span x and rise y:—

$$r = \left(y + \frac{x^2}{y}\right) \div 2;\ s = x\left(1 + \frac{2}{3}\,\frac{y^2}{x^2} - \frac{y^4}{8\,x^4}\right) \text{ nearly.} \quad \text{(8.)}$$

The load on any arc of the rib may be represented graphically in the following manner:—

In fig. 222, let A B be a quadrant, described about O with a radius representing that of the intrados. Let C be the point up to which the arch has been built, and D any other point in the intrados.

Conceive that the *half* of the radius A O represents w, the weight per foot of intrados.

From C draw C E ‖ A O; bisect C E in F, from which draw F H ‖ O B; draw D G ‖ A O; then will D G represent the normal pressure on each lineal foot of the rib at the point D; and the shaded area C D G F will represent the vertical component of the load on the rib between C and D, both in amount and in distribution; that is to say, [illegible] $= \frac{r}{2}$ (cos θ - cos [illegible]

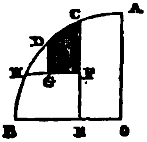

Fig. 222.

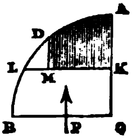

Fig. 223.

$$\tfrac{1}{2}\,A\,O : w :: D\,G : p$$

$$:: C\,D\,G\,F : P.$$

The point H is that below which the arch-stones cease to press on the rib, when the arch has been built up to the point C.

The case in which the rib is completely loaded, the arch being finished all but the key-stone, is represented by fig. 223. Bisect the vertical radius A O in K, and conceive A K to represent w; draw K L ‖ O B; L will be a point below which the stones do not press on the rib (supposing the arch to extend so far); and at that point $\theta = 60°$. Let D be any point in the intrados; draw D M ‖ A O; then

$$A\,K : w :: D\,M : p$$

$$:: A\,D\,M\,K : P;$$

and if D is the *springing* of the arch, A D M K represents the vertical load on the half rib, P_1. If the arrow P in the figure represents the position of one of the two supports of a girder rib, O P $= c$ in equation 7.

(3.) *Circular Arch of* 120° *and upwards.*—Because the arch-stones below the point where the inclination of the intrados to the horizon is 60°, do not press upon the rib when the load is complete, the value of P_1 for $\theta_1 = 60°$ applies also to all greater values of θ_1; it being understood that in every such case we are to make

$$x_1 = \sqrt{\frac{3}{4}} \cdot r = \cdot 866\, r;\ y_1 = \frac{r}{2};\ s_1 = 1\cdot 0472\, r; \ldots (9.)$$

whatsoever the actual rise and span of the arch may be. This gives the following results:—

$$P_1 = \cdot 6142\, w\, r; \ldots\ldots\ldots\ldots\ldots\ldots (10.)$$

$$M_1 = w\, r \left(\cdot 6142\, c - \frac{5\, r}{24} \right). \ldots\ldots\ldots\ldots\ldots (11.)$$

(4.) *Non-circular Arch.*—Find the two points at which the intrados is inclined 60° to the horizon; conceive a circular arc drawn through them and through the crown of the intrados, and proceed as in Case 2, calculating r and s by the formulæ 8, from x and y, the co-ordinates of each of the two points where the inclination of the intrados is 60°. The results will be near enough to the truth for practical purposes.

II. Striking-plates and Wedges.—These terms are applied to the apparatus by which the centre is lowered after the arch has been completed. Fig. 224 represents a pair of striking-plates, A and B, with a compound wedge C between them. The lower striking-plate B is a strong beam, suitably notched on the upper side, and resting on the top of the pier or row of posts which forms one of the supports of a centre; the upper striking-plate A, notched on the under side, forms the base of part of the frame of the centre; and the wedge C keeps the striking-plates A and B asunder, being itself kept in its place by keys or smaller wedges driven behind its shoulders. When the centre is to be struck, those keys are driven out; and the wedge C being driven back with a mallet, allows the upper striking-plate to descend. In fig. 221, p. 486, S, S, S, represent the ends of pairs of striking-plates resting transversely on the posts or piles which support the entire centre. In some centres, of which examples will be given, the striking-plates lie longitudinally. In the centres introduced by Hartley, each lagging can be struck separately by lowering the wedges or screws which support it; so that striking-plates to support the entire centre are unnecessary.

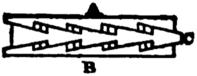

Fig. 224.

III. Framing of Centres.—The back-pieces which form the upper edge of the rib, are usually supported at points from 10 to 15 feet asunder. In some examples, however, those points are as close as 5 or 6 feet.

It is essential that a centre should possess stiffness so great that polygonal frames of many sides and timber arches are unfit forms for its ribs, because of their flexibility. Such forms have been used, but have caused great difficulty and even danger in the construction of the arch. The kinds of framework which have been found to succeed are of three kinds, viz.:—

(1.) Direct supports from intermediate points; to be always employed when practicable.

(2.) Inclined struts in pairs; } to be employed when intermediate points of support cannot be had close enough.
(3.) Trussed girders; }

(1.) *Direct Supports* in a very simple form are illustrated by fig.

221, p. 486. Several rows of piles support a series of pairs of striking-plates. On the upper striking-plates rest the ribs, each of which consists of the following parts:—A *sill* or horizontal beam, a series of vertical posts directly over the piles, horizontal braces or *wales*, *diagonal braces* between the posts, *oblique struts* near the upper ends of the posts, to support intermediate points in the back-pieces, and the *back-pieces*. Besides giving stiffness to the posts, the diagonal braces answer the purpose of supporting a given part of the rib in case the pile vertically below it should give way.

Fig. 225 is a skeleton diagram of Hartley's centre for the Dee Bridge at Chester, in which the greater number of the supports consisted of struts, radiating in a fan-like arrangement from iron sockets or shoes on the tops of temporary stone piers, of which there were four in the total span of 200 feet. The struts were stiffened by means of wales at distances of from 10 to 12 feet apart vertically. The duty of back-pieces was done by two thicknesses of 4½ inch planks. (See *Trans. Inst. Civ. Engs.*, vol. i.)

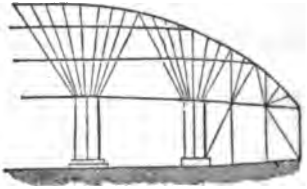

Fig. 225.

(2.) *Inclined Struts*, in pairs, are exemplified in fig. 226, which is a skeleton diagram of the centre of Waterloo Bridge. Each joint in the back-pieces, such as A, B, C, &c., was independently supported by a pair of struts of its own, springing from the striking-plates at F and F'. At each point where many of those struts intersected each other, such as H, I, and I', they were connected by abutting into one cast iron socket. At other points of intersection they were notched and bolted together. They were further stiffened by means of radiating pieces in pairs, whose positions are shown in the sketch. The striking-plates were longitudinal and inclined, and were supported on struts springing from the stepped bases of the stone piers of the bridge.

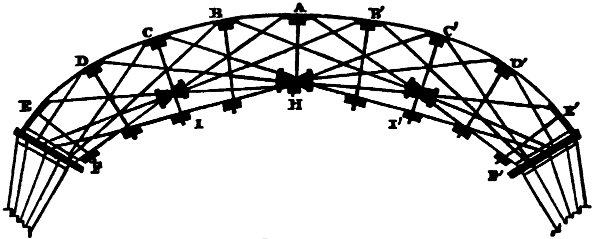

Fig. 226.

(3.) *Trussed Girders*, as applied to centres, are illustrated (in fig. 227) by the centre of London Bridge. The sketch shows that for about one-fourth of the span at each side the support was direct,

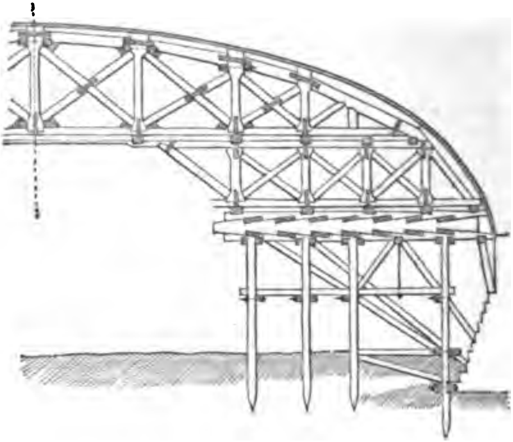

Fig. 227.

being given by vertical posts with diagonal braces between them; while across the middle half of the span the rib formed a diagonally-braced girder of great stiffness, its depth being about one-fourth of its span. The striking-plates were longitudinal and horizontal.

The ribs of a centre should be braced together transversely by horizontal and diagonal braces.

In framing centres it is desirable to use the pieces of timber in such a manner that they may be afterwards applied to other purposes.

(ADDENDUM to Article 174, p. 280, and Article 312, pp. 450 to 453.)

349 A. **Resistance of Timber to Torsion.**—The following are the results of some recent experiments by M. Bouniceau on the resistance of timber to twisting and wrenching, extracted from a paper in the "*Annales des Ponts et Chaussées*" for 1861. The co-efficients are modified so as to suit the formulæ of M. de St Venant for resistance to torsion, which are more correct than the ordinary formulæ employed in the original paper.

	Modulus of Rupture by Wrenching. f Lbs. on the Square Inch.	Modulus of Transverse Elasticity. C Lbs. on the Square Inch.
Red Pine of Prussia,	1,540	116,300
„ of Norway,	950	61,800
Elm,	1,390	76,000
Oak (of Normandy),	2,350	82,400
Ash,	1,460	76,000

The modulus of rupture by wrenching for oak, as stated above, is almost exactly the same with its resistance to shearing along the grain. (See tables at the end of the volume.) For red pine, the resistance to wrenching is the greater.

The following are the formulæ applicable to square bars:—

Let h be the breadth and thickness of the bar.

M, the moment of torsion required to wrench it asunder; then

$$M = \cdot 281 f h^3. \qquad \text{(1.)}$$

Also, let l be the length of the bar.

M′, any moment of torsion.

θ, the angle, stated in arc to radius unity, through which the bar is twisted by that moment; then

$$\theta = \frac{M' l}{\cdot 1405 \, C h^4}. \qquad \text{(2.)}$$

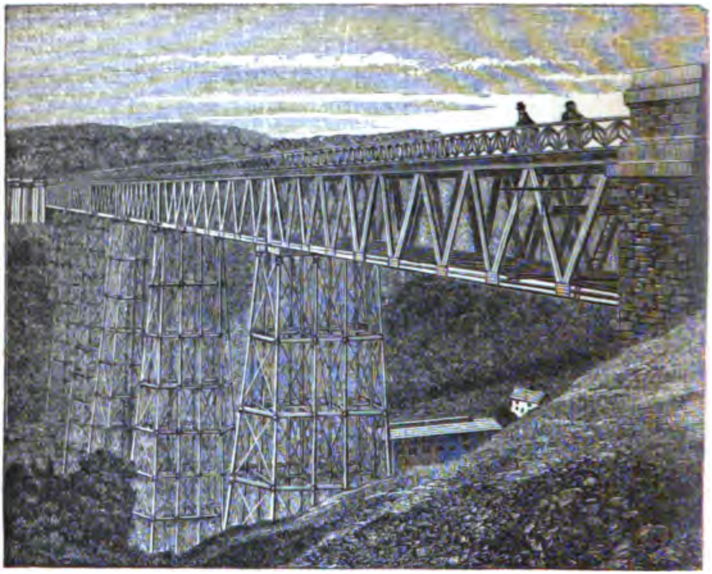

Fig. 228.—[The Crumlin Viaduct, from a Photograph.]

CHAPTER V.

OF METALLIC STRUCTURES.

Section I.—*Of Iron and Steel.*

350. **Sources and Classes of Iron in General.**—It would be foreign to the subject of the present treatise to enter into details as to the ores from which iron is obtained, and the processes of its manufacture. A brief summary, therefore, of those matters will alone be given, referring for more full information to such works as Fairbairn *On the Iron Manufacture;* Truran *On the Iron Trade;* Mushet's *Papers on Iron and Steel;* Karsten's *Handbuch der Eisenhuettenkunde;* Phillips's *Manual of Mineralogy.*

The chemical equivalent of iron is 28 times that of hydrogen.

The following are the most common conditions in which iron is found in its ores:—

			By Atoms.	By Weight.		Per centage of Iron.
I.	*Native Iron*, being iron nearly pure, or combined with from one-fourth to one-hundredth part of its weight of nickel. This is very rare, and is found in detached masses, which are known, or supposed, to have fallen from the heavens, ...					80 to 100
II.	Protoxide or Black Oxide of Iron, ...	Iron, ...	1	28	36	77·8
		Oxygen, ...	1	8		

Protoxide of iron is only found in combination with other substances.

			By Atoms.	By Weight.		Per centage of Iron.
III.	Peroxide or Red Oxide of Iron, ...	Iron, ...	2	56	80	70
		Oxygen, ...	3	24		
IV.	Magnetic Oxide of Iron = Peroxide + Protoxide, ...	Iron, ...	3	84	116	72·4
		Oxygen, ...	4	32		
V.	Hydrate of Peroxide of Iron =					
	Peroxide of Iron, 2 atoms, ...	Iron, ...	4	...112	187	60
		Oxygen, ...	6	... 72		
	Water, ...8 atoms, ...	Oxygen, ...	3			
		Hydrogen, ...	3	... 3		
VI.	Carbonate of Iron =					
	Protoxide of Iron, 1 atom, ...	Iron, ...	1	... 28	58	48·3
		Oxygen, ...	1	... 24		
	Carbonic Acid, ... 1 atom, ...	Oxygen, ...	2			
		Carbon, ...	1	... 6		

Iron is found combined with sulphur, forming what is called *Iron Pyrites;* but that mineral is not available for the manufacture of iron; and it forms a pernicious ingredient in ores, or in the fuel used to smelt them, because of the weakening effect of sulphur upon iron. The same is the case with *Phosphate of Iron.*

The most abundant foreign ingredients found mixed with compounds of iron in its ores are siliceous sand and silicate of alumina, or clay; next in abundance are the carbonates of lime and magnesia. Amongst other foreign ingredients, which, though not abundant, have an influence on the quality of the iron produced, are carbon, manganese, arsenic, titanium, &c. Of these manganese and carbon alone are beneficial; for manganese gives increased strength to steel, and carbon assists in reducing the ore; all the rest are hurtful.

The most common *Ores of Iron* are the following:—

I. *Magnetic Iron Ore*, consisting of magnetic oxide of iron, pure, or almost pure, and containing 72 per cent. of iron, is found chiefly in veins traversing the primary strata, and amongst plutonic rocks, and is the source of some of the finest qualities of iron, such as those of Sweden and the North-Eastern United States.

II. *Red Iron Ore* is peroxide of iron, pure or mixed. When pure and crystalline, it is called *Specular Iron Ore*, or *Iron-glance;* when pure, or nearly so, and in kidney-shaped masses, showing a fibrous structure, it is called *Red Hæmatite;* when mixed with less or more clay and sand, it is called *Red Ironstone* and *Red Ochre*. It is found in various geological formations, and is purest in the oldest. The purer kinds, iron-glance and hæmatite, produce excellent iron; for example, that of Nova Scotia.

III. *Brown Iron Ore* is hydrate of peroxide of iron, pure or mixed. When compact and nearly pure, it is called *Brown Hæmatite;* when earthy and mixed with much clay, *Yellow Ochre*. It is found amongst various strata, especially those of later formations.

IV. *Carbonate of Iron*, when pure and crystalline, is called *Sparry* or *Spathose Iron Ore;* when mixed with clay and sand, *Clay Ironstone;* when clay ironstone is coloured black by carbonaceous matter, it is called *Black-band Ironstone*. These ores are found amongst various primary and secondary stratified rocks, and especially amongst those of the coal formation.

The proportion of earthy matter in the ordinary ores containing carbonate of iron ranges from 10 to 40 per cent.

The iron of Britain is manufactured partly from hæmatite, but chiefly from clay ironstone and black-band.

The extraction of iron from its ores consists of a combination of processes, which may be described in general terms as follows:—If the iron is in the state of carbonate, the carbonic acid is expelled by the agency of heat, leaving oxide of iron; the earthy constituents of the ore are removed by means of the chemical affinity of other earths (especially lime), forming a glassy refuse called *Slag;* the oxygen is taken away from the iron by means of the chemical affinity of carbon; and in certain processes, carbon, combined with the iron, is taken away by means of the chemical affinity of oxygen. There are also processes whose object is to combine the iron with certain proportions of carbon. The substances employed in the extraction of iron from its ore may be thus classed,—the *ore* itself; the *fuel*, which produces heat by its combustion, and supplies carbon; the *air*, which supplies oxygen for the combustion of the fuel; the *flux* (generally lime), which promotes the fusion of the ore, and combines with its earthy constituents.

In some cases a substance is also used in order to remove sulphur and phosphorus from the ores and fuel. In this process (the invention of Mr. Calvert), chlorine, or some chloride, by preference common salt, is employed in such quantity that for every

$$\left.\begin{array}{r}\text{16 parts of sulphur}\\ \text{or 32 parts of phosphorus}\end{array}\right\}\begin{array}{l}\text{in the ore}\\ \text{or fuel,}\end{array}$$

there shall be 35 parts of chlorine;
and as common salt contains 23 parts of sodium to 35 of chlorine,

the proper proportion is 58 parts of common salt.

The metallic products of the iron manufacture are of three kinds; *malleable* or *wrought iron*, being pure or nearly pure iron; *cast iron* and *steel*, being certain compounds of iron with carbon.*

351. **Impurities of Iron.**—The strength and other good qualities of these products depend mainly on the *absence of impurities*, and especially of certain substances which are known to cause brittleness and weakness, of which the most important are, sulphur, phosphorus, silicon, calcium, and magnesium.

Sulphur and (according to Mushet) calcium, and probably also magnesium, make iron "*red short;*" that is, brittle at high temperatures; phosphorus and (according to Mushet) silicon make it "*cold short;*" that is, brittle at low temperatures. These are both serious defects; but the latter is the worse.

Sulphur comes in general from coal or coke used as fuel. Its pernicious effects can be avoided altogether by using fuel which contains no sulphur; and hence the strongest and toughest of all iron is that which is melted, reduced, and puddled either with charcoal, or with coal or coke that is free from sulphur. As to the artificial removal of sulphur, see the preceding Article.

Phosphorus comes in most cases from phosphate of iron in the ore, or from phosphate of lime in the ore, the fuel, or the flux. Mr. Calvert's method of removing it has already been mentioned above. The ores which contain most phosphorus are those found in strata where animal remains abound, such as those of the oolitic formation.

Calcium and *Silicon* are derived respectively from the decomposition of lime and of silica by the chemical affinity of carbon for their oxygen. The only iron which is entirely free from these impurities is that which is made by the reduction of ores that contain neither silica, nor lime, such as pure magnetic iron ore, pure hæmatite, or pure sparry iron ore.

If either of those earths be present in the ore, the other must be added as a flux, to form a slag with it; and a small portion of each of them will be deoxidated, the bases uniting with the iron. This is a defect of earthy ores for which no remedy is yet known.

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* According to some views recently set forth, *nitrogen* is one of the essential constituents of steel; but this wants confirmation.

The statements made relative to calcium are applicable also to magnesium.

The effect of aluminium upon iron is not known with certainty.

352. **Cast Iron** is the product of the process of *smelting* iron ores. In that process the ore in fragments, mixed with fuel and with flux, is subjected to an intense heat in a blast-furnace, and the products are *slag*, or glassy matter formed by the combination of the flux with the earthy ingredients of the ore, and *pig iron*, which is a compound of iron and carbon, either unmixed, or mixed with a small quantity of uncombined carbon in the state of plumbago.

The ore is often *roasted* or calcined before being smelted, in order to expel carbonic acid and water.

The proportions of ore, fuel, and flux are fixed by trial; and the success of the operation of smelting depends much on those proportions. The flux is generally limestone, from which the carbonic acid is expelled by the heat of the furnace; while the lime combines with the silica and alumina of the ore. If the ore contains carbonate of lime, less lime is required as a flux. If either lime or silica is present in excess, part of the earth which is in excess forms a glassy compound with oxide of iron, which runs off amongst the slag, so that part of the iron is wasted; and another part of that earth becomes reduced, its base combining with the iron and making it brittle, as has been stated in the preceding article; so that in order to produce at once the greatest quantity and best quality of iron from the ore, the earthy ingredients of the entire charge of the furnace must be in certain definite proportions, which are discovered for each kind of ore by careful experiment.

The total quantity of carbon in pig iron ranges from 2 to 5 per cent. of its weight.

Different kinds of pig iron are produced from the same ore in the same furnace under different circumstances as to temperature and quantity of fuel. A high temperature and a large quantity of fuel produce *grey cast iron*, which is further distinguished into No. 1, No. 2, No. 3, and so on; No. 1 being that produced at the highest temperature. A low temperature and a deficiency of fuel produce *white cast iron*. Grey cast iron is of different shades of bluish-grey in colour, granular in texture, softer and more easily fusible than white cast iron. White cast iron is silvery white, either granular or crystalline, comparatively difficult to melt, brittle, and excessively hard.

It appears that the differences between those kinds of iron depend not so much on the total quantities of carbon which they contain as on the proportions of that carbon which are respectively in the conditions of mechanical mixture and of chemical combination with the iron. Thus, grey cast iron contains *one* per cent., and sometimes less, of carbon in chemical combination with the iron, and from *one* to *three* or *four* per cent. of carbon in the state of plumbago in mechanical mixture; while white cast iron is a homogeneous chemical compound of iron with from 2 to 4 per cent. of carbon. Of the different kinds of grey cast iron, No. 1 contains the greatest proportion of plumbago, No. 2 the next, and so on.

There are two kinds of white cast iron, the *granular* and the *crystalline*. The granular kind can be converted into grey cast iron by fusion and slow cooling; and grey cast iron can be converted into granular white cast iron by fusion and sudden cooling. This takes place most readily in the best iron. Crystalline white cast iron is harder and more brittle than granular, and is not capable of conversion into grey cast iron by fusion and slow cooling. It is said to contain more carbon than granular white cast iron; but the exact difference in their chemical composition is not yet known.

Grey cast iron, No. 1, is the most easily fusible, and produces the finest and most accurate castings; but it is deficient in hardness and strength; and, therefore, although it is the best for castings of moderate size, in which accuracy is of more importance than strength, it is inferior to the harder and stronger kinds, No. 2 and No. 3, for large structures.

353. **Strength of Cast Iron.**—Something has been already stated as to the comparative strength of different kinds of cast iron. It may be laid down as a general principle, that the presence of plumbago renders iron comparatively weak and pliable, so that the order of strength among different kinds of cast iron from the same ore and fuel is as follows:—

Granular white cast iron.
Grey cast iron, No. 3.
„ „ No. 2.
„ „ No. 1.

Crystalline white cast iron is not introduced into this classification, because its extreme brittleness makes it unfit for use in engineering structures.

Granular white cast iron also, although stronger and harder than grey cast iron, is too brittle to be a safe material for the entire mass of any girder, or other large piece of a structure; but it is used to form a hard and impenetrable *skin* to a piece of grey cast iron by the process called *chilling*. This consists in lining the portion of the mould where a hardened surface is required with suitably shaped pieces of iron. The melted metal, on being run in, is cooled and solidified suddenly where it touches the cold iron; and for a certain depth from the chilled surface, varying from about $\frac{1}{8}$th to $\frac{1}{2}$ inch in different kinds of iron, it takes the white granular condition, while the remainder of the casting takes the grey condition.

Even in castings which are not chilled by an iron lining to the mould, the outermost layer, being cooled more rapidly than the interior, approaches more nearly to the white condition, and forms a *skin* harder and stronger than the rest of the casting.

The best kinds of cast iron for large structures are No. 2 and No. 3; because, being stronger than No. 1, and softer and more flexible than white cast iron, they combine strength and pliability in the manner which is best suited for safely bearing loads that are in motion.

As to the comparative strength of irons melted by the cold blast and by the hot blast, it appears from the experiments of Mr. Fairbairn and Mr. Hodgkinson, that with the same kind of ore and fuel, No. 1 cold blast is in general superior to No. 1 hot blast iron; No. 2 hot and cold blast are about equally good; No. 3 hot blast is in general superior to No. 3 cold blast; and the average quality of the iron on the whole is nearly the same with the hot as with the cold blast.

A strong kind of cast iron called *toughened cast iron*, is produced by the process, invented by Mr. Morries Stirling, of adding to the cast iron, and melting amongst it, from one-fourth to one-seventh of its weight of wrought iron scrap.

The manner in which the strength of cast iron depends on the absence of impurities from the ore and fuel has already been mentioned in Article 351, p. 497.

Various mixtures of different qualities of iron have been recommended by different engineers as materials for large castings. (On this point see the *Report on the Application of Iron to Railway Structures*, p. 265.) For example, Mr. Fairbairn recommended the following combination:—

Lowmoor, No. 3,	30	per cent.
Blaina, or Yorkshire, No. 2,	25	"
Shropshire, or Derbyshire, No. 3,	25	"
Good old malleable scrap,	20	"
	100	"

Sir Charles Fox recommended a combination of two-thirds Welsh cold blast iron, and one-third Scotch hot blast iron, the latter being manufactured from equal proportions of black-band and hæmatite ores. But both these and other engineers agreed in considering that the best course for an engineer to take in order to obtain iron of a certain strength for a proposed structure was, not to specify to the founder any particular mixture, but to specify a certain minimum strength which the iron should exert when tested by experiment.

The strength of cast iron to resist cross breaking was found by Mr. Fairbairn to be increased by *repeated meltings* up to the *twelfth*, when it was greater than at the first in the ratio of 7 to 5 nearly. After the twelfth melting that sort of strength rapidly fell off.

The resistance to crushing went on increasing after each successive melting; and after the *eighteenth* melting it was double of its original amount, the iron becoming silvery white and intensely hard.

The transverse strength of No. 3 cast iron was found by Mr. Fairbairn not to be diminished by raising its temperature to 600° Fahr. (being about the temperature of melting lead). At a red heat its strength fell to two-thirds.

The strength of cast iron of every kind is marked by two properties; the smallness of the tenacity as compared with the resistance to crushing, and the different values of the modulus of rupture of the same kind of iron in bars torn directly asunder, and in beams of different forms when broken across. These circumstances have already been referred to in Article 157, p. 235, Article 164, pp. 256 to 258, and Article 166, p. 261. The variations in the modulus of rupture for beams of different figures arise in all probability from the greater tenacity of the skin as compared with the interior of the casting; for an experiment on a bar torn directly asunder shows the least tenacity of its internal particles; while experiments on beams broken across show the tenacity of some layer which is nearer to or further from the skin according to the form of cross-section.

Intense cold makes cast iron brittle; and sudden changes of temperature sometimes cause large pieces of it to split.

The *proof strength* of cast iron has been shown to be about *one-third of the breaking load*, by experiments already mentioned in the note to p. 221. The usual *factor of safety* for the working load on railway structures of cast iron is *six*. (See Article 143, p. 222.)

In addition to the data in the tables at the end of the volume, the following table gives results as to the strength of cast iron, extracted and condensed from the experiments of Mr. Fairbairn and Mr. Hodgkinson. All the co-efficients are in lbs. on the square inch.

Kinds of Iron.		Direct Tenacity.	Resistance to Direct Crushing.	Modulus of Rupture of Square Bars.	Modulus of Elasticity.
No. 1. Cold blast, ...	from	12,694	56,455	36,693	14,000,000
	to	17,466	80,561	39,771	15,380,000
No. 1. Hot blast, ...	from	13,434	72,193	29,889	11,539,000
	to	16,125	88,741	35,316	15,510,000
No. 2. Cold blast, ...	from	13,348	68,532	33,453	12,586,000
	to	18,855	102,408	39,609	17,036,000
No. 2. Hot blast, ...	from	13,505	82,734	28,917	12,259,000
	to	17,807	102,030	38,394	16,301,000
No. 3. Cold blast, ...	from	14,200	76,900	35,881	14,281,000
	to	15,508	115,400	47,061	22,908,000
No. 3. Hot blast, ...	from	15,278	101,831	35,640	15,852,000
	to	23,468	104,881	43,497	22,733,000
No. 4. Smelted by coke without sulphur,		—	—	41,715	—
Toughened cast iron, ...	from	23,461	129,876	—	—
	to	25,764	119,457	—	—
No. 3. Hot blast after first melting,......		—	98,560	39,690	—
" " " twelfth melting,...		—	163,744	56,060	—
" " " eighteenth " ...		—	197,120	25,350	—

It is to be understood that the numbers in one line of the preceding table do not necessarily belong to the *same specimen* of iron, each number being an *extreme* result for the kind of iron specified in the first column.

The *modulus of rupture of cast iron by wrenching* is usually given as calculated by an erroneous formula. According to the correct formula, it is on an average of several experiments,

27,700 lbs. on the square inch.

If this be denoted by f', the *wrenching moments* of cast iron bars are

$$\left.\begin{array}{l} \text{for } \textit{round bars} \text{ of the diameter } h,\ \cdot196\, f'\, h^3 \\ \text{for } \textit{square bars} \text{ of the dimensions } h \times h,\ \cdot281\, f'\, h^3 \end{array}\right\} \text{in inch-lbs.}$$

354. **Castings for Works of Engineering.**—The strength and quality of the iron suitable for such castings has already been discussed.

As to appearance, it should show on the outer surface a smooth, clear, and continuous skin, with regular faces and sharp angles. When broken, the surface of fracture should be of a light bluish-grey colour and close-grained texture, with considerable metallic lustre; both colour and texture should be uniform, except that near the skin the colour may be somewhat lighter and the grain closer; if the fractured surface is *mottled*, either with patches of darker or lighter iron, or with crystalline patches, the casting will be unsafe;

and it will be still more unsafe if it contains air-bubbles. The iron should be soft enough to be slightly indented by a blow of a hammer on an edge of the casting.

Castings are tested for air-bubbles by ringing them with a hammer all over the surface.

Cast iron, like many other substances, when at or near the temperature of fusion, is a little more bulky for the same weight in the solid than in the liquid state, as is shown by the solid iron floating on the melted iron. This causes the iron as it solidifies to fill all parts of the mould completely, and to take a sharp and accurate figure.

The solid iron contracts in cooling from the melting point down to the temperature of the atmosphere, by $\frac{1}{96}$th part in each of its linear dimensions, or *one-eighth of an inch in a foot;* and therefore patterns for castings are made larger in that proportion than the intended pieces of cast iron which they represent.

In designing patterns for castings, care must be taken to avoid all abrupt variations in the thickness of metal, lest parts of the casting near each other should be caused to cool and contract with unequal rapidity, and so to split asunder or overstrain the iron.

Iron becomes more compact and sound by being cast under pressure; and hence cast iron cannon, pipes, columns, and the like, are stronger when cast in a vertical than in a horizontal position, and stronger still when provided with a *head*, or additional column of iron, whose weight serves to compress the mass of iron in the mould below it. The air bubbles ascend and collect in the head, which is broken off when the casting is cool.

Care should be taken not to cut or remove the skin of a piece of cast iron at those points where the stress is intense.

Cast iron expands in linear dimensions by about 1-900th, or ·00111, in rising from the freezing to the boiling point of water; being at the rate of ·00000617 for each degree of Fahrenheit's scale, or about ·0004 for the range of temperature which is usual in the British climate. Every structure containing cast iron must be so designed that the greatest expansion and contraction of the castings by change of temperature shall not injure the structure.

355. **Wrought or Malleable Iron** in its perfect condition is simply pure iron. It falls short of that perfect condition to a greater or less extent owing to the presence of impurities, of which the most common and injurious have been mentioned, and their effects stated, in Article 351, p. 497; and its strength is in general greater or less according to the greater or less purity of the ore and fuel employed in its manufacture.

Malleable iron may be made either by direct reduction of the ore, or by the abstraction of the carbon and various impurities from cast iron. The process of direct reduction is applicable to rich and pure ores only; and it leaves a slag or "cinder" which contains a large proportion of oxide of iron, and yields pig iron by smelting. The most economical and generally applicable process is that of removing the foreign constituents from pig iron; and for that purpose white pig iron (called "forge pig") is usually employed, partly because it retains less carbon on the whole than grey pig iron, and partly because it is unfit for making castings. The details of the process are very much varied; but the most important principle of its operation always is to bring the pig iron in a melted state into close contact with a quantity of air sufficient to oxidate all the carbon and silicon. The carbon escapes in carbonic oxide or carbonic acid gas; the silica produced by the oxidation of the silicon combines partly with protoxide of iron and partly with lime (which is sometimes introduced as a flux for it), and forms slag or "cinder." Chloride of sodium (common salt) is used to remove sulphur and phosphorus. In one form of the process this is accomplished by injecting jets of steam amongst the molten iron; the oxygen of the steam assists in oxidating the carbon and silicon, and the hydrogen combines with the sulphur and phosphorus. The surest method, however, of obtaining iron free from the weakening effects of sulphur and phosphorus is to employ ores and fuel that do not contain those constituents.

The most common form of the process of making malleable iron is *puddling*, in which the pig iron is melted in a reverberatory furnace, and is brought into close contact with the air by stirring it with a rake or "rabble." Some iron makers precede the process of puddling by that of "refining," in which the pig iron, in a melted state, has a blast of air blown over its surface. This removes part of the carbon, and leaves a white crystalline compound of iron and carbon called "refiners' metal." Others omit the refining, and at once puddle the pig iron; this is called "*pig boiling*." The removal of the carbon is indicated by the thickening of the mass of iron, malleable iron requiring a higher temperature for its fusion than cast iron. It is formed into a lump called a "loup" or "bloom," taken out of the furnace, and placed under a tilt hammer or in a suitable squeezing machine, to be "*shingled*;" that is, to have the cinder forced out, and the particles of iron welded together by blows or pressure.

The bloom is then passed between rollers, and rolled into a bar; the bar is cut into short lengths, which are fagotted together, reheated, and rolled again into one bar; and this process is repeated till the iron has become sufficiently compact and has acquired a fibrous structure.

In Mr. Bessemer's process, the molten pig iron, having been run into a suitable vessel, has jets of air blown through it by a blowing machine. The oxygen of the air combines with the silicon and carbon of the pig iron, and in so doing produces enough of heat to keep the iron in a melted state till it is brought to the malleable condition; it is then run into large ingots, which are hammered and rolled in the usual way. This process has been most successful when applied to pig iron that is free from sulphur and phosphorus, such as that of Sweden and Nova Scotia.

Strength and toughness in bar iron are indicated by a fine, close, and uniform fibrous structure, free from all appearance of crystallization, with a clear bluish-grey colour and silky lustre on a torn surface where the fibres are shown.

Plate iron of the best kind consists of alternate layers of fibres crossing each other, and ought to be nearly of the same tenacity in all directions.

Malleable iron is distinguished by the property of *welding:* two pieces, if raised nearly to a white heat and pressed or hammered firmly together, adhering so as to form one piece. In all operations of rolling or forging iron of which welding forms a part, it is essential that the surfaces to be welded should be brought into close contact, and should be perfectly clean and free from oxide of iron, cinder, and all foreign matter.

In all cases in which several bars are to be fagotted or rolled into one attention should be paid to the manner in which they are "*piled*" or built together, so that the pressure exerted by the hammer or the rollers may be transmitted through the whole mass. If this be neglected, the finished bar or other piece may show flaws marking the divisions between the bars of the pile (as is often exemplified in rails).

Wrought iron, although it is at first made more compact and strong by *reheating* and hammering, or otherwise working it, soon reaches a state of maximum strength, after which all reheating and working rapidly makes it weaker (as will afterwards be shown by examples). Good bar iron has in general attained its maximum strength; and therefore, in all operations of forging it, whether on a great or small scale, by the steam-hammer or by that in the hand of the blacksmith, the desired size and figure ought to be given with the least possible amount of reheating and working.

It is still a matter of dispute to what extent and under what circumstances wrought iron loses its fibrous structure and toughness, and becomes *crystalline* and brittle. By some authorities it is asserted that all shocks and vibrations tend to produce that change; others maintain that only *sharp* shocks and vibrations do so; and others, that no such change takes place; but that the same piece of iron which shows a fibrous fracture, if gradually broken by a steady load, will show a crystalline fracture, if suddenly broken by a sharp blow. The author of this work at one time made a collection of several journals of railway carriage axles, which, after running for two or three years, had broken spontaneously by the gradual creeping inwards of an invisible crack at the shoulder. The fracture of every one of these was wholly or almost wholly fibrous; while other axles from the same works, when broken by the hammer, showed some a fibrous and others a crystalline fracture.* It is certain, at all events, that iron ought to be as little as possible exposed to sharp blows and rattling vibrations.

It is of great importance to the strength of all pieces of forged iron that the *continuity of the fibres* near the surface should be as little interrupted as possible; in other words, that the fibres near the surface should lie in layers parallel to the surface. This principle is illustrated by the results of some experiments made by the author of this work on the fracture of axles. Two cylindrical wrought iron railway carriage axles, one rolled, the other fagotted with the hammer, of four inches in diameter, were taken; a pair of journals of two inches in diameter were formed on the ends of each axle, one journal being reduced to the smaller diameter entirely by turning, so that the fibres at the shoulder did not follow the surface, and the other as far as possible by forging with the hammer, only one-sixteenth of an inch being turned off in the lathe to make it smooth, so that the fibres at the shoulder followed the surface almost exactly. All the journals were then broken off by blows with a 16 lb. hammer; when those whose diameters had been reduced by turning broke off with the *first* blow; and of those which had been drawn down by forging, that of the rolled axle broke off with the *fifth* blow, that of the hammered axle with the *eighth*. (*Proceedings of the Inst. of Civil Engineers*, 1843.)

Another important principle in designing pieces of forged iron which are to sustain shocks and vibrations, is to avoid as much as possible abrupt variations of dimensions and angular figures, especially those with re-entering angles; for at the points where such abrupt variations and angles occur fractures are apt to commence. If two parts of a shaft, for example, or of a beam exposed to shocks and vibrations, are to be of different thicknesses, they should be connected by means of curved surfaces, so that the change of thickness may take place gradually, and without re-entering angles.

356. **Steel and Steely Iron.**—Steel, the hardest of the metals and the strongest of known substances, is a compound of iron with from

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* Full-sized drawings of the fractured surfaces of several of these axles are in the possession of the Institution of Civil Engineers.

0·5 to 1·5 per cent. of its weight of carbon. These, according to most authorities, are the only essential constituents of steel. (See Article 350, p. 497.)

The term "steely iron," or "semi-steel," may be applied to compounds of iron with less than 0·5 per cent. of carbon. They are intermediate in hardness and other properties between steel and malleable iron.

In general, such compounds are the harder and the stronger, and also the more easily fusible, the more carbon they contain; those kinds which contain less carbon, though weaker, are more easily welded and forged, and from their greater pliability are the fitter for structures that are exposed to shocks.

Impurities of different kinds affect steel injuriously in the same way with iron. (See Article 351, p. 497.)

There are certain foreign substances which have a beneficial effect on steel. One 2,000th part of its weight of silicon causes steel to cool and solidify without bubbling or agitation; but a larger proportion is not to be used, as it would make the steel brittle. The presence of manganese in the iron, or its introduction into the crucible or vessel in which steel is made, improves the steel by increasing its toughness and making it easier to weld and forge; but whether the manganese remains in combination with the iron and carbon in the steel, or whether it produces its effects by its temporary presence only, is not known with certainty.

Steel is distinguished by the property of *tempering;* that is to say, it can be hardened by sudden cooling from a high temperature, and softened by gradual cooling; and its degree of hardness or softness can be regulated with precision by suitably fixing that temperature. The ordinary practice is, to bring all articles of steel to a high degree of hardness by sudden cooling, and then to soften them more or less by raising them to a temperature which is the higher the softer the articles are to be made, and letting them cool very gradually. The elevation of temperature previous to the "annealing" or gradual cooling is produced by plunging the articles into a bath of a fusible metallic alloy. The temperature of the bath ranges from 430° to 560° Fahr.

It is supposed that hard steel is analogous to granular white cast iron, being a homogeneous chemical compound of iron and carbon; that soft steel is analogous to grey cast iron, and is a mixture of a carburet of iron containing less carbon than hard steel with another carburet containing more carbon; and that slow cooling favours the separation of those two carburets.

Steel is made by various processes, which have of late become very numerous. They may all be classed under two heads, viz., adding carbon to malleable iron, and abstracting carbon from cast iron. The former class of processes, though the more complex, laborious, and expensive, is preferred for making steel for cutting tools and other fine purposes, because of its being easier to obtain malleable iron than cast iron in a high state of purity. The latter class of processes is the best adapted for making great masses of steel and steely iron rapidly and at moderate expense. The following are some of the processes employed in making different kinds of steel:—

I. *Blister Steel* is made by a process called "*cementation*," which consists in imbedding bars of the purest wrought iron (such as that manufactured by charcoal from magnetic iron ore) in a layer of charcoal, and subjecting them for several days to a high temperature. Each bar absorbs carbon, and its surface becomes converted into steel, while the interior is in a condition intermediate between steel and iron. Cementation may also be performed by exposing the surface of the iron to a current of carburetted hydrogen gas at a high temperature. Cementation is sometimes applied to the surfaces of articles of malleable iron in order to give them a skin or coating of steel, and is called "*case-hardening*."

II. *Shear Steel* is made by breaking bars of blister steel into lengths, making them into bundles or fagots, and rolling them out at a welding heat, and repeating the process until a near approach to uniformity of composition and texture has been obtained. It is used for various tools and cutting implements.

III. *Cast Steel* is made by melting bars of blister steel in a crucible, along with a small additional quantity of carbon (usually in the form of coal tar) and some manganese. It is the purest, most uniform, and strongest steel, and is used for the finest cutting implements.

Another process for making cast steel, but one requiring a higher temperature than the preceding, is to melt bars of the purest malleable iron with manganese and with the whole quantity of carbon required in order to form steel. The quality of the steel as to hardness is regulated by the proportion of carbon. A sort of semi-steel, or steely iron, made by this process, and containing a small proportion of carbon only, is known as *homogeneous metal*.

IV. *Steel made by the air blast* is produced from molten pig iron by Mr. Bessemer's process (Article 355, p. 504) in two ways; either the blowing of jets of air through the iron is stopped at an instant determined by experience, when it is known that a quantity of carbon still remains in the iron sufficient to make steel of the kind required, or else the blast is continued until the carbon is all removed, so that the vessel is full of pure malleable iron in the melted state, and carbon is added in the proper proportion, along with manganese and silicon. The steel thus produced is run into large ingots, which are hammered and rolled like blooms of wrought iron.

V. *Puddled Steel* is made by puddling pig iron (Article 355, p. 504), and stopping the process at the instant when the proper quantity of carbon remains. The bloom is shingled and rolled like bar iron.

VI. *Granulated Steel* (the invention of Captain Uchatius) is made by running melted pig iron into a cistern of water, over a wheel, which dashes it about so that it is found at the bottom of the cistern in the form of grains or lumps of the size of a hazel nut, or thereabouts. These are imbedded in pulverized hæmatite, or sparry iron ore, and exposed to a heat sufficient to cause part of the oxygen of the ore to combine with and extract the carbon from the superficial layer of each of the lumps of iron, each of which is reduced to the condition of malleable iron at the surface, while its heart continues in the state of cast iron. A small additional quantity of malleable iron is produced by the reduction of the ore. These ingredients being melted together, produce steel.

There are other processes for making steel and steely iron of which the details are not yet publicly known.

357. **Strength of Wrought Iron and Steel.**—Wrought iron, like fibrous substances in general, is more tenacious along than across the fibres; and its tenacity is greater than its resistance to crushing. The effect of the latter difference on the best forms of cross-section for beams has already been considered in Article 164, pp. 256 to 259, and will be further illustrated in the sequel.

The ductility of wrought iron often causes it to yield by degrees to a load, so that it is difficult to determine its strength with precision.

Wrought iron has its longitudinal tenacity considerably increased by rolling and wire-drawing; so that the smaller sizes of bars are on the whole more tenacious than the larger; and iron wire is more tenacious still, as the figures in the table of tenacity at the end of the volume show.

Wrought iron is weakened by too frequent reheating and forging; so that even in the best of large forgings, the tenacity is only about *three-fourths* of that of the bars from which the forgings were made, and sometimes even less.

As to the *effect of heat on the strength of wrought iron*, it has been shown by Mr. Fairbairn (*Useful Information for Engineers*, second series):—

I. That the tenacity of ordinary *boiler plate* is not appreciably diminished at a temperature of 395° Fahr., but that at a dull red heat it is diminished to about three-fourths.

II. That the tenacity of good *rivet iron* increases with elevation of temperature up to about 320° Fahr., at which point it is about one-third greater than at ordinary atmospheric temperatures; and that it then diminishes, and at a red heat is reduced to little more than one-half of its value at ordinary atmospheric temperatures.

The resistance of iron rivets to shearing is nearly the same with the tenacity of the best boiler plates.

As to the strength of wrought iron to resist crushing, see Article 157, p. 237.

Numerous experiments have been made on the tenacity of steel; but its other kinds of strength have been very little investigated. Its tenacity, like that of bar iron, is increased by rolling and wire-drawing.

The experiments already quoted in the note to Article 142, p. 221, have shown that the *proof strength* of wrought iron is almost exactly *one-third* of the breaking load.

The tables at the end of the volume give only average or extreme results as to the strength of wrought iron and steel; and therefore the following tables are here annexed, in which more details are given, but still in a very condensed form, chiefly on the authority of Mr. Fairbairn, Mr. Hodgkinson, and Messrs. R. Napier and Sons:—

TABLE OF THE TENACITY OF WROUGHT IRON AND STEEL.

Description of Material.	Tenacity in lbs. per Square Inch. Lengthwise.		Crosswise.	Ultimate Extension.
MALLEABLE IRON.				
Wire—Very strong, charcoal,...............	114,000	Mo.		
Wire—Average,..........	86,000	T.		
Wire—Weak,.............	71,000	Mo.		
Yorkshire (Lowmoor),...	64,200	F.	52,490 F.	
„ from to	66,390 60,075	N.		0·20 0·26
Yorkshire (Lowmoor) and Staffordshire rivet iron,............	59,740	F.		0·2 to 0·25
Charcoal bar,..............	63,620	F.		0·2
Staffordshire bar,... from to	62,231 56,715	N.		·302 ·186
Yorkshire bridge iron,...	49,930	F.	43,940 F.	·04; ·029
Staffordshire bridge iron,	47,600	F.	44,385	·04; ·036
Lanarkshire bar,... from to	64,795 56,655	N.		·158 ·238

TABLE—*continued.*

Description of Material.	Tenacity in lbs. per Square Inch.				Ultimate
	Lengthwise.		Crosswise.		Extension.
Lancashire bar, ... from	60,110	N.			·169
to	53,775				·216
Swedish bar,......... from	48,232	N.			·264
to	47,855				·278
Russian bar, ... from	56,805	N.			·153
to	49,564				·133
Bushelled iron from turnings, ...	55,878	N.			·166
Hammered scrap,.........	53,420	N.			·248
Straps from various districts,... from	55,937	N.			·108
to	41,386				·048
Bessemer's iron, cast ingot, ...	41,242	W.			
Bessemer's iron, hammered or rolled, ...	72,643	W.			
Bessemer's iron, boiler plate,..................	68,319	W.			
Yorkshire plates,... from	58,487	N.	55,033	N.	·109; ·059
to	52,000		46,221		·170; ·113
Staffordshire plates, from	56,996	N.	51,251	N.	·04; ·034
to	46,404		44,764		·13; ·059
Staffordshire plates, best-best, charcoal,	45,010	F.	41,420	F.	·05; ·045
Staffordshire plates, best-best, from	59,820	F.	54,820	F.	·05; ·038
to	49,945	F.	46,470	F.	·067; ·04
Staffordshire plates, best,	61,280	F.	53,820	F.	·077; ·045
Staffordshire plates, common, ...	50,820	F.	52,825	F.	·05; ·043
Lancashire plates, ...	48,865	F.	45,015	F.	·043; ·028
Lanarkshire plates, from	53,849	N.	48,848	N.	·033; ·014
to	43,433		39,544		·093; ·046
Durham plates, ...	51,245	N.	46,712		·089; ·064

Effects of Reheating and Rolling.

Puddled bar, ...	43,904	C.
The same iron five times piled, reheated and rolled, ...	61,824	
The same iron eleven times piled, reheated and rolled, ...	43,904	

TABLE—*continued.*

Description of Material.	Tenacity in lbs. per Square Inch. Lengthwise.		Crosswise.		Ultimate Extension.
Strength of Large Forgings.					
Bars cut out of a large forged crank, ... from to	47,582 44,758	N.			·270 ·168
Bars cut out of large forgings, ...	33,600	M.			
STEEL AND STEELY IRON.					
Cast steel bars, rolled and forged, from to	132,909 92,015	N.			·052 ·153
Cast steel bars, rolled and forged, ...	130,000	R.			
Blistered steel bars, rolled and forged,...	104,298	N.			·097
Shear steel bars, rolled and forged, ...	118,468	N.			·135
Bessemer's steel bars, rolled and forged,...	111,460	N.			·055
Bessemer's steel bars, cast ingots, ...	63,024	W.			
Bessemer's steel bars, hammered or rolled,	152,912	W.			
Spring steel bars, hammered or rolled,.....	72,529	N.			·180
Homogeneous metal bars, rolled, ...	90,647	N.			·137
Homogeneous metal bars, rolled, ...	93,000	F.			
Homogeneous metal bars, forged, ...	89,724	N.			·119
Puddled steel bars, rolled and forged, ... from to	71,484 62,768	N.			·191 ·091
Puddled steel bars, rolled and forged,...	90,000	F.			
Puddled steel bars, rolled and forged,...	94,752	M.			
Mushet's gun-metal,...	103,400	F.			0·034
Cast steel plates,.... from to	94,289 75,594	N.	96,308 69,082	N.	·057; ·096 ·198; ·196

TABLE—*continued.*

Description of Material.	Tenacity in lbs. per Square Inch. Lengthwise.		Crosswise.		Ultimate Extension.
Cast steel plates,.... hard	102,900	F.			·031
soft	85,400				·031
Homogeneous metal plates, first quality,	96,280	N.	97,150	N.	·086; ·144
Homogeneous metal plates, second quality,	72,408		73,580		·059; ·032
Puddled steel plates,......... from	102,593	N.	85,365	N.	·028; ·013
to	71,532		67,686		·082; ·057
Puddled steel plates,......	93,600	F.			0·125

In the preceding table the following abbreviations are used for the names of authorities:—

C., Clay; F., Fairbairn; H., Hodgkinson; M., Mallet; Mo., Morin; N.,* Napier & Sons; R., Rennie; T., Telford; W., Wilmot.

The column headed "Ultimate Extension" gives the ratio of the elongation of the piece at the instant of breaking to its original length. It furnishes an index (but a somewhat vague one) to the ductility of the metal, and its consequent safety as a material for resisting shocks. The vagueness arises from the fact, that the elongation of a bar just before breaking consists not so much of a stretching of each particle as of a permanent *drawing out*, with a new arrangement of the particles; and that it bears no fixed proportion, even in the same material, to the original length, being proportionally greater for short than for long bars. It is probable that it depends on the thickness of the piece as well as on its length.

When two numbers separated by a semicolon appear in the column of ultimate extension (thus ·082; ·057), the first denotes the ultimate extension lengthwise, and the second crosswise.

358. **Resilience of Iron and Steel.**—In order to obtain an exact measure of the capacity of a material for resisting shocks *by tension*, the *modulus of elasticity*, as well as the tenacity, must be known; and then the *modulus of resilience* (whose nature and use have been explained in Article 149, p. 227, and Article 173, p. 279) can be computed in *inch-pounds to the cubic inch*, by dividing the *square of the proof tenacity* by the modulus of elasticity. For iron and steel, the proof tenacity may be estimated at *one-third of the ultimate tenacity.*

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* The experiments whose extreme results are marked N. were conducted for Messrs. R. Napier & Sons, by Mr. Kirkaldy. For details, see *Transactions of the Institution of Engineers in Scotland*, 1858-9.

The following table gives some examples of such computations:—

METAL UNDER TENSION.	Ultimate Tenacity.	Proof Tenacity.	Modulus of Elasticity.	Modulus of Resilience.
Cast iron—Weak, ...	13,400	4,467	14,000,000	1·425
„ Average, ...	16,500	5,500	17,000,000	1·78
„ Strong, ...	29,000	9,667	22,900,000	4·08
Bar iron—Good average,	60,000	20,000	29,000,000	13·79
Plate iron—Good average,	50,000	16,667	24,000,000?	11·57?
Iron Wire—Good average,	90,000	30,000	25,300,000	35·57
Steel—Soft, ...	90,000	30,000	29,000,000	31·03
„ Hard, ...	132,000	44,000	42,000,000	46·10

To express the power of resisting shocks *by compression*, the resistance to crushing might be substituted in these calculations for the tenacity; but owing to the indirect manner in which fracture by crushing takes place, the result would be of very doubtful accuracy.

359. **Corrosion and Preservation of Iron.**—On this point, see Article 330, p. 462, where some of the best methods of preserving the surface of iron from oxidation have already been mentioned.

Cast iron will often last for a long time without rusting, if care be taken not to injure its skin, which is usually coated with a film of silicate of the protoxide of iron, produced by the action of the sand of the mould on the iron. Chilled surfaces of castings are without this protection, and therefore rust more rapidly.

The corrosion of iron is more rapid when partly wet and partly dry than when wholly immersed in water or wholly exposed to the air. It is accelerated by impurities in water, and especially by the presence of decomposing organic matter, or of free acids. It is also accelerated by the contact of the iron with any metal which is electro-negative relatively to the iron, or in other words, has less affinity for oxygen, or with the rust of the iron itself. If two portions of a mass of iron are in different conditions, so that one has less affinity for oxygen than the other, the contact of the former makes the latter oxidate more rapidly. In general, hard and crystalline iron is less oxidable than ductile and fibrous iron.

Cast iron and steel decompose rapidly in warm or impure sea-water.

Pieces of iron which are kept constantly in a state of vibration oxidate less rapidly than those which are at rest; for example, the rails of a railway on which a constant traffic runs rust much more slowly than those on which there is little or no traffic.

(See Mallet "On the Corrosion of Iron," in the *Reports of the British Association* for 1843 and 1849.)

SECTION II.—*Of Iron Fastenings.*

360. **Rivets** are made of the most tough and ductile iron. (See "Rivet Iron," in the preceding tables of strength.) In order that a rivet may connect two or more layers of plates or flat bars firmly, and in order that the shearing stress brought to bear on the rivet by a force tending to pull the plates asunder may be uniformly distributed throughout the sectional area of the rivet, it is essential that the rivet should tightly fit its hole. The longitudinal compression to which the rivet is subjected during the formation of its head, whether by hand or by machinery, tends to produce that result.

The ordinary dimensions of rivets in practice are as follows:—

Diameter of a rivet for plates less than half an inch thick, about double the thickness of the plate.

For plates of half an inch thick and upwards, about once and a-half the thickness of the plate.

Length of a rivet before clenching, measuring from the head = sum of the thickness of the plates to be connected + $2\frac{1}{2} \times$ diameter of the rivet.

Inasmuch as the resistance of rivets to shearing is nearly the same with the tenacity of good iron plates (50,000 lbs. per square inch, or thereabouts), the distance apart of the rivets used to connect two pieces of plate iron together is regulated by the rule, that *the joint sectional area of the rivets shall be equal to the sectional area of plate left after punching the rivet holes.* This rule leads to the following algebraical formula:—

Let t denote the thickness of the plate iron,
d, the diameter of a rivet,
n, the number of rows of rivets,

it being understood that the rivets which form a row stand in a line perpendicular to the direction of the tension which tends to pull the plates asunder.

c, the distance from centre to centre of the adjoining rivets in one row; then

$$c = d + \frac{{\cdot}7854\, n\, d^2}{t}. \qquad \text{(1.)}$$

Each plate is weakened by the rivet holes in the ratio

$$\frac{c - d}{c} = \frac{{\cdot}7854\, n\, d}{t + {\cdot}7854\, n\, d}; \qquad \text{(2.)}$$

In "single-rivetted" joints, $n = 1$; in "double-rivetted" joints

$n = 2$, and the two rows of rivets form a zig-zag; in "chain-rivetted" joints, n may have any value greater than 1. A single-rivetted joint is weakened by unequal distribution of the tension in the ratio of 4 : 5.

Suppose that in a chain-rivetted joint the distance c from centre to centre of the rivets is fixed, so as not to weaken the plates below a given limit; then in order to find how many rows of rivets there should be,—in other words, how many rivets there should be in each file,—the following formula may be used:—

$$n = \frac{(c - d)\, t}{\cdot 7854\, d^2} \dots\dots\dots\dots (3.)$$

361. **Pins, Keys, and Wedges.**—These fastenings are, like rivets, themselves exposed to a shearing stress, while they serve to transmit a pull or thrust from one piece of an iron frame to another; and the rule for determining their proper sectional area is the same, with this modification only, that if a pin, key, or wedge is not held perfectly tight in its seat, the shearing stress, instead of being uniformly distributed throughout its sectional area, will be more intense at the central layer of the section than elsewhere, being distributed according to the laws explained in Article 168, p. 266.

The ratio in which the maximum intensity of the shearing stress exceeds its mean intensity is the quantity denoted by $q_0 \mathrm{A} \div \mathrm{F}$ in the table, p. 267; its two most important values in practice being the following:—

For rectangular keys and wedges,............ $\frac{3}{2}$;

For circular or elliptical pins,................ $\frac{4}{3}$;

and the sectional area of the fastening is to be increased in this proportion beyond what would be necessary if the stress were uniformly distributed.

In order that a wedge or key may be safe against slipping out of its seat, its angle of obliquity ought not to exceed the angle of repose of iron upon iron, which, to provide for the contingency of the surfaces being greasy, may be taken at about 4°. (Article 110, p. 172.)

362. **Bolts and Screws.**—If a bolt has to withstand a shearing stress, its diameter is to be determined like that of a cylindrical pin. If it has to withstand tension, its diameter is to be determined by having regard to its tenacity. In either case the effective diameter of the bolt is its least diameter; that is, if it has a screw on it, the diameter of the spindle inside the thread.

The projection of the thread is usually *one-half of the pitch;* and the pitch should not in general be greater than *one-fifth of the effective diameter*, and may be considerably less.

In order that the resistance of a screw or screw-bolt to rupture by stripping the thread may be at least equal to its resistance to direct tearing asunder, the length of the nut should be *at least one-half* of the effective diameter of the screw; and it is often in practice considerably greater; for example, once and a-half that diameter.

The head of a bolt is usually about twice the diameter of the spindle, and of a thickness which is usually greater than five-eighths of that diameter.

SECTION III.—*Of Iron Ties, Struts, and Beams.*

363. **Forms of Iron Bars.**—In designing ordinary structures of wrought iron, it saves time and expense to use iron bars of such forms of cross-section as are usually to be met with in the market. The variety of those forms continually increases with the demand for new shapes; but an engineer should avoid introducing new sections for bars into his designs, except when, by so doing, some important purpose is to be served, or some decided advantage to be gained.

Amongst the most common forms of rolled bars are the following:—

Round iron, with a circular cross-section.

Square iron, with a square cross-section.

Flat iron, with oblong rectangular cross-sections of various forms.

Half-round and convex iron, with one side cylindrical and the other flat.

Angle iron, with cross-sections shaped like an L, of various breadths, depths, and thicknesses.

T-iron, with cross-sections shaped like a T, with a table or flange and a rib of various dimensions.

Double T-iron, or H-iron, or I-shaped iron, with a web and two flanges, of various dimensions. The most common form of railway bars belongs to this class.

Channel iron, which is like a flat bar with flanges projecting from both of its edges, but in one direction only, so that if laid with the flat bar downwards it is like a trough or rectangular channel.

Bulb iron, which is like a flat bar with a cylindrical thickening along one or both edges.

The "Bridge Rail," fig. 229.

The "Barlow Rail," fig. 230.

Bars of a cross-shaped section are sometimes rolled; but the figure is unfavourable to soundness of the iron; and when this form of section is required, it is best to build it. In general, figures for iron bars which cannot be rolled without great distortion of the iron ought to be avoided, unless there are special reasons for using them.

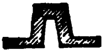
Fig. 229.

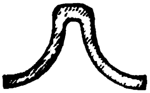
Fig. 230.

Angle bars and plates which exceed an inch in thickness are seldom so sound as those of less thickness. Where greater thicknesses are required, therefore, it is in general advisable to make them by building small thicknesses together.

364. **Iron Ties** ought in almost every case to be of malleable iron, as it has about three times the tenacity of cast iron.

A tie may consist either of one bar, or of several bars side by side, or of wires lying parallel in a bundle or spun into a rope; it may be in one length, or in two or more lengths joined together; if the lengths are numerous and short, they become *links*, and the whole tie a chain.

I. *Plate Iron Ties.*—The best mode of joining two lengths of a plate iron tie is by means of a fish-joint chain-rivetted. In fig. 231 are seen the ends of two lengths of a plate iron tie, meeting at the dotted line; they are connected by means of a fish-piece or *covering plate*, which is chain-rivetted to each of the pieces. The principles according to which the dimensions, number, and arrangement of the rivets are to be determined have been explained in Article 360, p. 515; and it has there also been shown to what extent the *effective sectional area* of the tie is diminished by the rivet holes, so as to be less than the total sectional area.

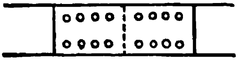
Fig. 231.

When a plate iron tie is built of several layers, they should *break joint* with each other; and at each joint there should be either a covering plate or a pair of covering plates, to transmit that share of the tension which belongs to the layer of plates in which the joint occurs.

II. *Tie-rods* or *Tie-bars* may be round, square, or flat, and may be made fast at the ends by pins passing through round eyes, by wedges driven into oval eyes or slots, or by screws and nuts. The proportions of these fastenings have been considered in Articles 361, 362, pp. 516, 517. Wedges and screws admit of being used to tighten the tie.

When an *eye* is formed on the end of a tie-bar, care should be taken that the sides of the eye are of sufficient strength. The tension is not uniformly distributed in them, being more intense at the inner side than at the outer. To allow for this, the sectional area may be made one-half greater than would be necessary if the tension were uniformly distributed.

III. *Compound Tie-bars* and *Flat Chains* are made of lengths or links, each consisting of flat bars placed side by side, and connected together by means of eyes and pins, as in the side view, fig. 232, and plan, fig. 232.* Whether it is desired to give stiffness to each individual bar or flexibility to the whole chain, the bars should be on edge. The numbers of bars in a compound link are odd and even alternately; thus, in the figure, the links A and C consist of odd numbers of bars, and B of an even number. The dimensions of the pin are to be found as in Article 361, p. 516, taking care to note at how many cross-sections it must give way at once if sheared across; for the stress is distributed amongst those cross-sections. In fig. 232* they are six in number. As to the eyes, see Division II. of this Article above.

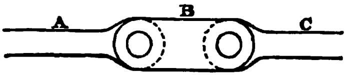

Fig. 232.

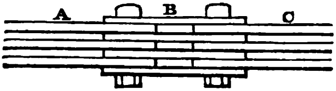

Fig. 232.*

IV. In *Oval-linked Chains* it is essential to strength that each link should be prevented from collapsing by a stay or cross-bar, as shown in fig. 233, when it appears by experiment that the tension is uniformly distributed over the cross-sections of the two sides of the link. When the stay is omitted, the strength of the chain is reduced in the proportion of

85 : 100, or nearly 6 : 7.

(Barlow, *On the Strength of Materials*, Article 143.)

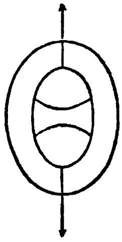

Fig. 233.

V. *Wire Cables* are sometimes made simply of a cylindrical bundle of parallel wires, *served* or bound together by a wire wound round the outside of the bundle. In constructing this sort of tie, great care is necessary in order to distribute the tension equally amongst the wires. The tension is equally distributed, without any special care, in *untwisted wire ropes* in which the wires are spun into strands, and the strands into ropes, without any rotation of individual wires, so that the fibres are all untwisted, and all equally strained. This is the strongest kind of iron tie for its weight; its tenacity, of about 90,000 lbs. per square inch of section, being equal to the weight of 27,000 feet of its own length, or thereabouts.

The best, and perhaps the only safe, mode of *making fast the end* of a wire rope is to make it form a turn or loop round a dead-eye, and splice it into itself: this is the only fastening which is as strong as the rope.

Wire cables require special care to protect them against oxidation.

VI. *Welded Ties.*—Iron ties have been lengthened by scarfing the ends of the two pieces together, heating them to a welding heat by a gas flame, and welding them together by an intense pressure. Data are wanting to determine precisely the strength of this sort of joint. In an experiment on the bursting of a cylindrical plate iron welded retort, the tenacity of the welded joint was found to be

30,750 lbs. per square inch;

or probably about 3-5ths of the tenacity of the plate iron

VII. In *proving Iron Ties*, they may safely be loaded with one-half of the instantaneous breaking load, without risk of permanent injury, the testing load being only applied once; although frequent application of the same load would at last break the tie.

365. **Cast Iron Struts and Pillars.**—Cast iron, from its great resistance to crushing, is peculiarly well suited for struts and pillars, especially those of moderate length. The best form for a cast iron strut or pillar containing a given quantity of material is that of a hollow cylinder. The laws of the strength of such pillars have already been fully explained in Article 158, pp. 236 and 237. The thickness of metal in them is seldom less than one-twelfth of the diameter.

Another form of cross-section commonly adopted for cast iron struts is that of a cross, fig. 234. The strength of such struts may be computed approximately by putting for the co-efficient a in equations 4 and 5 of Article 158, p. 237, a value greater than its value for a hollow cylinder, in the same proportion as a cross-shaped bar is more flexible than a hollow cylindrical tube of the same diameter and sectional area; that is to say, in the proportion of 3 to 1 nearly.

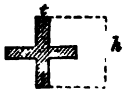

Fig. 234.

By similar reasoning, it appears that in the case of a hollow square cast iron strut, whose *diagonal* is equal to the diameter of the cylinder, the co-efficient a is to be increased in the ratio of 3 to 4.

Hence we have the following approximate formulæ for the *crushing load* of cast iron struts in lbs. *per square inch of sectional area*:—

Cross; diameter from end to end of a pair of arms $= h$;...................... $80{,}000 \div 1 + \frac{3\,l^2}{\cancel{400}\,h^2}$;

Hollow square; diagonal $= h$;............ $80{,}000 \div 1 + \frac{3\,l^2}{\cancel{800}\,h^2}$;

Hollow cylinder; diameter $= h$;......... $80{,}000 \div 1 + \frac{l^2}{\cancel{400}\,h^2}$.

The preceding formulæ refer to the case in which the struts are fixed in direction at the ends. When they are hinged at the ends, the second term of each divisor is to be made four times as great.

In order to give lateral stiffness to a flat-ended pillar, its ends should spread so as to form a capital and base, whose abutting surfaces should be "faced" in the lathe, or planed, to make them exactly plane and perpendicular to the axis of the pillar. For the same reason, when a cast iron pillar consists of two or more lengths, the ends of those lengths should be made truly plane and perpendicular to the axis of the pillar by the same process, so that they may abut firmly and equally against each other; and they should be fastened together by at least four bolts passing through projecting flanges.

366. **Wrought Iron Struts and Pillars.**—The principles of the strength of wrought iron struts have been explained in Article 158, pp. 237, 238. It appears from the formulæ deduced by Mr. Gordon from Mr. Hodgkinson's experiments, that while cast iron is the better material for a pillar whose length does not exceed a certain limit as compared with its diameter, wrought iron is the better material when the length exceeds that limit. For pillars with fixed ends, that limit, according to the formulæ, is about 26 times the diameter; for those with hinged ends, about 13 times; but from the nature of the calculation, those results must be regarded as roughly approximate only.

In order to stiffen wrought iron struts, they are made of various forms in cross-section, such as the angle iron, T-iron, double T-iron, channel iron, &c., already described. A very convenient form of cross-section is that of a cross. It is in general built by rivetting bars of simpler forms together; thus, it may be made up of two T-irons rivetted back to back, or four angle irons rivetted back to back, or (as in fig. 235) one flat bar A A, two narrower flat bars, B, B, and four angle irons, all rivetted together. This last form is that of the strut-diagonals of the Warren girders in the Crumlin Viaduct. A double T-shaped strut may either be a single bar, or may be built in a manner which will be described in treating of beams. The *Barlow Rail* (fig. 230, p. 518) is also a good form for struts.

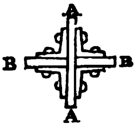

Fig. 235.

The stiffest form for a wrought iron strut is that of a *cell*, that is to say, a built tube, which may be cylindrical, rectangular, or triangular. Fig. 236 is a cross-section of a rectangular cell, with four plate iron sides connected together by angle irons and rivets. Fig. 237 is a triangular cell, running along the upper edge of a plate iron beam.

Fig. 238 shows a simple form of cross-section for a strut, being a segment of a circle. This might be further stiffened by rivetting a pair of angle irons along its edges.

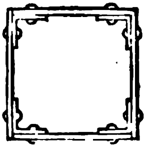
Fig. 236.

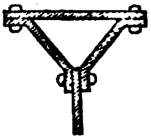
Fig. 237.

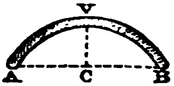

Fig. 238.

When a wrought iron strut is *hinged* at the ends, that generally takes place by its abutting at each end against a cylindrical pin, by which it is connected with some other piece of the framework, in the manner already described for tie-bars. To *fix* its ends in direction, as it seldom has large abutting surfaces, it is in general necessary to fasten it to the adjoining pieces of the structure by several bolts or rivets.

To insure the stiffness of a *built* strut, the bars of which it is built should break joint, like the layers of a built iron tie. The abutment of successive lengths against each other should be firm and equable; to insure which, every bar should have its ends made exactly plane and exactly perpendicular to its length. This is best done by a machine consisting of a pair of circular saws on one axis, at a clear distance apart equal to the intended length of the bars when cut: a bar being placed parallel to the axis, and moved towards it, has its two ends sawn off at once, in planes perpendicular to its length.

Mr. Gordon's formula for the ultimate strength of wrought iron struts, deduced from Mr. Hodgkinson's experiments, may be expressed as follows, P being the load, S the sectional area, l the length, and h the thickness:—

$$\frac{P}{S} = 36{,}000 \div 1 + \frac{a\,l^2}{h^2}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

where a has the value $\frac{1}{3{,}000}$ for struts fixed in direction at the ends, and of a solid rectangular section, h being the least dimension.

For other forms of cross-section, an approximate rule has already been given, to the effect that h is to be considered as representing the least dimension of a triangle or rectangle circumscribed about the bar; but in many cases it may be more satisfactory to take into account the least "radius of gyration" of the cross-section, as in the last article; and for that purpose the formula may be put in the following shape, r denoting that radius; that is to say, r^2 is the mean of the squares of the distances of the particles of the cross-section from a neutral axis traversing its centre of gravity in that direction which makes r^2 least:—

$$\frac{P}{S} = 36{,}000 \div 1 + \frac{l^2}{36{,}000\, r^2} \text{.................} (2.)$$

For hinged ends, 9,000 is to be substituted for 36,000.

The following are values of r^2 for different figures:—

I. Solid rectangle; least dimension $= h$;	$h^2 \div 12.$
II. Thin square cell; side $= h$;...	$h^2 \div 6.$
III. Thin rectangular cell; breadth b; depth h;......	$\frac{h^2}{12} \cdot \frac{h + 3b}{h + b}.$
IV. Thin triangular cell on the edge of a plate (fig. 237); base of triangle $= b$;	$b^2 \div 12.$
V. Solid cylinder; diameter $= h$;	$h^2 \div 16.$
VI. Thin cylindrical cell; diameter $= h$;	$h^2 \div 8.$
VII. Angle iron of equal ribs; breadth of each $= b$;	$b^2 \div 24.$
VIII. Angle iron of unequal ribs; greater b, less h;	$b^2 h^2 \div 12\,(b^2 + h^2).$
IX. Cross of equal arms;	$h^2 \div 24.$
X. H-iron; breadth of flanges b; their joint area B; area of web A;............	$\frac{b^2}{12} \cdot \frac{A}{A + B}.$
XI. Channel iron; depth of flanges + ½ thickness of web, h; area of web B; of flanges A;..............	$h^2 \cdot \left\{ \frac{A}{12(A+B)} + \frac{AB}{4(A+B)^2} \right\}.$
XII. Barlow rail; cross-section composed of two quadrants of radius R, measured to middle of thickness, connected by a table of sectional area = joint area of quadrants × ·273;	$R^2 \div 7$ nearly.

XIII. Pair of Barlow rails as above, rivetted base to base; ... $\Big\}$ $\cdot 393\ \mathrm{R}^2$.

XIV. Circular segment of radius R and length 2 R θ; .... $\Big\}$ $\left\{\frac{1}{2}+\frac{\cos\theta\sin\theta}{2\,\theta}-\frac{\sin^2\theta}{\theta^2}\right\}\mathrm{R}^2$.

367. **Cast Iron Beams.**—For the principles which are applicable to cast iron beams in common with beams in general, see Articles 169 to 179 A, pp. 230 to 296. The peculiar properties of cast iron as to strength, which have to be considered in designing beams, have been stated in Article 164, p. 257; Article 166, p. 261; Article 167, pp. 263 to 265; and in Article 353, pp. 499 to 502; and the precautions to be observed in designing these, as well as other castings, have been explained in Article 354, p. 502.

The most common and useful forms of cross-section for cast iron beams are the inverted ⊥-shaped (fig. 239), the trough-shaped (fig. 240), and the double T-shaped (fig. 241).

As to the transverse resistance of the ⊥-shaped section, see Article 163, Example VIII., p. 254. As to the proportionate area of flange and web which makes the tendency to break by crushing at B and tearing at C equal, see Article 164, Case I., p. 257; also the example in the same page. The same formulæ and example are applicable to trough-shaped beams, taking the two vertical ribs, B, B′, to be equivalent to one rib of the same depth and double the thickness.

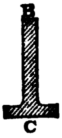

Fig. 239.

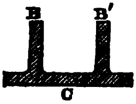

Fig. 240.

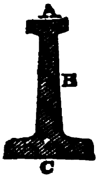

Fig. 241.

The thickness of the horizontal and vertical parts of these girders should be equal, or nearly equal, for the reason stated in Article 354, p. 503.

The double T-shaped beam is in general made of a figure introduced by Mr. Hodgkinson, with a view to making the strength equal above and below. As to the proportions of that section, see Article 164, Case II., p. 257; also the example in the same page. See also Hodgkinson *On the Strength of Cast Iron*.

In order to make the stretched table C large enough as compared with the compressed table A, it is necessary to make the former considerably thicker than the latter. This is reconciled with the rule as to avoiding abrupt changes of thickness in castings, by making the vertical web B of the same thickness with A at the top, and of a gradually increasing thickness towards the bottom, where it is nearly as thick as C.

Transverse ribs or feathers on cast iron beams are to be avoided, as forming lodgments for air-bubbles, and as tending to cause cracks in cooling.

Open-work in the vertical web is also to be avoided, partly for the same reasons, and partly because it too much diminishes the resistance to distortion by the shearing action of the load.

The various "*forms of equal strength*" in longitudinal sections, already described in Article 165, pp. 259, 260, are more easily executed in cast iron than in any other material, and are often employed in practice, especially those shown in figs. 141, 142, 143, 144, and 145.

The forms of horizontal section shown in figs. 142 and 144 are applied to the flanges or tables of double T-shaped girders of uniform depth.

In supporting a cast iron beam, provision must be made at one end for its expansion and contraction by heat and cold, which take place at the rate of about ·00111 for the 180° Fahr. between the ordinary freezing and boiling points of water, or ·0000062 nearly per degree of Fahrenheit's scale.

368. **Lengthened and Trussed Cast Iron Beams.**—It is seldom advisable to use a cast iron beam of a span so great that it cannot be cast in one length; but should it nevertheless be determined to do so, the following principles are to be observed in the construction of each junction.

Above the neutral axis the ends of the pieces should be true planes, abutting closely and equably against each other, and exactly perpendicular to the axis of the beam.

Below the neutral axis the pieces are to be connected by means of transverse flanges and wrought iron bolts, which will thus, at the joint, perform the duty of the lower or stretched table of the beam; and the total sectional area of those bolts should be such as to make, *not* their tenacity, but their *proportionate elongation by a given tension*, the same with that of the cast iron table for which they are a substitute. This condition will be approximately fulfilled by making the sectional area of the bolts in all about *one-half* of that of the cast iron table; when their tenacity will be more than sufficient.

Care should be taken so to arrange the bolts that the mean of the squares of their distances from the neutral axis of the section shall *not be less* than the corresponding quantity for the cast iron table whose duty they are to perform.

The same principles are to be followed in designing that sort of trussed cast iron beam in which a pair of wrought iron tie-rods are substituted for the whole or part of the lower table.

369. **Plain Wrought Iron Beams.**—For the principles which are applicable to wrought iron beams in common with beams in general, see Articles 169 to 179 A, pp. 230 to 296.

The most common and useful forms of section for wrought iron beams that are rolled in one piece are the T-shaped, and the I-shaped or double T-shaped, of which latter form fig. 242 is an example.

As to the resistance of cross-sections of those figures to cross-breaking, see Article 254, Examples VIII. and IX., pp. 254 to 256. As to the mode of fixing the proportions of such sections in order that they may be of equal strength against crushing and tearing, see Article 164, Cases III. and IV., p. 258, and the example in the same page; these being the cases applicable to a material in which the tenacity (denoted by f_b in the formulæ) is greater than the resistance to crushing (denoted by f_a).

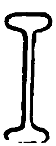

Fig. 242.

A plain wrought iron beam usually gives way under a transverse load by the compressed flange bending sideways; for that flange is in general so narrow, as compared with its length, that its condition is analogous to that of a long wrought iron strut. (See Article 158, p. 237.) The co-efficient f_a, therefore, which is the modulus of resistance of that flange, is not in general a constant quantity, but is less as the flange becomes narrower in comparison with the span of the beam.

From a reduction of the experiments of Mr. Fairbairn on wrought iron beams, given in his works, *On the Application of Iron to Building Purposes*, and *Useful Information for Engineers*, first series, it appears that the modulus in question may be computed with sufficient accuracy by the following formula, in which

l denotes the span of the beam, and
b, the breadth of its compressed flange:—

$$f_a \text{ (in lbs. on the square inch)} = \frac{36{,}000}{1 + \dfrac{l^2}{5{,}000\, b^2}} \ldots\ldots(1.)$$

The following table shows the result of applying this formula to variously proportioned beams, and of substituting its results in equations 10 and 11 of Article 164, p. 258; the value of the tenacity f_b being assumed to be 60,000 lbs. per square inch in each case. A_1 denotes the sectional area of the compressed flange; A_2 that of the vertical web; A_3 that of the stretched flange; h' the depth from centre to centre of the two flanges; M_0 the breaking moment in inch-lbs.:—

	f_t	$\frac{l}{b}$	f_c	A_1	$M_0 \div h'$.
I.	60,000	10	35,294	$\cdot 35\ A_2 + 1 \cdot 7\ A_3$	$14{,}118\ A_2 + 60{,}000\ A_3$
II.	60,000	20	33,333	$\cdot 4\ A_2 + 1 \cdot 8\ A_3$	$14{,}444\ A_2 + 60{,}000\ A_3$
III.	60,000	30	30,509	$\cdot 48\ A_2 + 1 \cdot 97\ A_3$	$14{,}916\ A_2 + 60{,}000\ A_3$
IV.	60,000	40	27,273	$\cdot 6\ A_2 + 2 \cdot 2\ A_3$	$15{,}455\ A_2 + 60{,}000\ A_3$
V.	60,000	50	24,000	$\cdot 75\ A_2 + 2 \cdot 5\ A_3$	$16{,}000\ A_2 + 60{,}000\ A_3$

The preceding results are made applicable to the T-shaped section simply by making $A_3 = 0$.

In cases in which a beam is liable to be strained alternately in either direction, the section is to be made similar above and below, so that $A_1 = A_3$; the beam tends to give way in every case by lateral bending of the flange which is compressed for the time, and f_c is the modulus of rupture; and the expression for the breaking moment assumes the simplified form,

$$M_0 = f_c\, h' \left(\frac{A_2}{6} + A_1 \right) \text{.....................(2.)}$$

The *expansion* of wrought iron beams by heat is very slightly greater than that of cast iron beams, being about ·0012 for the 180° between the ordinary freezing and boiling points of water, or about ·0000067 per degree of Fahrenheit.

370. **Plate and Box Beams.**—This class of wrought iron beams comprises various cross-sections, built of plates and bars rivetted together in various ways, but all virtually equivalent to double T-shaped sections, and having their strength dependent on the same principles. The following are examples:—

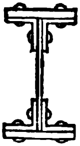

Fig. 243.

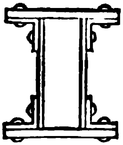

Fig. 244.

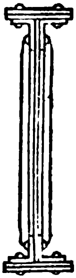

Fig. 245.

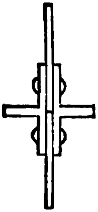

Fig. 246.

Fig. 243 is a plate beam having a single plate for the vertical web, while each of the horizontal ribs or flanges consists of a flat bar and a pair of angle irons, rivetted to each other and to the vertical web.

Fig. 244 is a "box beam," in which there is a double vertical web. The advantage of this construction is to give additional breadth, and therefore additional lateral stiffness and additional strength for resisting thrust, to the compressed table or flange.

Fig. 245 is a plate beam of greater dimensions than fig. 243. The horizontal ribs or flanges contain more than one layer of flat bars, and the web, which consists of plates with their largest dimension vertical, is stiffened by vertical T-iron ribs at the joints of those plates, as shown in the horizontal section, fig. 246.

To give still greater stiffness and strength to the upper or compressed horizontal rib, it is sometimes a cylindrical tube or "cell;" sometimes a rectangular cell, as in fig. 236, p. 522; sometimes a triangular cell, rivetted to the upper edge of the vertical web, as in fig. 237, p. 522; and in some cases a line of plates bent into an inverted segmental trough, as shown in the cross-section, fig. 238, p. 522, has been made fast at its summit V, by angle-irons and rivets, to the upper edge of the vertical web.

In fixing the dimensions of the parts, and computing the strength, of beams of this class, the rules of the preceding article are all applicable, having regard to the following special principles:—

I. The several tiers or layers of pieces of which the beam is built should break joint as much as possible.

II. *Upper Horizontal Rib.*—The several lengths of the pieces composing the upper horizontal rib should abut closely and truly against each other, having end surfaces made exactly perpendicular to the axis of the beam, as already described for wrought iron struts in Article 366, p. 522. In using equation 1 of Article 369, p. 526, to compute the modulus of rupture by crushing, f_c, the following are the divisors by which $\frac{l^2}{b^2}$ is to be divided:—

For a flat upper horizontal rib, or a triangular cell, ...	5,000
For a square cell, ...	10,000
For a cylindrical cell or an inverted semicircular trough (diameter $= b$),	7,500
For an inverted segmental trough, subtending the angle 2ι to radius unity,	$\frac{7{,}500}{\sin^2 \iota}\left(1 - \frac{\sin 2\iota}{2\iota}\right)$.

III. *Lower Horizontal Rib.*—The several lengths of plates or bars of which the lower horizontal rib consists are to be connected with each other by covering-plates and rivets as prescribed for wrought iron ties in Article 364, p. 518; and the symbol A_2 in the formulæ of Article 369, and of the other articles there referred to, is to be understood to stand, not for the total sectional area of the lower rib, but only for the *effective sectional area* left after making the proper deduction for rivet-holes, according to the principles explained in Article 364, p. 518, and Article 360, p. 515.

For the best plate iron, the value of the modulus of tenacity, f_t, is on an average about 50,000 lbs. per square inch. The following are the results of substituting that value for 60,000 in the computations of the table in Article 369, p. 527:—

	f_t	$\frac{l}{b}$ For a Flat Upper Rib.	f_c	A_1	$M_0 \div h'$
I.	50,000	10	35,294	·21 A_2 + 1·41 A_3	10,784 A_2 + 50,000 A_3
II.	50,000	20	33,333	·25 A_2 + 1·5 A_3	11,111 A_2 + 50,000 A_3
III.	50,000	30	30,509	·32 A_2 + 1·64 A_3	11,584 A_2 + 50,000 A_3
IV.	50,000	40	27,273	·41 A_2 + 1·83 A_3	12,121 A_2 + 50,000 A_3
V.	50,000	50	24,000	·54 A_2 + 2·08 A_3	12,667 A_2 + 50,000 A_3

IV. *Vertical Web.*—The thickness of the vertical web is seldom made less than $\frac{3}{8}$ inch, and, except in the largest beams, is in general more than sufficient to resist the shearing stress. In those beams in which it becomes necessary to attend specially to the power of the vertical web to resist the shearing action of the load, the amount of that shearing action is to be computed for a sufficient number of cross-sections by the formulæ of Article 161, Case IX., pp. 247, 248, 249, and its greatest intensity, for an assumed thickness of web, by the formula of Article 168, Case VIII., p. 267. (In many cases, however, it is sufficiently accurate to assume the shearing stress to be entirely borne by the vertical web, and uniformly distributed throughout its section.) It is then to be considered that the shearing stress at the neutral axis is equivalent to a pull and a thrust of equal intensity inclined opposite ways at 45°, and that the vertical web tends to give way by buckling under the thrust; so that its ultimate resistance in lbs. per square inch is given by the following expression:—

$$\frac{36{,}000}{1+\dfrac{s^2}{3{,}000\,t^2}};\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(1.)}$$

in which t is the thickness of the plates of the web, and s the distance measured along a line inclined at 45° to the horizon, between two of its vertical stiffening ribs; or, if it has no such ribs, between the upper and lower horizontal ribs. The intensity of the shearing action of the working load should not exceed one-sixth of the resistance given by the above formula.

V. *Longitudinal Variations of Section.*—Inasmuch as the bending moment of the load diminishes from the middle of the beam towards the ends, and the shearing force from the ends towards the middle, according to principles stated in Article 161, pp. 245 to 249, the transverse sections of the horizontal ribs may be diminished from the middle towards the ends, and that of the vertical web from the ends towards the middle, so as to make the resistance to bending and shearing respectively vary according to the same law.

VI. *Vertical Ribs.*—Each vertical rib is to be considered either as a suspending-piece from which a portion of the load hangs, or as a pillar on which a portion of the load lies, according as the load is hung from or supported upon the beam; and its transverse section must be made sufficient for the duty so thrown upon it, according to the principles of Article 364, p. 518, or Article 366, p. 521, as the case may be; and regard must be had to the fact, that a large rolling load, such as that upon one of the wheels of a locomotive engine, may be concentrated upon one vertical rib.

Above each of the *points of support*, the vertical ribs must either be placed closer or made larger, so that they may be jointly capable of safely bearing, as pillars, the entire share of the load which rests on that point of support.

A pair of vertical T-iron ribs rivetted back to back through the web-plates may be held to act as a pillar of cross-shaped section. (Article 366, Case IX., p. 323.)

(*Note as to Diagonal Ribs.*—It is obvious that the best position of the stiffening ribs would be diagonal, sloping upwards from the ends of the beam towards the middle at angles of 45°; but this would involve inconvenience and expense in workmanship, and would cause the plates for the web to be cut into awkward and complex figures).

VII. *Rivets.*—The principles which regulate the number and dimensions of the rivets that connect the lengths of the stretched horizontal rib together have been sufficiently explained in the passages referred to in Division III. of this Article.

The rivets which connect one division of the web with an adjoining vertical rib should be capable of withstanding safely the greatest shearing action of the load at the joint in question.

The shearing action on the rivets which connect one of the horizontal ribs with a given division of the web is to the vertical shearing action of the load at the middle of that division very nearly as the length of the division in question is to its depth.

VIII. *Camber.*—In order that a built wrought iron beam may become nearly straight under its working load, it should be constructed in such a manner that, if unloaded, it would have a "camber" or upward convexity equal to the anticipated working deflection.

Formulæ for that deflection are formed by putting the intensity of the *working stress* for the quantities denoted by f' in equations 13 and 13 A of Article 169, p. 275.

The modulus of elasticity for plate beams, according to the best authorities, is considerably less than for continuous iron bars, its value in lbs. per square inch being

$$\mathrm{E} = \text{from } 16{,}000{,}000 \text{ to } 18{,}750{,}000,$$
$$\text{or say, on an average, } 17{,}500{,}000.$$

The equation 13 A, above referred to, being that suitable for cross-sections of equal strength, is as follows, putting $l \div 2$ instead of c, the half span:—

$$\text{deflection } v_1 = \frac{n'' \, (f_a' + f_b') \, l^2}{4 \, \mathrm{E} \, h}; \;\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

Taking 6 as the factor of safety, we may make, with sufficient accuracy for the present purpose,

$$f_a' + f_b' = (f_a + f_b) \div 6 = 84{,}000 \div 6 = 14{,}000.$$

For an uniformly-loaded beam, with an uniform cross-section, $n'' = \frac{5}{12}$; if the cross-section varies along the beam so as to be of uniform strength and uniform depth, $n'' = \frac{1}{2}$. Assuming the latter to be nearly the case, we have

$$\text{working deflection } v_1 = \frac{14{,}000}{8 \times 17{,}500{,}000} \cdot \frac{l^2}{h} = \frac{l^2}{10{,}000 \, h}; \;(3.)$$

or *a third proportional to the depth and the hundredth part of the span.*

For example, an ordinary proportion is $l = 15\,h$; then $v_1 = \frac{3\,l}{2000}$.

371. **Great Tubular Girders.**—This term is applied to hollow plate iron girders so large that the traffic of a bridge can pass through the interior. A great tubular girder differs from a magnified box-beam, inasmuch as the former requires special appliances for stiffening the upper and lower horizontal tables.

Fig. 247 is a skeleton diagram of a cross-section of the class of tubular girder used in the Conway and Britannia bridges, in which the top and bottom are *cellular*, consisting of plates so disposed as to form rows of square or nearly square cells, like that in fig. 236,

p. 522. In order that these cells may be sufficiently stiff, the width of each of them should not exceed *thirty times the thickness of the plates* of which they are made. The joints of the cells are connected and stiffened by means of covering plates outside as well as angle irons within. The breadth is fifteen feet.

The two vertical webs, or sides, B, exactly resemble the vertical

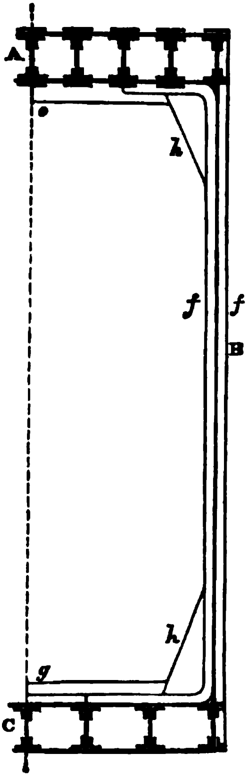

Fig. 247.

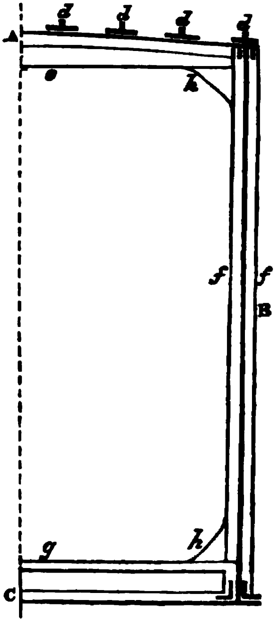

Fig. 248.

web already described in Division IV. of the last article, being composed of plates set up on end, and connected by means of pairs of vertical T-iron ribs, *f*, *f*. The horizontal joints of the side plates are made fast by covering strips. The lateral steadiness of the connection between the horizontal tables and the vertical webs is assisted by means of gussets, *h*, *h*, and horizontal prolongations of the inner T-iron ribs. The top and bottom are further stiffened by transverse ribs, *e*, *g*, one at every third set of vertical ribs.

Fig. 248 shows one-half of a kind of cross-section for a great tubular girder 16 feet broad, in which the top and bottom consist of one or more layers of plates rivetted close together, and stiffened by means of projecting ribs, instead of by a cellular structure. The girders of the Victoria Bridge over the St. Lawrence belong to this class. A is the upper table; it is slightly arched, having a radius of curvature equal to about six times its breadth, and is stiffened by the longitudinal T-iron ribs *d*, *d*, *d*, *d*, about 2 feet 3 inches apart, and by the transverse ribs *e*, about 7 feet apart. B is one of the sides, with vertical T-iron ribs in pairs, *f*, *f*, about 3½ feet apart. C is the bottom, consisting of a sufficient thickness of plates, with covering strips; it is stiffened, so far as it needs stiffening, by its connection with the transverse joists *g* of the platform, which are double T-shaped plate beams, one at every 7 feet; *h*, *h*, are gussets to stiffen laterally the connection between the sides and the top and bottom.

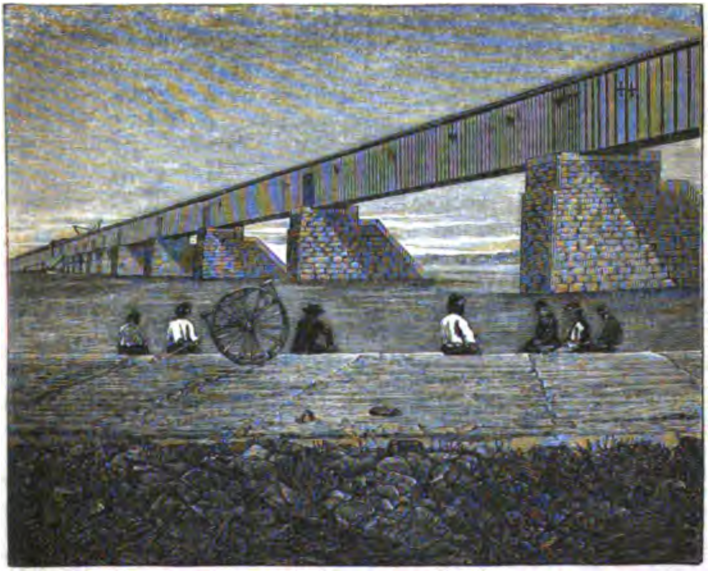

Fig. 249.—[The Victoria Bridge, from a Photograph.]

The whole of the principles of construction and strength stated in the preceding article for plate and box-beams are applicable to great tubular girders. In applying to them the principle of Division VI. of that Article, p. 530, so far as it relates to the strength and stiffness required for the vertical ribs at the points of support, it may be found necessary greatly to enlarge those ribs, and to give them the form of double T-shaped plate girders standing on end, and tapering from below upwards.

To illustrate the relative proportions of the parts of which a tubular girder is composed, the following statement shows those proportions for the tubes of the Conway Bridge:—

The quantities of iron in the top and bottom are nearly equal; for though the top, being compressed, has a larger *effective* section than the bottom, which is stretched, the total section of the bottom is increased so as to be nearly equal to that of the top, by the greater dimensions of the covering plates required at the joints.

The two sides together contain nearly the same quantity of iron with the top.

The distribution amongst the various parts is as follows:—

	Top. Per cent.	Sides. Per cent.	Bottom. Per cent.	
Plates,............................	61·0	51·3	61·1	
Angle iron and T-iron,...........	29·3	37·2	14·9	
Covers,............................	3·8	5·4	19·7	
Rivet-heads, ...	5·9	6·1	4·3	
	100·0	100·0	100·0	
Proportion per cent. of effective to total section,.....	87·7	43·0	72·2	Without deducting rivet-holes
Proportion per cent. of effective to total section of bottom, deducting *one-seventh* for rivet-holes,......			61.9	

(See Fairbairn *On Tubular Bridges;* Clark *On the Britannia and Conway Bridges;* Hodges *On the Victoria Bridge;* Stephenson "On Iron Bridges," in the *Encyc. Brit.*)

372. **Erection of Iron Girders—Continuous Girders.**—In the erection of iron girders, two methods may be followed: the girder may be built on the ground, and lifted to its position, or it may be built in its position on a scaffolding. The former method was adopted with the girders of the Britannia Bridge, each of which was floated on pontoons to a position between the piers directly below its permanent position; the faces of the piers having recesses to admit the ends of the girder. It was then lifted by means of

chains, hanging from the cross-heads of the plungers of a pair of hydraulic presses on the top of the piers, through a height equal to the stroke of the plungers (6 feet). As the beam rose, the recesses below its ends were built up with brickwork, which formed a pair of temporary supports for it while the plungers were lowered and the chains shortened, in order to raise it through the height of another stroke, and so on. The girders of the Victoria Bridge were built upon a scaffolding in their final position, all the pieces of which they were made having been shaped and punched in England.

The use of scaffolding, when practicable, is in general the easiest and least expensive method, especially if the girders are very large. In the case of the Britannia Bridge it was prohibited, lest it should interrupt the navigation of the strait.

In order to give an iron girder of several spans the property of *continuity* over its piers, the following method has been practised:—Suppose two lengths of the girder to have been erected, and to be still discontinuous over the pier where they meet. Each of those lengths is bent by its own weight; and their adjoining ends, instead of standing in parallel vertical planes, lean away from each other. The further end of one of the lengths is now tilted up, by means of a hydraulic press, a lifting jack, or some such suitable machine, until the two adjoining ends meet accurately; when they are made fast to each other by fishing, bolting, rivetting, or other suitable means of connection. The further end of the girder that has been tilted up is now lowered into its proper place; and the same process is followed for each joint where continuity over a pier is required.

As to the effect of continuity over the piers upon the strength and stiffness of a girder, see Article 178, p. 287, and especially Method II. of that Article, pp. 288 to 292.

The continuity of a girder must not be carried throughout a greater length than is consistent with a proper provision for its expansion and contraction by changes of temperature. An iron girder can have only one fixed support; all the rest must be on roller beds or slides; and in the case of a girder continuous over piers the best position for the fixed point of support is near the middle of its length. The largest continuous iron girders yet erected are those of the Britannia Bridge; they are 1,511 feet in length, and rest on three piers and two abutments, forming four spans; they have a fixed support on the central pier, and rest on rollers at the other four points of support, so that the length of metal which expands and contracts at each side of the fixed support is 755½ feet.

From what has been stated respecting the mode of connecting the lengths of a continuous girder, it is obvious that, previous to the making of the connection, each length bends under its own weight as a separate girder, and that the whole of its top should be stiffened to resist compression. After the connection has been effected, the top of each girder assumes a state of tension, and the bottom a state of compression, from the piers to the points of *contrary flexure* (p. 291, equation 10). Hence both the top and the bottom of a girder which is to be continuous over the piers are to be stiffened by means of cells or ribs, so as to be capable of resisting either compression or tension. Such is the case in the Britannia Bridge, where the girders are cellular both above and below. (See the authorities cited in p. 534.)

It may sometimes be desired to make the girder *imperfectly continuous over the piers*, for the following reason:—It has already been shown in Article 178, Method II., equations 6 and 7, pp. 289, 290, that if w be the fixed and w' the rolling load per unit of length, the moments of flexure are respectively,

$$\text{over a pier, } -\mathrm{M}_1 = \frac{2\,w + w'}{24} \cdot l^2; \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

$$\text{in the middle of a span, } \mathrm{M}_0 = \frac{w + 2\,w'}{24}\, l^2; \ \ldots\ldots\ldots (2.)$$

$$\text{the sum of which, or } \frac{w + w'}{8} \cdot l^2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

is simply the moment of flexure in the middle of a separate girder. The effect of the operation, then, already described, by which a girder is made continuous over the piers, consists in relieving the middle of each span of the girder of bending action to the amount denoted by the expression (1), and transferring that amount of bending action to the parts over the piers. If, as is the case in tubular bridges of the largest class, the rolling load is less than the fixed load, (1) is greater than (2); but the most advantageous method of employing the strength of the material, is to make the bending actions at mid-span and at each pier equal to each other, each of them being one-half of the expression (3); that is to say,

$$\frac{w + w'}{16} \cdot l^2. \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

To effect this result, an *imperfect continuity* is to be produced in the following manner:—

Observe the angular opening between the end surfaces of a pair of lengths of the girder as they lean from each other before being connected; denote it by θ, then compute the following quantity:—

$$\frac{\frac{w+w'}{16}}{\frac{2w+w'}{24}} \quad 4$$

$$\frac{w-w'}{4\,w+2\,w'}\cdot\theta;\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.)$$

and let this be the angle between the faces of a wedge-shaped filling piece to be introduced into the opening between the ends of the lengths of girder before connecting them, so that the opening may be reduced in the ratio of the expression (4) to the expression (1). Then tilt up the further end of one of the lengths until the joints fit, and connect them.

If the rolling load is to be neglected in making this adjustment, the expression (5) becomes simply $\theta \div 4$, so that the angular opening is to be reduced to 3-4ths, and the further end of one of the lengths tilted up to 3-4ths of the height which would have been necessary for perfect continuity. The result is, that the moments of flexure at mid-span and at the piers become equal to each other for a fixed load. This was the method followed at the junction over the central pier of the Britannia Bridge.

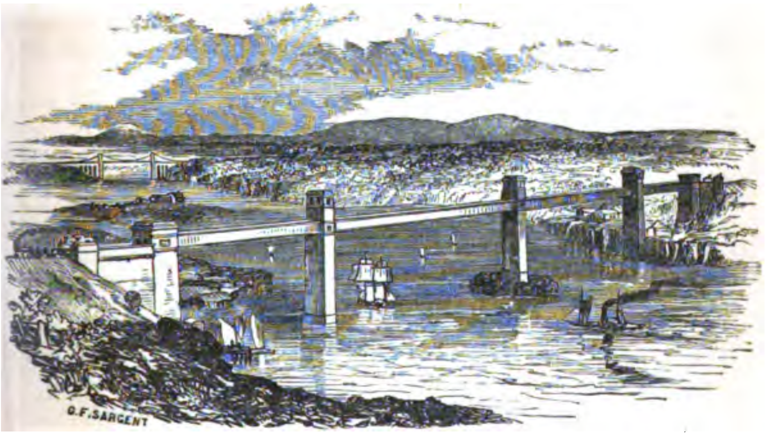

Fig. 250.—[The Britannia Bridge.]

373. **Effect of Wind on Tubular Girders.**—The pressure of the wind against one side of a tubular girder acts like an uniformly distributed load tending to bend it sideways, and producing a bending moment and maximum stress whose values are as follows:—

Let w'' be the pressure per lineal inch of girder, found by multiplying the intensity of the pressure of the wind by the depth of the side of the girder.

l, the span;
b, the breadth, from centre to centre of the vertical sides.
A_1 and A_3, as before, the sectional area of the top and bottom; and A_2, the joint sectional area of the sides.

Then the greatest bending moment is,

$$M = \frac{w'' l^2}{8} \text{ at the middle for a separate single span girder; (1.)}$$

$$M = \frac{w'' l^2}{12} \text{ at each pier for a girder continuous over the piers; (2.)}$$

and the greatest stress is

$$p_1 = M_1 \div b \left(\frac{A_2}{2} + \frac{A_1 + A_3}{6} \right) \text{ ....(3.)}$$

The greatest pressure of wind ever observed in Britain* was 55 lbs. on the square foot, = ·382 lb. on the square inch.

374. **Plain Arched Iron Ribs** are usually made of cast iron, but wrought iron is sometimes employed. The present article is confined to those iron arches which have not, or do not depend for their stiffness upon, diagonal bracing in their spandrils, so that the disfigurement of each rib is resisted either by its own stiffness alone, or by that stiffness combined with the stiffness of a horizontal girder directly above the rib.

The whole theory of the action of a load on an arched rib has already been given in Article 180, pp. 296 to 314, with the exception of some cases which have come to the author's knowledge since that part of this work was in type, and which will be treated of in this and in some subsequent articles.

Cases in which the arched rib is so braced by means of the spandril-framing that a special theory is required for them, will be treated of in the next section.

Reference will now be made to those parts of preceding articles where the formulæ to be used in computing the strength of arched ribs are to be found.

The usual form of section is the I or double T-shape, with equal flanges above and below, the thickness of the web being equal, or nearly equal. The depth (denoted by *h* in the formulæ) is not generally to be computed by means of a formula, but is to be found, either by a series of trials, or by adopting an empirical rule, such as making it from 1-40th to 1-60th of the span. The ratio *q*, to be used in the formulæ for strength and stiffness, is to be computed for such a section by means of the expression in Case XI. of Article 179, p. 295.

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* By the late Dr. Nichol, at Glasgow Observatory.

The neutral axis of the rib should be a parabola; for which, in arches of small rise as compared with the span, an arc of a circle may be substituted without material detriment to the stability of the arch.

The rib may be made of *uniform stiffness* by increasing the sectional area from the crown to the springing in the proportion of the secant of the inclination, as explained in Article 180, Problem II. When the rib is not of uniform stiffness, but of *uniform section,* the computation by means of the formulæ gives the area of section *at the crown of a rib of uniform stiffness of the same strength,* and this must be augmented in the proportion of the secant of the inclination of the neutral axis at the springing to radius, in order to obtain the *uniform area of section* required for the proposed rib of uniform section.

CASE I.—When the rib has flat ends firmly bedded against immovable skew-backs (as is usually the case with cast iron arches), the case is that of Article 180, Problem IV.; and the first step in the calculation is to compute the quantity denoted by B, by means of equation 30, p. 305 (which requires the following correction:—for $\frac{``4\,k^2"}{3\,l^2}$ read $\frac{``16\,k^2"}{3\,l^2}$).

Should the skew-backs, through the yielding of piers or abutments, spread asunder when the arch is loaded, the case is that of Problem V., and B is to be computed by means of equation 40, p. 308 (in which also for $\frac{``4\,k^2"}{3\,l^2}$, read $\frac{``16\,k^2"}{3\,l^2}$). In using this last equation, a sectional area, A_1, has to be assumed.

To allow for the straining effects of rise and fall of temperature, proceed as follows:—Let t denote the greatest probable deviation of the temperature from that at which the bridge is to be erected; E, the modulus of elasticity of the material; p_0, the intended mean intensity of the greatest thrust at the crown of the rib; and e, the co-efficient of expansion, whose value is

·0000067 per degree of Fahrenheit, or
·0000120 per centigrade degree,

then, in computing B, by means of equation 30 or equation 40, the following additional term is to be introduced into the factor within the brackets:—

$$\mp \frac{e\,t\,E}{p_0}; \qquad (1.)$$

the sign $\left\{\mp\right\}$ being used according as t denotes $\left\{\begin{matrix}\text{rise}\\ \text{fall}\end{matrix}\right\}$ of temperature.

The mean intensity of thrust p_0 may be unknown; in which case a provisional calculation of the horizontal thrust and area of section must be made without allowing for the effects of change of temperature, in order to obtain an approximate value of p_0.

When B has been computed, the next step is to compute, by the formulæ 36, p. 370, the proportions r_1 and r'_1 of the span which must be loaded with a rolling load, in order to make the thrust and tension respectively the greatest possible.

Should the sectional area be fixed, the greatest thrust p_1 and the greatest tension p'_1, are then to be computed by means of equations 37 and 38 respectively, p. 370.

Should the sectional area have to be fixed by computation, transpose, in these equations, the symbols A and p; they then become formulæ for computing the sectional area, if the greatest safe working thrust and tension respectively be put for p_1 and p'_1: and the *greater* of the two values of A_1 is to be adopted for the area at the crown of a rib of uniform stiffness.

To find the total horizontal thrust H when the stress is greatest, use equation 31, p. 306. The quantity p_0 in the expression (1) above has for its value $H \div A_1$.

The greatest total horizontal thrust is found by making $r = 1$.

The approximate formulæ, 37 A, 38 A, p. 370, and 31 B, 33 B, 37 B, 38 B, p. 308, may be used in the cases there explained.

CASE II.—When the rib is fixed at the crown to a *horizontal girder*, see Problem VIII., pp. 313, 314.

CASE III.—The beam may be vertically *hinged at the ends*, by having them rounded, and supported by hollow cylindrical bearings, so that they resemble trunnions or journals. This case falls under Problem VI.; and the first step in the calculation is to compute the value of the quantity C by means of equation 51, p. 310 (in which for "$\frac{4\,k^2}{3\,l^2}$," read "$\frac{16\,k^2}{3\,l^2}$").

To allow for the effect of changes of temperature, introduce into the factor of C within the brackets, the expression (1) of this Article, already explained.

The most severe stress occurs very nearly when one half of the span is loaded. Under that condition,

To find the total horizontal thrust H, use equation 52 A, p. 311;

To find the greatest moment of flexure M′, use equation 57 A, p. 312;

To find the greatest intensity of stress if the sectional area has been fixed, use equation 58, p. 311;

To find the required sectional area at the crown, make $p_1 =$ the greatest safe working intensity of thrust; and use the following formula:—

$$A_1 = \frac{1}{p_1}\left(\frac{M'}{q\,h} + H\right). \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

In Cases I., II., and III., *to find the greatest deflection*, see Problem VII., pp. 312, 313.

Case IV.—*Rib hinged at the crown and at the ends.*—The hinging of the rib at the crown, as well as at the ends, has been proposed by M. Mantion (*Annales des Ponts et Chaussées*, 1861), but has never yet been executed. This mode of construction would have the great advantage of annulling the straining effect both of changes of temperature, and of the yielding of the piers. The formulæ for computing the greatest stress in this case are deducible from those of Problem VI., pp. 311, 312, by making $C = 0$; and they are as follows:—

Let l be the span } of the neutral line in inches;
k, the rise }
w_0, the fixed load per lineal horizontal inch;
w, the rolling load per lineal horizontal inch;

then the greatest intensity of stress occurs when one half of the rib is loaded with the rolling load; and in that condition the total horizontal thrust is,

$$H = \frac{l^2}{8\,k}\left(w_0 + \frac{w}{2}\right); \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

the greatest moment of flexure, which acts downwards on the loaded half and upwards on the unloaded half of the rib, is

$$M' = \frac{l^2\,w}{64}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

the greatest intensity of thrust occurs at the outer flange of the loaded and the inner flange of the unloaded half of the rib, and has the following value:—

$$p_1 = \frac{1}{A_1}\left(\frac{M'}{q\,h} + H\right) = \frac{l^2}{8\,A_1}\left\{\frac{w}{8\,q\,h} + \frac{1}{k}\left(w_0 + \frac{w}{2}\right)\right\}; \ldots(5.)$$

and the greatest intensity of tension, if any, occurs at the inner flange of the loaded and outer flange of the unloaded rib, and has the following value:—

$$p'_1 = \frac{1}{A_1}\left(\frac{M'}{q\,h} - H\right) = \frac{l^2}{8\,A_1}\left\{\frac{w}{8\,q\,h} - \frac{1}{k}\left(w_0 + \frac{w}{2}\right)\right\} \ldots\text{(6.)}$$

To proportion the depth of the rib h to its rise k, so that the greatest tension may bear any given ratio to the greatest thrust, make,

$$\frac{q\,h}{k} = \frac{w}{8\,w_0 + 4\,w}\cdot\frac{p_1 - p'_1}{p_1 + p'_1}; \ldots\ldots\ldots\ldots\text{(7.)}$$

(for the value of q, as before, see Article 179, Case XI., p. 295.)

The *greatest total horizontal thrust* occurs when the rib is loaded over its whole span; and its amount is,

$$H_1 = \frac{l^2}{8\,k}\,(w_0 + w). \ldots\ldots\ldots\ldots\text{(8.)}$$

In many of the older examples of cast iron arched bridges, the ribs consist of a large number of small cast iron open-work panelled frames, acting as voussoirs, and bolted, dowelled, or otherwise connected together; but this mode of construction is deficient in strength and stability; and in later and better examples the ribs are made in as few and as long pieces as is practicable, and these are made to abut firmly and accurately against each other at planed surfaces, and are connected by means of transverse flanges and bolts. In cast iron arches of moderate size each rib usually consists of two lengths only, bolted together at the crown. In Southwark Bridge the ribs consist of pieces of 20 feet in length, whose ends abut, not directly against each other, but against transverse plates, which serve to bind the several parallel ribs of the bridge together crosswise, and through which the flanges of the lengths of the ribs are bolted together. In the new Westminster Bridge each rib consists of five pieces, the side pieces being of cast iron, and the middle piece of wrought iron.

The subject of iron arched ribs will be further considered in treating of *braced iron arches* in the next section.

SECTION IV.—*Of Iron Frames.*

375. **Iron Platforms.**—A platform in which timber planking is supported by iron girders, or girders and joists, requires no remarks beyond those which have already been made in Article 336, pp. 465 to 468, regard being had to the difference of the material of

the joists. As to the weight of platforms with their loads, see p. 466.

As to the distribution of the load of the platform of a railway bridge amongst the girders, see Article 341, pp. 475 to 477.

Iron may be used as the covering of a platform in various forms.

I. The *Barlow Rail* is a good form of section for supporting very heavy loads. (See fig. 230, p. 518; also Article 366, Example XII., p. 523.) When proportioned as directed in the example cited (that is to say, the sectional area of the table = the joint area of the quadrantal wings × ·273), it has the following properties:—

Let R be the radius of the quadrantal wings measured to the middle of their thickness.

t, their thickness, then,

$$\left.\begin{array}{l}\text{Sectional area of Barlow Rail} = 4\ \mathrm{R}\ t \text{ very nearly;}\\ \text{The neutral axis is nearly at the middle of the depth; and}\\ \text{Breaking moment} = \dfrac{2}{7} f_c\, \mathrm{R} \times \text{area} = \dfrac{8}{7} f_c\, t\, \mathrm{R}^2 \text{ nearly;}\end{array}\right\}(1.)$$

f_c being the modulus of rupture by crushing or buckling of the top table, or probably from 30,000 to 35,000 lbs. per square inch.

II. *Corrugated Iron* in sheets is suited to bear light loads, provided the sheets are so supported that the bending action of the load takes place in a plane parallel to the ridges and furrows. Iron laths or strips must be rivetted across the ridges and furrows to prevent them from spreading. These may be at distances apart equal to about twice the breadth of the corrugations.

Let b denote the breadth of a sheet of corrugated iron, t its thickness, h the depth from ridge to furrow; then its breaking moment is approximately,

$$\frac{4}{15} f_c\, h\, b\, t; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

f_c being the modulus of rupture by crippling at the compressed part.

III. *Bending Moment of the Load on a Plate.*—When a rectangular plate is supported on two parallel edges, the bending moment exerted by a load placed upon it is the same as that exerted by the same load on a beam of the same span.

When a rectangular plate is firmly supported at all its four edges by joists and girders, the bending moment is diminished. If the plate is square, the bending moments exerted in planes parallel to its two dimensions are equal to each other; if it is oblong, the greatest bending moment is exerted in a plane parallel to the breadth, or lesser dimension of the plate, the tendency being to split it lengthwise at the middle of its breadth.

The following formulæ are founded on a theory which is only approximately true, but which nevertheless may be considered to involve no error of practical importance:—

Let W denote the total load.
l, the length of the plate, between the supports of its ends.
b, its breadth, between the supports of its side edges.
M, the greatest bending moment.

Case I.—Square plate, load uniformly distributed;

$$M = \frac{W b}{16}. \quad \text{(3.)}$$

Case II.—Square plate, load collected in the centre;

$$M = \frac{3 W b}{16}. \quad \text{(4.)}$$

Case III.—Oblong plate, load uniformly distributed;

$$M = \frac{W l^4 b}{8 (l^4 + b^4)}. \quad \text{(5.)}$$

Case IV.—Oblong plate, load collected in the centre; l less than 1·19 b;

$$M = \frac{3 W l^4 b}{8 (l^4 + b^4)}. \quad \text{(6.)}$$

Case V.—Oblong plate, load collected in the centre, l equal to or greater than 1·19 b;

$$M = \frac{W b}{4}; \quad \text{(7.)}$$

being the same as for a plate supported at the *side edges only.*

Case VI.—Circular plate, of the diameter b, supported all round the edge, load uniformly distributed;

$$M = \frac{W b}{6 \pi} = \cdot 053 \, W b. \quad \text{(8.)}$$

Case VII.—Circular plate, load collected in the centre;

$$M = \frac{W b}{2 \pi} = \cdot 159 \, W b. \quad \text{(9.)}$$

IV. *Cast Iron Flooring Plates.*—The breaking moment of these plates is to be made greater than the bending moment of the working load in the ratio of a suitable factor of safety (such as *six*). They should be strengthened by means of vertical ribs or feathers on the upper side; and then the moment of resistance may be computed as for a trough-shaped or ⊥-shaped girder. (Article 367, p. 524; Article 163, pp. 264, 265.)

V. *Buckled Wrought Iron Plates* (the invention of Mr. Mallet) are plates of various figures (usually square or oblong), having a slight convexity in the middle, and a flat rim round the edge, called the "fillet;" and are the best form yet devised for the iron covering of a platform. They are usually placed so that the convex part is compressed, and the flat fillet stretched, and when they give way under an excessive load, it is usually by the crushing or crippling of the convex part.

Let l be the length of that section of a buckled plate at which the greatest bending moment is exerted (according to the principle stated at the beginning of Division III. of this Article, p. 543); h, the depth of curvature at the centre of the plate; t, its thickness, all in inches. Then the moment of resistance is nearly

$$\frac{4}{15} f_a \, l \, h \, t; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10.)$$

f_a being a modulus of rupture by crushing or crippling the plate at its crown or most convex part. From published results of experiment, it appears that for a plate 36 inches square, including the fillet, which is 2 inches broad, with a curvature of 1·75 inch, and $\frac{1}{4}$ inch thick, made fast all round the edges, the crushing load distributed over the plate is about 18 tons; whence, according to Case I.,

$$f_a = 21{,}600 \text{ lbs. per square inch nearly.}$$

This co-efficient, like that expressing the resistance of wrought iron struts to crushing (see Article 366, p. 522, equation 1), probably varies with the proportion of the thickness of the plate to its breadth, having for its maximum value 36,000; but sufficient experiments have not yet been published to show the law of its variation precisely. According to the table of safe loads for buckled plates 3 feet square, published by the inventor, the safe load varies nearly as the square of the thickness; this would make the co-efficient f_a vary nearly in the simple ratio of the thickness for plates of equal breadth, and of proportionate thicknesses, within the limits of those mentioned in the table, which are ·048 inch and ·375 inch. The factor of safety adopted being 4 for a steady load, and 6 for a moving load, the safe loads given in the table for a plate 3 feet square, $\frac{1}{4}$ inch thick, and with 1·75 inch of curvature, are 4·5 tons for a steady load, and 3 tons for a moving load. The buckled plates used by Mr. Page for the platform of the New Westminster Bridge measure 84 inches by 36, with a curvature of $3\frac{1}{2}$ inches, and thickness of $\frac{1}{4}$ inch; they bear 17 tons on the centre without giving way. According to Case V., this corresponds to a maximum thrust at the convex part of 17,920 lbs. per square inch.

The square form of buckled plates, supported and fastened at all the four edges, is the most favourable to strength.

376. **Iron Roofs.**—An iron roof may either be made entirely of iron, or the framework may be of iron and the covering of some other material. As to the construction and weight of various sorts of covering for roofs, see Article 337, p. 468. To the sorts of covering there described there may now be added *buckled iron plates*, already described in the last article; the thicknesses suited for roofing being from 1-20th to 1-10th of an inch.

The framework of iron roofs consists of parts analogous to those already described in treating of the framework of timber roofs in Articles 338, 339, pp. 469 to 475, with the exception, that in roofs covered with sheet iron, whether plain, corrugated, or buckled, the "common rafters" are unnecessary; the covering being supported on horizontal T-iron or angle iron bars, which act as laths or as purlins, and which are themselves supported on the principal rafters. Those principal rafters, and the trusses to which they belong, are placed at regular distances of from 2 feet 6 inches to 7 feet apart; the average distance is about 5 feet.

The general designs of those frames or trusses are analogous to those used in timber roofs; and in the computation of the thrusts and pulls along the several pieces, the same formulæ are applicable. (See Article 339, pp. 469 to 475.) In iron roof trusses, however, there is seldom a *tie-beam;* the principal tie being usually a single rod, supported at one or more points, and having no transverse load except its own weight between the supported points.

To the examples of roof trusses given in Article 339, may be added the following, which illustrates a kind of secondary trussing peculiar to iron roofs as distinguished from timber roofs:—1 2 3 is

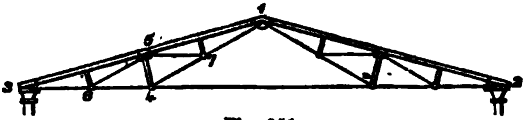

Fig. 251.

the *primary truss*, consisting of the two rafters 1 2, 1 3, and the principal tie-rod 2 3. To find the stresses on its pieces, conceive *half* the weight of a division of the roof to be concentrated at the point 1, and proceed as in Article 339, Case I., p. 469.

1 3 4 is a *secondary truss*, supporting the middle point 5 of the rafter by the aid of the strut-brace 4 5. Conceive *one quarter* of the weight of a division of the roof to be concentrated at 5, and proceed as in Article 119, figs. 102, 103, p. 182.

5 6 3, 1 7 5, are *smaller secondary trusses*, similar to 1 3 4, but of half the dimensions, and each sustaining *one-eighth* of the weight of a division of the roof.

The points of support of the rafter may thus be multiplied to any required extent.

The total or resultant stress on each portion of each bar of the truss is to be determined by the aid of the principle of Article 1 1, p. 184.

Thus the pull on the middle division of the great tie-rod is simply that due to the primary truss, 1 2 3. The pull on 4 7 is simply that due to the secondary truss 1 4 3. The pulls on 5 7 and 5 6 are simply those due to the smaller secondary trusses 1 5 7, 5 6 3. The pull on 1 7 is the sum of those due to the trusses 1 4 3 and 1 7 5. The pull on 6 4 is the sum of those due to the trusses 1 2 3 and 1 4 3. The pull on 6 3 is the sum of those due to the trusses 1 2 3, 1 4 3, and 5 6 3. The thrust on each of the four divisions of the rafter 1 3 is the sum of three thrusts, due to the primary truss, the larger secondary truss, and one of the smaller secondary trusses respectively.

As to the effect of *cambering the principal tie* by bracing it up to the top of the truss, see Article 119, fig. 100, p. 181.

In the *construction of iron roof-trusses* the rafters are usually made of T-shaped or H-shaped iron bars, and the struts of T-iron or angle iron bars, or any convenient form for resisting thrust. As to the strength of struts of these and other figures, see Article 366, pp. 521 to 524. The divisions of a rafter, and also the struts, may be considered as *hinged at the ends.* For the struts, cast iron is sometimes employed. (See Article 365, p. 520.) The smaller ties are usually round or square rods; the larger ties are sometimes flat bars set on edge. The foot of a rafter may be connected with the end of the great tie-bar by a gib and key traversing an oblong slot (Article 361, p. 516); or the foot of the rafter may abut into a cast iron shoe, to which the tie-rod may be fastened by a key, a pin, or a screw and nut. (Article 362, p. 516.) The oblique and vertical ties, or suspending-pieces, generally have *jaws* or forks at their upper ends, where they are hung from the rafters by means of pins, and screws at their lower ends, where they are connected with the struts and with the great tie-bar by means of pinching nuts. A central vertical suspending-rod is called a "king-bolt;" lateral vertical suspending-rods are called "queen-bolts."

A roof may have arched iron ribs instead of rafters. As to their strength, see Article 374, pp. 537 to 542. As to the stress on a semicircular rib, see the formulæ for such ribs when made of timber, Article 345, pp. 481, 482.

A simple and light roof for moderate spans is made by using bent sheets of corrugated iron so as to act at once as a covering and an arch, the thrust at the foot being resisted by horizontal tie-rods. As to the strength of corrugated iron, see Article 375, p. 543.

377. **Iron Braced Girders—General Design.**—Iron trusses or braced girders are analogous, in their figure and in the action of the load upon them, to the timber "bridge trusses" already described in Article 341, p. 475, and the same formulæ are to a great extent applicable to both. The chief differences are, that pieces which act alternately as struts and as ties are more frequently found in iron than in timber trusses; and that in iron trusses figures frequently occur which resemble those of timber trusses inverted, so that the ties become struts and the struts ties.

For the distribution of the load amongst a set of parallel bridge-girders, see pp. 475 to 477.

The following are examples of the general designs of iron braced girders:—

I. *Triangular Truss.*—(See fig. 252.) This exactly resembles the triangular timber truss, fig. 207, p. 470, inverted; B B being a strut, supported in the middle by the strut D, and the tie-rods A and C. The stress on each of its pieces may be computed by means of the formulæ 1, p. 470, substituting thrust for tension, and tension for thrust.

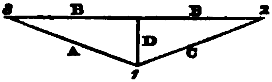

Fig. 252.

If each of the divisions, B, B, of the horizontal strut acts also as a *beam*, supporting a distributed load, the greatest intensity of thrust amongst its particles is to be computed by the formula (already given for arched ribs),

$$p = \frac{1}{\mathrm{A}}\left(\frac{\mathrm{M}}{q\,h} + \mathrm{H}\right); \qquad \text{(1.)}$$

in which H is the horizontal thrust, computed as in p. 470;

M, the bending moment;

A, the sectional area of the strut B B;

h, its depth;

q, a factor depending on its figure, as to which, see Article 179, p. 295.

By transposing p and A in equation 1, above, it becomes a formula for computing the required sectional area, p being made equal to the greatest working thrust per square inch.

II. *Trapezoidal Truss.*—(See fig. 253.) This resembles the trapezoidal timber truss, fig. 209, p. 470, inverted; B B B being a horizontal strut, supported by vertical struts K, K; A, F, and C, are the principal ties; G, G, tie-braces, which act only when the points 5 and 6 are unequally loaded.

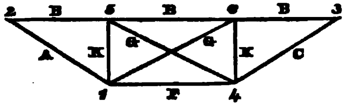

Fig. 253.

The greatest stress on the principal pieces takes place when both points 5 and 6 are fully loaded.

Let W denote the greatest load on each of these points (including *one-quarter* of the weight of the truss itself);
- c, the half-span of the truss;
- x, the distance of each of the points 5 and 6 from the middle of the span;
- k, the depth of the truss, measured from the centre of the horizontal strut B B B to the centre of the horizontal tie F;
- H, the total thrust along B B B, and total tension along F;
- T, the total tension on each of the inclined ties, A, C; then

$$\mathrm{H} = \mathrm{W}\,(c-x) \div k;\ \mathrm{T} = \sqrt{(\mathrm{H}^2 + \mathrm{W}^2)}. \ \ldots\ldots\ldots\ldots (2.)$$

To find the greatest amount of tension, S, on each of the diagonal braces, G, G, let W′ be the *greatest excess* of the load on either of the points, 5, 6, above the load on the other point; then (as in equation 4, p. 477),

$$\mathrm{S} = \left(\frac{\mathrm{G}}{k} + \mathrm{W}'\frac{c-x}{2\,c\,k}\right) \cdot \sqrt{4\,x^2 + k^2}. \ \ldots\ldots\ldots\ldots (3.)$$

G being the weight of one of the braces.

The greatest thrust on each of the vertical struts K, K, is given by the expression,

$$\mathrm{V} = \mathrm{W}' \cdot \frac{c-x}{2\,c} + \frac{\mathrm{B}}{4} + \mathrm{K} + \mathrm{G}; \ \ldots\ldots\ldots\ldots (3\ \mathrm{A}.)$$

in which B denotes the weight of the horizontal strut B B B, and K that of the upright itself.

III. *Zig-zag Truss, or Warren Girder.*—This girder consists of upper and lower horizontal booms, the former of which acts as a strut, and the latter as a tie, in resisting the bending action of the load; of a series of diagonal braces forming a zig-zag, which resist the shearing action of the load by thrust and tension alternately; and in some cases of a series of vertical suspending-rods to hang cross-joists from the upper row of joints, such as 1, 3, 5, N – 1, in fig. 255.

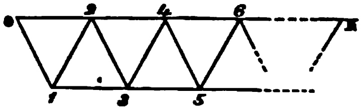

Fig. 254.

Fig. 254 represents the general design of a Warren girder suited for supporting a platform above it, at the points marked with the even numbers, 2, 4, 6, &c. (as in the Crumlin Viaduct, fig. 228, p. 494). Fig. 255 represents the general design of a Warren girder suited for supporting a series of cross-joists below it, hung from all the joints, 1, 2, 3, 4, 5, 6, &c., N – 1.

The actions of the load on this girder are computed by the method already explained in Article 160, pp. 230 to 243, as applied to a beam loaded at detached points. When every joint is equally loaded (as in fig. 255), the formula for the bending moment at any cross-section is that of Article 161, Case VIII., p. 247. In computing the shearing force, regard must be had to the action of a travelling load, as explained in Article 161, Case IX., pp. 247, 248.

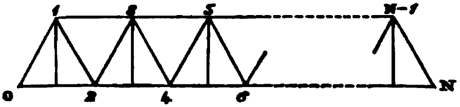

Fig. 255.

The most convenient method of computing the stress on each piece of a Warren girder is by means of a series of additions and subtractions, general formulæ being only used to check the accuracy of the results.

The first step is to number all the joints of the girder, as in the figures, designating one of the points of support as 0, and the other as N; N being the number (always even) of equal horizontal divisions into which those joints divide the span. Let n denote the number affixed to any particular joint;

l, the span of the girder;

k, its depth, from centre to centre of the horizontal booms;

s, the length of each diagonal brace;

$$\left(=\sqrt{k^2+\frac{l^2}{\mathrm{N}^2}}\right);$$

F_n, the shearing action at a cross-section between the joints n

and $n+1$; thus, F_0 is the shearing action between 0 and 1; F_1, between 1 and 2, &c.

H_n, the pull or thrust, as the case may be, upon that piece of a horizontal boom which lies between the joints $n-1$ and $n+1$; that is, opposite the joint n; for example, H_1 is the tension on 0 2 in fig. 255, or the thrust on 0 2 in fig. 254; H_2 is the thrust on 1 3 in fig. 255, or the tension on 1 3 in fig. 254; and so on.

The most severe bending action at each cross-section takes place when the girder is loaded over the whole span; the most severe shearing action at any given cross-section, when the larger segment of the span is loaded and the shorter unloaded; therefore the former supposition must be made in computing the stress on the horizontal booms, and the latter in computing the stress on the diagonal braces.

Two cases may be distinguished; that of fig. 255, in which the load is applied at every joint, and that of fig. 254, in which the load is applied at the joints marked with even numbers only. The former, though the more complex in construction, is the simpler in calculation, and is therefore taken first.

Case I.—*Each joint loaded.* Let the fixed part of the load on each joint be w, the rolling part w'; so that

$$W = (w + w')(N-1), \text{......................}(4.)$$

is the full load of the girder.

To find the Horizontal Stresses.

Compute the supporting pressure (F_0) at each of the points 0 and N by taking half the full load; that is to say,

$$F_0 = \frac{W}{2} = (w + w') \cdot \frac{N-1}{2}. \text{ ....}(5.)$$

Then compute the first term, and by successive subtractions of the quantity $\frac{l}{N k}(w + w')$, all the other terms, of the following series, which is that of the shearing actions, each multiplied by the ratio of the length of one horizontal division of the span to the depth of the girder.

$$\left.\begin{array}{l} \frac{l}{N k} F_0; \\ \frac{l}{N k} F_1 = \frac{l}{N k} F_0 - \frac{l}{N k}(w + w'); \\ \frac{l}{N k} F_2 = \frac{l}{N k} F_1 - \frac{l}{N k}(w + w'); \\ \quad \&c. = \&c. \end{array}\right\} \text{.............}(6.)$$

Then the series of horizontal stresses on the several divisions of the booms are to be computed by successive *additions*, as follows:—

$$\left.\begin{aligned} H_1 &= \frac{l}{N k} F_0; \\ H_2 &= H_1 + \frac{l}{N k} F_1; \\ H_3 &= H_2 + \frac{l}{N k} F_2; \\ \&c. &= \&c. \end{aligned}\right\} \quad \text{......................(7.)}$$

The test of the accuracy of this series of calculations is, that for the middle horizontal piece, whose number is N ÷ 2, it should give a result agreeing with that of the following formula:—

$$H_{\frac{N}{2}} = \frac{N l}{8 k} (w + w'). \quad \text{......................(8.)}$$

This is the maximum value of H, which has equal values for pieces equally distant from the middle piece.

The value of H for any particular piece whose number is n may be tested, if required, by the following formula:—

$$H_n = \frac{l}{N k} (w + w') \cdot \frac{n (N - n)}{2}. \quad \text{................(9.)}$$

To find the Diagonal Stresses due to the fixed part of the Load.

Let the stress produced by the fixed part of the load on the diagonal brace which lies between the joints n and $n+1$ be denoted by T_n. This will be a pull or a thrust alternately, according as the brace in question slopes downwards or upwards towards the middle of the span. The values of this stress are computed by a series of subtractions of the constant difference $\frac{s w}{k}$ as follows:—

$$\left.\begin{aligned} T_0 &= \frac{s w}{k} \cdot \frac{N-1}{2}; \\ T_1 &= T_0 - \frac{s w}{k}; \\ T_2 &= T_1 - \frac{s w}{k}; \\ \&c. &= \&c. \end{aligned}\right\} \quad \text{......................(10.)}$$

and the accuracy of the calculations is tested by the rule, that for the braces adjoining the middle joint of the girder, the result should be $s\,w \div 2\,k$.

To find the Diagonal Stresses due to the rolling part of the Load.

These stresses are proportional to the series of "triangular numbers," 0, 1, 3, 6, 10, &c., which result from the successive addition of the natural numbers, 1, 2, 3, 4, &c.; and they are to be computed as follows:—By successive additions of the common difference $\frac{s\,w'}{\mathrm{N}\,k}$, form the following arithmetical series, containing $\mathrm{N}-1$ terms,

$$\frac{s\,w'}{\mathrm{N}\,k};\ 2\,\frac{s\,w'}{\mathrm{N}\,k};\ 3\,\frac{s\,w'}{\mathrm{N}\,k};\ \&\text{c.}\ \ldots\ \frac{(\mathrm{N}-1)\,s\,w'}{\mathrm{N}\,k};\ \ldots\ldots(11.)$$

the accuracy of the additions being tested by the direct computation of the last term of the series. Then compute the following series of N stresses, by beginning with 0, and adding successively the terms of the preceding series;

$$\left.\begin{array}{l} \mathrm{S}_0 = 0; \\ \mathrm{S}_1 = \dfrac{s\,w'}{\mathrm{N}\,k}; \\ \mathrm{S}_2 = \dfrac{s\,w'}{\mathrm{N}\,k} + 2\,\dfrac{s\,w'}{\mathrm{N}\,k}; \\ \&\text{c.} = \&\text{c.} \end{array}\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12.)$$

and test the accuracy of the calculation by the direct computation of the last term, viz.:—

$$\mathrm{S}_{\mathrm{N}-1} = \frac{(\mathrm{N}-1)\,s\,w'}{2\,k}.\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13.)$$

Divide this series of N terms into two halves, and range the terms of the second half beside those of the first half *in inverted order;* thus

$$\left.\begin{array}{ll} \mathrm{S}_0 & \mathrm{S}_{\mathrm{N}-1} \\ \mathrm{S}_1 & \mathrm{S}_{\mathrm{N}-2} \\ \mathrm{S}_2 & \mathrm{S}_{\mathrm{N}-3} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{S}_n & \mathrm{S}_{\mathrm{N}-n-1} \\ \&\text{c.} & \&\text{c.} \end{array}\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots(14.)$$

Then for any given diagonal, whose number is n (that is, which lies between the joints n and $n+1$), the quantity corresponding to that number in the first column (S_n) will be the greatest stress produced by the rolling load, of the *contrary kind* (thrust or pull), to that produced by the fixed load; and the quantity in the same line of the second column (S_{N-n-1}) will be the greatest stress produced by the rolling load, of the *same kind* with that produced by the fixed load.

To find the greatest resultant Stress on each Diagonal Brace.

(1.) For the braces which slope *upwards* towards the middle of the span, take the sum of the stress due to the fixed load, and the greatest stress of the *same* kind due to the rolling load, as expressed by the formula,

$$T_n + S_{N-n-1}; \quad \text{.........................(15.)}$$

the result will be the most severe stress, and will be a *thrust*.

(2.) For the braces which slope *downwards* towards the middle of the span, make the same calculation; the result will be the *greatest stress*, and will be *tension*.

But when a piece of wrought iron is exposed alternately to tension and thrust, the thrust, although less than the tension, may be *more severe*, on account of the smaller capacity of the material for resisting it. To ascertain whether this is the case for any particular brace sloping downwards towards the middle of the span, compare the *tension* produced by the fixed load (T_n) with the *greatest thrust* produced by the rolling load (S_n); and if the latter is the greater, the excess

$$S_n - T_n, \quad \text{..........................(16.)}$$

will be the greatest thrust to be borne by the brace in question.

Case II.—*The joints marked with even numbers loaded, the others unloaded.* In this case the full load is expressed as follows:—

$$W = (w + w') \cdot \left(\frac{N}{2} - 1\right). \quad \text{...................(17.)}$$

To find the Horizontal Stresses

compute the supporting pressure at the point 0 as follows:—

$$F_0 = \frac{W}{2} = (w + w') \cdot \left(\frac{N}{4} - \frac{1}{2}\right). \quad \text{..............(18.)}$$

Then compute the following series, of which the terms are equal by pairs, each pair being less than the preceding by the difference $\frac{l}{N k}(w + w')$:—

$$\left.\begin{array}{l}\frac{l}{\mathrm{N}\,k}\,\mathrm{F}_1=\frac{l}{\mathrm{N}\,k}\,\mathrm{F}_0;\\ \frac{l}{\mathrm{N}\,k}\,\mathrm{F}_3=\frac{l}{\mathrm{N}\,k}\,\mathrm{F}_2=\frac{l}{\mathrm{N}\,k}\,\mathrm{F}_1-\frac{l}{\mathrm{N}\,k}\,(w+w');\\ \frac{l}{\mathrm{N}\,k}\,\mathrm{F}_5=\frac{l}{\mathrm{N}\,k}\,\mathrm{F}_4=\frac{l}{\mathrm{N}\,k}\,\mathrm{F}_3-\frac{l}{\mathrm{N}\,k}\,(w+w');\\ \qquad\qquad \&c.,\quad \&c.,\quad \&c.\end{array}\right\}\ \ldots.(19.)$$

The test of the accuracy of these calculations is, that for the division adjoining the middle joint, the result should be as follows:—

If the middle joint is loaded; that is, if $\frac{\mathrm{N}}{2}$ is even, $\frac{w+w'}{2}$;

If the middle joint is unloaded; that is, if $\frac{\mathrm{N}}{2}$ is odd; 0.

The series of horizontal stresses are computed by successive additions, precisely as in Case I., viz.:—

$$\mathrm{H}_1=\frac{l}{\mathrm{N}\,k}\,\mathrm{F}_0;\ \mathrm{H}_2=\mathrm{H}_1+\frac{l}{\mathrm{N}\,k}\,\mathrm{F}_1;\ \&c.\ \ldots\ldots\ldots(20.)$$

The test of the accuracy of this series of calculations is, that for the middle horizontal piece it should give a result agreeing with that of one or other of the following formulæ:—

If N ÷ 2 is even,

$$\mathrm{H}_{\frac{\mathrm{N}}{2}}=\frac{\mathrm{N}\,l}{16\,k}\,(w+w');\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(21.)$$

If N ÷ 2 is odd,

$$\mathrm{H}_{\frac{\mathrm{N}}{2}}=\frac{(\mathrm{N}^2-4)\,l}{16\,\mathrm{N}\,k}\,(w+w').\ \ldots\ldots\ldots\ldots\ldots\ldots(22.)$$

To find the Diagonal Stresses,

the calculations are the same as in Case I., with the following modifications:—

Throughout all the calculations, $\frac{\mathrm{N}}{2}$ is to be substituted for N; that is to say, the girder is to be treated as having $\frac{\mathrm{N}}{2}$ instead of N divisions.

Each of the series (10), (12), has $\frac{\mathrm{N}}{2}$ instead of N terms.

The series (11) has $\frac{\mathrm{N}}{2}-1$ instead of $\mathrm{N}-1$ terms.

If N ÷ 2 is odd, there will be a middle term in the series (12); and when the second half of the series is ranged in inverted order beside the first half, as in the table (14), that middle term is to be written at the bottom of each column.

Each of the results denoted by T and S in the series (10) and (14), and by their sums and differences in the formulæ 15 and 16, applies to a *pair* of adjacent diagonal braces, one sloping upwards and the other downwards towards the middle of the span. Thus

T_0, S_0, $S_{\frac{N}{2}-1}$, apply to the braces 0 and 1;

T_1, S_1, $S_{\frac{N}{2}-2}$, " " " 2 and 3;

and generally,

T_n, S_n, $S_{\frac{N}{2}-n-1}$, apply to the braces $2n$ and $2n+1$.

The ordinary angle of inclination of the braces in the Warren girder is 60°; in which case some labour of calculation is saved by the fact that the length of a brace, s, is equal to the distance from joint to joint along one of the booms, $2l \div N$.

Example of Case II.—Suppose the design of the girder to be as in fig. 254, and to consist of 17 equilateral triangles, so that $N = 18$;

$$\frac{l}{N k} = \frac{1}{\sqrt{3}} = \cdot 57735; \frac{s}{k} = 1 \cdot 1547;$$

also let the loads on each of the points 2, 4, 6, 8, 10, 12, 14, 16, be respectively

fixed, $w = 12{,}000$ lbs.

rolling, $w' = 18{,}000$ lbs.

This is nearly the case for a railway bridge girder of 160 feet span, supporting half the load of a line of rails. The supporting pressure is

$$F_0 = (w + w') \cdot \left(\frac{N}{4} - \frac{1}{2}\right) = 120{,}000 \text{ lbs.}$$

The following table shows the calculation of the horizontal stresses:—

$\frac{l}{N k}(w+w')$ Lbs.	n	$\frac{l}{N k} F$ Lbs.	H Lbs.
	0,18	69282·0	
17320·5	1,17	69282·0	69282·0 thrust
	2,16	51961·5	138564·0 pull
17320·5	3,15	51961·5	190525·5 thrust
	4,14	34641·0	242487·0 pull
17320·5	5,13	34641·0	277128·0 thrust
	6,12	17320·5	311769·0 pull
17320·5	7,11	17320·5	329089·5 thrust
	8,10	0	346410·0 pull
	9	0	346410·0 thrust (Middle piece)

The verification of the last result is,

$$H_9 = \frac{(N^2 - 4)\,l}{16\,N\,k}(w + w') = 20 \times 17320{\cdot}5 = 346410.$$

The following are the calculations of the stresses on the braces, due to the fixed load:—

$$T_0 = \frac{s\,w}{k}\left(\frac{N}{4} - \frac{1}{2}\right) = 13856{\cdot}4 \times 4 = 55425{\cdot}6$$

$\frac{s\,w}{k}$	T	n, for braces to which the results are applicable as	
Lbs.	Lbs.	Thrust.	Pull.
	55425·6	1,16	0,17
13856·4			
	41569·2	3,14	2,15
13856·4			
	27712.8	5,12	4,13
13856·4			
	13856·4	7,10	6,11
13856·4			
	0	8,9	

The following table shows the calculation of the greatest stresses produced by the rolling load:—

$\frac{2\,s\,w'}{N\,k}$		S	n, for braces to which the results are applicable as	
Lbs.	Lbs.	Lbs.	Thrust.	Pull.
		0	0,17	1,16
	2309·4			
2309·4		2309·4	2,15	3,14
	4618·8			
2309·4		6928·2	4,13	5,12
	6928·2			
2309·4		13856·4	6,11	7,10
	9237·6			
2309·4		23094·0	8,9	9,8
	11547·0			
2309·4		34641·0	10,7	11,6
	13856·4			
2309·4		48497·4	12,5	13,4
	16165·8			
2309·4		64663·2	14,3	15,2
	18475·2			
		83138·4	16,1	17,0

The verification of the accuracy of the additions is given by the following calculation:—

$$\frac{s\,w'}{k}\left(\frac{N}{4}-\frac{1}{2}\right)=20784{\cdot}6\times 4=83138{\cdot}4.$$

The following table shows the combined actions of the fixed and rolling loads on the braces; S′ denoting, for brevity's sake, the smaller value of S for the given brace. Thrusts are denoted by *t*, pulls by *p*:—

n	T	S	S′	T + S	S′ − T
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
0,17	55425·6 *p*	83138·4 *p*	0 *t*	138564·0 *p*	
1,16	55425·6 *t*	83138·4 *t*	0 *p*	138564·0 *t*	
2,15	41569·2 *p*	64663·2 *p*	2309·4 *t*	106232·4 *p*	
3,14	41569·2 *t*	64663·2 *t*	2309·4 *p*	106232·4 *t*	
4,13	27712·8 *p*	48497·4 *p*	6928·2 *t*	76210·2 *p*	
5,12	27712·8 *t*	48497·4 *t*	6928·2 *p*	76210·2 *t*	
6,11	13856·4 *p*	34641·0 *p*	13856·4 *t*	48497·4 *p*	0
7,10	13856·4 *t*	34641·0 *t*	13856·4 *p*	48497·4 *t*	0
8,9	0	23094·0 *p*	23094·0 *t*	23094·0 *p*	23094·0 *t*

The accuracy of the numbers in the columns headed T + S and S′ − T may be checked by setting down the former in direct order, and the latter in inverted order, and taking their second differences, which ought to be constant, and equal to $2\,s\,w' \div N\,k$; that is, in the present case, 2309·4. The following is the process:—

	First Diff.	Second Diff.
138564·0		
	32331·6	
106232·4		2309·4
	30022·2	
76210·2		2309·4
	27712·8	
48497·4		2309·4
	25403·4	
23094·0		2309·4
	23094·0	
0		

It appears from the values of S′ − T that in the example chosen the two middle braces alone act alternately as struts and ties under a rolling load.

It is unnecessary to give a numerical example of the calculations in Case I.; for they differ from those in Case II. only in being more simple.

IV. An *Iron Lattice Girder* consists essentially of a pair of horizontal booms to resist the bending action of the load, and of two series of diagonal braces, inclined opposite ways, usually at 45°, to resist the shearing action. There may also be upright ribs, one at each loaded point, and one or more at each point of support, to distribute the load and the supporting pressures amongst the diagonal braces. Although these upright pieces are not absolutely essential, except at the points of support, it is advisable not to omit them. Their strength is to be fixed according to the same principles with that of the upright ribs of plate girders; Article 370, Division VI., p. 530.

To compute the stresses on the pieces of a lattice girder, one of its points of support is to be designated as 0 and the other as N, (N being the number of divisions into which the loaded points divide it); and the loaded points are to be numbered consecutively from 1 to N − 1.

In computations respecting the *shearing* action of the load, F_0 is to designate the shearing action in the division of the girder between 0 and 1, F_1 between 1 and 2, &c., and, generally, F_n between n and $n+1$; but in computations respecting the *horizontal stresses*, which depend on the bending action, H_1 is to denote the stress on the booms *at* a vertical section traversing the point 1, &c.

This being understood, the calculation of the thrusts and pulls on the horizontal booms is to be proceeded with precisely as for Case I. of a zig-zag girder; formulæ 5 to 9, pp. 551, 552.

To find the stress on the lattice work, compute the two series of quantities $T_n + S_{N-n-1}$, $S_n - T_n$, for the several divisions of the girder, as for a zig-zag girder, Case I., formulæ 10 to 16, pp. 552 to 554, and assume each of those forces to be equally distributed amongst the lattice bars that traverse the division of the girder to which it belongs. This assumption of equal distribution is not exact; but its errors are not of practical importance.

If the loaded points are numerous and near each other, the girder may be treated as an uniformly loaded beam, Article 161, Case VI., p. 246; and Case IX., p. 247.

V. *Zig-zag and Lattice Continuous Girders.*—In both these classes of girders the effect of continuity over the piers may be computed as follows:—

Calculate the horizontal stress on each division of the girder, when fully loaded, on the supposition that it is *discontinuous* at the piers; and let H_m be the result thus obtained for the middle horizontal bar. Then

$$H_r = \frac{2\,w + w'}{3\,w + 3\,w'}\,H_m \text{ .... (23.)}$$

will be the *tension on the upper boom*, and *thrust on the lower boom* of a continuous girder, over the piers, when its spans are alternately loaded and unloaded; and the *difference* between this and the stress H_a already calculated for any given bar of the girder supposed discontinuous, will be the stress on that bar when the girder is continuous and loaded on alternate spans; that is to say,

$$H_a - H_r \text{ ....} (24.)$$

When this expression is negative (that is, when H_r is the greater term), the kind of stress is reversed. When it is $= 0$, it indicates a point of contrary flexure.

378. **Iron Braced Girders—Construction.**—I. *General Remarks.* Various iron trusses or braced girders have been made, in which the struts are of cast iron and the ties of wrought iron, advantage being thus taken of the greater resistance of cast iron to crushing and of wrought iron to tearing; but the greater pliability and brittleness of cast iron, and the rapid diminution of its resistance to crushing as the proportion of length to diameter increases (as to which see Article 366, p. 521), have led to the general employment of wrought iron for the struts as well as for the ties, care being taken that the struts are of figures suited to resist a thrust, by having sufficient lateral stiffness. When a piece acts alternately as a strut and as a tie, it must have sufficient total sectional area, and sufficient stiffness, to resist the greatest thrust that can act along it; and at the same time its effective sectional area, deducting rivet-holes, must be sufficient to resist the greatest tension which can act along it. As to the figures, construction, and mode of connection, of iron ties and struts, see Articles 364, 365, and 366, pp. 518 to 524; and as to the fastenings by which they are connected together, see Articles 360 to 362, pp. 515 to 517.

II. The *Trapezoidal Truss*, already treated of in the preceding Article, p. 549, and represented in fig. 253, was used on an enormously large scale by the second Brunel in the railway viaduct over the Wye at Chepstow, the largest span of which, of about 300 feet, is crossed by two parallel and similar girders, of the following construction:—The horizontal strut B B B is a cylindrical plate iron tube 9 feet in diameter, and $\frac{5}{8}$ths of an inch thick, stiffened by transverse circular partitions or "diaphragms" at intervals. It is supported at the ends upon cast iron saddles, resting on cast iron pillars. The effective depth of the truss, denoted by k in the formulæ, is about 50 feet. Each of the principal ties, A, F, C, and of the diagonal braces, G, G, consists of a pair of flat-linked chains, attached to the sides of the tube, and sufficiently far apart at the level of the bottom of the truss to leave room for the traffic on a line of rails between them. The vertical struts K, K, are rectangular frames with openings sufficient for the same traffic. The points 1 and 4 form the intermediate supports of a pair of ordinary plate iron girders, whose ends rest on the piers; so that for those girders the span of 305 feet is subdivided by the aid of the truss into three spans; the central span being 124 feet, and the side spans each 90½ feet. Below the four plate girders, thus hung from the two great trusses, are attached the cross joists of the roadway. The two tubes are braced together horizontally to increase their lateral stiffness.

The total fixed load of this structure *for one line of rails* is about 1½ tons per lineal foot, or 3,360 lbs. The greatest rolling load may be taken, as is usual in railway bridges, at one ton per foot, or 2,240 lbs.

The following are the proportions per cent. in which the fixed load is distributed:—

Effective section of tube, ...	20	per cent.
Other parts and appendages of tube,...	15	„
Main chains, ...	23	„
Diagonal chains, ...	5	„
Upright frames, saddles, &c., ...	9	„
Plate girders, joists, and roadway,......	28	„
	100	„

With the fixed and rolling loads above-mentioned, the thrust along the tube is about 3,600 lbs. per square inch of section; so that the factor of safety is considerably more than six for both fixed and rolling loads.

III. *Warren or Zig-zag Girders.*—In the earlier examples of these girders, the upper horizontal strut or boom was a tube, cylindrical inside, and on the outside resembling a cylinder with four flat projections above, below, and at each side, for the convenience of attaching the diagonals to it. The strut braces were of cast iron, and cross-shaped. In later examples the upper boom and strut braces are made of wrought iron; the upper boom being either like a trough-shaped girder built of flat bars and angle iron, with the flanges downwards, or like a box-beam, and the strut-braces H-shaped, or cross-shaped, as shown in fig. 235, p. 521. The main tie or lower boom, and the tie-braces, consist of flat links set on edge; as to which, see Article 364, Divisions II. and III., pp. 518, 519. The joints of the lower boom, and its connections with the braces, are made by means of large cylindrical pins. Such pins also connect the braces with the upper boom, whose side plates or bars form a channel into which the ends of the braces enter and fit. The joints of the horizontal tie may also be made by fishing and rivetting. The whole structure has the advantage of being easily carried in pieces to its intended site, and there put together.

If the platform is hung below the girders, lateral stability is to be given to them by making the vertical suspending-pieces, whereby the cross joists are hung from the higher joints of the girders, of an I-shaped form of section, and equal, or nearly equal in stiffness, to the platform joists. When the platform is supported above the girder, lateral stiffness is to be given by the horizontal diagonal bracing of the platform, and also by vertical transverse diagonal bracing between the girders; and for this purpose rods of from 1 inch to 1½ inch in diameter are in general sufficient. (For details of various Warren girders, see Humber *On Iron Bridges.*)

From the manner in which the parts of a zig-zag girder are connected together, it is evident that its diagonal strut-braces, and the several divisions of its horizontal boom, are to be treated as *struts hinged at the ends.* (See Article 366, p. 523.)

IV. *Lattice Girders.*—The forms and modes of construction applicable to the upper and lower booms of the Warren girder are also applicable to those of the lattice girder. The diagonal strut-braces are made of any convenient shape that is well suited to resist thrust; their greatest breadth should be placed *transversely*, because in the longitudinal plane of the girder, they are stiffened by being bolted or rivetted to the tie-braces at each intersection. The holes made for that purpose weaken the tie-braces, and are to be allowed for in computing their strength. In the Boyne Viaduct, the strut diagonals of the lattice girders are themselves formed like small lattice beams, consisting of a pair of T-iron ribs connected together by small diagonal braces.

379. **Iron Bowstring Girders.**—The most common kind of iron bowstring girder (fig. 256) consists of a cast or wrought iron arch or bow, springing from two shoes or sockets, which are tied

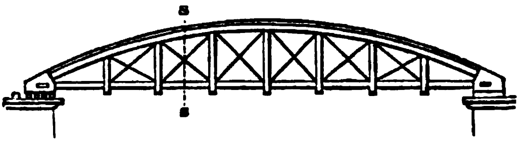

Fig. 256.

together by a horizontal tie; the cross joists of the platform are suspended from the arch by vertical suspending-pieces, which at the same time support the weight of the tie; and stiffness to resist a rolling load is given by means of diagonal tie-braces.

The proper figure for the centre line or neutral curve of the bow is a parabola; but a circular segment is often used in practice. The cross-section of the bow, like that of the upper boom of a lattice girder, must be of a form suited for resisting thrust. A cylindrical tube is the strongest form; an inverted trough-shape, either cast, or built of plates and angle bars, is convenient for the attachment of the suspending-pieces. These have usually an I-shaped section, with the greatest breadth transverse, to give them lateral stability; and for the same purpose they widen towards the bottom, where they are rivetted to the ends of the plate or box beams that form the cross joists. The main tie is best made of parallel flat bars on edge, and is made fast to the shoes at each end by gibs and cotters; the diagonal braces are round or flat rods. The stress on each part is found as follows:—

Case I.—The girder stiffened by diagonal braces.

Let l be the span, measured along the centre line of the main tie, between the ends of the centre line of the bow;

k, the rise from centre line to centre line;

$$\left.\begin{array}{l} w\text{, the fixed load, and} \\ w'\text{, the rolling load,} \end{array}\right\}\text{ per unit of length of span.}$$

Then the tension of the main tie, and the horizontal thrust at the crown of the bow, are given very nearly by the formula

$$H = (w + w')\, l^2 \div 8\, k. \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

The thrust at any other point of the arch varies nearly as the secant of its inclination; or, to express it in symbols, let x be the horizontal distance of the point in question from the middle of the span; then the thrust is,

$$\sqrt{\left\{H^2 + (w + w')^2\, x^2\right\}}. \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

At the springing, for x put $l \div 2$.

Let N be the number of parts into which the vertical pieces divide the span, so that there are N − 1 of those pieces; the greatest tension on any one of them is nearly

$$\frac{(w'' + w')\, l}{N}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

w'' being the fixed or dead load per unit of length, exclusive of the weight of the bow.

It is possible that when the girder is partially loaded with a travelling load, some of the upright pieces, which, with a uniform load, act as ties, may be made to act as struts. To find whether this is the case, number the uprights from one end of the girder;

let n be the number of any given upright; compute the value of the expression

$$\frac{l}{\mathrm{N}}\left(w' \cdot \frac{n\,(n+1)}{2\,\mathrm{N}} - w''\right); \ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

if this is positive, it gives the greatest thrust on the upright in question; if null or negative, it shows that the upright never acts as a strut.

The greatest tension produced by a rolling load on any given diagonal brace is given by the expression,

$$\frac{w'\,l\,s}{y} \cdot \frac{n\,(n+1)}{2\,\mathrm{N}^2}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

where s is the length of the given brace, y the difference of level of its ends, and n and $n+1$ the numbers of the uprights between which it is placed.

Case II.—The girder without diagonal braces. In this case the action of a rolling load must be resisted by the stiffness of the bow. The greatest stresses on the tie and on the suspending-pieces are given by the expressions (1) and (3) respectively; but the bow becomes virtually *an arched rib hinged at the ends*, as to which see Article 374, Case III., p. 540, and Article 180, Problem VI., pp. 310 to 312. In computing the quantity C by equation 51 of the last-mentioned Article, p. 310, the effect of change of temperature is *not* to be considered, because the bow and tie expand equally; and the term denoted by $a\,\mathrm{E}\,\mathrm{A}_1 \div l$ in that equation is to be replaced by $u_a \div u_b$, denoting the ratio which the greatest safe shortening of the bow bears to the greatest safe lengthening of the tie. If they are both of wrought iron, this may be assumed as approximately $= \frac{1}{2}$.

Case III.—*Bowstring Suspension Bridge.*—(Fig. 257.) In this class of bridge, of which the greatest example is that erected by the second Brunel over the Tamar at Saltash, the tie hangs in a catenary curve, and assists the bow in supporting the vertical pieces. The bow is a wrought iron oval tube stiffened by transverse diaphragms; the tie consists of a pair of chains.

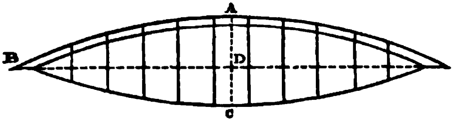

Fig. 257.

In fixing the proportions of a bridge of this kind, it is advisable to make the horizontal thrust due to the weight of the bow balance the horizontal tension due to the weight of the tie, independently of any additional load; and for that purpose, the common horizontal chord of the two arcs should divide the greatest vertical distance between their lines of resistance, A C, at the point D, into segments proportional to their weights; that is to say,

$$\begin{array}{ccccc} \text{Weight of bow} & : & \text{weight of tie} & : & \text{sum of weights} \\ :: \quad AD & : & DC & : & AC = k. \end{array}$$

The formulæ applicable to this case are the same with those for Case I. or Case II., according as the girder is enabled to resist a travelling load by means of diagonal braces, or by the stiffness of the bow alone.

380. **Braced Iron Arches.**—This term is applied to arches in which the arched rib and horizontal rib are so connected together by zig-zag braces (as in fig. 258), or by lattice work, in the intervening spandril, that each half-arch, together with its spandril, forms one stiff frame or truss. The best examples of this kind of arch are made of wrought iron; amongst them may be mentioned, the railway bridge over the Theiss at Szegedin, by M. Cezanne (*Annales des Ponts et Chaussées*, 1859), which consists of eight arches of 41·418 mètres in span (135·88 feet), and the bridge of the Paris and Creil railway over the Canal Saint-Denis, by M. M. Salle and Mantion (*Annales des Ponts et Chaussées*, 1861), consisting of a single arch of dimensions which will presently be stated. Each of those structures consists of four parallel frames, one under each rail; each frame consists of a curved rib, a straight horizontal rib, and a zig-zag series of braces, alternately vertical and sloping; these pieces are built of plate and bar iron, rivetted together so that the whole frame acts like one piece. In the Theiss bridge, all the pieces are I-shaped in section, consisting of a middle web with flanges or tables connected to it by angle irons and rivets; in the Paris and Creil railway bridge, the horizontal rib is I-shaped, the braces are cross-shaped, and the curved rib consists of a vertical web, with four Barlow rails rivetted to it, two on each side; and for about one-eighth of the span on each side of the crown, the horizontal rib and curved rib have no opening between them, so that the same vertical web serves for both. The four parallel frames of the arch are stiffened transversely by two sets of T-shaped

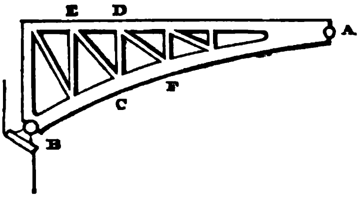

Fig. 258.

diagonal stays, one connecting the horizontal ribs together, and the other the curved ribs.

It has already been stated in Article 374, p. 540, that M. Mantion proposes, as the best mode of constructing an iron arch, to have hinges at the crown and at the springing, as at A and B, fig. 258. The arches of the Paris and Creil railway bridge are hinged at the springing, but continuous at the crown; those of the Theiss bridge are continuous at the crown, and have flat abutting surfaces at the springing; nevertheless, from the smallness of those surfaces as compared with the other dimensions of the arch, it is probable that the arches of this bridge also act nearly as if they were hinged at the springing.

The following are some of the principal dimensions of the Paris and Creil railway bridge:—

The length of each semi-arch is divided into ten equal divisions horizontally; there are in each spandril eight vertical and six diagonal braces; for two divisions and a-half adjoining the crown there are no braces, the curved and straight ribs having one web in common.

	Mètres.	Feet.
Span between axis of bearings,	44·846	147·14
Rise,	4·85	15·91
		Inches.
Depth of curved rib (=span ÷ 66 nearly),	0·680	26·77
" of straight rib,	0·300	11·81
" of combined rib at crown,	0·705	27·76
" of braces, four longest at each end,	0·200	7·87
" of braces, remainder,	0·150	5·91
Breadth, of T-shaped transverse braces of curved ribs,	0·150	5·91
Depth,	0·080	3·15
Breadth, of T-shaped transverse braces of straight ribs,	0·125	4·92
Depth,	0·060	2·36
Length, of semi-cylindrical bearings for ends of curved ribs,	0·302	11·89
Diameter,	0·200	7·87
Length, of cast iron abutting-plate, which carries semicylindrical bearing,	1·40	53·11
Breadth,	1·00	39·37
Mean thickness,	about ·080	about 3·15

Areas of Cross-section.		Square Millimetres.	Square Inches.
Combined rib at crown,		59,400	92·07
Curved rib in five divisions adjoining the crown,	from	77,650	120·36
	to	42,050	75·18
Curved rib—Remainder,		35,950	55·72
Horizontal rib,	from	23,200	35·96
	to	9,700	15·04
Braces,	from	7,482	11·60
	to	11,505	17·83
Transverse diagonal stays,	from	2,760	4·28
	to	1,665	2·58

The horizontal ribs perform the duty of beams in supporting cross joists; for besides the joists that lie directly above the uprights of the spandril, there are two intermediate joists at equal distances resting on the horizontal ribs in each space between a pair of uprights.

The following is the load of the structure:—

	Total for Double Line. Kilogrammes.		Total for Double Line. Lbs.		Lbs. per Lineal Foot of Single Line.	
Wrought iron framework,.........		125,000		275,578		936
Dead load exclusive of iron frame, viz.:—						
Timber platform, ...	45,000		99,208		337	
Ballast,..............................	45,125		99,483		338	
Rails,..................................	6,875	97,000	15,157	213,848	52	727
Total dead load, ...		222,000		489,426		1,663
Greatest working live load, estimated at 4,000 Kilogrammes per mètre of single line, on 90 mètres,............		360,000		793,663		2,697
Total greatest working load,......		582,000		1,283,089		4,360

The following are the methods of computing the stresses on the several pieces of a braced iron arch:—

For the uprights and sloping braces, use the same rules as for the suspending-rods and diagonal braces of a bowstring girder. (See Article 379, p. 563). For the arch proceed as follows:—

Case I.—*When the arch is hinged both at crown and springing,* the most severe stress on the arc and on the horizontal rib are determined as follows, with an approximation sufficient for practical purposes (see fig. 259):—

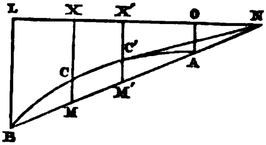

Fig. 259.

Let w be the dead load per lineal foot;
w', the live load per lineal foot;
c, the *half* span, } of the centre line of the arched rib in
k, the rise, } feet.

Then the horizontal thrust due to the dead load is,

$$H_0 = \frac{w\,c^2}{2\,k}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

and to a live load over the whole span,

$$H_1 = \frac{w' c^2}{2 k}; \text{.........................} (2.)$$

and if this be the most severe way of loading the arch, the required sectional area at any given point of the arched rib, where its inclination is i, will be,

$$A = \frac{(H_0 + H_1) \sec i}{f}; \text{.......................} (3.)$$

f being the *safe working* thrust on the material; or say about 6,000 lbs. per square inch.

To ascertain the effect upon the curved and straight ribs, of loading one-half of the arch with the live load, and leaving the rest unloaded, either a geometrical or an algebraical method may be followed. For the geometrical method, let A B, fig. 259, be the centre line of the curved rib, O L that of the straight rib; join A B with a straight chord. Let X C M be any vertical ordinate. Then the stress along the horizontal rib at X is,

$$\frac{H_1 \cdot M C}{2 C X}; \text{..........................} (4.)$$

and this is tension when X is in the unloaded half of the span, and thrust when it is in the loaded half.

The horizontal component of the greatest stress arising from a rolling load on half the span, at the point C in the arched rib, occurs when C is in the unloaded half of the rib, and is as follows:—

$$\frac{H_1 \cdot M X}{2 C X}; \text{..........................} (5.)$$

and should this prove *greater* than H_1, that is to say, should M X be greater than 2 C X, the expression (5) is to be substituted for H_1 in equation 3; but should M X be not greater than 2 C X, equation 3 is to be left unaltered.

To find the point of *greatest* horizontal stress in the unloaded half of the beam, produce the straight lines L O, B A, till they meet in N, from which draw N C′ touching the curve A C B; C′ will be the point sought.

The algebraical formulæ for the expressions (4) and (5) are as follows:—

Let O A $= a$; O X $= x$; then,

$$\frac{H_1 \cdot M C}{2 C X} = \frac{H_1 (k c x - k x^2)}{2 (a c^2 + k x^2)}; \text{..............} (4 A.)$$

$$\frac{H_1 \cdot MX}{2\,CX} = \frac{H_1 (a\,c^2 + k\,c\,x)}{2\,(a\,c^2 + k\,x^2)}. \quad \ldots\ldots\ldots\ldots (5\text{ A.})$$

The value of x which makes the last expression a maximum is given by the equation

$$OX' = x' = c \left\{ \sqrt{\frac{a}{k} + \frac{a^2}{k^2}} - \frac{a}{k} \right\}. \quad \ldots\ldots\ldots\ldots (6.)$$

It is to be observed that the processes expressed by the formulæ 4, 5, 4 A, 5 A, 6, are applicable only to the *openwork* parts of the frame. Where the horizontal rib and the arched rib are connected by a web, so as to form virtually *one rib*, that rib is to be conceived to be under the combined action of the thrust $H_0 + \frac{H_1}{2}$, and the bending moment

$$M' = \frac{H_1 \cdot MC}{2} = \frac{H_1\,k\,(c\,x - x^2)}{2\,c^2}. \quad \ldots\ldots\ldots\ldots (7.)$$

Let h be the depth of the compound rib, and q a co-efficient depending on its form of section, as given in pp. 294, 295. Then its sectional area is given by the equation

$$\left.\begin{aligned} A' = \frac{1}{f'}\left(\frac{M'}{q\,h} + \frac{H_1}{2} + H_0\right) = \frac{H_1}{2\,f'}\left(\frac{k\,(c\,x - x^2)}{q\,h\,c^2} + 1\right) \\ + \frac{H_0}{f'}; \end{aligned}\right\} (8.)$$

and *if this area is greater than that given by equation* 3, it is to be adopted.

Case II.—*When the rib is continuous at the crown*, the *exact* determination of the state of stress at different points becomes a problem of almost impracticable complexity; but an approximate solution, sufficient to determine what sectional area is required at and near the crown, in order to resist the straining effects of deflection, yielding of the piers, and changes of temperature, may be obtained as follows:—

Compute a series of values of the expression $q\,m'\,h^2$, as explained in Article 180, p. 302, equation 17, for a series of equidistant cross-sections of the entire iron frame, and use the *mean* of all those values to compute the quantity C by the following formula:—

$$C = \frac{15\,q\,m'\,h^2}{8\,k^2}\left(1 + \frac{4\,k^2}{3\,c^2} + \frac{2\,a\,E\,A_1}{c} \mp \frac{e\,t\,E}{p_0}\right); \ldots (9.)$$

in which a is the enlargement of span due to yielding of the piers per lb. of thrust; A_1, an assumed approximate sectional area of the curved rib; E, the modulus of elasticity; $\pm t$, the extreme $\left\{\begin{matrix}\text{rise}\\ \text{fall}\end{matrix}\right\}$ of temperature; e, the co-efficient of expansion per degree (see p. 539); p_0, the intended mean intensity of thrust at the crown.

Then calculate a moment of flexure as follows:—

$$M'' = \frac{(w + w')\,c^2}{2} \cdot \frac{C}{1 + C} = (H_0 + H_1)\frac{C\,k}{1 + C}; \quad (10.)$$

let h_0 be the depth of the rib *at the crown*, and q_0 the value of q for the same point; then the corrected sectional area at the crown will be,

$$\left.\begin{aligned} A = \frac{1}{f}\left(\frac{M''}{q_0\,h_0} + H_0 + H_1\right) = \frac{H_0 + H_1}{f} \\ \left(\frac{C\,k}{(1 + C)\,q_0\,h_0} + 1\right). \end{aligned}\right\} \quad \ldots\ldots(11.)$$

When the horizontal rib of a braced iron arch acts also as a beam, the sectional area required to resist at once the direct stress and the bending action is to be computed according to the principle of Article 374, Case III., equation 2, p. 540.

381. **Iron Piers.**—An iron pier for supporting arches or girders may consist of any convenient number of hollow cylindrical pillars, either vertical or raking, each pillar being made of pieces of a convenient length, turned or planed at the ends, and united by a projection and socket, and also by flanges or lugs and bolts, as explained in Article 365, p. 521, and the several pillars being connected together by horizontal and diagonal braces. For the method of determining the stress on each pillar and brace, see Article 348, pp. 484, 485. Each *length* of a pillar between a pair of braced points may be considered as a strut *hinged at the ends*, and its strength computed accordingly. (See Article 365, p. 521).

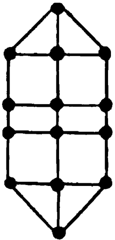

Fig. 260.

As an example of piers constructed in this manner, may be taken those of the Crumlin Viaduct (fig. 228, p. 494), in which the greatest height of the rails above the valley is about 200 feet. Each pier consists of fourteen cast iron columns, in lengths of 17 feet, with an uniform external diameter of 12 inches, and a thickness of metal ranging from one inch at the base to $\frac{5}{8}$ inch at the top. The two centre columns are vertical; the remainder rake in such a manner that while the base of the highest pier measures 60 feet by 27, the top of each pier measures 30 by 18. The longitudinal and transverse horizontal braces are cast iron beams, I-shaped in section, and 12 inches deep; their flanges are 5 inches broad. The diagonal braces in vertical and raking planes are flat bars measuring 4 inches by ¾ inch; there are also horizontal diagonal braces, which are round rods of 2 inches diameter. Each column has a foot or base from 3 feet to 5 feet high, spreading to 3 feet square, and resting on a foundation of masonry to which it is bolted and joggled. (See Humber *On Iron Bridges.*)

Wrought iron struts of suitable figures may be used instead of cast iron pillars in the construction of piers; and, like them, they are to be considered as hinged at the points which are fixed by the bracing. (See Article 566, p. 521.)

In some cases a pier is made of a single row of hollow cylindrical cast iron pillars, or even of a single such pillar; in which case the greatest intensity of tension and of thrust are to be computed as follows:—Let P be the vertical load of one pillar; H, the horizontal thrust applied to it, at a height of Y above its base, or above the horizontal section at which the stress is to be calculated; d, the mean between the external and internal diameters of the pillar; A, its sectional area (= 3·1416 d × thickness of metal); then

$$\text{greatest intensity of} \left\{ \begin{matrix} \text{thrust} \\ \text{tension} \end{matrix} \right\} = \frac{1}{\mathrm{A}} \left(\frac{4\,\mathrm{H}\,\mathrm{Y}}{d} \pm \mathrm{P} \right) \text{ nearly. (1.)}$$

Cases in which the bending moment arising from the thrust differs from H Y will be considered further on.

When a pillar simply rests on a firm base, without being imbedded in the soil like a pile, it is advisable so to proportion it that there shall be no tension at any point of its base; and for that purpose the diameter at the base should not be less than that given by the following formula:—

$$d = \frac{4\,\mathrm{H}\,\mathrm{Y}}{\mathrm{P}}. \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.)}$$

As examples of piers of this class, may be taken those used for the bridges of the Bombay and Baroda railway, by Lieutenant-Colonel Kennedy (see *Civil Engineer and Architects' Journal*, September, 1861), each consisting of three hollow cylindrical vertical cast iron pillars, connected together by horizontal and diagonal braces, with the addition, in powerful currents, of a pair of raking struts of the same dimensions and construction with the pillars, making angles of 30° with the vertical. The pillars are cast in lengths of 9 feet, and are 2 feet 6 inches in external diameter, and 1 inch thick; the lengths are connected together by flanges and bolts. For the part above ground the flanges are external, and have each 12 bolts of 1 inch diameter; for the part below ground, they are internal, and have each 10 bolts; and the diameter above-mentioned has been adopted as the least which will easily admit of a workman's going inside to fasten the bolts of the internal flanges. In foundations in earth the lowest length forms a screw-pile, with a screw 4 feet 6 inches in diameter, by means of which the pillar is screwed from 20 to 45 feet into the ground according to the softness of the material. Further mention of such piles will be made in a subsequent chapter, under the head of "Timber and Iron Foundations." When the ground consists of rock, each pillar is inserted into a cylindrical hole about 2 feet deep, and fixed there with cement. The three vertical pillars stand at distances of 14 feet from centre to centre. The horizontal braces are of T-iron, of a sectional area between 5 and 6 square inches; the diagonal braces are of angle iron, of a sectional area between 3 and 4 inches; each brace is fastened to lugs on the pillars, and tightened at one end by a gib and cotter.

The piers just described have lateral stiffness sufficient to withstand the pressure of a current, or of bodies borne along with it; but they are not adapted to bear the thrust of an arch, unless it be one of very small size. The superstructure of the bridges in which they are used consists of Warren girders.

In some lately erected bridges, the cast iron columns which form the piers are cylinders of 7 feet, 9 feet, 10 feet, and upwards, in diameter, and from 1 to 2 inches thick, filled with concrete or with rubble masonry. The mode of sinking such cylinders will be described under the head of "Timber and Iron Foundations." They are capable of withstanding a considerable thrust from an arch.

For example, in the Theiss bridge, mentioned in Article 380, p. 565, each pier consists of two cylinders, side by side:—

The diameter of each cylinder is 3 mètres, or	9·843 feet.
The thickness,............................ about	1·38 inch.
The depth of the springing of the arches below the centre line of the horizontal ribs, ...	$Y' = 18·93$ feet.
The height of the springing of the arches above the base of the pier, ...	$Y = 65·4$ feet.
The greatest thrust against a column occurs when one of the arches springing from it is fully loaded, and the other unloaded; in this case the vertical load *on one column* is, ...	$P = 368,000$ lbs.
And the excess of the thrust of the loaded over that of the unloaded arch,	$H = 361,500$ lbs.

M. Cezanne, in his account of this bridge before referred to, considers the column as a *vertical beam*, acted upon by the pressure H at the springing of the arch, which is resisted by the thrust of the *horizontal rib of the unloaded arch* at the top of the column, and by that of the foundation at its base, so that the bending moment, instead of being H Y, as in equation 1 of this Article, is

$$\frac{\mathrm{H\,Y\,Y'}}{\mathrm{Y+Y'}};\dots\dots\dots\dots(3.)$$

and the greatest

$$\left.\begin{matrix}\text{thrust}\\ \text{tension}\end{matrix}\right\}=\frac{1}{\mathrm{A}}\left(\frac{4\,\mathrm{H\,Y\,Y'}}{(\mathrm{Y+Y'})\,d}\pm\mathrm{P}\right).\dots\dots\dots(4.)$$

According to these principles, the greatest intensities of the stress in the cylinders of the Theiss bridge are,

Thrust, about 4,300 lbs. per square inch.
Tension, about 730 „ „

382. **Suspension Bridges.**—I. *Figure, Weight, Arrangement, and Loading of Chains or Cables.*—The whole theory of the action of an uniformly distributed load on a suspension bridge, when the suspending-rods are vertical, has been given in Article 125, pp. 188 to 191, and when the suspending-rods are oblique, in Article 126, pp. 191 to 194.

It is advisable to make the factor of safety for the fixed load *three*, and that for the rolling load *six;* but in many actual suspension bridges the factors are much less.

When, for reasons of practical convenience, each chain is made of uniform sectional area, that area must be proportioned to the greatest pull; that is to say, to the pull at the points of support (or at the highest point of support, if their heights are unequal); but a saving may be made, both of load and of material, by making the sectional area of the chain at different points vary as the pull; that is to say, as the secant of the angle of inclination of the chain. The weights of sections of the chain, extending over *equal horizontal distances*, will in this case vary as the *squares* of the secants of their angles of declivity.

The following formulæ show both the absolute and comparative weights of chains of uniform section and of uniform strength, to a degree of approximation sufficient for practical purposes:—

Let x be the half-span of the chain; y, its depression, both in feet; the ordinary proportions of x to y range from $4\frac{1}{2} : 1$ to $7\frac{1}{2} : 1$.

Let C be the weight of a chain of the length x, and of a cross-section sufficient to bear safely the greatest working horizontal tension H.

C', the weight of a *half-span* of the chain of *uniform section*.

C", the weight of a *half-span* of the chain of *uniform strength*; then,

$$C' = C \cdot \left(1 + \frac{8}{3}\frac{y^2}{x^2}\right) \text{ nearly. } \ldots\ldots\ldots\ldots\ldots (1.)$$

$$C'' = C \cdot \left(1 + \frac{4}{3}\frac{y^2}{x^2}\right). \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

The error of the first formula is in excess, and does not exceed 1-3000th part in any case of common occurrence in practice.

The value of C in the above formulæ may be taken as follows:—

$$\text{For wire cables of the best kind, } C = \frac{H\,x}{4500}; \ldots\ldots (3.)$$

$$\text{For cable-iron links, } C = \frac{H\,x}{3000}; \ldots\ldots\ldots\ldots (4.)$$

it being understood that the last formula gives the *net* weight only; in other words, the weight exclusive of the additional material in the eyes and pins by which the links are connected together.

About *one-eighth* may be added to the net weight of the chains for eyes and fastenings.*

As to the structure and mode of connection of flat-linked chains and wire cables, see Article 364, Divisions III., V., pp. 519, 520.

The smallest number of chains or cables in a suspension bridge is two, one to support each side of the roadway. In other cases there are from two to four parallel sets of chains, each consisting of two or more chains in the same vertical plane. For example, in the Menai Bridge, there are sixteen chains, in four sets of four.

The equal distribution of the load amongst a set of chains which hang in one vertical plane may be effected in different ways; one being to distribute the suspension-rods equally amongst them. In order that this plan may be effective, all the chains should be of exactly equal span, depression, and dimensions, so that they may all be affected alike by changes of temperature and of load.

---

* A great improvement in the manufacture of bars for bridge chains, introduced by Messrs. Howard and Ravenhill, consists in a process of rolling them with enlarged heads on their ends, so that the eyes can be made without forging or welding.

Here may be mentioned the test applied by Mr. Page to the bars used for the chains of Chelsea Suspension Bridge (which test has been omitted from its proper place in Article 357). Each bar was subjected to a tension of the intensity of 13½ tons (or 30,240 lbs.) per square inch; and if, after the removal of the stress, the length of the bar was found to be *permanently* increased by more than 1-400th inch per foot (or 1-4800th), it was rejected. The ultimate tenacity of bars which withstood this test was found to be 31 tons, or 69,440 lbs. per square inch.

The method which insures the most accurately equal distribution of the load on two chains, is that used by I. K. Brunel in the Hungerford Bridge, and represented in fig. 261; A is a suspension-rod, hanging from the middle of a small wrought iron lever, B, of equal arms; the ends of that lever are hung by rods C, D, from the two chains E, F, each of which bears exactly half the load of the rod A.

In Chelsea Bridge the rods C and D are dispensed with; and the lever B becomes a sort of scalene triangle, whose two upper angles are supported, one by one, on the joint-pins of one chain, the other by a pin resting on the top of the other chain, while from its lowest angle hangs the rod A.

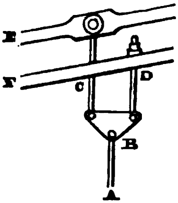

Fig. 261.

Each suspending-rod should have its length capable of adjustment, by means of a screw, arranged according to convenience.

The ordinary distance between the suspending-rods is from 5 to 12 feet; and each of them carries one end of a cross joist of the platform.

When a suspension bridge consists of several bays or spans, the chains of all of them must form portions of equal and similar parabolas (the parabola being considered a sufficiently close approximation to the true curve in which the chain hangs, as already explained in Article 128, pp. 197, 198).

II. *Platform.*—On this point see what has already been stated as to timber platforms in Article 336, pp. 465 to 468, and iron platforms, in Article 375, pp. 542 to 546.

The platform of a suspension bridge is usually cambered, or slightly arched upwards.

III. *Piers and Saddles.*—As to the properties of different methods of supporting the chains on the tops of the piers, see Article 125, Problem VI., p. 191. Unless the pier has considerable stability, the second construction there described, viz.:—That in which the chains are made fast by pins to a truck, supported on rollers on a level base or platform, is to be preferred, as insuring that the load on the pier shall be exactly vertical. From good practical examples it appears that the length of the platform on which the rollers rest may be about one-fiftieth part of the span of one bay of the chains. The truck should be of wrought iron.

In some cases *hinged cast iron piers* have been used, each of which has the chains made fast to its upper end, while, at its lower end, it is capable of turning through a small angle in a vertical plane, about a horizontal axis, so as to lean slightly inwards or outwards as the distribution of the load varies.

Mr. P. W. Barlow has proposed, in order to diminish or prevent the disfigurement of a suspension bridge of many bays, when one bay is loaded and the adjoining bays unloaded, that the ends of the chains of each bay should be made fast to the top of a wrought iron pier, constructed like a plate girder set on end, and having strength and stability sufficient to resist the excess of horizontal tension in the loaded bay above that in the unloaded bay.

Let w' denote the greatest *travelling* load per foot of span;

x, the half-span of a bay;
y, the depression of each chain;

then the excess of horizontal tension in question is

$$H' = \frac{w' x^2}{2 y}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.)$$

and this being multiplied by the depth of any given horizontal section of the pier below the point of attachment of the chains, gives the bending moment at that section. The vertical stress produced by that moment, compressive at one side of the pier and tensile at the other, is combined with the compressive vertical stress produced by the total load, whose amount is as follows:—

Let w be the fixed load, per foot of span;
W'', the weight of the pier itself, above the given horizontal section; then the load is

$$P = W'' + (2 w + w') x. \quad \ldots\ldots\ldots\ldots\ldots\ldots(6.)$$

As to the combined action of the load and bending moment, see Article 381, p. 571.

IV. *Abutments—Anchoring Chains.*—The term "Abutment" is applied to those masses, whether of masonry or of natural rock, to which the extreme ends of the chains are made fast, and by whose stability the tension of the chains is resisted. For example, in fig. 262 (which bears a general resemblance to an abutment of Hungerford Bridge), a pair of chains enter an opening in the abutment at A, in a direction nearly horizontal. At B their direction is changed to one more steeply inclined, by the aid of a saddle, which presses against the masonry in front of it. The

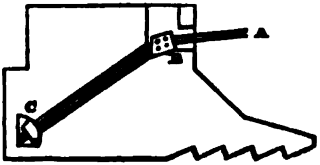

Fig. 262.

chains traverse a sloping tunnel or passage in the abutment, and finally pass through holes in the "anchoring plates" of cast iron at C, where they are fixed by keys or wedges; the anchoring plates press against a pair of transverse cast iron girders imbedded in the masonry.

Except that the tendency is to upset or to slide forwards instead of backwards, the principles of the stability of the abutment of a suspension bridge are precisely the same with those of the abutment of an arch; that is to say, the weight of the abutment must be sufficient to prevent it by friction from sliding on its base; its weight and thickness must be sufficient to prevent it from upsetting; and the centre of resistance of its base must not deviate from the centre of figure by more than a safe fraction of the thickness. As to ordinary foundations for such abutments, see Articles 235 to 239, pp. 377 to 382; as to the stability of the abutments, see Articles 263, 264, pp. 396 to 401. The resistance to sliding forward may be increased by making the base of the abutment, or part of it, slope so as to be perpendicular, or nearly so, to the resultant pressure, as in the front part of the abutment in fig. 262.

When piles are used in the foundation, they should be driven as nearly as possible in the direction of the resultant pressure. (See Section II. of the next chapter.)

The saddles by the aid of which the direction of a chain within its abutments is changed, do not require rollers, though they must be capable of sliding to an extent sufficient to admit of the expansion and contraction of the chain. This has been effected by making them rest on a bed about 4 or 5 inches thick, consisting of layers of asphalted felt.

As wire cables, from their great extent of surface, require more care in order to prevent them from rusting than bars, it is generally considered advisable that chains made of bars should always be used *within the abutments* of suspension bridges, although to the outer ends of such chains wire cables of equal strength may be attached. The cavities and passages containing these anchoring chains and their fastenings ought to be accessible for purposes of examination, painting, and repair.

V. *Oscillations and Means of Checking them.*—A suspension bridge consisting simply of abutments, piers, chains, vertical suspending-rods, and load, is free to oscillate both vertically and horizontally, the vertical oscillations consisting in a wave-like motion of the chains and platform. Every impulse applied to the bridge causes a series of oscillations of extent proportional to the impulse, which go on until they are gradually extinguished by friction; and the application of a series of impulses at intervals which are commensurable with the periodic time of oscillation of the bridge causes the extent and the consequent straining effect of the oscillations to go on continually increasing; so that a long series of successive impulses of very small amount, occurring at regular intervals, may be sufficient to endanger or destroy a very strong suspension bridge. Such is known to be the effect of the regular tread of soldiers in marching; and, therefore, when they approach a suspension bridge, they must be instructed to break into an irregular step.

Storms of wind cause oscillations, both vertical and horizontal, which have sometimes proved very destructive.

Although the oscillation of suspension bridges cannot be wholly prevented, it may be very efficiently checked by means of a system of oblique stays, which may either be external to the framework of the bridge, or be contained within it.

As an example of a mixed system of external and internal stays may be taken that of the Niagara Suspension Bridge. In it there are 120 stays, which may be described as "guy-ropes;" they are iron ropes, each of a sectional area which is about 1-200th part of the joint sectional area of the four main cables; some of them extend obliquely downwards from the saddles on the top of the piers to the platforms; others extend obliquely downwards and sideways from the lower platform to various points of the rocks on which the piers stand. The upper, or railway platform, and the lower, or road platform, constitute respectively the top and bottom of a tubular lattice girder 24 feet broad and 18 feet deep, with timber booms and uprights diagonally braced both horizontally and vertically with iron. (See fig. 265, p. 581.)

Every well-constructed suspension bridge has its platform stiffened horizontally by diagonal bracing; as to the action of which, see Article 336, p. 467. Vertical diagonal bracing is very generally used to give vertical stiffness: this will be considered more in detail further on.

In order to stiffen two suspension bridges in the Isle of Bourbon, the elder Brunel tied the platforms down to a set of inverted chains (called "counter-chains") whose total sectional area is about one-third of that of the main chains.

Suspension bridges with sloping rods are stiffer than those with vertical rods.

VI. *Bracing to resist a heavy Travelling Load.*—Various methods have been proposed, and partially tried, to enable a suspension bridge to resist the action of a heavy travelling load, such as a railway train, without undergoing more disfigurement than a girder. In order to make such methods effect their purpose completely in bridges of several bays, the chains must be made fast to piers of sufficient strength and stability, as described in p. 576.

(1.) *Auxiliary Girders.*—These are a pair of straight girders of any convenient construction (such as the plate, the zig-zag, or the lattice) hung from the chains by the suspending-rods, and supporting the cross joists of the platform. A sketch of an auxiliary girder is shown in fig. 263. It should be not merely *supported* at each end, but *fastened down*, as there are certain positions of the rolling load which tend to lift one of its ends. It should not, however, be *fixed in direction* there. In order to enable it to act with the greatest efficiency, it should be *hinged* at the middle of the span, which may be effected by making it in two halves, connected together by means of a cylindrical pin of dimensions sufficient to bear the shearing stress, which will presently be stated. The object of this is to annul the straining action which would otherwise arise from the deflection and expansion of the chain.

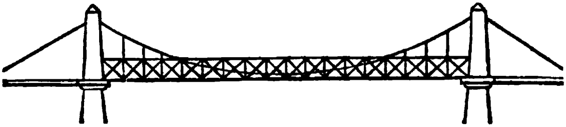

Fig. 263.

This precaution having been observed, the greatest bending action on the auxiliary girder will be that due to *half the rolling load*, upon a girder of *one-half of the span of the chain;* and the greatest shearing action, which will take place at the central pin, and at each point of support, will be equal to *one-eighth* of the rolling load over the whole span. That is to say, in symbols,

Let w' be the greatest rolling load per unit of span;
x, the *half-span;*
M, the moment of the greatest bending action on the auxiliary girder;
F, the greatest shearing force; then

$$M = \frac{w' x^2}{16}; \qquad (7.)$$

$$F = \frac{w' x}{4}. \qquad (8.)$$

Each half of the auxiliary girder is accordingly to be designed as if for a girder of the span x, under an uniformly distributed load of the intensity $w' \div 2$; regard being had to the fact that such load acts alternately upwards and downwards, so that each piece of the girder must be capable of acting alternately as a strut and as a tie, under equal and opposite stresses.

If the girder is not hinged, but continuous, at the middle of the span, it should be made capable of bearing a bending action whose moment is

$$M = \frac{w' x^2}{14}; \quad \text{……………………} (9.)$$

and not to go into unnecessary nicety of calculation, the cross-section capable of resisting that moment may be continued uniformly throughout the *middle half* of the stiffening girder.*

(2.) *By Diagonally-Braced Pairs of Chains.*—This system is represented in fig. 264. In order that the two chains may be affected alike by the expansive action of heat, their curvatures should be equal; in other words, their vertical distance apart should be the same throughout the whole span. If that vertical distance be made equal to *half the depression* of each chain, no additional material will be required in the chains beyond what is necessary to support a travelling load over the whole span. The diagonal braces should be capable of acting as struts and ties alternately, under stresses computed as for an auxiliary girder. Material would be saved by this mode of stiffening, as compared with the auxiliary girder; but it would probably be less efficient and durable, as the alteration of the curvature of the chains by heat and cold would tend to strain and loosen the joints of the braces.

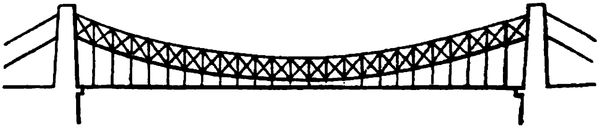

Fig. 264.

(3.) *By Diagonal Bracing between the Chains and Platform.*—This case is to be treated as a braced iron arch inverted, with the action of each force reversed; so that the formulæ of Article 380, pp. 567 to 570, and of Article 379, equations 3, 4, 5, pp. 563, 564, may be applied. This method of stiffening will not be efficient, unless the weight of the platform bears such a proportion to the rolling load as to prevent any suspending-rod from being subjected to thrust; and that such may be the case, the result of equation 4, p. 564, should be negative, or nothing, for each such rod. It is also open to the same objections with method (2.)

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* See, on this question, the *Civil Engineer and Architects' Journal* for November and December, 1860; also, *A Manual of Applied Mechanics*, second edition, p. 375.

(4.) *By Tension Ribs.*—Mr. E. A. Cowper has proposed to use, instead of flexible chains or cables, stiff wrought iron ribs, like inverted arches.

The theory of the action of the load on such "tension ribs" is precisely the same as in the case of ordinary arched ribs (see Article 374, p. 537), except that every force is reversed, tension being substituted for thrust, and thrust for tension (if any). In order to annul the straining action of the yielding of the piers, and of changes of central deflection and temperature, those ribs should be hinged at the middle and at the points of support; in which case all the formulæ of Article 374, Case IV., p. 541, become applicable to them, with the modification stated above.

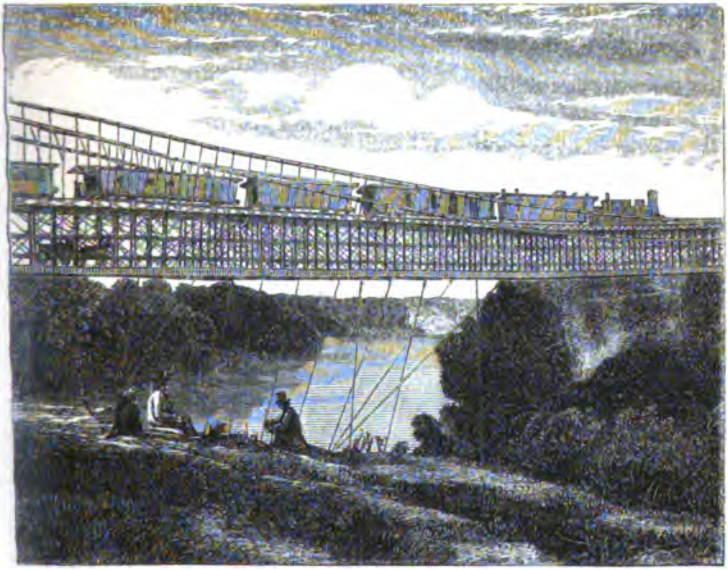

Fig. 265.—[Niagara Falls Bridge, from a Photograph.]

VII. *Table of the Principal Dimensions of some Suspension Bridges.*

Bridge and Engineer.	Spans of Chains. Feet.	Depression of Complete Spans. Feet.	Span ÷ Depression	Number of Chains or Cables.	Total Effective Sectional Area. Square Inches.	Mean Net Weight of Chain per Foot of Span. Lbs.	Gross Fixed Load per Foot of Span. Lbs.	Thickness of Piers at Level of Platform. Feet.	Thickness of Abutment at Level of Platform. Feet.	Breadth of Platform over all. Feet.
Union (round rods), (Brown.)	449	30	14·9	6	38	129	...	17·5	...	18
Menai (flat links),... (Telford.)	570	43	13·26	16	260	880	...	29	...	28
Chelsea (flat links), (Page.)	183 348 183	29	12	4	at centre. 214 at piers. 230	767	...	19 (cast iron.)	77	47
Pesth (flat links),.... (Tierney Clark.)	298 666 298	47·6	14	4	at centre. 486 at piers. 507	1690	9892	30	140	46
Bamberg (flat links),	211	14·14	14·9	4	40·2	137	1581	15·25	...	30·5
Freiburg (wire),...... (Chaley.)	870	63	13·84	4	49	167	760	20	...	21·25
Niagara Falls (wire), (Rœbling.)	821·3	upper cables. 54 lower cables. 64	15·21 12·83	4	241·6	820	2032	...	...	24
Niagara (wire),...... (Serrel.)	1040	75	13·87	10	42	142	1581	15·25	...	30·5

383. **Proportion of Weight to Load in Iron Bridges.**—In Article 167, p. 263, the general principles have been explained according to which the weight of a beam intended to carry a given load can be approximately determined before designing the beam. The examples there given, however, are applicable to simple beams only, in which every portion of the material directly contributes to the resistance to the bending and shearing action of the load. In large iron bridges there are many parts which do not directly contribute to that resistance, but which, being necessary for the connection or staying of the parts which do so, are essential parts of the structure; and they increase its weight in a proportion which ranges in various practical examples from once and a-half to double.

To deduce from practical examples a formula for computing the probable ratio which the weight of the superstructure of a bridge of a given design will bear to the external load, the following data, from an existing bridge of similar design, are required:—

l, the span in feet;
B, the gross weight of the superstructure, either in all or per foot of span;
s_2, the factor of safety applicable to that weight (say 3);
W′, the greatest working travelling load (either over all or per foot of span) consistent with a proper factor of safety s_1 (say 6). This is not to be taken from the *actual* travelling load, but computed as follows:—Let W be the calculated breaking load; then

$$W' = \frac{W - s_2 B}{s_1} \text{.} \qquad (1.)$$

From these data compute the following quantity:—

$$L = l \left(1 + \frac{s_1}{s_2} \frac{W'}{B}\right); \qquad (2.)$$

then, for any other bridge of similar design and proportions, the probable proportion of the weight of the superstructure to the greatest working travelling load is given by the formula,

$$\frac{B}{W'} = \frac{s_1}{s_2} \cdot \frac{l}{L - l} \qquad (3.)$$

If $s_1 = 6$ and $s_2 = 3$, these formulæ become as follows:—

$$L = l \left(1 + 2 \cdot \frac{W'}{B}\right); \qquad (4.)$$

$$\frac{B}{W'} = \frac{2l}{L - l} \qquad (5.)$$

The following are some examples of values of L:—

	L Feet.
For tubular bridges, not continuous; the depth about 1-16th of the span (as the Conway Bridge); the effective section two-thirds of the whole iron,	614
For tubular bridges, mean depth about 1-16th of the span, continuous over piers; l in the formulæ denoting the span of the greater or intermediate bays (as the Britannia Bridge),	760
Warren girder bridges, not continuous, with cast iron struts; depth about 1-15th of the span,	670
Warren girder bridges, not continuous, with the frame entirely of wrought iron; depth about 1-10th of the span,	900
Iron arched bridges; rise about 1-9th of the span,	630
Wire cable suspension bridge; the depression 1-14th of the span; the cables 4-10ths of the weight of the superstructure; ultimate tenacity of the wire 90,000 lbs. per square inch (as Niagara Falls Bridge),	2000

In designing railway bridges, W′ is in general assumed to be one ton, or 2,240 lbs. per lineal foot of a single line. For bridges not carrying railways, the most severe moving load may be assumed to be that of a closely packed crowd, as stated in Article 335, p. 466; that is, 120 lbs. per square foot of platform, so that in such cases,

$$W' = 120 \text{ lbs.} \times \text{breadth of platform in feet.}$$

For a bridge with two platforms, one carrying a road and the other a railway, those two loads are to be combined.

SECTION V.—*Of Various Metals and Alloys.*

384. **Lead** is used in engineering works as a covering for roofs (as to which, see Article 337, p. 468), as a material to fasten iron cramps into masonry, by filling up the cavities between them, and sometimes as a means of distributing the pressure on the beds of arch-stones (as to which, see Article 277, p. 414). As to its tenacity and heaviness, see the tables at the end of the volume. It melts at a temperature of about 630° Fahrenheit. When a fresh surface of

lead is exposed to air or water, it becomes coated in a short time with a thin grey film of oxide, which protects the metal against further oxidation, unless some acid be present capable of dissolving the oxide.

385. **Zinc** is used for covering roofs (see Article 337, p. 468), and also for coating pieces of iron to protect them against oxidation. (See Article 330, p. 462.) A fresh surface of zinc, when exposed to the air, becomes coated with a thin film of oxide, which protects the metal against further oxidation, unless an acid be present to dissolve the oxide. The coating with zinc, or "galvanizing," as it is called, of thin pieces of iron, such as sheets and wires, makes them more ductile, and a little less tenacious than before. It is effected by carefully cleansing the surface of the iron, and placing it in contact with a solution of a compound of oxide of zinc and potash; the negative pole of a galvanic battery is connected with the piece of iron, the positive pole with a plate of zinc immersed in the solution. Zinc melts at a temperature which is estimated to be about 700° Fahrenheit. At a temperature somewhat above a red heat it evaporates, and is then highly combustible.

386. **Tin—Alloys of Tin.**—Tin melts at 426° Fahrenheit. It resists oxidation better than any of the more common metals, except gold and silver. It enters readily into combination with iron; and it is by immersing well-cleansed sheets of iron in melted tin that "tin plate" or tinned iron is prepared, the iron being coated with a layer of an alloy of iron and tin, which passes gradually into pure tin at its outer surface. Although tin is very soft and ductile, most of its alloys with other metals are harder than either of the component metals.

387. **Copper.**—As to the tenacity of copper, which differs considerably according to the manner in which the metal has been treated, see the table at the end of the volume. It is diminished to about two-thirds by a temperature of 600° Fahrenheit.

Copper resists oxidation well, owing to the formation over its surface of a film of verdigris, or carbonate of copper, which protects the metal. This property, together with its great strength, makes it an useful material for fastenings of timber work and masonry in situations where iron would be rapidly oxidated, and where the cost of copper fastenings, being from six to eight times that of iron fastenings, can be afforded.

As to the use of sheet copper for covering roofs, see Article 337, p. 468.

388. **Bronze.**—Although the term "Brass" is popularly applied to all the alloys of copper, those in which it is combined with tin are more properly called *Bronze*. These compounds are harder than copper, to a degree increasing with the quantity of tin

which they contain, up to a proportion which gives the maximum of hardness.

In order that bronze may be of good quality, as regards accuracy of the figure of castings, soundness, and strength, a general principle, applicable to all alloys, should be observed in its composition,—the quantities of the ingredients should bear definite atomic proportions to each other. When this rule is not observed, the metal produced is not a homogeneous compound, but a mixture of two or more different compounds in irregular masses, shown by a mottled appearance of the castings when broken; and these masses being different in expansibility and elasticity, tend to separate from each other.

The following is a list of some of the principal alloys of copper and tin, in which the chemical equivalents of those metals are assumed to be respectively,

Copper,	31·5
Tin,	59

Composition.				
By Atoms.		By Weight.		
Copper.	Tin.	Copper.	Tin.	
6	1	189	59	Bell-metal: hard and brittle: contracts in cooling from its melting point, 1-63d. .
14	1	441	59	Hard bronze.
16	1	504	59	Bronze, or gun-metal: contracts in cooling from its melting point, 1-130th.
18	1	567	59	Softer bronze.

As the table of tenacity at the end shows, bronze, or gun-metal, is twice as tenacious as good ordinary cast iron, and as tenacious as copper in bolts, while at the same time it is harder than copper. It is much used in machinery. Lead is sometimes present in it as an adulteration, and is very injurious to its strength and durability.*

389. **Brass,** properly speaking, is the general name of the alloys of copper with zinc. They are weaker than copper or bronze, but are useful from their fusibility and ductility. The following is a table of the principal alloys of copper and zinc, in which the chemical equivalents assigned to those metals are,

Copper,	31·5
Zinc,	32·5.

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* For information as to the alloys of copper, tin, zinc, and lead, see a paper in the *Manchester Memoirs* for 1860, by Mr. Crace Calvert and Mr. Johnson.

Composition. By Atoms. Copper.	Composition. By Atoms. Zinc.	Composition. By Weight. Copper.	Composition. By Weight. Zinc.	
6	1	189	32·5	Hardened copper.
4	1	126	32·5	Malleable brass.
2	1	63	32·5	Ordinary brass: contracts in cooling from its melting point, 1-60th. Tenacity, see table at end of volume.
1	1	31·5	32·5	Prince Rupert's metal: very hard.

389 A. **Gold—Auriferous Steel.**—According to some recent experiments, steel is increased in toughness by being alloyed with a small quantity of gold.

(Addendum to Article 357, p. 512.)

389 B. **Strength of Wrought Iron and Steel,**—

Description of Material.	Tenacity in lbs. on the Square Inch.		Ultimate Extension in Fractions of Length.
Coleford Gun-metal—			
Weakest,	108970	F.*	·190
Strongest,	160540		·030
Mean of ten sorts,	137340		·072

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* For details, see the *Mechanics' Magazine* for the 11th October, 1861, or the *Engineer* of the same date.

CHAPTER VI.

OF VARIOUS UNDERGROUND AND SUBMERGED STRUCTURES.

SECTION I.—*Of Tunnels.*

390. **Tunnels in General.**—As tunnels, compared with open excavations, are an expensive and tedious class of works, and as they form inconvenient portions of a line of communication, the engineer should study to avoid the necessity for them as far as possible.

As to the setting out of tunnels, see Article 70, p. 114.

The nature of the strata through which a proposed tunnel is to pass should be carefully ascertained, not only by means of borings and shafts, but in some cases also by means of horizontal or nearly horizontal mines or *drifts*, along the intended course of the tunnel. Shafts and drifts will be further described in the ensuing articles.

The most favourable material for tunnelling is rock that is sound and durable without being very hard. Great hardness of the material increases the time and cost of tunnelling, but gives rise to no special difficulty. A worse class of materials are those which decay and soften by the action of air and moisture, as some clays do; and the worst are those which are constantly soft and saturated with water, such as quicksand and mud.

In choosing the site of a tunnel, regard should be had, not only to the nature of the material, and to the shortness and directness of the tunnel, but to the facility for getting access to its course at intermediate points by means of shafts and drifts.

The engineer should, as far as possible, avoid curved tunnels, especially those in which the curvature is so sharp or so extensive as to prevent daylight from being seen through from end to end.

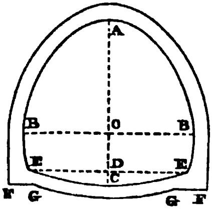

Fig. 266.

As to the figures of tunnels which require a lining of brickwork or masonry to prevent fragments of rock from falling from the roof, or to sustain the pressure of earth, and as to the strength and stability of that lining, see Article 297 A, pp. 433 to 435. Fig. 266 is an example of the elliptic form described in that article, with an inverted arch E C E at the floor. The parts F G, G F, of the base, which directly bear the side-walls and their load, are horizontal. O is the centre of the ellipse E B A B E, B B the minor axis, A O C about three-fourths of the major axis.

Tunnels made in rock that is so sound as not to require a lining of masonry or brickwork to prevent pieces of it from falling in, may be made, if the rock is igneous, of almost any shape that is most convenient for the traffic. The elliptical or horse-shoe form already described, is, however, generally adopted for the sides and top, the floor being level. In stratified rocks, the strongest form for the roof is that of a pointed arch; though a flat roof has been used where the rock consists of thick layers, and has few natural joints.

In ordinary tunnels, measured within the masonry or brickwork, the dimensions of most common occurrence are—

	Height.	Width.
For single lines of railway,	20 ft.	15 ft.
For double lines of railway,	24 ft.	from 24 ft. to 30 ft.
For navigable canals,......	from 14 ft. to 30 ft.,	from 14 ft. to 30 ft.

The *smallness* of tunnels for water-conduits and drains is limited by the least dimensions of the space in which miners can work efficiently; that is, about $4\frac{1}{2}$ feet high and 3 feet wide.

The best source of information on the construction of tunnels is Mr. F. W. Simms's work *On Practical Tunnelling.*

391. **Shafts or Pits.**—Shafts or pits are sunk for three purposes: to ascertain the nature of strata to be excavated, as already mentioned in Article 187, p. 331, when they are called *trial shafts;* to give access to a tunnel when in progress, for the purpose of carrying on the work, removing the material excavated, admitting fresh and discharging foul air, and pumping out water, when they are called *working shafts;* to admit light and fresh air at intervals to, and remove foul air from, a tunnel when completed, when they are called *permanent shafts.*

I. *Trial Shafts* are in general sunk at or near the centre line of the proposed tunnel. Their transverse dimensions are fixed mainly with a view to convenience in sinking them. Six feet is an ordinary diameter for a round trial shaft; six feet by four are ordinary dimensions for rectangular shafts. The shape is regulated by the material to be used in lining the shaft, being rectangular in timbered shafts, and cylindrical in those that are *steined* or lined with stone or brick.

The number and distance apart of trial shafts are to be determined after previous boring, in the same manner as for a deep cutting (Article 187, pp. 331, 332); that is to say, no general rule can be laid down on the subject; but the engineer must, to the best of his judgment, sink such shafts as are necessary in order to give him an accurate knowledge of the strata to be excavated.

II. *Working Shafts* may be either rectangular or round. Their usual transverse dimensions range from 6 feet to 9 feet; the greater diameter is advantageous, because of its admitting of large quantities of material being raised and lowered at a time. Their distance apart varies, in ordinary cases, from 50 to 300 yards. In some cases, however, it has been found necessary to place them as close as 20 or 30 yards apart, for the purpose of discharging foul air; while in other cases the height of the ridge to be tunnelled through has rendered the sinking of shafts impracticable for very long distances. An extreme example of the last case is the tunnel now in progress through Mont Cenis, which, when complete, will be eight miles long, and which must be excavated entirely from the two ends, without the aid of shafts.

The range of working shafts of a tunnel may lie either along its centre line, or in a line parallel to the centre line, at an uniform distance to one side. When the latter system is adopted, the object is to keep the shafts clear of the excavation and building of the tunnel, with which they are connected by cross drifts.

When a working shaft is to be used in order to drain the tunnel of water as the work proceeds, it is sunk to such a depth below the bottom of the excavation as to form a sufficient reservoir for water, called a "*sump*," from which the water is raised by a windlass and buckets, or by a pump. The most convenient form of bucket is one that is hung in a stirrup by a pair of trunnions whose axis nearly traverses the centre of gravity of the bucket. When lowered, the bucket is held upright by a catch; and after it has been raised, the removal of the catch allows it to be easily tilted over, in order to discharge the water.

III. *Permanent Shafts* are in general working shafts that have been made permanent parts of the structure, the brick lining of each being supported on a permanent *curb*, or suitably formed ring of brickwork, or of cast iron, surrounding a circular orifice in the roof of the tunnel. The top of each shaft is protected by being surrounded with a wall, and covered with a grating.

Permanent shafts are occasionally met with of a diameter as great as, or greater than, that of the tunnel. For example, the shafts at the ends of the Thames tunnel are 50 feet in diameter; the tunnel itself consisting of a pair of archways, each 14 feet in clear width, and the entire width of passages and brickwork being 37½ feet.

IV. *Sinking Shafts in sound rock* is performed simply by the operations of blasting and quarrying, as already described in Article 207, p. 344. In order to be safe from the effects of explosions, the workmen should ascend to a height of 50 or 60 feet above the bottom of the shaft (if it is so deep), before each blast is fired. The noxious fumes produced by the powder may be partially dispersed or absorbed by dashing in a bucket of water; but a more efficient plan of ventilation, especially in deep shafts, is either to extract the foul air through a sheet iron tube leading up to a furnace or to an exhausting fan, or to blow fresh air down by means of a fan through such a tube.

Ventilating apparatus is indispensable when foul air (such as carbonic acid gas, or "choke damp") or inflammable gas ("fire damp") is disengaged from the strata that are traversed by the shaft.

When water flows into the shaft, it is to be collected at the bottom in a "sump" or well of smaller diameter than the shaft, and raised by buckets, or by pumping, either to the surface of the ground or to some drift through which it can be discharged.

V. *Sinking Timbered Shafts.*—A shaft sunk through soft materials, or through loose rock, must be lined with timber, masonry, or brickwork.

The principal pieces in the timbering of a shaft, as well as in the timbering of drifts, tunnels, and underground excavations in general, may be distinguished into *props*, which are struts or posts, either vertical or raking, and usually of round timber; *sills* and *bars*, being horizontal pieces, sometimes round and sometimes squared; and *cleading* or *boards*. Props are combined with sills or bars into framework simply by abutting joints at their ends, which are made fast in their places by the aid of spikes called "*brobs*," of the shape shown in fig. 267, and usually about 6 inches long. Fig. 268 represents the foot of a prop resting on a sill, and made fast with four brobs, of which three are shown. The shape of the head of a brob enables it to be knocked out as easily as it is driven in.

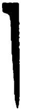
Fig. 267.

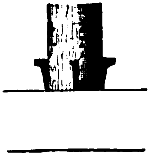
Fig. 268.

Fig. 269 is a section of a square-timbered shaft of about 9 feet square. The timbering consists of horizontal square frames or "*settings*," one at every six feet of depth or thereabouts, each made of four square sills of 12 inches × 12 inches, supported by round props of 8 or 9 inches diameter, and clad outside with vertical "*poling boards*" of 3 inch deal. The shaft having been sunk and timbered as far as the earth will stand for a time vertical, the further sinking is effected as follows:—In the centre of the bottom of the shaft a small pit is dug, at the bottom of which, at A, is laid a small platform of boards; then by cutting notches in the sides of the pit, "*raking props*," such as those shown by dotted lines, are inserted;

their lower ends abutting against a "*foot-block*" at A, and their upper ends against the lowest setting, so as to give it a temporary support. The pit is then enlarged to the dimensions of the shaft above; vertical poling boards are set up against its sides, with their upper ends behind the temporarily supported square setting, and their lower ends behind a new square setting, laid on the bottom of the excavation; vertical props are inserted between those settings, and made fast; the raking props and their foot-blocks are taken away; a new small pit is dug, and so on as before. Care should be taken that the earth presses firmly against the poling boards. Should streams of water come in through the chinks between the boards, the tendency of those streams to carry with them particles of sand, and so to leave cavities in the earth, may be counteracted by stuffing straw behind the boards.

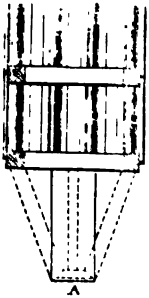

Fig. 269.

VI. *Sinking Stone or Brick-lined Shafts* (which are usually cylindrical) may be effected in two ways; by "*underpinning*," or by a "*drum-curb*."

To sink a shaft by *underpinning*, it is first dug as deep as the earth will stand vertical. At the bottom of the excavation is laid a "*curb*;" that is, a flat ring, whose internal diameter is equal to the intended clear diameter of the shaft, and its breadth equal to the thickness of the brickwork (usually 9 inches). It is made of oak or elm planks 3 or 4 inches thick, either in one layer fished at the joints with iron, or in two layers breaking joint, and spiked or screwed together. On this, to line the first division of the shaft, a cylinder of brickwork is built in hydraulic mortar or cement. In the centre of the floor is dug a small pit, as described in Division V. of this article, at the bottom of which a platform and foot-blocks support raking props, which are inserted to give temporary support to the curb with its load of brickwork; the pit is enlarged to the diameter of the shaft above; on the bottom of the excavation is laid a new curb, on which is built a new division of the brickwork, giving permanent support to the upper curb; the raking props and their foot-blocks are removed; a new pit is dug, and so on as before. Care should be taken that the earth is firmly packed behind the brickwork, and that the shaft is carried down truly vertical.

A *Drum Curb* (fig. 270), which may be made of timber or of cast iron, consists essentially of a flat ring for supporting the brickwork, and of a vertical hollow cylinder or drum, of the same outside diameter as the brickwork, supporting the ring on its upper edge, and bevelled to a sharp edge below. The drum may be strengthened if necessary by an additional ring, and its connection with the rings made more secure by brackets, as shown in the figure.

When the shaft has been sunk as far as the earth will stand vertical, the drum-curb is lowered into it, and the building of the brick cylinder commenced, care being taken to complete each course of bricks before laying another, in order that the curb may be equally loaded all round. The earth is dug away from the interior of the drum; and this, together with the gradually increasing load of brickwork, causes the sharp lower edge of the drum to sink into the earth; and thus the digging of the shaft at the bottom, the sinking of the drum-curb, and the brick lining which it carries, and the building of the brickwork at the top, go on together.

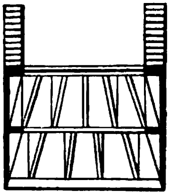

Fig. 270.

Great care must be taken so to regulate the digging that the shaft shall sink vertically.

Should the friction of the earth against the outside of the shaft at length become so great as to stop its descent, before the requisite depth is attained, a smaller shaft may be sunk in the interior of the first shaft. A shaft so stopped is said to be "earth-fast."

VII. *Temporary Support of Working Shafts.*—When a working shaft is sunk in the centre line of an intended tunnel, it is obvious that the completion of the excavation for the tunnel will remove the support from below the lining of the shaft, which support will only be replaced when the arching of the tunnel is completed.

There are two modes of giving temporary support to the shaft, from below and from above.

Support from below is given, if the ground is solid enough, by means of a pair of strong parallel sills, say 15 inches square, and 10 feet longer than the intended span of the tunnel. Each of these is sent down the shaft in three pieces, which are inserted into small horizontal drifts running at right angles to the line of tunnel, about 3 or 4 feet above its intended roof, and are there scarfed together. The drifts are then rammed up. The distance between the two sills is equal to the clear width of the shaft. They support a square frame, which supports the lowest curb of the part of the shaft to be carried.

Should the material be too soft to admit of this mode of support, the two sills (each of which may now be in one piece) are to be laid on the surface of the ground over the mouth of the shaft across the line of tunnel, and somewhat closer together than the width of the shaft. The lower end of the shaft is carried by a strong wooden frame, which is hung from the two sills by means of four wrought iron suspending-rods or chains.

The part of the shaft thus temporarily supported is generally lined with brick; the part below the temporary support is lined with timber, which is removed in the course of the excavation of the tunnel.

392. **Drifts, Mines, or Headings**, are small horizontal or inclined underground passages, made in order to explore the strata in the line of an intended tunnel, to drain off water, and to facilitate the ranging of the line and levels and setting out of the works (see Article 70, p. 114), the access of the workmen, and the transport of materials; and for the last-mentioned purpose they are often furnished with small temporary railways.

I. *Positions of Principal Headings.*—The working shafts of a tunnel are almost always connected together by means of a heading, which accordingly runs either along or parallel to the centre line of the tunnel. In some cases the heading runs along the centre line, while the working shafts lie at one side, and are connected with the main heading by cross headings.

When a tunnel runs through a steep hill, near or parallel to one of the sides of the hill, cross headings opening above ground at the hillside may be used instead of working shafts; but such cases seldom occur.

In tunnelling through soft and wet ground the most convenient level for the principal heading is at or near the bottom of the tunnel. In hard and dry materials it may be placed near the roof. Other positions will be mentioned farther on.

II. *The least Dimensions of a Heading* in which miners can conveniently work are about 3 feet broad and 4½ or 5 feet high.

III. *Headings in Solid Rock* are driven by blasting and quarrying, as to which, see Article 207, p. 344.

Machinery has been used for driving headings. The most remarkable is that employed at the tunnel now in progress through Mont Cenis (see p. 590). It consists of a number of horizontal jumpers, driven at the rate of about 200 blows per minute, by machinery which is moved by compressed air, conveyed into the mine through a pipe. The air is supplied and compressed by hydraulic machinery near the outer end of the mine. As in jumping by hand, each jumper makes a portion of a turn after each blow. By the use of eight jumpers for six hours, from 60 to 70 holes of 3 feet long and 1¼ inch diameter are made in the face of the rock at the end of the mine; those holes are then used for blasting the rock by gunpowder; and the progress at present made is about a yard in length in ten hours, of a heading measuring from 10 to 13 feet in breadth and from 6½ to 10 feet in height. The air used to drive the jumping machines is said to be sufficient to ventilate the mine.

IV. *Timbered Headings.*—Headings in loose and soft materials are lined with timber, the principal parts of the timbering being, as in other cases of the timbering of excavations, horizontal pieces, props, and poling boards. Fig. 271 is a longitudinal section of a heading in earth proceeding in the direction shown by the arrow. The frames, or "settings," are placed at from 2 to 3 feet apart, and are made of round timber 5 or 6 inches in diameter, so that the pieces can be easily handled by one man. The section shows the ground-sills resting in grooves cut in the floor, the props standing on them, the upper horizontal pieces, called "cap-sills," resting on the props, and the poling boards driven between the settings and the sides and top of the excavation. These boards are usually from $\frac{3}{4}$ inch to an inch thick. In running sand and other soft and wet materials, poling boards are laid under the bottom sills also, so as completely to enclose the heading; and straw is packed behind the boards, to keep sand from running in through the chinks. The operations of carrying the heading forward are as follows:—Drive a set of poling boards forward into the earth, between the last setting and the forward ends of the last set of poling boards; then excavate the earth within the new set of boards, and insert a new setting, and so on.

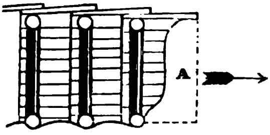

Fig. 271.

V. *Precautionary Borings.* — In driving a mine through ground in which it is possible that cavities containing large quantities of water may be encountered, borings ought to be carried forward both directly and obliquely in advance of the mine, in order that the neighbourhood of such cavities may be ascertained in time to guard against sudden outbreaks of water into the mine, and that the water accumulated in them may escape by degrees and without danger through a bore-hole. This precaution is especially necessary in approaching old pits, mines, or tunnels, which are very generally found to be full of water.

VI. The *Cost and Labour of Mining*, all things included, such as blasting, timbering, removing water, lights, temporary rails and waggons, &c., vary from five times to twenty times the cost and labour of excavating the same quantity of the same material in the open air.

393. **Tunnels in Dry and Solid Rock** are in general excavated by driving a heading immediately below the intended roof of the tunnel, from which heading the excavation is extended sideways and downwards by blasting and quarrying.

These operations require labour to the extent of from three-fourths of a day's work to three days' work of a miner per cubic yard of rock, according to its hardness, being considerably more than is required in the open air.

The following data, on the authority of Becker, show the distribution per cent. of the cost of excavating a railway tunnel in Jura limestone, which required 1·15 days' work of a miner to excavate each cubic yard:—

Workmen's wages,	45	per cent.
Blasting powder,	15	"
Fuses,	3	"
Lamp oil,	8	"
Boring tools,	29	"
	100	"

This tunnel advanced at the rate of about a foot per day.

394. **Tunnels in Dry Fissured Rock** require brick or stone arching within, to guard against the fall of portions of the roof. The most convenient way to make them is in general to commence at a heading running along close below the roof of the excavation; to extend the excavation sideways and downwards to the floor at each side of the tunnel, leaving a wall of rock standing in the middle. This wall is used as a pier to support temporary props (should such be required) for the roof of the excavation, and also to support the centres for the arching, which is carried forward as close behind the excavation as the convenience of working will admit. When the arching is complete, and the centres struck, the central wall of rock is cut away.

All hollows between the brickwork and the rock should be carefully filled with concrete.

The labour of executing brickwork in tunnels (including cost of lights) is about double of that of executing the same quality of brickwork above ground.

395. **Tunnels in Soft Materials,** whether such as are soft from the first, or such as become soft by exposure to air and moisture, like some kinds of clay, require timbering to support the sides and top of the excavation, constructed on the same principles with that of headings.

In such tunnels a principal heading is in general required at the level of the floor, for purposes of drainage.

The excavation of the tunnel is carried on in various ways; that which will here be described is the method of which a detailed account is given by Mr. Simms in his *Practical Tunnelling,* as having been practised at Blechingley tunnel and Saltwood tunnel; the former in blue shale, and the latter in sand.

The tunnel is executed in *lengths*, each of about 12 or 15 feet. These are designated as follows, in the order in which they are executed:—

Side lengths, on each side of a working shaft.

Leading lengths, in prolongation of the tunnel from the side lengths.

Junction lengths, where two portions of the tunnel meet midway between two shafts.

Shaft lengths, directly under the working shafts.

The first operation in commencing a side length, leading length, or junction length, is to drive a heading at the top of the excavation, whose roof must be 1½ or 2 feet above the intended top of the brickwork.

From that heading the excavation is extended sideways and downwards by a process exactly like that of driving a heading, as shown in fig. 272, which is a cross-section of the excavation, after it has been extended a short distance to the right of the top heading. The earth is supported by poling boards, which are supported by strong horizontal timbers called *bars*, 8 or 10 inches in diameter. The after ends of these bars are supported,—

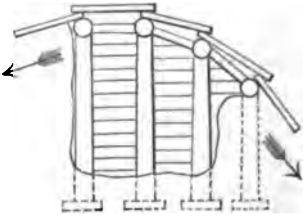

Fig. 272.

In side lengths, by props resting on the framework of the working shaft;

In all other lengths, by the top of the arch of the previous length;

And they are kept asunder by four or five short struts between each pair. The forward ends of the bars rest on props, each of which stands on a foot-block.

Fig. 273 is a longitudinal section, showing the timbering of the excavation of a length of the tunnel when complete; the pieces being numbered in the order in which they are put in.

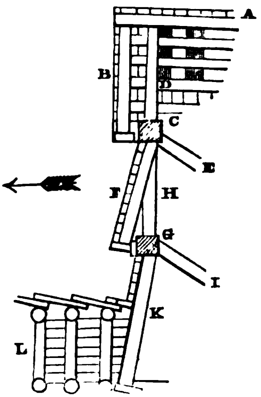

Fig. 273.

A are bars already mentioned, covered with poling boards.

B, props resting on foot-blocks, and covered with poling boards. When the excavation has been carried down to the level of these foot-blocks, there is inserted—

C, a strong sill (say 13 inches square), sent down in two pieces and scarfed together. It extends completely across the excavation, and 1½ or 2 feet into the earth at each side; and at first rests on the earth.

D, props inserted so as to rest on the sill C and support the bars A. Places are now cut to receive

E, struts, 2 or 3 in number, 10 inches in diameter, or thereabouts, whose forward ends abut against the sill C, and their backward ends in side lengths against the timbering of the shaft, and in other lengths against notches in the completed brickwork.

The excavation being by degrees carried down, there are inserted—

F, raking props below the sill C, standing on foot-blocks, and covered in front with poling boards. When the excavation has been carried down to the level of the foot-blocks, there is inserted—

G, a lower sill, similar to C; and this is ultimately supported and kept in its place by struts I and raking props K, in the same manner with C.

L is part of the bottom heading.

The bottom of the excavation is formed with great accuracy to receive the invert, or inverted arch, which forms the base of the brickwork, the levels being set out as described in Article 70, p. 116. The invert and side walls are built according to moulds, as described in Article 252, p. 392; and the arch of the roof upon centres, consisting of three ribs under each length. The best centres have ribs of iron, with screws under each laggin. The centres are usually supported on cross sills, which are themselves supported partly by posts resting on the floor, and partly by their ends being inserted into holes in the side walls, which are built up after the centres are struck.

After the brickwork of a length has been built, most of the crown bars which lie above the arch can be pulled forward so as to serve for the next length; those which resist this must be left. All spaces between the brickwork and the earth must be carefully rammed up.

The following was the distribution of the cost of Blechingley tunnel, according to Mr. Simms:—

MATERIALS.		Per Cent.
Bricks,	30½	
Cement,	11	
Timber,	11½	
Ironwork,	2½	
Miscellaneous,	6½	
Carried forward,	——	62

Brought forward,		62
LABOUR.		
Mining—Shafts, heading, &c.,	3¼	
„ Tunnelling,	15¾	
		19
Brickwork,		12
MISCELLANEOUS EXPENSES,		
Such as tunnel entrances, culvert, machinery, buildings, inspection, &c.,		7
		100

The total cost per yard forward was about £72; the clear dimensions of the tunnel being 24 feet × 24 feet, and the brickwork from 1 foot 10½ inches to 3 feet thick.

The form of cross-section is that already given in fig. 266, p. 588.

396. **Tunnel Fronts—Drainage.**—A tunnel front consists of spandril walls above the arch, and wing walls at each side, like those of a bridge. To secure the end of the arch against the tendency of the slope of earth above it to push it outwards, it may be tied back by longitudinal iron rods to a horseshoe-shaped curb of cast iron, built into the brickwork at a distance back from the front about equal to the height of the tunnel.

A tunnel which has no invert may be drained by means of a pair of side drains, like a cutting; but where there is an invert, the main drain should be a central culvert, of which the invert itself may form the floor.

A catch-water drain should divert the surface water which might otherwise flow over the tunnel front.

397. **Tunnelling in Mud.**—The celebrated tunnel of the elder Brunel under the Thames consists of a rectangular mass of brickwork laid in cement, 37·5 feet broad and 22 feet high, containing a pair of parallel horseshoe archways, each 14 feet span and 17 feet high, which are connected together by small cross archways at intervals. The least thicknesses of the brickwork are, at the crown of the arch 2½ feet, at the base of the invert 2½ feet, at the sides 3 feet, in the central wall 3½ feet. The whole mass of brickwork rests on a base of elm planks 3 inches thick.

In driving this tunnel, the place of the timbering of the excavation described in Article 395, was supplied by a machine called a "shield," which was pushed on in advance of the brickwork at a distance of about eight feet. The shield was of the same dimensions with the mass of brickwork. It consisted of twelve equal and similar divisions standing vertically side by side, and capable of being pushed forward to a short distance independently of each other. Each division consisted of a cast and wrought iron frame about 3 feet broad (to allow a small space between the frames), containing three stages for workmen. It had two cast iron feet, resting on the floor of elm planks; on these feet it was supported by a pair of hinged legs of lengths adjustable by screws. It had an iron roof extending back to the brickwork, and a pair of jack-screws at the top and bottom, abutting against the front end of the brickwork, to push it forward. The several frames were connected together by hinged arms, nearly vertical, to enable them to afford support to each other when required. The spaces at each side of the shield, extending back from the face of the excavation to the brickwork, were guarded by iron plates. Each frame had in front of it, extending from top to bottom, a range of poling boards; each poling board was 3 inches thick and 6 inches broad, and was pressed against the material in front by a pair of small jack-screws abutting against the frame.

The following is an outline of the process of excavation with this apparatus:—Take out the uppermost poling board in front of a frame, cut away about 3 inches of the stuff in front, replace the poling board, with its screws now abutting, not against the frame directly behind it, but against the two frames at each side of that; screw it forward till it again presses against the earth. Proceed in this way till the whole range of poling boards have been advanced three inches, their screws abutting against the two frames at each side of their proper frame; shorten the legs of the latter frame so as to lift its feet, and advance them; then push forward the frame *six inches* by means of its large abutting screws: it is supposed to have been previously three inches behind the adjoining frames, and is now three inches in advance of them. Repeat the whole operation of advancing the poling boards, restoring their screws to their proper frame. This entire operation having been performed on six alternate frames at the same time, the same operation is performed on the other six alternate frames, and so on until the whole shield has been advanced far enough to admit of a new ring of brickwork being built. In order to advance the brickwork behind a given frame of the shield, the poling boards of that frame must have their screws abutted against the adjoining frame, so that the great abutting screws may be removed from the front of the part of the brickwork which is in progress.

The shafts at the two ends of the tunnel have been mentioned in Article 391, p. 590. The Rotherhithe shaft was sunk 38 feet on a drum-curb, and about as much farther by underpinning; the Wapping shaft was sunk to its whole depth, 72 feet, on a drum-curb.

(For details of the Thames tunnel, see Weale's *Quarterly Papers on Engineering.*)

The Thames tunnel cost £1,137 per yard forward, or nearly £12 5s. per cubic yard of its entire bulk.

SECTION II.—*Of Timber, Iron, and Submerged Foundations.*

398. **General Principles—Submerged Foundations.**—Foundations which can be executed by the use of earthwork and masonry alone have already been treated of in Chapter III. of this Part, Section IV., Articles 235 to 239, pp. 377 to 382. The present section relates to those foundations which involve the use of structures in timber and iron, and of operations under water.

The general principles already explained with reference to ordinary foundations, viz., that the base should be as nearly as possible perpendicular to the resultant pressure, and that the centre of pressure should not deviate from the centre of figure of the base beyond certain limits, are applicable to the foundations considered in the present section also. The mathematical expression of those principles has been given in Articles 236, 237, pp. 377 to 380.

In calculations respecting the stability of structures whose foundations are submerged in water, it is to be borne in mind that the pressure of the water on the immersed part of the structure has the same effect as if the weight of that part were diminished by an amount equal to the weight of an equal volume of water; that is, as if the heaviness of the immersed part of the structure were diminished by 62·4 lbs. per cubic foot. (See Article 107, Division IV., p. 165.)

399. **Foundations on Timber Platforms** are employed where the ground is too soft and wet for the expedients mentioned in Article 239, p. 381. The best European timber for such platforms is elm or oak. Beams of from 10 inches to 1 foot square are laid about 3 feet apart, in two layers, crossing each other so as to form a grating, the space between them is filled with concrete, and above them is laid a layer of planking, 3 or 4 inches thick, on which the building rests. Another mode of constructing such platforms is to lay several layers of planks and pin them together. In order that timber platforms may be durable, they should be constantly wet.

400. **Foundations on Iron Platforms** are of too recent introduction to be yet capable of reduction to general principles. As a practical example, however, of a platform of this kind, may be cited the cast iron invert lately substituted for a stone invert in a lock at Grangemouth, as described in a paper by the engineer, Mr. Milne (see *Transactions of the Institution of Engineers in Scotland* for 1859-60). The lock is 30 feet broad; the depth of water 18 feet 6 inches; the

side walls about 8 feet thick and 20½ feet high; the invert in question consists of a series of trough-shaped cast iron girders, lying close together side by side, and bolted to each other through their vertical sides; each of them is 2 feet broad, 21 inches deep at the springing, 12 inches deep at the centre of the invert, and 2 inches thick; each of their vertical sides has a flat horizontal flange at the top, 4 inches broad. The trough-shaped interior of each girder is filled with concrete, covered with a layer of bricks laid in cement.

401. **Short Piles** are driven in order to compress and consolidate the soil. They are usually of round timber, from 6 to 9 inches in diameter, and from 6 to 12 feet long, and are planted as close to each other as is practicable without causing the driving of one pile to make the others rise. The outside row of piles should be driven first, then the next within, and so on to the centre. The mass of consolidated soil and piles thus produced may be regarded, as respects the relation between its bulk and the load that it can bear, in the same light as if a trench had been dug of the same volume, and filled with a stable material; as to which, see Article 239, p. 381. On the top of the piles may be placed either a platform, a layer of concrete, or both.

402. **Bearing Piles** act as pillars, each supporting its share of the weight of the building. They may either be driven through the soft stratum until they reach a firm stratum and penetrate a short distance into it; or, if that be impracticable, they may be supported wholly by the friction of the soft stratum. It appears from practical examples that the limits of the safe load on piles are as follows:—

For piles driven till they reach the firm ground, 1,000 lbs. per square inch of area of head.

For piles standing in soft ground by friction, 200 lbs. per square inch of area of head.

The diameters of long piles range from 9 inches to 18 inches, and should never be less than 1-20th of the length. Their distance from centre to centre averages about 3 feet, and is seldom less than 2½ feet.

The best material for them is elm, which should be chosen as straight-grained as possible. The bark should be removed, and knots or rough projections smoothed off.

Piles should be driven with the butt or natural lower end of the timber downwards. It is roughly sharpened to a point whose length is from 1½ times to twice its diameter; and should stones or other hard materials occur in the strata to be pierced, the point must be fitted with a "shoe" of cast or wrought iron, fastened on with spikes. The weight of these shoes averages about 1-100th part of that of the piles.

To prevent the head of a pile from being split or bruised by the blows of the "*ram*" used in driving it, it is bound with a wrought iron hoop.

Pile-driving engines are of various kinds. The simplest is the "*ringing engine*," in which the ram, weighing about 800 lbs., and moving between timber guides, is attached to one end of a rope which passes over a pulley. The other end of the rope branches out into a number of smaller ropes, each held by a man, in the proportion of one man for each 40 lbs. weight of the ram, or thereabouts. The men, pulling all together, lift the ram 3 or 4 feet, and on a given signal, let go all at once, so as to drop it on the head of the pile. It is found that they work most effectively when, after every 3 or 4 minutes of exertion, they have an interval of rest; and under these circumstances they can give about 4,000 or 5,000 blows per day.

In the "*monkey engine*," the ram, weighing about 400 lbs., and held by a staple in a pair of tongs, is drawn up 10 feet, 15 feet, or higher if necessary, by means of a windlass; at the top of the lift the handles of the tongs come in contact with two inclined planes which cause them to let the ram fall; the tongs are then lowered, and have jaws so shaped that on reaching the staple at the top of the ram they lay hold of it again. The windlass may be driven by men, horses, or steam power.

The steam hammer is sometimes used for driving piles; and also an engine somewhat on the same principle, in which the ram is lifted by the pressure of compressed air. In such machines rams of great weight are sometimes used, such as 1 ton, or a ton and a-half.

Piles may be driven in a direction either vertical or raking, according to the position of the guides between which the ram slides. That direction should be parallel to that of the pressure which they are to resist.

When the head of a pile is to be driven below the reach of the stroke of the ram, the blow is transmitted from the ram to the pile by means of an intermediate short post of timber called a "*punch*," or "*dolly*."

According to some of the best authorities, the test of a pile's having been sufficiently driven is, that it shall not be driven more than *one-fifth* of an inch by *thirty blows* of a ram weighing 800 lbs. and falling 5 feet at each blow; that is to say, by a series of blows whose total mechanical energy amounts to

$$30 \times 800 \times 5 = 120,000 \text{ foot-pounds.*}$$

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* The following formulæ show the relation between the blow required to drive a pile a given depth, and the greatest load that it will bear without sinking further, supposing it to be supported by an uniformly distributed friction against its sides.

Piles are *drawn*, when required, by means of the hydraulic press.

When a firm stratum, into which the points of a set of piles are driven, underlies a stratum so soft that their lateral stability is doubtful, a mass of loose stones may be thrown in round them to give them the steadiness which they want.

After the driving of a set of piles has been completed, their heads are to be sawn off to the height required for the support of the platform.

The soft ground round the tops of the piles is then to be scooped out to a depth which in ordinary cases ranges from 3 to 5 feet, and the space filled with hydraulic concrete, laid in layers not exceeding 1 foot deep.

The platform supported by the piles consists of a grating of beams of 10 or 12 inches square, called *string-pieces* and *cross-pieces*, half-notched into each other over the heads of the piles, to which they are fixed by treenails, and covered with planking 3 or 4 inches thick. The spaces between the beams of the grating are to be filled with hydraulic concrete. The beams on the top of the outermost rows of piles are usually made so deep that their upper surfaces are flush with that of the planking, which is *rabbeted* into them; that is, sunk in a groove. Those beams are in this case called the *capping*.

Piles may be driven into rock by first jumping holes in it of a little less diameter than the piles.

For *cast iron* piles, the best form is that of a tube. To prevent their being broken by the blows of the ram in driving them, a

Let W be the weight of the ram.
h, the height from which it falls.
x, the depth through which the pile is driven by the *last* blow.
P, the greatest load it will bear without sinking farther.
S, the sectional area of the pile.
l, its length.
E, its modulus of elasticity.

Then the energy of the blow is thus employed:—

$$W h = \frac{P^2 l}{4 E S} \text{ (employed in compressing the pile) } + P x \text{ (employed in driving it);}$$

and consequently,

$$P = \sqrt{\left(\frac{4 E S W h}{l} + \frac{4 E^2 S^2 x^2}{l^2}\right)} - \frac{2 E S x}{l}.$$

Piles are usually driven until P, as computed by this formula, is between 2,000 and 3,000 lbs. per square inch of the area S; and as their working load ranges from 200 to 1,000 lbs. per square inch, the factor of safety against sinking is from 3 to 10. The factor of safety against direct crushing of the timber should not be less than 10.

timber punch is interposed between the head of the ram and the pile. The best mode, however, of driving them, is by the aid of the screw, which will be mentioned in the next article.

403. **Screw Piles,** the invention of Mr. Alexander Mitchell, are piles which are screwed into the stratum in which they are to stand. The pile may be either of timber or iron, and that it may admit of being easily turned about its axis, should be cylindrical, or at all events octagonal. The screw blade, which is fixed on at the foot of the pile, is usually of cast iron, and seldom makes more than a single turn. Its diameter is from twice to eight times that of the shaft of the pile, and its pitch from one-half to one-fourth of its diameter. The best mode of driving screw piles is to apply the power of men or of animals, walking on a temporary platform, directly to levers radiating from the heads of the piles.

As an example may be cited the cast iron piles already mentioned in Article 381, p. 572, as being used in the piers of railway bridges in India. Each of these was screwed into the ground by means of four levers, each 40 feet long, and each having eight bullocks yoked to it. According to this example, the greatest working load upon each screw of 4 feet 6 inches in diameter, *exclusive* of the earth and water above it, is nearly as follows:—

Pier 25 tons + superstructure 12 + train 30 = 67 tons = 150,080 lbs., being at the rate of nearly
100 lbs. per square inch of the horizontal projection of the screw-blade.

As these piles are screwed from 20 to 45 feet into the earth, the weight of earth above each screw-blade may be taken as ranging from 14 lbs. to 31 lbs. per square inch; so that the load on each screw blade, exclusive of the weight of earth above it, ranges from 3 times to 7 times that weight, and including the weight of earth, from 4 times to 8 times; results which correspond with the theory of Article 237, p. 379, if the angles of repose of the earth be assumed to range from about 28° to about 19°.

The chief uses of screw piles are to form the vertical supports of platforms of openwork piers, whether of timber or iron, and of such structures as harbour-jetties and lighthouses, and to fasten down permanent mooring-chains in harbours.

404. **Sheet Piles** are flat piles, which, being driven successively edge to edge, form a vertical or nearly vertical sheet, for the purpose of preventing the materials of a foundation from spreading, or of guarding them against the undermining action of water. They may be made either of timber or of iron.

Timber sheet piles are planks having a projection or feather along one edge, and a corresponding groove along the opposite edge.

They are of any breadth that can readily be procured, and from $2\frac{1}{2}$ to 10 inches thick, and are sharpened at the lower end to an edge, which, in stony ground, may be shod with sheet-iron.

When a space is to be enclosed with sheet-piling, a range of *guide-piles* is first driven, being long rectangular piles at regular intervals apart of from 6 to 10 feet: these are driven to the same depth as bearing-piles. To the opposite sides of these, near the top, are notched or bolted a pair of parallel *string-pieces* or *wales*: these are horizontal beams, from 5 to 10 inches square, notched on the guide-piles to such a depth as leave a space between them of a width equal to the thickness of the sheet-piles. If the sheet-piles are to stand more than 8 or 10 feet above the ground, a second pair of wales is required near the level of the ground.

The sheet-piles are driven between the wales to about half the depth of the guide-piles, beginning with the sheet-piles next the guide-piles, and working towards the middle of each space between a pair of guide-piles; so that the last or central sheet-pile acts as a wedge to tighten the whole.

In *iron sheet-piling* the guide-piles may be either tubular, or of a form of section like a trough-girder set on end (fig. 240, p. 524). The sheet-piles are also like trough-girders set on end, being plates stiffened by vertical ribs on the inner side. Their side edges are so formed as to make over-lapping joints, and their lower edges are wedge-shaped.

For example, in the foundations of Chelsea Bridge, the cast iron guide-piles are tubular, flat on the outer side, semi-cylindrical on the inner side, 12 inches in external diameter, and 1 inch thick, and are 27 feet long; the sheet-piles are cast iron plates, 10 feet long, from 6 to 7 feet broad, and 1 inch thick, stiffened by vertical ribs, which are from 4 to 6 inches deep, and from 10 to 20 inches apart, and by one horizontal rib of about the same dimensions at the upper edge.

405. **Timber and Iron-Cased Concrete Foundations.**—In foundations of this class the building rests on a mass of concrete (as to the strength and dimensions of which, see Article 239, pp. 381, 382), that mass being cased with sheet-piling of timber or iron, such as that described in the preceding article.

The sheet-pile casing is constructed first, and is sufficiently braced, transversely and diagonally, to enable it to resist the pressure to which it may be exposed, whether of water and mud from without, or of concrete, while in the soft state, from within. The soft material within the casing is then scooped out.

The concrete should be that described in Art. 230, p. 374, as *strong hydraulic concrete*, or "*beton*," and should be laid in layers of about a foot thick, each layer being either well rammed or thrown in from a stage at least 10 feet high. Time should be given for the concrete to become firm before a heavy load is placed on it; for it has been shown by recent observations that intense pressure retards the setting of concrete.

The casing, besides facilitating the excavation of a bed for the concrete, serves to protect it afterwards from injury by such causes as the wearing action of a river current. When the casing is of iron, it is capable of bearing also a share of the load.

Sometimes a timber or iron-cased mass of concrete is combined with a system of bearing-piles, as described in Article 402, pp. 602 to 604.

406. **Iron Tubular Foundations** consist of large hollow vertical cast iron cylinders, filled with rubble masonry or concrete, such as have already been partly described in Article 381, pp. 572, 573.

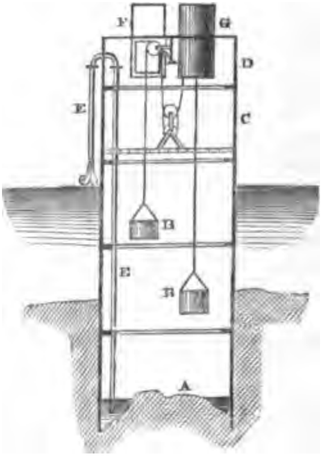

Fig. 274.

The general construction of such cylinders and the mode of sinking them are shown by the vertical section, fig. 274. Amongst the auxiliary structures and machinery not shown in the figure are, a temporary timber stage from which the pieces of the cylinder can be lowered, and on which the excavated material can be carried away; and a steam engine to work a pump for compressing air.

The following were the dimensions of the engine and pump used to supply air to a cylinder of the Theiss bridge formerly referred to; the diameter being 9·84 feet, and the greatest depth below the surface of the water to which the cylinder was sunk, 66 feet, corresponding to an absolute pressure of 3 atmospheres (or 2 atmospheres, that is to say, 29·4 lbs. on the square inch, above the ordinary atmospheric pressure). This is said to be the greatest pressure under which the excavators can work without danger to their health.

Diameter of steam cylinder (high pressure),......	8·67 inches.
„ of air-cylinder,............................	11·82 „
Length of stroke (the same in both cylinders),...	0·656 foot.
Number of revolutions per minute,..........from	100 to 120.
Horse power,.....................................from	10 to 12.

From these data it appears that the volume of air supplied, measured at the ordinary atmospheric pressure, was from 100 to 120 cubic feet per minute. It appears that the number of persons within the cylinder at one time was from six to eight.

The cylinder consists of lengths of about 9 feet, united by internal flanges and bolts. The joints are cemented and made air-tight with a well-known composition, consisting of

Iron turnings,.............................	1,000	parts	by weight.
Sal-ammoniac,.............................	10	„	„
Flour of sulphur,..........................	2	„	„

Water enough to dissolve the sal-ammoniac.

In some examples each joint is made tight by means of a ring-shaped cord of vulcanized indian rubber, lodged in a pair of grooves on the faces of the flanges.

The lowest length, A, of the cylinder, has its lower edge sharpened, that it may sink the more readily into the ground. The intermediate lengths, B, B, and the uppermost length, C, have flanges at both edges, upper and lower. The portion D, at the top, forms the "bell." The lower edge of the bell has an internal flange by which it is bolted to the cylinder below; its upper end is closed, and may be either dome-shaped, or flat, and strengthened against the pressure of the air within by transverse ribs, as in the figure. In the example shown the bell is made of wrought iron boiler plates.

D is a siphon, 2 or 3 inches in diameter, through which the water is discharged by the pressure of the compressed air.

F and G are two cast iron boxes, called "air-locks," by means of which men and materials pass in and out. Each of them has at the top a trap door, or lid opening downwards from the external air, and at one side, a door opening towards the interior of the bell, and is provided with stop cocks communicating with the external air and with the interior of the bell respectively, which can be opened and closed by persons either within the bell, within the box, or outside of both. These may be called the escape cock and the supply cock.

The bell is provided with a supply pipe and valve for introducing compressed air, a safety valve, a pressure gauge, and a large escape valve for discharging the compressed air suddenly when required.

At the lower flange of the division C is a timber platform, on which stands a windlass.

The apparatus is represented as working in a stratum of earth or mud, covered with water.

The operation of sinking a cylinder is analogous to that of sinking a shaft with a drum-curb. (Article 391, p. 592.) The first operation is to lower the lowest length A of the cylinder, till it rests on the earth, with as many intermediate lengths B as are sufficient to reach a foot or two above the top water-level, and one additional length C, all bolted together. Then the bell is bolted on. The whole cylinder sinks to a depth depending on the material on which it rests. The engine then forces in air until the water is expelled from the cylinder. Workmen, with tools and buckets, can now pass in and out through the boxes or air-locks. To pass in, the operation is as follows:—Shut the supply-cock of the box, if not shut already; open the escape-cock—should there be compressed air in the box, it will be discharged; open the trap-door and enter the box; shut the trap-door and fasten it; shut the escape-cock; open the supply-cock; in a few minutes the box will be filled with compressed air at the same pressure with that in the bell; open the side door and pass into the bell. To pass out, the operation is as follows:—Should the escape-cock of the box be open, shut it; should the supply-cock be shut, open it; the box will soon be filled with compressed air, if not full already; open the side-door, enter the box, close the side-door, shut the supply-cock, open the escape-cock; when the air has fallen to the external pressure, open the trap-door and pass out. Some of the workmen (generally two) descend by a ladder or a bucket to the bottom of the cylinder, dig away the earth from its interior, and put it into buckets, which are raised by a set of men working the internal windlass, and sent through the air-locks, whence they are removed by an external windlass, not shown in the figure.

So soon as the excavation has been carried down to the level of the lower edge of the cylinder, the miners carry their tools and the lower division of the siphon E up to the platform; the whole of the workmen leave the bell; the great supply-valve is shut, and the great escape-valve opened, so that the whole of the compressed air escapes. The cylinder being deprived of the support arising from the pressure of the compressed air against the top of the bell, sinks to a depth usually varying from one to two yards. When it has given over sinking, the great escape-valve is shut, and the great supply-valve opened, and the operation goes on as before, until it becomes necessary to put on an additional length of cylinder. This is done, while the pressure within and without are equal, by unbolting and taking off the bell, putting on a new length of cylinder on the top of C, which now becomes an intermediate length, removing the platform and windlass up to the new length, putting an additional length into the siphon, and replacing and bolting on the bell.

In the case taken as an example, each cylinder was sunk by gangs of nine men working six hours at a time; and the earth (sand and clay) was removed at the rate of 15 buckets, each containing ·09 of a cubic yard, per hour; that is,

$$15 \times \cdot 09 = 1 \cdot 35 \text{ cubic yard per hour, by nine men;}$$
$$\text{or } \cdot 15 \text{ cubic yard per man per hour;}$$
$$\text{or } 6\tfrac{2}{3} \text{ hours of one man per cubic yard.}$$

The total volume of earth which has to be removed ranges, according to the stiffness of the material, from *once* to *three times* the volume formed by multiplying together the sectional area of the cylinder and the depth to which it is sunk.

Care must be taken to keep the cylinder upright as it descends, by means of stays.

When the sinking of the cylinder has been completed, it is filled with masonry, or with hydraulic concrete; as to which, see Article 405, p. 606. About one-half of the building is performed in the compressed air; the remainder, with the cylinder open at the top, the bell being removed.

Care should be taken to pack the concrete or masonry well below, and to bed it firmly above, each of the pairs of internal flanges.

In very soft materials it is sometimes necessary to drive a set of bearing piles in the interior of each cylinder, in order to support the concrete and masonry.

The earliest mode of sinking iron tubular foundations was that invented by Dr. Potts, in which the air is *exhausted* by a pump from the interior of the tube, which is forced down by the pressure of the atmosphere on its closed top. This method is well suited for sinking tubes in soft materials that are free from obstacles which the edge of the tube cannot cut through or force aside, such as large stones, roots, pieces of timber, &c.*

407. **Foundations made by Well-sinking** are in some respects analogous to iron tubular foundations. They are suitable for a soft and wet stratum with a firm stratum below it. They are made by sinking a sufficient number of cylindrical stone or brick-lined shafts, each on a drum-curb (see Article 391, p. 592), through the soft stratum, until the firm stratum is reached. These shafts are

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* The method of sinking cylinders for foundations, by the aid of compressed air, was first employed at the bridge over the Medway at Rochester, executed from the designs of Sir William Cubitt.

It was at first intended that the tubes should be sunk by the exhaustive process; but the remains of an old timber bridge, imbedded in the mud at the bottom of the river, rendered that impracticable; and the compressive process was then invented by the contractors' agent, Mr. Hughes.

then filled with rubble masonry, or with brickwork, so that each of them becomes a solid cylindrical pillar.

408. **Caissons.**—A caisson is a sort of flat-bottomed boat in which the foundation-courses and lower part of some structure which is to stand in water, such as a bridge-pier, are built, and floated to their intended site. The bottom of the caisson is a horizontal timber platform, fitted to form a permanent part of the foundation, as described in Article 399, p. 601. The sides are vertical, and are capable of being detached from the bottom. A seat is prepared for the platform, by excavation alone, by laying a bed of concrete, by driving a set of piles, or otherwise, as the occasion may require. The caisson is moored over that seat, and when the building within it has been carried to a sufficient height, it is gradually sunk, by slowly admitting the water, until the platform rests on its bed. The sides are then detached and removed.

The usual method of connecting the sides with the bottom is as follows:—The main supports of the bottom consist of a number of parallel transverse beams whose ends project beyond the sides; across the upper edges of the sides are laid an equal number of similar beams, into which the uppermost wales or longitudinal pieces of the sides are so notched as to be kept by the beams from being forced together by the pressure of the water. The projecting ends of the upper set of beams are connected with those of the lower set by long vertical iron bolts, outside the caisson, having a hook and eye joint a little above the lower beams; and by unfastening these the sides are at once detached from the bottom.

The dimensions of the timber used in the bottom are usually about the same as for a foundation-platform (Article 399, p. 601); those of the framework of the sides may be computed according to the principles of the strength of materials, so as to bear safely the greatest pressure of the water.

In an example described by Becker, used in building a bridge pier, the caisson was about 63 feet long, 21 feet broad, and 15 feet deep over all, the masonry within being about 18 feet broad. The cross beams were 10 inches square, and about 2 feet 10 inches apart from centre to centre; the upright standards of the sides were 10 inches square, and 5 feet 8 inches from centre to centre.

A caisson that has been sunk, but which still has the sides attached, may be floated again, in order to rectify its position, if necessary, by closing the valve for the admission of water, and pumping the water out.

409. **Dams for Foundations** are made for the purpose of excluding water from a space in which a foundation or some such structure is to be made. The materials principally used in them are timber, iron, and clay puddle, as to which last, see Article 206,

p. 344. Hydraulic concrete also is occasionally used, as to which, see Article 230, p. 374.

I. *Clay Dams.*—In still water of a depth not exceeding 3 or 4 feet, and on moderately firm ground, a clay puddle embankment forms a sufficient dam; care being taken, before commencing it, to dig a trench for its foundation, so as to remove loose and porous material from the surface of the ground.

II. *Coffer Dams.*—In greater depths, the essential part of an ordinary dam consists of two parallel rows of main piles and sheet piles (see Article 404, p. 605), enclosing between them a vertical wall of clay puddle. The upper wales of the two rows of piles are tied together by cross beams, which support a stage of planking for the use of the workmen. The main piles in one row are from 4 to 5 feet apart. The ground is excavated between the rows of sheet piles until a sufficiently firm bottom is reached, and the puddle rammed in layers.

The common rule for the thickness of a coffer dam is to make it equal to the height above ground, if the height does not exceed ten feet; and for greater heights, to add to ten feet one-third of the excess of the height above ten feet.

When the height exceeds twelve or fifteen feet, or thereabouts, three, and sometimes four or more, parallel rows of sheet piling are driven, thus dividing the thickness of the dam into two, three, or more equal divisions, each of six feet thick, or thereabouts; the outermost division is made of the full height, and the heights of the inner divisions are made less, so as to form a series of steps.

It appears from experience that a thickness of from two to five feet of clay puddle is sufficient to make a coffer dam water-tight; the additional thickness given by the rules above mentioned is required for stability, and the more so that the timber framework cannot be stiffened inside by diagonal braces between the rows of girder piles; for such braces would conduct streams of water along their sides through the puddle.

Another mode of obtaining stability is to make the dam simply of sufficient thickness to exclude the water, and to support it from within against the pressure of the water by means of sloping struts, abutting at their upper ends against the main piles of the inner face of the dam, and at their lower ends, in soft ground, against piles driven for that purpose, and in hard ground, against foot-blocks.

Let b be the breadth, in feet, of the division of the dam sustained by one such strut.

x, the depth of water,

w, the weight of a cubic foot of water,

being 62·4 lbs. for fresh, and 64 lbs. for salt water.

Then, by the principles of Article 107, p. 166, equation 9, the total pressure of the water against that division of the dam is

$$P = w\,b\,x^2 \div 2; \text{......................}(1.)$$

and the moment of that pressure, relatively to a horizontal axis at the level of the ground is

$$M = w\,b\,x^3 \div 6. \text{......................}(2.)$$

Let h be the height above the ground at which the strut abuts against the dam, and i its inclination to the horizon; the thrust along the strut is

$$T = M \sec i \div h; \text{......................}(3.)$$

and the scantling required to bear that thrust safely may be computed by the principles of Article 158, p. 238, equations 6, 7, 8.

When a coffer dam is to be exposed to waves, add together the greatest depth of still water in front of it, and *twice* the greatest height to which the crest of a wave rises above the level of still water, and put the sum for the greatest depth to which the dam is to be adapted (x in the formulæ). In shallow water on exposed parts of the coast, this amounts very nearly to making x equal to double the greatest depth of still water.

In firm ground impervious to water, planks laid horizontally on edge between a double row of guide piles may be substituted for sheet piling. The least thickness suitable for such planks is about 2½ inches; and with guide piles five feet apart this is sufficient for a depth of about six feet; for greater depths, the thickness must increase in proportion to the square root of the depth.

For a rocky bottom, the following construction has been used by Mr. David Stevenson (see *Trans. Inst. of Civil Engineers*, vol. III.; also *Encyc. Brit.*, Article "Inland Navigation"):—Two parallel rows of vertical iron rods, three feet apart, were jumped into the rock to a depth of fifteen inches, to answer instead of guide piles; inside these rods, and supported by them, were two vertical linings of planks laid on edge horizontally, between which clay puddle was rammed; outside the iron rods were horizontal timber wales five feet apart vertically, or thereabouts; these were bolted together in pairs, through the dam, to which stability was given by means of inclined timber struts, as already described.

III. *Caisson Dams.*—Another mode of constructing a dam on a rocky bottom is to use a number of caissons, or flat-bottomed boats, suitably formed, so as to enclose the space which is to be guarded by the dam; when these have been floated to their proper places and moored, they are to be gradually sunk until they begin to rest on the bottom; two rows of main piles, running respectively along the outer and inner faces of the enclosure of caissons, are now to be lowered vertically side by side until their lower ends rest firmly on the bottom, and bolted in that position to the sides of the caissons; the loading of the caissons, by means of stones or other heavy materials, and by admitting water, is now to be proceeded with, until either the whole or a considerable part of their weight rests on the main piles. A framework is thus formed, resting on the bottom by means of the main piles. A third row of piles, or posts, suitably framed to the inner row of main piles, is now to be set up parallel to and within them; and between these two rows, the dam, properly speaking, is to be formed in the manner already described, with two linings of planks and a puddle wall. When the dam is removed, because of the foundation or other work within it being finished, or because the work is to be interrupted, the caissons are to be unloaded and pumped dry, and floated away, so as to be available at a future time for the resumption of the same work, or the execution of another, as the case may be.

As to dams of this class, see Mr. Hodges's account of the Victoria Bridge.

Caissons, or boats, capable of being floated and grounded at will, as above described, are suitable where it is necessary, not to make a water-tight dam, but merely to obtain protection from a current that would otherwise impede or injure the work. (See Stevenson *On American Engineering*, Chapter VIII.)

IV. *Crib-Work Dams* are used where timber is abundant and cheap. Crib-work consists of a series of layers of logs, laid alternately lengthwise and crosswise, notched and pinned to each other at their intersections: the distance apart of the logs in each layer is three or four times their diameter. A skeleton frame of any required dimensions having been formed in this manner, is floated to its intended site, and there loaded with stones laid upon platforms supported by some of the upper layers of logs, until it sinks. It can then be used in the same manner and for the same purposes as the caisson dams of Division III. (On the subject of crib-work, see Stevenson *On American Engineering*; Hodges on the *Victoria Bridge*.)

V. *Wicker-Work Dams* will be mentioned further on.

410. **Excavating, under Water, Dredging, and Blasting.**—Processes have already been described by which excavations are made under the water-level by the aid of some apparatus for excluding the water from the site of the excavation, such as iron cylinders filled with compressed air (Article 406, p. 607), or coffer dams (Article 409, p. 612). The present article relates to the making of such excavations by tools or mechanism, without excluding the

water. Cases in which the currents of the water itself are made available for that purpose will be considered in a later chapter.

I. *Protection of the Excavation.*—When an excavation is made under water in order to deepen a channel, it seldom requires to be protected; but when it is made with a view to the construction of a foundation, and there are loose materials, either in the ground excavated, or suspended in the water, it must be guarded against currents in the water, which otherwise would sweep those materials into it and fill it up. This may be done by caissons (Article 407, Division III., p. 613), cribs (Article 409, Division IV., p. 614), or by an enclosure of sheet piling, whether timber or iron (Article 404, p. 605); and if the excavation is for the purpose of making a piled or concrete foundation, the sheet piling may afterwards form the permanent casing of that foundation. (Article 405, p. 606.)

II. *Dredging by Hand* is performed by means of an implement called a "spoon," or "spoon and bag." It consists of a pole, at one end of which is fastened an iron ring, steeled at the forward edge, and forming the mouth of a bag of strong leather or coarse canvas. The ring is hung by a rope tackle capable of being wound up by means of a crab, and the further end of the pole is held by a man. As the rope is wound up the spoon is dragged forward along the bottom, against which the man who holds the pole causes the edge of the ring to press, scooping earth into the bag, until it arrives directly below the crab, when it is hauled up and emptied into a punt or mud barge.

In small depths of water, such as four or five feet, the labour and cost of this operation are not much greater than those of excavating similar materials on dry land. In greater depths the operation becomes more laborious and costly, nearly in proportion to the depth; and in depths of more than ten feet it is not applicable.

Another kind of hand dredge has a sort of sheet iron scoop instead of the ring and bag, and is suitable for rough and stony materials.

III. The *Dredging Machine* consists essentially of a pair of parallel chains, driven by pullies so as to move up the upper side and down the under side of an inclined plane, and carrying in soft ground a series of buckets, and in stiff ground buckets and rakes alternately; the rakes to break up the ground and the buckets to lift it. The upper end of the inclined plane is hinged, so that the lower end adapts itself to the level of the bottom. The machine works in a well in the middle of the after part of a strong barge, over the stern of which the buckets empty themselves into a punt or mud boat. The ordinary prime mover is a steam engine; but small dredging machines are also used, which are worked by hand. According to Mr. David Stevenson, a steam dredge of sixteen horse-power will, under favourable circumstances, raise about 140 tons of stuff per hour (that is, about 100 or 110 cubic yards); and the cost ranges from an amount nearly equal to that of excavation in similar material on land (say about 8d. per cubic yard for sand and gravel) to about half that amount (or nearly 4d.). In general, the larger and more powerful the machine, the less is the cost of dredging.

IV. *Blasting Rock* in shallow water is nearly similar to the same operation on land, as to which see Article 207, p. 344. In general, proportionately more powder must be used than on land; for under water, it is desirable to shiver the rock into pieces that can be removed by dredging. In a good example of such operations, described by Mr. Edwards in the *Proceedings of the Inst. of Civil Engineers*, vol. IV., the weight of rock loosened was about between 5,000 and 6,000 times that of the powder exploded.

In deep water, the diving bell must be used in preparing the blasts.

V. *Removing Large Stones.*—Boulders and blocks of stone which are too large to be lifted by the dredging machine may either be split or blasted into smaller pieces, or may be attached, with the aid of diving apparatus, by means of a lewis (Article 251, p. 391), to a boat, and so lifted and carried away.

411. **Diving Apparatus** (I.) *for a single diver* consists essentially of a metallic *helmet*, usually spherical, and made of copper, enclosing the diver's head and resting on his shoulders, connected at its base with an air and water-tight dress, provided with a long flexible tube and valve, opening inwards, for supplying air from a compressing pump above water, an escape valve for foul air, opening outwards, about the level of the diver's chest, and some glazed openings (usually three in number), at the level of his eyes. Each of these openings should be furnished with a water-tight valve, which the diver can instantly close in the event of the glass being broken. The air feed-pipe enters at the back of the helmet, and the air is conducted thence by arched passages over the diver's head to points near the glazed eye-holes. By this arrangement the entrance of water is prevented, in the event of the feed-pipe bursting. To overcome the buoyancy of the apparatus, and enable the diver to sink, his waterproof dress is loaded with about a hundredweight of lead, part in the soles of the shoes, part fastened to the breast and back. He usually hauls himself up by means of a rope; but should he wish to ascend suddenly he has only to close the escape-valve, when the air inflates the waterproof dress and causes him to float to the surface. If necessary, he carries a bull's-eye lantern, air and water-tight, and supplied with air in the same manner with the helmet; the chimney has a flexible discharge-

pipe ascending to the surface, with a valve opening outwards. This lamp is required more especially in turbid water.* In America a diving helmet has been used made wholly of glass.

II. The *Diving Bell* commonly used is shaped like a rectangular box with rounded corners, measuring about six feet by four feet horizontally, and five feet high, two inches thick in the top and upper part of the sides, and increasing to three and a-half inches or thereabouts at the lower edge, for the sake of stability. It usually weighs about five tons, and displaces three and a-half tons of water, or thereabouts, when quite filled with air: the difference is the load on the crane and windlass by which it is lowered and raised. It has a number, not usually exceeding twelve, of bull's eyes, or glazed holes in the top to admit light; they are eight or ten inches in diameter, and the glass about two inches thick. The flexible feed-pipe for supplying compressed air is about three inches in diameter. If the quantity of air required be calculated according to the data already stated as to the supply of foundation-cylinders (Article 406, p. 608), or according to the usual practice in public buildings, it should amount to about twelve cubic feet, measured at atmospheric pressure, *per man per minute*. Signals may be made by persons in the bell to those at the pumps and crane by pulling cords and ringing bells.

III. The *Diving Boat* (of which different kinds have been invented by Dr. Payerne and others) is a diving bell on a large scale, conveniently shaped for being moved about, and provided with a magazine of compressed air, contained in a casing surrounding the working chamber or bell. This magazine answers the purpose of the air-bladder of a fish, by enabling those within the bell to make it sink and rise at will; for by injecting water with a forcing-pump into the magazine, the boat becomes heavier, and sinks; and by opening an escape-cock at the bottom of the magazine, the water is forced out by the compressed air, and the boat becomes lighter and rises.

412. **Embanking and Building under Water.** (See also Article 205, p. 344.)—Embankments under water may be made by tipping in the material from a stage supported on posts or on screw piles, or from boats; a moveable inclined plane or shoot being used to direct the material to the spot where it is to fall. Stones and gravel are in general the only materials whose stability can be relied on when exposed to currents in the water; and the diameter of the smallest pieces should not be less than about one twenty-fourth part of the velocity of the current in feet per second. When

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* See description of Heinke's Diving Apparatus, in the *Civil Engineer and Architects' Journal* for September, 1860.

the outside of an embankment is formed with stones, the interior may be filled with smaller and softer materials. In water not agitated by waves an embankment of loose stones will stand at a slope ranging from that of 1 to 1 to that of 2 to 1; but where it is exposed to waves, it must be faced with blocks set by hand, with the aid of diving apparatus, if necessary, the least dimension of any block in the facing being not less than two-thirds of the greatest height of a wave from trough to crest. Further remarks on this will be made in a later chapter.

A loose stone embankment may be protected against waves and currents by means of wooden crib-work.

Hydraulic concrete can be laid under water simply by pouring it into an excavation, or into a space enclosed with a timber or iron casing, the surface of each layer, in deep water, being levelled and smoothed with the aid of diving apparatus. Regular masonry, whether consisting of stones, or of large blocks of hardened concrete, requires the aid of diving apparatus during the whole process of building.

For the facing of sea-works exposed to the action of waves in deep water, such as breakwaters, enormous blocks of hydraulic concrete are sometimes used, measuring from 12 to 27 cubic yards in volume. For the protection of these against the corroding action of sea-water, a method has lately been introduced of coating them all over, to a thickness of about three inches, with asphaltic concrete, composed of two parts of asphaltic mastic (Article 234, p. 376) and three of broken stone. (See a paper by M. Léon Malo, in the *Annales des Ponts et Chaussées*, 1861.)

In ashlar masonry which is to be exposed to very violent shocks from the waves, such as that of lighthouses, the stones, besides being fastened together by metal cramps, are sometimes bonded also by dove-tailing, in the manner shown in plan by fig. 275, which represents part of a course of a lighthouse. This was first practised by Smeaton at the Eddystone lighthouse. Its chief use is to resist the tendency which the stones at the face of a wall have to *jump out* immediately after receiving the blow of a wave. Stones of different courses are sometimes connected by *tabling*, which consists in making flat projections on the beds of the stones which fit into corresponding recesses in the beds of those above and below them.

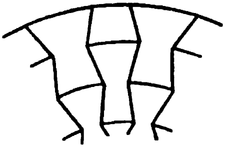

Fig. 275.

PART III.

OF COMBINED STRUCTURES.

CHAPTER I.

OF LINES OF LAND-CARRIAGE.

SECTION I.—*Of Lines of Land-Carriage in General.*

413. **General Nature of Works.**—The works which constitute a line of land-carriage (exclusive of the buildings and machinery by the aid of which the traffic is carried on) may be divided into PERMANENT WAY and FORMATION; the permanent way being that part of the structure which directly bears the traffic, and the formation, the whole of the rest of the works, whose object is to make and preserve a suitable passage for the permanent way across the country. In a restricted sense, the word *formation* or *forming* is applied to the base or surface on which the permanent way directly rests.

As the methods of constructing the works which constitute the FORMATION, in the widest sense, have been described in the preceding part of this treatise, it is only necessary in the present chapter to enumerate them (referring to the places where they are described in detail), and to state the principles according to which they are adapted to particular lines of conveyance. They may be thus classed:—

I. *Earthwork*, consisting of cuttings and embankments, to make passages through hills and over valleys respectively. (See Part II., Chapter II., p. 315.)

II. *Fences.*—As to temporary fences, see Article 189, p. 333. Permanent fences will be again referred to.

III. *Drains*, which are treated of in the same chapter in their relation to earthwork. As to the masonry of large drains, see Article 297 A, p. 433.

IV. *Retaining Walls.*—(See Articles 265 to 275, pp. 401 to 413.)

V. *Level Crossings* of other lines of communication will be again mentioned further on.

VI. *Bridges*, which may be classed according to their purposes, or according to their materials.

The purpose of a bridge may be—

A. To cross over or under some existing line of communication. which it is either impracticable or inexpedient to cross on the level. In this case there are in general certain minimum dimensions fixed by law or by agreement for the passage to be allowed for the existing line, which will be again referred to.

B. To cross a valley, in which an embankment would be impracticable, or too expensive, or otherwise inexpedient. In this case the bridge is called a *viaduct*.

C. To cross a stream, river, estuary, strait, or other piece of water. The principles to be observed in this case, in order that the current may not be impeded, nor the navigation (if any) injured, will be referred to in a subsequent chapter.

The materials of a bridge may be—

a. Masonry or brickwork; as to which, see Part II., Chapter III., p. 349, and in particular, Section VIII., p. 413.

b. Timber; as to which, see Part II., Chapter IV., p. 437, and in particular, Article 336, p. 465, and Articles 341 to 349, pp. 465 to 492.

c. Iron; as to which, see Part II., Chapter V., p. 494.

As to the ordinary *foundations* of bridges, see Part II., Chapter III., Section IV., p. 377; and as to the more difficult kinds of foundations, see Chapter VI., Section II., p. 601.

VII. *Tunnels;* as to which, see Part II., Chapter VI., Section I., p. 588.

The PERMANENT WAY of a line of land-carriage is either a *road*, a *railway*, or a *tramway*; the essential distinctions being that a *road* presents a firm surface of a certain breadth, which can be traversed by vehicles over all its parts and in all directions; a *railway* confines vehicles to certain definite tracks, by means of rails on which specially formed wheels run; a *tramway* is intermediate between these, and consists of flat rails laid on a part of the surface of a road, and so formed that vehicles with wheels suited for an ordinary road can run upon them when required.

414. **Selection of Line and Levels.**—The selection of the position of a line of conveyance depends on statistical and commercial, as well as mechanical considerations; but although the former come frequently under the notice of the engineer, they are foreign to the proper subject of this treatise.

In a purely engineering point of view, the object to be aimed at in laying out the course of a line of communication is to convey the traffic with the least expenditure of motive power consistent with due economy in the construction of the works. Economy of motive power is promoted by low summit-levels, flat "*gradients*" (as the rates of declivity of lines of land-carriage are called), low summit-levels, easy curves, and a direct line; but limitations to the height of summits, the steepness of gradients, and the sharpness of curves, limit also the power of adapting the line to the inequalities of the ground, and so economizing works.

The data required by the engineer in order to enable him to select a line, and the means of obtaining these data, have been stated in Part I., Chapter I., Article 11, p. 9, and further explained in subsequent articles of that part; and as regards borings, pits, and mines, in Part II., Article 187, p. 331, and Articles 391, 392, pp. 589 to 595. The general character of the inequalities of the ground, or "features of the country," and the modes of representing them, have been described in Part I., Articles 58, 59, 60, pp. 93 to 98.

A projected line of communication may either be limited to the connection of two points in the same valley, or it may have to connect points in two or more valleys, by crossing the ridges between them. In the former case, there is no summit-level to cross; in the latter, there may be one or more summit-levels. In general, the best point for crossing a ridge is the lowest *pass* (see Article 58, pp. 94, 95) which occurs in the district to be traversed; but cases may arise in which a higher pass is to be preferred to a lower, because of its being more easily accessible, or because of its offering greater facilities for cutting or tunnelling. The ridge ought to be crossed as nearly as possible at right angles.

When a line of communication has to cross a great valley, the following principles are to be observed as far as practicable:—To choose a narrow part of the valley; to cross the deepest part of it as nearly at right angles as possible; to find, if possible, firm ground for the foundation of a viaduct, or of a high embankment.

The principle of crossing obstacles as nearly as possible at right angles applies to bridges over rivers, and over or under other lines of communication. The cost of a skew bridge increases nearly as the square of the secant of the obliquity.

When a line of communication runs along one side of a valley, the obstacles which it has to cross are chiefly the small branch valleys that run into the main valley, and the promontories or ends of branch ridge-lines that jut out into the main valley between the branch valleys. In this case the greatest economy of works is attained by taking a serpentine course, concave towards the main valley in crossing the branch valleys, and convex towards the main valley in going round the promontories, except where narrow necks in the promontories and narrow gorges in the branch valleys enable a more direct course to be taken with a moderate amount of work.

In ascending the head of a steep valley towards a high pass, it may be sometimes necessary to take a serpentine or even a zigzag course in order to obtain a sufficiently easy gradient, independently of considerations of economy of work. In a few instances a projecting promontory or spur of a mountain has been made available for the ascent of a line of conveyance to a pass, by laying out the line in a spiral course, each coil of the spiral passing first through the promontory by a tunnel, and then winding round outside of it.

It is obviously difficult to lay out a line of conveyance so as at once to accommodate the traffic which passes along the lower part of a large and deep valley, and that which passes over the pass at its head; for in order to reach the summit easily, the line must quit the lower part of the valley at a certain distance from the pass, and ascend gradually along the sides of the hills, so that in some cases a branch line may be required for the lower part of the valley.

In the formation of all lines of conveyance, it is advisable to avoid long reaches of level line in cutting, as being difficult to drain. (See Article 192, p. 335.)

As to crossing a great plain, see Article 203, p. 342. In this case the level of the line of communication is generally fixed so as to be sufficiently high above the highest water-level of floods.

415. The **Ruling Gradient** of a line of communication means the steepest rate of inclination which prevails generally on the line; being exceeded only on exceptional portions, where auxiliary motive power can be provided, or where the loads to be conveyed up the ascent are lighter than on other portions of the line. The economy with which the works can be constructed depends mainly on the steepness admissible for the ruling gradient.

Two things are chiefly to be considered in fixing a ruling gradient: the motive power available in ascending, and the avoidance of waste of power in descending.

Let W denote the greatest gross load to be dragged up an ascent;
f, the proportion of the resistance to the load on a level;
i, the sine of the angle of inclination of the ascent; then

$$(f+i)\,W,$$

is the greatest resistance to be overcome in ascending the ruling gradient; and this should not exceed the greatest tractive force which the prime mover is capable of exerting. Let P be that force; then

$$\left.\begin{array}{l} (f+i)\,W \text{ should not be greater than } P;\text{ or, in} \\ \quad\text{other words,} \\ i \text{ should not be greater than } \dfrac{P}{W}-f. \end{array}\right\}\ldots(1.)$$

The fulfilment of this condition is essential. Another condition, which it is desirable to fulfil, if possible, is, that no mechanical energy shall be wasted through the necessity of using brakes, or of backing the prime mover, in order to prevent excessive acceleration of speed in descending the ruling gradient; and to fulfil this condition

i should (if possible), not exceed f. ....(2.)

The co-efficient of resistance f differs very much for different sorts of permanent way. In each case it consists of two parts; one arising from friction, and constant at all speeds, and another arising from vibration, and increasing with the velocity; so that it may have different values in the formulæ 1 and 2; that in formula 1 corresponding to the *least speed* of ascent consistent with the convenience of the traffic, and that in formula 2 corresponding to the *greatest speed* of descent consistent with safety.

When the traffic is heavier in one direction than in another, the ruling gradient in the direction of the ascent of the lighter traffic may be the steeper.

As a general consequence of these principles, it is obvious that the less the proportion of the resistance on a level to the load, the flatter must be the ruling gradient, and the flatter the ruling gradient is the heavier are the works, and the more difficult is it to lay out the line. Such, for example, is the case with railways, as compared with roads. In railways additional expense and difficulty are occasioned by the necessity of certain limitations as to the sharpness of the curves; but these will be explained in Section IV.

SECTION II.—*Of Roads.*

416. **Resistance of Vehicles and Ruling Gradients.**—The vehicles capable of being used on roads may be distinguished into sledges and wheel-carriages. The only cases in which sledges are suitable vehicles for roads are those in which the surface is either too soft or too steep to admit of the use of wheel-carriages with safety. Their resistance on roads has not been determined precisely by experiment.[*]

The resistance of wheel-carriages on roads consists of a constant part, and a part increasing with the velocity. According to General Morin, its proportion to the gross load is given approximately by the following formula:—

$$f = \{a + b\,(v - 3{\cdot}28)\} \div r; \text{ ....(1.)}$$

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[*] The resistance of an iron-shod sledge on hardened snow is stated by Kossak to be about 1-30th of the gross load.

where r is the radius of the wheels *in inches*, v the velocity in feet per second, and a and b two constants, whose values for carriages with springs are as follows:—

	a	b	f for Wheels of 18 Inches Radius.	
			$v=14·67$	$v=7·33$
For good broken stone roads,	·4 to ·55	·025	·038 to ·046	·028 to ·036
For pavements,……. from	·27	·068	·060	·030.
to	·39	·03	·041	·028.

For carriages without springs, the constant b is about $3\frac{1}{2}$ times greater than for those with springs.

The following table is founded chiefly on experiments by Sir John Macneill:—

	f
Stone pavement,…………………………	1-68th = ·015
Broken stone road on a firm foundation,	1-49th = ·020
Broken stone road on a foundation of flints,………………………………	1-34th = ·029
Gravel road,………………………………	1-15th = ·067
Soft sandy and gravelly ground,………..	1-7th = ·143

Telford estimated the average resistance of carriages on a level part of a good broken stone road at *one-thirtieth* of the gross load; and according to the principle expressed in Article 415, equation 2, he assigned 1 in 30 as the ruling gradient which ought, *as far as possible*, to be adopted on a turnpike road.

If the tractive force which a horse can exert steadily and continuously at a walk be estimated at 120 lbs., the adoption of a ruling gradient of 1 in 30, the resistance on a level being 1-30th of the load, insures that each horse shall be able to draw up the steepest declivity of the road a gross load of

$$120 \times 30 \div 2 = 1,500 \text{ lbs.}$$

A horse can exert, for a short time, an effort two or three times greater than that which he can keep up steadily during his day's work; and thus steeper ascents for short distances may be surmounted.

In the roads laid out by Telford, the ruling gradient of one in 30 is adhered to, wherever it is practicable to do so; and sometimes considerable circuits are made for that purpose. Occasionally, however, he found it necessary to introduce steeper gradients for a short distance, such as 1 in 20, or 1 in 15.

417. **Laying out and Formation of Roads in General.**—Heavy

works of earth and masonry seldom occur on lines of road, which are often, throughout the greater part of their extent, made on the natural surface of the ground. In this case the operation of forming the road consists simply in digging, in ground that is level across, a drain at each side of the road, and in ground that has a sidelong slope, a drain at the uphill side; throwing the earth from the drains on the track of the intended road, so as to raise it slightly above the adjoining ground, and levelling any small inequalities that occur in its course. According to M'Adam,* this is all that is required preparatory to laying the covering or "metal" of the road, even in swampy ground. According to other authorities, it is advisable, in marshy ground, to prepare a foundation for the road by means resembling those employed in embanking over soft ground (Article 204, p. 342); for example, by digging a trench 2 or 3 feet deep, and filling it with clean sand or gravel, as a base for the road; or by spreading a layer of dried peat, or of fascines, so as to form a sort of raft to float on the morass. When *fascines* are used for this purpose, they will rapidly decay unless they are constantly wet. They consist of bundles of twigs, 20 feet long, or thereabouts, and from 9 to 12 inches in diameter, and are laid in layers alternately lengthwise and crosswise, and fastened with pegs, until a bed is formed about 18 inches deep, over which gravel is spread.

418. **Breadth and Cross-section.**—For the ordinary breadth of the carriageway of a turnpike or main road, about 30 feet is a sufficient width, with 5 or 6 feet additional for a footway at one side.

For cross-roads smaller widths are sufficient, such as 20 feet for the carriageway, and 5 feet for the footway.

The widths prescribed by law in Britain for those parts of public roads which are interfered with by railways are as follows:—

Turnpike roads,..................................	35 feet.
Public carriage roads (not turnpike),........	25 „

For the widths of roadways in populous towns and their neighbourhood no general rule can be laid down.

In some of those cases in which the traffic is greatest, the width of carriageway is about 50 feet, with a pair of footways, each from 10 to 15 feet wide.

The carriageway should have a slight rise or convexity in the middle, in order that water may run off it towards the sides; and for that purpose from 4 inches to 6 inches is sufficient. This convexity should be given to the *formation*, so that the thickness of covering may be uniform. The footways should be nearly level with the highest part or crown of the carriageway, and have a very slight slope towards the carriageway.

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* See M'Adam *On Roads*, ninth edition, 1827.

For the form of cross-section of the convexity of the carriageway, Telford recommends a very flat ellipse; but Mr. Walker prefers two straight lines, connected by a short curve at the crown. He advises, also, that every part of a road should, as far as practicable, have a slight declivity longitudinally, to facilitate drainage.

419. **Drainage and Fencing.**—The side-drains, which have already been mentioned, are similar to those of pieces of earthwork, as to which, see Articles 190, 193, 202, pp. 304, 305, 342. A depth of 2 or 3 feet is in general sufficient for them; and when they are open ditches, they may be from 3 to 4 feet wide at the top. If covered, they may consist of earthenware tubes of 6 inches diameter, or thereabouts, on an average, or built culverts of about 12 inches square. Roadways in towns are in general drained into underground sewers.

The *gutters* or *channels* run along each side of the carriageway, and are usually about 3 inches deep. They collect the surface-water from the road, and discharge it into the side-drains through transverse tubes, which pass below the fences and footway.

Mitre drains are small underground tile drains or tubes, diverging obliquely from the centre line of the roadway at intervals of 60 yards or thereabouts, and leading, with a declivity of about 1 in 100, into the side-drains.

In towns the channels discharge their water into the sewer through passages called *gully-holes*, sometimes having horizontal openings covered by gratings, sometimes vertical openings in the curb of the footway. In order to prevent the escape of foul air through them, they are provided with siphon traps, or with valves opening inwards.

When a road is drained by an open ditch, the fence should be between the ditch and the road. The permanent fences of roads are usually either hedges or walls. According to Telford, they should not exceed 5 feet in height, in order that the sun and wind may have free access to the road to dry it.

420. **Broken Stone Roads.**—The true principles of the construction of roads covered with broken stone were discovered by John Loudon M'Adam, and are fully described in his work *On Roads* already referred to.

The stone, or "*road metal,*" should be hard, tough, and durable. (On these points, see Part II., Section I., pp. 349 to 361.) The best materials are granite (p. 355) and trap-rock, or whinstone (p. 356). Hard compact limestone (p. 359) may also be used, and gravel composed of flints (p. 357); but all flints should be broken into angular pieces, as if for making concrete.

The stones are broken down by means of a hammer with a steeled face, into smaller and smaller pieces, until at length they are reduced to pieces roughly approximating to a cubical shape, and not exceeding 6 ounces in weight; which, on an average, is the weight of a cube of stone of 1·6 inches in the side. M'Adam directed each road inspector to carry a small balance, so as to be able to test the weight of a few stones from each heap. Another mode of testing that the stone is broken small enough is by means of a brass ring $2\frac{1}{2}$ inches in diameter, through which no piece should be too large to pass.

Beside breaking all gravel into angular pieces, it should be screened, to clear it of earth.

The road metal, thus prepared, is to be evenly spread over the road with a shovel and rake, in three successive layers of between 3 and 4 inches deep, each layer being left to be partly consolidated by traffic before another is laid; and thus is formed a firm, compact bed of angular fragments of stone about 10 inches thick, which is impervious to water, or nearly so, and which soon acquires a smooth surface.

According to M'Adam, 10 inches is the greatest thickness of metal required for any road, from 5 to 9 inches being often sufficient; and his practice was to lay the metal simply on the natural ground, with no preparation except levelling inequalities and digging drains, as described in Article 417, p. 625.

According to the practice of Telford, before laying down the metal, a foundation or "*bottoming*" is laid, consisting of pieces of durable, but not necessarily hard stone, measuring from 4 to 7 inches in each dimension. The largest of those pieces are set by hand, with their largest sides resting on the formation, and between these the smaller pieces are packed, so as to form a compact layer about 7 inches deep in the centre of the road, and 4 inches deep at the sides, part of the convexity being made in this manner. Above this bottoming the metal is spread as already described.

A broken stone road is repaired by slightly loosening the surface with a pick, and spreading uniformly over it a layer of metal. Unless the surface is first loosened, the new metal will not *bind* or consolidate with the old. The practice of repairing roads by patches, called "darning," is bad.

In order to make the traffic on a broken stone road easier when it is first laid, a layer of sand and gravel, called "*blinding*," is sometimes spread over it; but this practice is a bad one, for the sand and gravel work their way between the fragments of stone, and prevent their ever forming so compact a mass as they ought to do.

When mud forms on the surface of a road, it is to be removed by scraping; but if the road is well made of good materials, little of that work is required.

Wheels of small diameter are the most destructive to roads.

According to Telford, the load on a broken stone roadway ought not to exceed one ton on each wheel: the tire of the wheel, for a load of one ton, being four inches broad. The limitation as to load agrees with general practice; but the breadth of four inches for a load of one ton per wheel appears to be only necessary for vehicles without springs; for those properly provided with springs, a breadth of from 2 to 2½ inches is sufficient under any load of ordinary occurrence, provided they are to run on firm and compact roadways only. On soft and loose roadways an additional breadth of wheel prevents the resistance from being so great as it would be with narrow wheels. The consolidation of a broken stone road may be hastened by rolling it with a cast iron roller weighing from 1 to 3 tons if drawn by horses; in France, steam-rollers weighing 10 tons have lately been introduced. (See *Annales des Ponts et Chaussées*, 1861.)

421. **Stone Pavements.**—The *foundation* of a stone pavement may consist either of a layer of hydraulic concrete, or of rubble masonry set in hydraulic mortar, from 6 to 9 inches deep;

Or of three successive layers of broken stone road metal, each about 4 inches deep, consolidated by allowing the traffic to run upon them for a time;

Or of three well-rammed layers of gravel, each 4 inches deep, with a layer of sand about 1 inch deep on the top.

The best materials for stone pavements are syenite and granite, the hardest that can be found; and the next, trap or "whinstone." Stones of a laminated structure are to be avoided if possible; and should it be absolutely necessary to use them, they are to be set with their beds or laminæ on edge.

Paving-stones should be roughly squared, special care being taken that they do not taper downwards. They are to be set in regular courses, running across the roadway, and breaking joint with each other. In order that the stones may not tend to cant or tilt over, their depth in a vertical direction should be somewhat more than double their horizontal breadth: for the same reason, the length should be equal to the depth, or not much greater. The dimensions usually adopted are—

Breadth (in a direction along the roadway),......	4 inches.
Depth (in a vertical direction),......................	9 „
Length (in a direction across the roadway),...........................	from 9 to 12 „

Paving-stones are sometimes made to taper slightly *towards the top*, so that their joints are close below, and open to the extent of an inch or thereabouts above, the wedge-formed spaces thus left being filled with gravel, or chips of stone, imbedded in bituminous cement. (Article 234, p. 376.) This gives a more* secure footing to horses than a close-jointed pavement.

Small pieces of granite, nearly cubical, and measuring about 4 inches each way, have been used at Euston Square Station. They rest on a layer of sand 1 inch deep, and three layers of gravel mixed with chalk, each 4 inches deep, and are set as close as possible.

Paving-stones are rammed into their places with a wooden rammer or beetle, weighing about 55 lbs. A small steam-hammer has been sometimes used for this purpose.

They may be covered, when first laid, with a blinding of sand and fine gravel, about an inch or an inch and a-half deep, to fill the joints by degrees.

Their joints may be made water-tight by being laid in cement or hydraulic mortar; or in iron-turnings, which rust, and make a sort of cement with the sand and gravel of the blinding that works its way into the joint; or in a bituminous cement, or by being *grouted* with hydraulic lime in a semi-fluid state after being laid.

Rubble or *Boulder Pavement* consists of stones of irregular shapes set in a bed of sand or gravel. It causes great resistance to vehicles, is liable to irregular sinking, and requires frequent repair.

The chief disadvantage attending the use of well-made stone pavement in towns is its liability to be disturbed for the purpose of laying gas and water-pipes and small sewers. One method of obviating this is to provide "*side-trenches*" to contain those underground works, being narrow excavations lined at the sides with brick walls, and situated under the outer edge of the foot-pavement, by the flags of which they are covered. The wall of the side-trench next the roadway is strengthened against the pressure of the earth by means of transverse walls, with openings in them for the passage of sewers and pipes; and between those transverse walls the longitudinal wall is slightly arched horizontally, like the retaining wall in fig. 176, p. 412. The other longitudinal wall of the side-trench forms the back of a row of cellars under the foot-pavement. The side walls of the cellars are in a line with the transverse walls of the side-trench, and act as buttresses to give it stability. In an example given in Mr. Newlands's Report* for 1848, the side-trench is 13 feet deep from surface of footway to foundation, 2½ feet wide inside, and has cross walls at every 7 feet; the brickwork is one brick, or 9 inches thick. It contains an oval sewer-pipe of 27 × 18 inches, a 10 inch water-pipe, and a 10 inch gas-pipe. Sewers which are large enough to be traversed by men may be repaired by getting access to them through subterranean passages leading into them from trap-doors in the foot-pavement.

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* See *Reports of the Borough Engineer of Liverpool* (Mr. James Newlands).

Another method of obviating the necessity for raising the pavement of the carriageway is to have a "*sub-way*" or tunnel under the street, containing the sewer and the gas and water-pipes. This method has hitherto been tried for short distances only.

422. **Footways of Roads.**—The dimensions and figure of footways have been treated of in Article 418, p. 625. In country roads the principles of their construction are the same with those of a broken stone road, except that smaller and less hard materials are used, such as gravel, engine ashes, slag broken small, fragments of burnt clay, &c.; and that from 2½ to 4 inches is a sufficient thickness. The footway should have a declivity of about 2 inches towards the channel, its lowest edge being not more than 9 inches above the bottom of the channel, and its side towards the channel being formed either by a slope of from 1 to 1 to 1½ to 1, or by a *curb-stone* set on edge, from 4 to 6 inches thick. To consolidate footways, a cast iron roller may be used, weighing from ¼ to ½ a ton.

In streets the footways have a foundation of concrete, broken stone, gravel, or sand, and are covered with flagstones, usually from 1½ to 4 inches thick, being thinnest for the strongest material. The best materials are those which are hardest, toughest, and least pervious to water, such as hornblende slate, the harder kinds of clay slate, and gneiss; strong sandstone and compact limestone rank after these; soft, brittle, and porous stones are unsuitable.

423. **Bituminous or Asphaltic Pavements** consist of a thin layer of what has been described in Article 234, p. 376, as "Bituminous Concrete," laid on a foundation of broken stone. The formation of the roadway has a convexity of 1-100th of the breadth.

The foundation consists of road metal, as described in Article 420, p. 626, laid in a layer of 4 inches deep for the carriageway, and 2 inches deep for the footway, and consolidated with a rammer of 55 or 56 lbs. weight, or with a cast iron roller.

The covering, which is about 1½ inch thick for a carriageway, and ¾ inch thick for a footway, consists of a mixture of road metal or gravel and "bituminous mortar." The proportions of its ingredients have been given in Article 234, p. 376. The order in which they are to be combined is the following:—Having melted the bitumen, add the asphalt broken small, then the resin oil, then the sand, and lastly the broken stone. To test the composition, a specimen of it is cooled in water to the temperature of about 80°; a piece of plank, having two four-sided pyramidal points of iron on the under side, is laid with one point resting on a formerly-tried standard sample, and the other on the new sample; a man stands on the middle of the plank, when the impressions on the standard sample and new sample should be of equal depth. That depth should be about 3-10ths of an inch for carriageways and 2-10ths for footways, the latter requiring the stronger material. Should it prove too hard, bitumen and resin oil are to be added; should it prove too soft, asphalt and sand. The covering is laid on the roadway in the hot state, in rectangular sections; its surface is sprinkled with sand, and the surplus sand swept off, and it is then left to cool. No artificial asphalt is equal to natural asphalt for making roads.

To make bituminous roadways cold, asphalt is to be broken as for road metal, spread about 2 inches deep, wet all over with coal-tar, and rammed with a 56 lb. beetle.

To repair the surface of a bituminous roadway, dissolve one part of bitumen in three of pitch oil or resin oil; spread 10 ounces of the mixed oil over each square yard of roadway, and sprinkle on it 2 lbs. of asphalt in powder; then sprinkle the surface with sand, and sweep away the loose sand.

Good bituminous pavements under constant traffic should wear at the rate of about 1-40th of an inch a-year.

424. **Plank Roads** are useful in newly settled countries in which timber is abundant.

The *formation* of a plank road is made by digging two parallel ditches about 16 feet clear apart, and throwing the earth dug out on the space between, so as to raise it slightly. One-half of the track thus formed—that is to say, a breadth of 8 feet—is left with an earthen surface, sloping towards the nearest ditch with a fall of 6 inches. On the other half, the planked covering, also 8 feet broad, is made in the following manner:—Imbedded in the ground are two parallel lines of longitudinal wooden sleepers, 4½ feet apart from centre to centre. Each sleeper is from 10 to 20 feet long, 12 inches broad, and 4 inches deep, or thereabouts; under the joints of those sleepers are laid short connecting sleepers, of the same scantling and 3 feet long. The earth is well rammed between and beside the sleepers, until it is flush with their upper sides, across which are laid the planks, 8 feet long and 3 inches thick. The edge of the planking next the earthen division of the roadway should not be straight and smooth, but should have alternate projections and recesses of about 3 feet long and 3 inches deep, to give a hold for carriage wheels, so that they may easily be drawn on to the planking from the soft part of the road. The planked part of the roadway is broad enough for one carriage only; and when two carriages meet or pass each other, the lighter must make way for the heavier by moving to the earthen division of the road.

425. **Wooden Pavement** consist of prismatic blocks of wood, rectangular or hexagonal, and about 6 inches deep, set with the fibres vertical, upon a firm foundation of broken stone. It was at one time much used in Britain, especially in London, but was abandoned on account of its slippery surface and rapid decay.

425 A. **Cast Iron Pavements** made of plates have been tried, but have proved very dangerous to horses. A pavement consisting of hexagonal cast iron cells filled with gravel is now under trial.

SECTION III.—*Of Tramways.*

426. **Stone Tramways.**—The best kind of stone tramway consists of a pair of parallel ranges of oblong blocks of granite, about $4\frac{1}{2}$ feet apart from centre to centre, with their upper surfaces forming part of the surface of a road, each block being from 2 to 4 feet long, about 10 or 12 inches broad, and of the same depth with the rest of the covering of the roadway. The upper surfaces of these stone tram-rails are slightly hollowed towards the middle of their breadth, to the depth of half an inch, or thereabouts, in order that carriage-wheels may tend to remain on them.

427. **Iron Tramways,** as formerly made, consisted of flat bars of iron for the wheels of carriages to run upon, with a rising ledge or flange at one side, to prevent the wheels from leaving the track. The iron tramways now in most common use are those which, like the stone tramways of the last article, form part of the surface of a road or street. The tram-rails used for this purpose are oblong plates of cast iron, 3 or 4 feet long; they are usually 7 or 8 inches broad, and about $1\frac{1}{2}$ inch thick; they are slightly concave towards the middle of their breadth, like stone tram-rails; and at each joint the ends of two adjoining plates rest on a square plate of cast iron of the same thickness. A combination of the tram-rail and edge rail, lately introduced, will be mentioned further on.

SECTION IV.—*Of Railways.*

428. The **Resistance of Vehicles on a Level** is here considered in so far only as it affects the ruling gradient. The resistance to the motion of a loaded carriage on a level railway is nearly proportional to the gross load, and its proportion to that load may be expressed either as a fraction, or as a certain number of lbs. per ton, the latter being a common mode of expressing it for practical purposes.

Let f be the proportion of the resistance on a level to the gross load, expressed as a fraction; then

$$\text{resistance in lbs. per ton} = 2{,}240 f \text{ ....(1.)}$$

It is true that the part of the resistance which is due to the displacement and friction of the air must depend, not on the load, but on the dimensions and figures of the vehicles; but our experimental knowledge of the laws of the resistance of the air to bodies so large as railway carriages is scarcely sufficient to enable us to calculate that resistance separately with such precision as to make the result of the computation practically useful.*

The co-efficient of resistance on a level, f, consists of two parts; one representing the effect of friction, which is independent of the speed; the other representing the effect of concussion and of the resistance of the air, which increases with the speed. The law according to which the latter part of the co-efficient of resistance increases is still uncertain, owing to the irregularities of the results of experiment. According to one formula (founded on experiments by Mr. Gooch), it is insensible up to a speed of about 10 miles an hour, and then increases nearly in the simple ratio of the excess of the speed above that limit. According to another formula (that of Mr. D. K. Clark), it is nearly proportional to the square of the speed; and both those formulæ agree, in a rough way, with experiment.

The following are the formulæ in question, in each of which V denotes the velocity in miles an hour:—

$$\text{Co-efficient of resistance, } f = \cdot 00268 \left(1 + \frac{V - 10}{20}\right); \text{ (2.)}$$

$$\text{Resistance in lbs. per ton, } 2{,}240 f = 6 \left(1 + \frac{V - 10}{20}\right); \text{ (2 A.)}$$

$$\text{Co-efficient of resistance, } f = \cdot 00268 \left(1 + \frac{V^2}{1{,}440}\right); \text{...(3.)}$$

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* The following are two alternative formulæ by the late Mr. Wyndham Harding and Mr. Scott Russell, in which separate expressions are given for the resistance of the air. In the first formula that resistance is assumed to be proportional to the area of frontage of the train; in the second, to its volume.

T denotes the weight of the train, *in tons.*
V, its velocity, in miles an hour.
A, its area of frontage, in square feet.
B, its volume, in cubic feet; then

$$\text{resistance in lbs.} = \left(6 + \frac{V}{3}\right) T + \frac{V^2 A}{400}; \text{ or}$$

$$= \left(6 + \frac{V}{15}\right) T + \frac{V^2 B}{50{,}000}.$$

$$\text{Resistance in lbs. per ton, } 2{,}240\, f = 6 + \frac{V^2}{240}. \quad (3\ \text{A})$$

Carriages have been made and used in which the co-efficient of resistance was as small as ·002, or about 4½ lbs. per ton, at velocities not exceeding 12 miles an hour, the resistance being sensibly constant at such velocities.*

The formulæ 2, 2 A, 3, 3 A, are applicable to good railway carriages with springs, in trains drawn by an engine at an uniform speed on a well-made line, in good repair, with easy curves, and in moderately calm weather; the experiments on which they are founded having been made under those circumstances, and the resistance determined by means of a dynamometer between the engine and the train.

Another mode of determining the resistance of a carriage on a railway is to start it off at a considerable speed, and allow it to come gradually to rest by its own resistance; but in this mode of experimenting, although the friction is the same as in the other mode, the resistance arising from concussion is considerably less, because much of the vibration originates with the engine.†

The absence of springs augments that part of the resistance which increases with the velocity; but wagons without springs are used only at very low speeds.

The following are some examples of resistances per ton at different speeds, calculated by the two formulæ respectively:—

Speed in miles an hour, $V =$	10	15	20	30	40	50	60
f by equation 2,...............	·00268	·00335	·00402	·00536	·00670	·00804	·00938
2,240 f by equation 2 A,....	6	7½	9	12	15	18	21
f by equation 3,...............	·00287	·00310	·00342	·00435	·00565	·00733	·00938
2,240 f by equation 3 A,....	6·4	6·9	7·7	9·7	12·7	16·4	21

Mr. D. K. Clark considers that his experiments indicate that the resistance on a level, given by equations 3, 3 A, is liable to be

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[*] See Rankine *On Cylindrical Wheels on Railways*; also Wood *On Railroads*.

[†] To ascertain the resistance of a vehicle by experiments on its gradual retardation, stones or other marks are to be dropped from the carriage at equal intervals of time (say of t seconds each), and the distances between those successive marks measured.

Let x_1, x_2, x_3, x_4, &c., be those distances in feet. Then the velocities at the end of

t, $2\,t$, $3\,t$, &c., seconds, are nearly,

$$v_1 = \frac{x_1 + x_2}{2\,t},\ v_2 = \frac{x_2 + x_3}{2\,t},\ v_3 = \frac{x_3 + x_4}{2\,t},\ \text{\&c., in feet per second.}$$

Let v_n and v_{n+1} denote the velocities at the end of $n\,t$ and $(n+1)\,t$ seconds respectively. Then the co-efficient of resistance at the end of $n\,t$ seconds is nearly,

$$f = \left\{(v_n - v_{n+1}) \div 32{\cdot}2\right\} \mp i \text{ according as the gradient is } \begin{cases}\text{ascending}\\ \text{descending.}\end{cases}$$

exceeded in the following proportions, from various occasional causes:—

From a road ill laid, or in bad repair, ...	40	per cent.
From resistance on curves, ...	20	„
From strong side winds, ...	20	„
Total, ...	80	„

It may be held, however, that all those causes of increased resistance are seldom combined at one time and place; and that 50 per cent. is a liberal allowance for *contingent resistances.*

The friction of good ordinary mineral wagons, at low speeds, may be estimated as ranging from

8 lbs. per ton, or ·00353
to 10 lbs. per ton, or ·00446

and as being on an average about

9 lbs. per ton, or ·00402, or 1-250th nearly.

429. **Proportion of Gross to Net Load.**—In the following statement the ordinary proportions of the weight of goods and mineral wagons to the loads which they carry are given on the authority of Mr. D. K. Clark; and from those proportions are deduced the proportions of gross to net load in goods and mineral trains:—

	Wagon ÷ Net Load.	Gross Load ÷ Net Load.
Well made open wagons, ...	$\frac{1}{2}$	$1\frac{1}{2}$
Well made covered wagons, ...	$\frac{3}{4}$	$1\frac{3}{4}$
Clumsy wagons, ...	1	2

In computing the gross load to be drawn behind a locomotive engine which has a tender, the weight of the tender (from 10 to 15 tons) is to be added to that of the wagons and their load.

Passengers without luggage may be estimated at about 15 or 16 to the ton, and with luggage, about 10 to the ton (but this last is an uncertain estimate). In a passenger train the gross load may be roughly estimated at about three times the net load, with carriages suited for locomotive railways and high speeds, weighing when empty 5 or 6 tons for a carriage capable of carrying 20 or 30 passengers. In light carriages on horse-worked railways the gross load needs not exceed double the net load.

430. The **Tractive Force** which the prime movers on railways exert will here be considered so far as it is connected with the question of gradients. The prime movers commonly employed on

railways are, gravity, horses, fixed steam engines, and locomotive steam engines. The strength of men and the force of the wind have also been employed, but in isolated experiments only.

I. *Gravity* either assists or opposes the other kinds of motive power on all inclined parts of a railway. It may act as the sole motive power on a descending gradient that is sufficiently steep.

The only case in which gravity acts as a *tractive force* on a railway is that of a "*self-acting inclined plane*," on which a train of loaded wagons descending draws up a train of empty wagons. Let i be the sine of the inclination of the plane, f the co-efficient of resistance of the wagons, T the weight of a train of empty wagons, W the net load of a train. Then the available tractive force at an uniform speed is

$$(i - f)\,(\mathrm{T} + \mathrm{W}).$$

The rope, according to present practice, is of iron wire, and usually endless, and lies on a series of sheaves or pullies 7 yards apart. The weight of each sheave is between 20 and 30 lbs.; the weight of the rope (allowing 6 as the factor of safety) *per foot of its length* should be 1-4500th of the greatest working tension. Let R be the weight of the rope and pullies; their total resistance is usually estimated at about 1-20th of their weight, and the resistance of the train of empty wagons is $(i + f)$ T. In order that the tractive force may simply balance the resistances, we must have

$$(i - f)\,(\mathrm{T} + \mathrm{W}) = \frac{\mathrm{R}}{20} + (i + f)\,\mathrm{T}\,; \;\ldots\ldots\ldots\ldots(1.)$$

and the inclined plane will not work unless the inclination is steeper than that given by solving the above equation; that is to say,

$$i \text{ must be greater than } \left\{\frac{\mathrm{R}}{20} + f\,(\mathrm{W} + 2\,\mathrm{T})\right\} \div \mathrm{W}\ldots.(2.)$$

Assume $f = \cdot 004$, $\mathrm{T} = \mathrm{W} \div 2$; then

$$i \text{ must be greater than } \left\{\frac{\mathrm{R}}{20\,\mathrm{W}} + \cdot 008\right\}\ldots\ldots(2\,\text{A.})$$

The excess of steepness above this limit causes an excess of tractive force above resistance, which produces accelerated motion. The acceleration may be allowed to go on so long as the velocity does not exceed a safe limit; so soon as that limit has been attained, the surplus tractive force must be counteracted by the use of the brake, so that the tractive force and increased resistance may exactly balance each other, and the velocity become uniform.*

II. *Horses.*—An animal produces its greatest day's work when working for eight hours per day, and with a certain definite speed and tractive force.

Let P_1 denote the tractive force corresponding to the greatest day's work;
P, any other tractive force;
V_1, the speed corresponding to the greatest day's work;
V, any other speed;
T, the time, in hours per day, during which the exertion is kept up.

Then the following formula is approximately true, for efforts and speeds not greatly differing from P_1 and V_1, and for times not greatly exceeding eight hours per day:—

$$\frac{P}{P_1} + \frac{V}{V_1} + \frac{T}{8} = 3. \qquad \text{.......................(3.)}$$

For a good average draught horse the following data are nearly correct:—

$P_1 = 120$ lbs.
$V_1 = 3{\cdot}6$ feet per second, or about $2\frac{1}{2}$ miles an hour.

For a high-bred horse of average strength and activity it is difficult to assign the values of P_1 and V_1, for want of sufficient data. The following values agree in a general way with some of the results of experience in the traction of stage coaches and of light railway carriages:—

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* It is seldom necessary to enter into detailed calculations as to the effect of acceleration on a self-acting inclined plane. It may sometimes, however, be desirable to do so, where the declivity is so slight that there is a doubt whether the velocity attained will be sufficiently great to enable a pair of trains to traverse the inclined plane without inconvenient delay.

To find the time occupied in traversing the plane unimpeded, let M denote the total weight of the rope and sheaves, and of both trains, together with one-half of the weight of the pullies. Let F denote the *excess* of the tractive force above the resistance. Let L be the length of the plane; then

$$\text{time in seconds} = \sqrt{\frac{L}{16}\frac{M}{F}} \text{ nearly.}$$

The mean velocity may of course be found by dividing L by this time and the greatest velocity acquired is double of the mean velocity.

$P_1 = 64$ lbs.
$V_1 = 7{\cdot}2$ feet per second, or about 5 miles an hour.

The following are examples:—

T, hours per day,.........	4	4	4	1	1	1	1
V, miles an hour, ...	5	$7\frac{1}{2}$	10	5	$7\frac{1}{2}$	10	$12\frac{1}{2}$
P, tractive force, $= 64\left(3 - \frac{T}{8} - \frac{V}{5}\right)$,...	96	64	32	120	88	56	24

It may be observed that the preceding data and calculations have reference to *average* speeds, and that the horse may occasionally be required to exert from once and a-half to double the efforts above stated, provided that he is allowed to slacken his speed during the increased effort, and that the additional exertion is kept up for a short time only.

III. *Fixed Steam Engines* are employed for the most part on short distances, where the speed is moderate and the inclination steep. Their power is usually applied to an endless rope *running* on sheaves, like that of a self-acting inclined plane (p. 636). The steam engine is placed at the top of the ascent, and drives a large horizontal cast iron pully, from 5 to 10 feet in diameter, having three or four grooves in its rim. This is called the *driving pully*. At a short distance in front of that pully (that is, in the down-hill direction) is a pully one or two feet smaller in diameter, and with one groove fewer in its rim. This is called the *straining pully:* it rests on a small four-wheeled truck, and is pulled away from the driving pully by a chain and weight, the weight being sufficient to give the requisite tension to the rope, which is carried round the grooves of the two pullies. At the foot of the inclined plane the rope passes round a third horizontal pully, as large as the driving pully.

The engine works to the best advantage, and the rope is least strained, when one train is ascending and another descending at the same time.

The greatest tension on the rope is found as follows:—

Let P denote the greatest tractive force required to overcome gravity, and the friction of the train, rope, and sheaves, calculated as in p. 636. About *one-third* of this will be the tension required at the descending side of the rope, to give sufficient "*bite*" or adhesion between it and the driving pully, so that the greatest working tension at the ascending side of the rope will be about.

$$1{\cdot}33\ \mathrm{P};^{*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

and its weight per foot, if it is made of strong charcoal iron wire, should be 1-4500th of this. The pull upon the axis of the straining pully should be about

$$2{\cdot}74\ \mathrm{P}.^{*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

To find the indicated horse-power of the engine, let v be the velocity of the rope in feet per second (= velocity in miles an hour × 1·466); then

$$\mathrm{I\cdot H\cdot P\cdot} = \frac{1{\cdot}25\ \mathrm{P}\,v}{550} = \frac{\mathrm{P}\,v}{440}; \ldots\ldots\ldots\ldots (6.)$$

the multiplier 1·25 being introduced on the supposition that the friction of the steam engine wastes one-fifth of the indicated power.

Another mode of transmitting the power of the fixed engine to the train is to employ the engine, by means of pumps or of a fan, to exhaust the air from a tube, along which a piston is propelled towards the engine-station, by the excess of the atmospheric pressure behind it above the diminished pressure in front of it. For details as to this mode of propulsion, see *Proceedings of the Institution of Civil Engineers* for 1844. It was practised for some years on some short lines of railway, the tube being about 15 inches in diameter, but was eventually given up in favour of locomotive engines. Its use has of late been revived for the purpose of conveying parcels through tubes.

IV. *Locomotive Engines.*—The tractive force of a locomotive engine is in general limited, not by the power which the engine is capable of exerting—for that is almost always more than sufficient to draw any load that it ever has to convey—but by the "*adhesion*," as it is called, or force which prevents the driving wheels from slipping on the rails.

The adhesion is equal to the weight which rests on the driving wheels, multiplied by a co-efficient which depends on the condition of the surface of the rails; being greatest when they are clean and dry, and least when they are wet and greasy, or covered with ice.

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[*] These calculations are made on the supposition that the co-efficient of friction between the wire rope and the driving pully is ·15, that there are three grooves in the driving pully, and that the tension is made just sufficient to prevent slipping. In practice, however, it is not uncommon to strain the rope till the tension at the descending side is equal to the tractive force; and in that case

greatest tension at the ascending side = 2 P;
pull on the axis of the straining pully = 5·7 P.

On an average, the adhesion of a locomotive engine may be estimated at about one-seventh of the load on the driving wheels; for by sprinkling sand on the rails when they are slimy, or if they are icy, directing jets of steam on them, it may in general be prevented from falling below that amount.

In order that the rails may be able to bear the load on the driving wheels without damage, it is considered advisable that the load *on each wheel* should not in ordinary cases exceed 5 tons = 11,200 lbs. According to this rule the limits of load on the driving wheels, and of tractive force, are

		Load on Driving Wheels.		Adhesion.
(1.)	For engines with one pair of driving wheels,...........	10 tons	= 22,400	3,200
(2.)	„ two pairs of driving wheels, coupled,	20 „	44,800	6,400
(3.)	„ three pairs of driving wheels, coupled,	30 „	67,200	9,600
(4.)	„ four pairs of driving wheels, coupled,	40 „	89,600	12,800

Locomotive engines are seldom made with fewer than six wheels. Those which are intended for the propulsion of comparatively light trains at high speeds have but one pair of driving wheels of from 5½ to 7 feet, and sometimes, on the broad gauge, 8 feet in diameter. The best position for the shaft of that pair of wheels is nearly under the centre of gravity of the engine, in which case, by proper adjustment of the springs, it can be made to bear any proportion of the weight from ⅓ to ½. In Mr. Crampton's form of engine, however, that shaft is placed altogether behind the boiler of the engine, in order to obtain a large diameter of wheel along with a low centre of gravity. Engines for goods trains of moderate weight have also usually six wheels, two pairs of which are driving wheels, of 5 feet diameter, and are coupled together. For heavier goods and mineral trains engines are used having all six wheels coupled, and usually of smaller diameter, such as 4½ or 4 feet, and in some cases of engines for ascending steep inclined planes, 3½ or 3 feet. According to Mr. D. K. Clark, the usual weights of locomotive engines are as follows:—

ENGINES WITH SEPARATE TENDERS.

(*The Tender weighs from* 10 *to* 15 *tons.*)	Tons.
Narrow gauge passenger locomotives, six-wheeled, with one pair of driving wheels,	19 to 23
Do. do. do. unusually heavy,	24 to 27
Broad gauge passenger locomotive, eight-wheeled, with one pair of driving wheels 8 feet in diameter,..............................	35
Goods locomotive, from four to six wheels coupled,..	27 to 32

TANK ENGINES, CARRYING THEIR OWN FUEL AND WATER.	Tons.
For light traffic on branch lines,..................	12 to 20
For heavy traffic on steep inclined planes,......	40

In comparing the tractive force of a locomotive engine, as limited by adhesion, with the resistance and gravity of the train which it is to draw, it is obvious that the resistance due to friction and concussion *of the engine itself* is to be left out of account; for that resistance does not constitute a backward pull on the engine, tending to make the driving wheels slip.

It appears, then, that the *available tractive force* of a locomotive engine in ascending a given inclined plane, which must be at least equal to the resistance of the heaviest train that it has to draw, is to be found by subtracting from the adhesion that component of the weight of the engine which acts as a resistance to its ascent; that is to say,

Let E denote the total weight of the engine;
q E, that part of the weight which rests on the driving wheels;
i, the sine of the inclination of the railway;
P, the available tractive force; then

$$P = \left(\frac{q}{7} - i\right) E \text{ ....(7.)}$$

The following are examples:—

		q.	$\frac{q}{7} - i$.
Passenger engines, one pair of driving wheels,..................................	from	·33	·048 — i
	to	·5	·071 — i
Goods engines, two pairs of wheels coupled,................................	from	·67	·095 — i
	to	·75	·107 — i
Goods engines, all wheels coupled,.............		1	·143 — i

The weight of the tender has in some instances been made to produce adhesion, and increase the available tractive force, by communicating motion from the hindmost axle of the engine to those of the tender by means of gearing-chains and pullies; but this method is suitable to low speeds only.

431. **Ruling Gradients of Railways.**—The general nature of a ruling gradient, and of the principles according to which it is determined, have been explained in Article 415, p. 622.

Self-acting inclined planes and *fixed engine inclined planes* are exceptional cases, which are not comprehended under the general principles according to which ruling gradients are determined.

Horse-power is applicable to lines of short length and light traffic only. The tractive force which a horse can exert under various circumstances has been stated in Article 430, Division II., p. 637. The mean resistance of the goods and mineral wagons on a level may be taken at 1-250th of the gross load, or 9 lbs. per ton; and if the passenger carriages are carefully constructed, in a manner specially suited to the traffic, their resistance may be taken at 1-500th, or $4\frac{1}{2}$ lbs. per ton, on a level straight line, and ·00268, or 1-373d, or 6 lbs. per ton, on a level line with a moderate proportion of curves in its course. It appears from experience that gradients of from 1 in 100 to 1 in 70 may be surmounted without auxiliary power, provided they do not extend to a distance of more than two miles, or thereabouts, at a stretch, and that the horse is not urged to a higher speed than he naturally assumes; but for longer ascents it is advisable, if possible, to limit the steepness to 1 in 200. On ascents of from 1 in 50 to 1 in 40, or steeper, either the load drawn by one horse on other parts of the line should be divided between two, or an auxiliary horse should be harnessed to each carriage or train, and the speed should not exceed a walking pace. Steep ascents for very short distances may sometimes be surmounted by taking a "race" at them.

In fixing the ruling gradient of a *locomotive* railway, it is not to be supposed that rules deduced from the general principles already explained are to be held as absolutely binding. Their proper use is to guide the engineer, when no cause exists sufficient in his judgment to warrant a deviation from them.

This being understood, it appears that there are four things to be adapted to each other,—the greatest load of a train, the least speed of conveyance in ascending declivities, the description of engine, and the ruling gradient; that is, the steepest gradient up which the ordinary traffic is conveyed by the ordinary engines of the line, without the aid of auxiliary engines specially adapted to steep inclined planes. The adaptation of those four things to each other is indicated by the following equation, in which Mr. Clark's formula, Article 428, equation 3, p. 633, is adopted for the resistance of the train:—

Let E denote the weight of the engine (see Article 430, p. 640).

q E, the part of that weight which rests on the driving wheels (see Article 430, p. 641).

T, the gross weight of the train and tender (if there is a tender). As to the proportion of gross to net load, see Article 429, p. 635; as to the weight of the tender, see Article 430, p. 640.

V, the least speed in miles per hour at which the conditions of the traffic will admit of the ruling gradient being ascended.

i, the sine of the ruling gradient $\left(\text{whose inclination in ordinary terms will be described as 1 in } \frac{1}{i}\right)$; then

$$\left(\frac{q}{7}-i\right) \mathrm{E}=\left\{ \cdot 00268\left(1+\frac{\mathrm{V}^2}{1,440}\right)+i\right\} \mathrm{T}. \ldots\ldots (1.)$$

From this equation are deduced the following formulæ: given q, i, V, to find the ratio of the weight of the engine to that of the train and tender; also the reciprocal of that ratio:—

$$\mathrm{E} \div \mathrm{T}=\frac{\cdot 00268\left(1+\frac{\mathrm{V}^2}{1,440}\right)+i}{\frac{q}{7}-i}; \ldots\ldots\ldots\ldots (2.)$$

$$\mathrm{T} \div \mathrm{E}=\left(\frac{q}{7}-i\right) \div\left\{\cdot 00268\left(1+\frac{\mathrm{V}^2}{1,440}\right)+i\right\}; \quad (3.)$$

given E, q, T, i, to find the speed of ascent V;

$$\mathrm{V}=733 \sqrt{\left\{\left(\frac{q}{7}-i\right) \frac{\mathrm{E}}{\mathrm{T}}-\cdot 00268-i\right\}}; \ldots\ldots (4.)$$

given E, q, T, V, to find the ruling gradient i;

$$i=\left\{\frac{q\,\mathrm{E}}{7}-\cdot 00268\left(1+\frac{\mathrm{V}^2}{1,440}\right) \mathrm{T}\right\} \div(\mathrm{E}+\mathrm{T}). \quad (5.)$$

According to Mr. Clark's allowance of 50 per cent. for occasional or contingent resistances referred to in Article 428, p. 635, ·00402 may occasionally have to be substituted for ·00268 in the preceding formulæ.

The following may be taken as examples of the results of such computations, the formula employed being equation 3:—

	Example.	I.	II.	III.	
	Speed in miles an hour,	24	18	12	
		Tons.	Tons.	Tons.	
	Weight of engine,	20	30	30	
	Number of driving or coupled wheels,	2	4	all	
		Tons.	Tons.	Tons.	
	Load on driving or coupled wheels,...	10	21	30	
	Weight of tender,	10	12	15	
Ascending gradient.	1 in 50,	33	91	145	Gross load of train.
	1 in 80,	63	154	238	
	1 in 100,	79	191	293	
	1 in 133·3,	104	245	373	
	1 in 200,	142	332	505	

The thing to be principally considered in fixing a ruling gradient is the traffic. This having been ascertained, so as to determine the probable gross load of the several descriptions of passenger and goods trains, and the speed at which they are to run up the steepest parts of the line, the ruling gradient is to be fixed so that engines with not more than about 5 tons of load on each wheel may be able to draw the trains.

It is in general bad economy to incur heavy works in order to ease the ruling gradient, merely for the sake of enabling light engines to convey a heavy traffic; but where the traffic is light, and moderate gradients can be obtained without heavy works, light engines may be used with advantage.

431 A. The **Action of Brakes** may have to be considered in connection with questions respecting gradients.

The immediate effect of applying brakes is to stop wholly or partially either some or all of the wheels of the train, so that they slide instead of rolling on the rails; and the increased resistance thus produced stops the movement of the train in the course of a *time* proportional directly to the speed and inversely to the resistance, and of a *distance* proportional directly to the square of the speed and inversely to the resistance. The *distance* in the course of which the train is stopped is of more importance practically than the time, and is found as follows:—

Let f' be the proportion which the resistance produced by the brakes bears to the weight of the train;

v, the speed in feet per second; then

$$\text{distance in feet on a level} = v^2 \div 64\cdot4\, f'. \quad \text{........(1.)}$$

For practical purposes it is more convenient to state the velocity in miles an hour. Let V denote that velocity; then

$$\text{distance in feet on a level} = \mathrm{V}^2 \div 30 f' \text{ nearly.} \quad \text{....(2.)}$$

There are self-acting brakes, operated upon by the buffers, by mechanism worked by steam, or otherwise, which act on all the wheels at once.* For such brakes it may be considered that

$$f' = \cdot 14 \text{ nearly.} \quad \text{......................(3.)}$$

It is not considered desirable to stop a train much more suddenly than these brakes do, lest an injurious shock should be produced.

For ordinary brakes, worked by hand in carriages called "brake vans," the value of f' may be estimated as ranging

$$\text{from about } \cdot031 \text{ to } \cdot023; \quad \text{.................(4.)}$$

---

* See Mr. Fairbairn's *Report to the British Association on Brakes*, 1859.

so that they stop the trains in distances ranging from 4½ to 6 times the distances required by brakes that act on all the wheels.

The following are some of the results of those data, calculated in round numbers, as precision of calculation is useless in this case:—

Speed in miles an hour,.........	10	20	30	40	50	60
Distance in feet required for stopping the train on a level, with brakes acting on all the wheels, ...	24	96	216	384	600	864
With ordinary brakes,... from	108	432	972	1728	2700	3888
to	144	576	1296	2304	3600	5184

On a gradient ascending at the rate of 1 in $1 \div i$, the resistance available for stopping the train becomes $f' + i$, and it is stopped in so much the shorter distance.

On a gradient *descending* at the rate of 1 in $1 \div i$, the resistance available for stopping the train is diminished to $f' - i$, and the distance required for stopping it becomes,

$$\text{distance on a level} \times f' \div (f' - i). \quad \text{............(5.)}$$

432. **Gradients with Auxiliary Power.**—Where an inclined plane occurs steeper than the ruling gradient, auxiliary power may be applied either by attaching an additional locomotive to each train, or by having special locomotives of great weight and power to draw the trains up, or by using a fixed engine and rope. The economy of the use of auxiliary power depends mainly on the constancy with which it can be kept at work; and this depends on the nature of the traffic.

The locomotive engines used for this purpose are usually tank engines, in order that the weight producing adhesion may be as great as possible.

433. **Power Exerted by Locomotive Engines.**—Besides drawing the train, the locomotive engine has to overcome the resistance of its own wheels and axles, and of its own mechanism. If the power or mechanical energy expended in overcoming this additional resistance, while the engine travels over a given distance, be divided by that distance, there is obtained the additional train-resistance which would be equivalent to the resistance of the engine; and this being added to the resistance of the tender and train, gives the *gross resistance* of the engine, tender, and train. Various rules have been proposed and tried for computing the additional resistance of the engine.

The following rule is founded on the principle that the resistance of the engine consists of two parts; the first, being the resistance of the engine *as a carriage*, is the same with that of a train of the same weight; the second, being the resistance caused by the strain on the mechanism, bears a certain proportion to the whole resistance of the engine and train, whether arising from friction, concussion, or gravity; and that proportion appears to be about one-third. That principle is expressed by the following formula for the gross resistance R, of an engine whose weight is E, drawing a tender and train whose gross weight is T, at the speed of V miles an hour up an incline of 1 in $1 \div i$:—

$$R = \frac{4}{3}(T+E)\left\{ \cdot 00268\left(1+\frac{V^2}{1,440}\right)+i\right\}; \ldots\ldots(1.)$$

or if R is in lbs., and T and E in tons,

$$R = (T+E)\cdot\left\{8+\frac{V^2}{180}+2,987\,i\right\}.^* \ldots\ldots(2.)$$

Under unfavourable circumstances ·00402 may occasionally have to be substituted for ·00268, and $12+\frac{V^2}{120}$ for $8+\frac{V^2}{180}$.

For a descending gradient, each term in i is to be subtracted instead of added.

The energy exerted by the engine per minute, in foot-pounds, is the product of the effort or gross resistance in pounds and speed in feet per minute; that is to say,

$$88\,V\,R; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

(one mile an hour being 88 feet per minute). The *indicated horse-power* is

$$\frac{88\,V\,R}{33,000}=\frac{V\,R}{375}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

Let A be the area of each of the two pistons of the engine, in square inches; p, the *mean effective pressure*, in lbs. on the square inch; c, the circumference of the driving wheels, in feet; l, the length of stroke of the pistons, also in feet; then

---

[*] The above formula differs from that of Mr. D. K. Clark in the following respects:—One-third is added to the whole resistance of the engine and train, considered as carriages, whether arising from friction, concussion, or gravity; whereas, in Mr. Clark's formula, one-third is added to the friction, two-fifths to the resistance from concussion, and nothing to the resistance from gravity, the result being as follows:—When E and T are tons and R in lbs.

$$R = \left(8+\frac{V^2}{171}+2,240\,i\right)(T+E).$$

$$\left.\begin{array}{l} 2\,p\,A = \dfrac{c\,R}{2\,l};\ \text{and} \\ \therefore\ p = \dfrac{c\,R}{4\,l\,A}. \\ \text{The } \textit{mean speed} \text{ per minute of the pistons is } 176\,l\,V \div c; \\ \left.\begin{array}{l}\text{The } \textit{volume swept through} \\ \quad \textit{by the piston per minute,} \\ \quad \textit{in cubic feet,} \ldots\ldots\ldots\ldots \end{array}\right\} \dfrac{352\,l\,V\,A}{144\,c} = \dfrac{22\,l\,V\,A}{9\,c}. \end{array}\right\} \ldots(5.)$$

EXAMPLE.	I.	II.	III.
Speed in miles an hour,......	24	18	12
Ascending gradient, ...	1 in 133·3	1 in 133·3	1 in 80
Weight of engine,...	20 tons	30 tons	30 tons
Weight of tender,............	10 „	12 „	15 „
Weight of train,..............	104 „	245 „	238 „
Circumference of driving wheel,....................	20 feet	15 feet	14 feet
Stroke of pistons, ...	2 „	2 „	2 ft. 2 in.
Area of each piston, ...	200 sq. in.	226 sq. in.	253 sq. in.
Effort or gross resistance, ...	4,502 lbs.	9,241 lbs.	13,057 lbs.
Mean effective pressure in lbs. on the square inch,	56·3 „	76·7 „	83·4 „
Mean speed of pistons, feet per minute,.........	422·4	422·4	328·85
Volume swept through by pistons in cubic feet per minute,....................	1173·3	1325·9	1155·5
Indicated horse-power,......	288	444	418

The mean effective pressure of steam in the cylinder is regulated by the effort required to overcome the resistance, as shown by the formulæ and calculations just given. The pressure of the steam in the boiler exceeds the mean effective pressure in the cylinder in a proportion depending on the extent to which the steam is worked expansively, and various other circumstances. It usually ranges in practice from 80 lbs. to 140 lbs. per square inch above the atmospheric pressure. In some cases engines have been worked at a pressure of 200 lbs. per square inch. The most common pressures at present are from 100 to 120 lbs.

The speed at which the engine runs when exerting a given effort is regulated by the quantity of steam at the required pressure which the boiler is capable of producing; which quantity depends on the quantity of fuel that can be burned in the furnace in a given time, and the efficiency of that fuel in producing steam.

The consumption of fuel by locomotive engines, per indicated horse-power per hour, may be estimated as ranging from 3 to 5 lbs., and the evaporation from 7 to 9 lbs. per lb. of fuel. The whole area of heating surface in ordinary engines varies from 800 to 2,000 square feet; and the area of heating surface for each lb. of fuel burned per hour varies from about half a square foot to 1½ square foot, and is on an average about one square foot.

The action of the blast-pipe gives to the locomotive engine the power of adapting its consumption of fuel to the work which it has to perform, within certain limits. Hence the rapid consumption of fuel by heavy and powerful engines, in ascending steep inclined planes, is to a great extent compensated by the saving which takes place in descending.

The details of the construction of locomotive engines, and of the action of the fuel and steam in them, belong to the subject of mechanical engineering, and cannot be comprehended in the present work. For information respecting these matters, see Mr. D. K. Clark's work *On Railway Machinery*, and that of the author *On Prime Movers*.

434. **Curves.**—I. *Additional Resistance on Curves.*—Curves on a line of railway increase the resistance to an extent which is somewhat uncertain. From experiments made by Lieutenant David Rankine and the author on light passenger carriages with truly cylindrical wheels, having a resistance of ·002, or 4½ lbs. per ton, on a level straight line, it appeared that the additional resistance was

$$\left.\begin{array}{lll}\text{in fractions of the load,} & 3\cdot3 & \div \text{ radius in feet;}\\ \text{or,} & \cdot000625 & \div \text{ radius in miles;}\\ \text{or in lbs. per ton,} & 1\cdot4 & \div \text{ radius in miles.}^{*}\end{array}\right\}\ (1.)$$

So long as the practice prevailed of *tapering* the tires of wheels to a considerable extent, the resistances on curves and straight lines appear to have been nearly equal; the tapered or conical form somewhat diminishing the resistance on curves, while it considerably increased that on straight lines, by causing the carriages to move in a serpentine course, and so augmenting the resistance due to concussion. But since the taper of the wheels has been in some cases done away with, and in others made very slight (about 1 in 40), the resistance on straight lines has become sensibly less than on curves.

Some experiments on American railways by Mr. Latrobe give, for the resistance due to curvature, the following results:—

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[*] Rankine *On Cylindrical Wheels*, 1842.

$$\left.\begin{array}{lrl} \text{in fractions of the load,} & 1{\cdot}36 & \div \text{ radius in feet;} \\ & \text{or, } {\cdot}000258 & \div \text{ radius in miles;} \\ \text{or in lbs. per ton,} & 0{\cdot}578 & \div \text{ radius in miles.} \end{array}\right\} (2.)$$

The smallness of these results, compared with those given in the formulæ (1), may perhaps be owing to the use, on the American railways, of "bogey" carriages, which will be mentioned further on.

II. *Adaptation of Vehicles to Curves.*—Both engines and carriages are adapted by the Americans to sharply curved lines of railway by means of the "bogey"—a small four-wheeled truck, with its wheels as close together as possible, capable of turning about a pivot into various positions relatively to the carriage or engine which it supports, and having a platform on which the carriage is supported by rollers. A long passenger carriage is supported on two bogeys, one near each end; a locomotive engine has one bogey under the leading end, the after end being supported on one or two pairs of driving wheels. An improvement has lately been introduced in the construction of bogey engines, which consists in placing the pivot about which the bogey turns, midway between the centre of the bogey and the middle point between the axles of the driving wheels. By the aid of these contrivances engines and carriages are enabled to pass round curves of radii as small as 3½ chains (231 feet). On British railways, curves of sharper radii than 10 chains are of rare occurrence.

III. *Cant of Rails of a Curve.*—This term denotes the transverse slope which is given to the surface of the rails of a curve, in order to counteract the tendency of the carriages to go straight forward, and so to leave the curve. That tendency arises from three causes, —from the centrifugal force, from the parallelism of the axles, and from the slip of the wheels.

Let v be the velocity of a train in feet per second, moving round a curve of the radius r in feet, then its *centrifugal force* bears to its weight the proportion of

$$\frac{v^2}{32{\cdot}2\, r} : 1; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

and this is the ratio which the *cant*, or elevation of the outer above the inner rail of the curved line of rails, must bear to the GAUGE, or transverse distance between the rails.

If V be the speed in miles an hour,

$$\text{cant for centrifugal force} = \text{gauge} \times \frac{V^2}{15\, r} \text{ nearly} \ldots (4.)$$

The clear gauge is—

On British narrow gauge railways, ……	4 feet 8½ inches.
On British broad gauge railways, ………	7 feet.
On Irish railways, ………………………	5 feet 3 inches.

One-half of the cant should be given by raising the outer rail above the level of the centre line, the other half by depressing the inner rail. It is impossible to adjust the cant alike for all speeds; but it is best to adapt it nearly to the highest speed of ordinary occurrence on the line.

For example, suppose that speed to be 40 miles an hour; then the values of the cant for centrifugal force, in inches, are as follows for different gauges:—

Gauge.	Cant for Centrifugal Force, in Inches.
4 feet $8\frac{1}{2}$ inches.	6,000 ÷ radius in feet.
5 " 3 "	6,720 ÷ radius in feet.
6 " 0 "	7,680 ÷ radius in feet.
7 " 0 "	8,960 ÷ radius in feet.

The tendency to leave the line which arises from the axles being parallel, instead of radiating from the centre of the curve, cannot easily be distinguished from that due to the next cause.

The tendency to leave the line which arises from the slip of the wheels is produced in the following way:—The outer rail of any given arc of the curve is longer than the inner rail in the ratio of

$$\text{radius} + \text{gauge} : \text{radius};$$

and while the inner wheel rolls over a given arc of the inner rail, the outer wheel, if it be of the same diameter with the inner wheel, has to slip over a distance equal to the difference of the lengths of the rails. Thus is produced an additional resistance to the advance of the outer wheel of each pair of wheels, tending to make the front end of the carriage swerve outwards. The taper or coning of the wheels was devised to prevent this tendency, by causing the outer wheel to run on a portion of its tire of larger diameter than that on which the inner wheel runs at the same time. It has the disadvantage already mentioned, however, of increasing the oscillation or sideward swinging of trains on straight lines. The tendency to swerve may be corrected in cylindrical wheels by means of an additional cant, which throws the larger proportion of the weight on the inner rail. The additional cant required for that purpose was determined experimentally by Lieutenant David Rankine and the author for carriages moving on a narrow gauge line at speeds of from 3 to 12 miles an hour, and found to be independent of the velocity, and inversely proportional to the radius of the curve; being given by the following formula:—

$$\text{additional cant in feet} = 600 \div \text{radius in feet}; \ldots (5.)$$
$$\text{,, ,, in inches} = 7{,}200 \div \text{radius in feet}; (5\text{ A.})$$

but such additional cant is probably rendered unnecessary by the use of bogeys.

IV. *Method of Easing Changes of Curvature.*—Every change of curvature should be accompanied by a change in the cant of the rails; and as changes of the cant of the rails can only be made by degrees, changes of curvature should be gradual also, whether they occur at the junctions of curves with straight lines, or of curves of reverse curvature, or of different radii, with each other.

Two methods of setting out curves with gradual changes of curvature have been practised—one, invented by Mr. Gravatt about 1828 or 1829, consists in the use of the *curve of sines* instead of the circle for railway curves; the other, invented by Mr. William Froude about twenty years ago, consists in adhering to the use of the circle throughout the greater part of the extent of each curve, but introducing at each end of each curve a small portion of a curve approximating to the *elastic curve,* for the purpose of making the change of curvature by degrees. The proper place for referring to these methods would have been Article 63, p. 101, which relates to the ranging and setting out of curves; but although they have been so long in use, no account of either of them was published until after that part of this book was in type. For details respecting them, see *Transactions of the Institution of Engineers in Scotland,* vol. iv., 1860-61.

The curve of sines is that which gives the most gradual variation of curvature; but its use involves the abandonment of circular arcs, which are the most easily and rapidly set out of all curved lines. As Mr. Froude's "*curve of adjustment*" (as it may be called) is easily combined with circular arcs, the most convenient mode of applying it to practice will now be described.

Suppose that a portion of a line of railway, consisting of a curve, or of a series of curves, but beginning and ending with straight lines, is to be set out in such a manner that all changes of curvature shall be gradual.

(1.) Begin by ranging the centre line as a series of circular arcs, according to Method I. of Article 63, p. 102, and marking it with poles or temporary stakes.

(2.) Determine the *length* to be adopted for the "*curves of adjustment*" as follows:—Compute the *cant* required for each of the circular arcs, according to the rules of Division II. of this article, and the several *changes of cant;* observing that the change of cant between a straight line and a curve is simply the cant of the curve; that if two adjacent curves are curved in the same direction, the change is the difference of cant; and that if they are curved in reverse directions, the change is the sum of the two cants.

Multiply the *greatest change of cant* by the reciprocal of the *steepest gradient of adjustment;* that is, the greatest *difference of inclination* which can conveniently be given to the outer and inner rail in changing the cant. The result will be the length of each of the curves of adjustment.

According to Mr. Froude, the gradient of adjustment should not exceed 1 in 300. Then,

$$\text{Length of curve of adjustment} = 300 \times \text{greatest change of cant.} \quad (6.)$$

(3.) Compute, for each circular arc of the series, the *shift* as follows:—

$$\text{Shift} = (\text{length of curve of adjustment})^2 \div 24 \text{ radius.} \quad (7.)$$

Then shift the poles by which a given circular arc is marked inwards (that is, towards the centre of curvature of the arc) through the distance computed by the above formula. For example, in fig. 276, let A B, B C, be a pair of consecutive circular arcs, marked by poles, and joining each other at their point of contact B. Let B E, B F, be the *shifts* proper to those two arcs respectively, as computed by equation 5; after all the poles have been shifted, they will mark the arcs D E, F G, having a gap between them at E F, equal to the sum of the two shifts, if the arcs are curved in reverse directions, or the difference of the shifts, if the arcs are curved in the same direction. Straight lines are not to be shifted; so that where a curve joins a straight line, the gap is simply the shift of the curve.

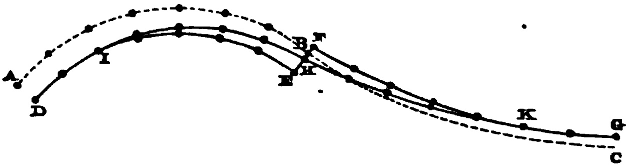

Fig. 276.

(4.) Set out the "*curve of adjustment*" I H K as follows:—For its middle point, bisect the gap E F in H. For its ends I and K, lay off E I and F K, each equal to half its length, as computed by equation 6. For intermediate points in the division I H, lay off ordinates at right angles from a series of points in the circular arc I E, proportional to the cubes of the distances from I; and for intermediate points in the division K H, lay off ordinates at right angles from a series of points in the circular arc K F, proportional to the cubes of the distances from K.

The following is a formula for calculating those ordinates:—

Let a denote the length I K of the curve of adjustment;
b, the gap E F, or sum of the shifts;
x, the distance, measured on the circular arc, of any point from I or from K, as the case may be;
y, the ordinate; then

$$y = \frac{4\,b\,x^3}{a^3}. \qquad (8.)$$

Example.—A curve of 20 chains radius (= 1,320 feet), with cant suited to a speed of 40 miles an hour on a narrow gauge line, is to be connected with a straight line.

Cant (see p. 650) = 500 feet ÷ 1,320 = ·3788 foot;
Length of curve of adjustment, a = ·3788 × 300 = 113·6 feet;
Shift for circular arc = $(113{\cdot}6)^2 \div 24 \times 1{,}320$ = ·407 foot.

(As the arc is to join a straight line, this is also the width of the gap b.)

$$\text{Formula for ordinates, } y = \frac{4 \times {\cdot}407\,x^3}{(113{\cdot}6)^3} = {\cdot}000{,}001{,}11\,x^3.$$

The easing or "humouring" of changes of curvature is performed by rail-layers by the eye, with considerable accuracy, in the case of reverse curves, where a "bit of straight" has been set out to connect the two circular arcs; but no such approximate process is possible at junctions of curves with straight lines, or of curves of unequal radii, curved in the same direction.

V. *Combination of Curves and Gradients.*—As curvature of the line increases the resistance of trains and the danger of jumping off the line at high speeds, it is advisable to avoid very sharp curves on steep gradients, and on parts of the line where the speed is to be very high.

Where sharp curves necessarily occur in the course of a steep ascent, it is advisable, instead of adopting an uniform gradient, to make it slightly steeper on the straight parts of the line, and slightly flatter on the curved parts, in order that the resistance of an ascending train may be as nearly as possible uniform. In the absence of more precise data, formula 1 of Division I. of this Article, p. 648, may be used to compute the resistance due to curvature for engines and carriages without bogeys, and formula 2, p. 649, for those with bogeys.

VI. *Additional Problems in Setting out Curves.*

Problem First.—*To find the radius of a circular arc which shall* *successively touch three straight lines*, B D, D E, E C, fig. 277, measure the middle straight line D E, and the acute angles at D and E. Then

$$\text{Radius} = \text{D E} \div \left(\tan\frac{1}{2}\text{D} + \tan\frac{1}{2}\text{E}\right); \text{.........(9.)}$$

which having been computed, the curve can be set out by Method I. of Article 63, p. 102.

PROBLEM SECOND.—*To set out the curve of sines, or harmonic curve*, proposed by Mr. Gravatt. This curve may be used with advantage where there is clear ground and sufficient time to range it. Let B A, C A, be the two straight tangents to be connected by means of the curve, cutting each other in A. Lay out the straight line A D E, bisecting the angle B A C, and choose in it a point D for the curve to traverse. Lay off the distances.

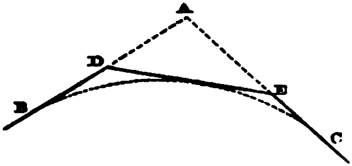

Fig. 277.

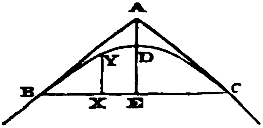

Fig. 278.

$$\text{A B} = \text{A C} = 2{\cdot}75193\ \text{A D} \cdot \sec\frac{1}{2}\text{B A C}; \text{.....(10.)}$$

then B and C will be the ends of the curve. Conceive the chord B E C to represent a semicircle stretched out straight, and divided into 180 degrees, and lay off ordinates at right angles to it proportional to the sines of arcs marked upon it: the ends of those ordinates will be points in the curve. The middle or longest ordinate is,

$$\text{D E} = 1{\cdot}75193\ \text{A D}; \text{..................(11.)}$$

Let x denote any abscissa B X, measured from one of the ends of the curve; y, the corresponding ordinate X Y; then

$$y = \text{D E} \cdot \sin\frac{90^\circ \times x}{\text{B E}}; \text{.................(12.)}$$

the value of the half-chord B E being,

$$\mathrm{B\,E} = 2{\cdot}75193\ \mathrm{A\,D} \cdot \tan \frac{1}{2}\ \mathrm{B\,A\,C}. \qquad (13.)$$

The sharpest curvature occurs at D, where the radius of curvature is

$$\mathrm{D\,E} \cdot \tan^2 \frac{1}{2}\ \mathrm{B\,A\,C}. \qquad (14.)$$

If the cant of the rails at D is adapted to this radius, the cant at any other point may be determined with sufficient accuracy for practice by making it vary simply as the ordinate y. The form of the curve is nearly, though not exactly, that of an elastic bow of uniform section, bent by means of a string connecting its ends.

Problem Third.—*To connect a circular arc and a straight line, or two circular arcs, which do not touch or cut each other, by means of an elastic curve* (Mr. Froude's curve of adjustment). Fig. 276, p. 652, may be taken to illustrate the case where two arcs curved in reverse directions are to be connected; fig. 279, to illustrate that in which two arcs curved in the same direction are to be connected.

Find the pair of points at which the arcs or lines to be connected are nearest to each other. This is best done by first finding two pairs of points at which the lines to be connected are at equal distances apart; the pair of points required will be midway between those two pairs of points. Let E and F be the pair of points thus found; measure the *gap* E F, then calculate the *half-length of the curve of adjustment* by means of the following formula, in which r and r' denote the radii of the arcs to be connected:—

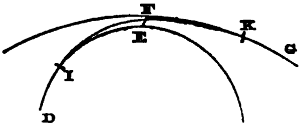

Fig. 279.

$$\mathrm{E\,I} = \mathrm{F\,K} = \sqrt{\left\{ 6\ \mathrm{E\,F} \div \left(\frac{1}{r} \pm \frac{1}{r'} \right) \right\}}; \qquad (15.)$$

the sign + or — being used in the denominator, according as the directions of curvature are reverse or similar. If one of the lines to be connected is straight, $1 \div r'$ is to be made $= 0$; so that the formula becomes

$$\mathrm{E\,I} = \mathrm{F\,K} = \sqrt{6\ \mathrm{E\,F} \cdot r}. \qquad (16.)$$

(The ends of the curve of adjustment may also be determined approximately, by finding the two pairs of points at which the distance between the lines to be connected is 4 E F.)

The curve of adjustment is now to be set out by ordinates, as in Division IV. of this Article, p. 652.

VII. *Enlargement of Gauge on Curves.*—In order to enable trains to pass more easily round curves, it is the practice of some engineers to make the gauge about half an inch wider on them than on straight lines; so that supposing, for example, that the wheels have half an inch of "*play*," or "*clearance*," on straight lines, they will have an inch on curves.

VIII. *Legal Limitations to the Sharpness of Curves.*—According to the Railway Clauses Consolidation Act of 1845, the power of diminishing the radii of curves below the length marked on the parliamentary plan of a railway is thus limited:—if the radius of any curve, as shown on the parliamentary plan, exceeds half a mile (2,640 feet), it may be shortened to any extent which does not reduce it below half a mile; but no radius of such a curve is to be shortened to less than half a mile, nor is any radius shown as less than half a mile to be shortened to any extent. These restrictions may be dispensed with by the Board of Trade upon sufficient cause being shown.

435. **Laying out and Formation of Railways in General.**—Subject to the principles respecting gradients and curves which have been explained in the preceding articles of this section, the general principles of the selection of the course and the formation of a line of railway are those which have already been stated in Articles 413, 414, pp. 619 to 622, and the other parts of this work referred to in those articles.

The following principles are specially applicable to railways:—

I. The *Breadth of Formation or Base* depends upon the gauge, or clear distance between the rails of a track, the number of tracks, the clear space between them, the clear space left outside of them for projections of carriages and for men on foot, and the additional space required for the slopes of the "*ballast*," the side drains, &c. The following are examples:—

Single Line.	Narrow Gauge.		Irish Gauge.		Broad Gauge.	
	Ft.	In.	Ft.	In.	Ft.	In.
Clear space outside of rail,	4	0	4	0	4	0
Head of rail,	0	2½	0	2½	0	2½
Gauge,	4	8½	5	3	7	0
Head of rail,	0	2½	0	2½	0	2½
Clear space outside of rail,	4	0	4	0	4	0
Least breadth of top of ballast; and least width admissible for archways, &c., traversed by the railway,	13	1½	13	8	15	5
Spaces for slopes of ballast, and benches beyond them, on embankments, from	3	10½	4	4	9	2
to	8	10½				
Total breadth of top of embankments, from	17	0	18	0	24	7
to	22	0				

DOUBLE LINE.	Narrow Gauge.		Irish Gauge.		Broad Gauge.	
	Ft.	In.	Ft.	In.	Ft.	In.
Clear space outside of rail,........................	4	0	4	0	4	0
Head of rail,..	0	2½	0	2½	0	2½
Gauge,..	4	8½	5	3	7	0
Head of rail,..	0	2½	0	2½	0	2½
Middle space (called the "*six feet*,")...........	6	0	6	0	6	0
Head of rail,..	0	2½	0	2½	0	2½
Gauge,..	4	8½	5	3	7	0
Head of rail,..	0	2½	0	2½	0	2½
Clear space outside of rail,........................	4	0	4	0	4	0
Least breadth of top of ballast; and least width admissible for archways, &c., traversed by the railway,......................	24	3	25	4	28	10
Spaces for slopes of ballast and trenches beyond them, on embankments,...... from	3	9	4	8	9	2
to	8	9				
Total breadth of top of embankments, from	28	0	30	0	38	0
to	33	0				

Cuttings are sometimes made of a width at the formation level equal to that of the embankments on the same line; in other cases they have an additional width given to them, amounting sometimes to as much as 9 feet, in order that there may be the more space for the side drains. On the whole, the most common breadths of base for both embankments and cuttings, on narrow gauge lines, are

for single lines, 18 feet,
for double lines, 30 feet.

Arches over the railway are seldom made of the minimum spans shown by the foregoing tables, except in the case of tunnels. Bridges over narrow gauge lines are usually of the following spans:

over a single line, from 16 to 18 feet;
over a double line, from 28 to 30 feet;

and the same breadths are applicable to cuttings with retaining walls, and rock cuttings with vertical or nearly vertical sides.

II. The *Formation Level* is from 1½ to 2 feet, or thereabouts, below the intended level of the rails, according to the depth of the permanent way (see Article 66, p. 112), and is marked by a line of some distinctive colour on the working section.

III. *Side Slope.*—The formation or base is sometimes made to fall from the centre towards the sides at the rate of about 1 in 60, to facilitate drainage.

IV. *Cross Drains.*—Where the nature of the soil makes cross drains necessary, they may be made by digging small trenches across the base from 7 to 9 inches deep, and from 3 to 5 yards apart, and filling them with broken stone.

V. *Positions of Stations relatively to Gradients.*—Although it may sometimes be absolutely necessary to have stations in the course of steep gradients, the engineer should, as far as possible, avoid that necessity, because of the difficulty and inconvenience of stopping descending trains, starting ascending trains, and shifting carriages at stations so placed. There is an advantage in having a station at a summit level, because the gradients facilitate the starting and stopping of trains in both directions.

VI. *Legal Limits to Powers of Deviation.*—In Britain, the ordinary limits of deviation as to the situation of a railway are 100 yards in the country and 10 yards in towns to either side of the centre line, as marked on the parliamentary plan, and such limits are marked on the plan. Wider limits, in special cases, are granted by special enactment upon sufficient cause being shown; and the limits may be restricted, at the discretion of the promoters, to any extent consistent with the execution of the work. The ordinary limits of deviation of the level of a railway are 5 feet in the country and 2 feet in towns above and below the level of the upper surface of the rails, as shown on the parliamentary section. Further deviations of level require the sanction of owners of property affected by them, except in the case of embankments and viaducts, which may be lowered to any extent consistent with leaving sufficient headroom for roads.

Gradients less steep than 1 in 100 may be made steeper to an extent not exceeding 10 feet per mile; gradients of 1 in 100, or steeper, may be made steeper to an extent not exceeding 3 feet per mile; gradients may be made flatter to any extent.

As to curves, see Article 434, p. 656. On all these points, see the *Railways' Clauses Consolidation Act*, 1845.

436. **Crossings and Alterations of other Lines of Conveyance.**—I. *General Explanations.*—When the course of a railway crosses that of a previously existing line of land-carriage, the railway may either be carried over or under the existing line by means of a bridge, or across it on the level of its surface. When the line of conveyance to be crossed is a canal or a river, the railway must be carried either over or under it. In order to facilitate such crossings, it may be necessary to alter the level or divert the course of existing lines of conveyance; and in some cases a diversion of the course of an existing line of conveyance may be required independently of any crossing; and for all those purposes, cuttings and embankments are required. The parts of a road whose levels are altered for the purpose of carrying the railway across it, are called the *approaches* of the crossing.

The information which must be given on the section of a proposed railway respecting such alterations of existing lines of communication has already been referred to in Article 14, pp. 14, 15. Proposed diversions of such lines should be shown on the plan, and proposed alterations of width noted. But, as formerly stated, with regard to all matters connected with the preparation of parliamentary plans, reference should be made to the standing orders of parliament themselves, and not to any second-hand account of their provisions.

As to working sections of alterations of existing lines of communication, see Article 66, p. 113.

II. *Legal Limitations affecting Crossings of existing Lines of Conveyance.*—In order fully to understand those limitations, as they are regulated by law, in Britain, the statutes called "Railways' Clauses Consolidation Acts" must be consulted. The following is an outline of the more important of the limitations:—

A. *Level Crossings* of public carriage roads are not lawful unless individually authorized by the special act relating to the particular railway; and in order that they may be so authorized, the engineer must be prepared to show cause for using them instead of bridges, and to prove that they are consistent with the public safety. All level crossings must be provided with gates, which, in ordinary, are to be kept shut across the road. In the case of level crossings of public roads, those gates are to be capable of being closed across the railway when the passage along the road is open; and there must be a lodge or box for a gatekeeper, and a proper system of signals.

B. *Over Bridges* (as bridges for carrying roads over the railway are called) are to have a clear width of roadway between the parapets,

for a turnpike road,.................of 35 feet,
for any other public carriage road, of 25 feet,

provided the average width of the road between its fences throughout a distance of 50 yards on each side of the centre line of the railway is not less than 35 feet or 25 feet, as the case may be. Should the average width of the road be less than the limit abovementioned, the roadway of the bridge may be made of a width equal to such existing average width, provided that in no case shall the roadway be made of a less clear width than,

for a turnpike road,...................... 30 feet,
for any other public carriage road, ... 20 „

and that, if at any future period the road should be widened, the railway company shall be obliged to widen the bridge to 35 feet or 25 feet as the case may be.

For *private roads* the prescribed least width is 12 feet; but this may be altered by special agreement between the proprietor of the road and the promoters of the railway.

The *parapets* of all over bridges are to be at least 4 feet high, and the fences of their approaches at least 3 feet high.

C. *Under Bridges* (as bridges for carrying roads under the railway are called) are subject to the same conditions as to width of roadway with over bridges; and those conditions fix the least span of the arch. Its height is subject to the following conditions:—

For a *turnpike road* the clear headroom is to be at least,
at the springing of the arch, 12 feet;
throughout a breadth of 12 feet in the middle
of the archway, 16 feet.

For any other *public carriage road* the clear headroom is to
be at least,
at the springing of the arch, 12 feet;
throughout a breadth of 10 feet in the middle
of the archway, 15 feet.

For a *private road* the clear headroom is to be at least,
throughout a breadth of 9 feet in the middle
of the archway, 14 feet.

D. The *inclination* of an altered road is not to be made steeper.
for a turnpike road, than 1 in 30;
for any other public carriage road,... } than 1 in 20;
for a private road, than 1 in 16;

provided that the undertakers of the railway shall not be obliged to make the inclination of the altered road easier than its original "mesne" inclination, or than the original "mesne" inclination of the road within a distance of 250 yards from the point where it crosses the centre line of the railway.

(No rule is prescribed for computing the "mesne" inclination of the road; but the following appears to be as little open to objection as any that can be devised. Add together all the rises and all the falls of the portion of the road in question, and divide their sum by its length.)

E. The rules of the general act may be modified or set aside in particular cases by special enactment, upon sufficient cause being shown; and in the case of crossings of private roads, conditions may be settled by agreement between their proprietors and the promoters of the railway.

F. A sufficient *temporary road* must be provided until the permanent road is complete. In many cases it is most convenient to divert the road, and use the original roadway as a temporary road.

G. Works in tidal waters must be sanctioned by the admiralty.

III. The *Least Dimensions of Under Bridges* are virtually fixed by the rules above-mentioned. The following are examples, in which the arches are treated as segmental, that being the best form in the present case. The rise is given as computed by Mr. Cotton in his work *On Railway Engineering in Ireland.*

The power to make roadways of less than the prescribed widths of 35 and 25 feet in certain cases is of no avail as regards under bridges, or the abutments of over bridges, because of the liability to enlarge the bridges at a future time.

Bridge under Railway and over	Turnpike Road.	Public Carriage Road.
	Feet.	Feet.
Span, ...	35·00	25·00
Rise, ...	4·50	3·53
Clear headroom in centre, ...	16·50	15·53
Radius of intrados, ...	36·28	23·90
Thickness of arch-ring, ...	2·50	2·00
Depth of coating of puddle, ...	0·50	0·47
Depth of permanent way, say	2·00	2·00
Total height, roadway to rails, ...	21·5	20·00
To allow for additional depth in skew bridges, the above heights may be increased to	23·00	21·00
For *iron under bridges*, the above heights may be diminished to	20·00	18·00

As to the *least dimensions of over bridges*, see Article 290, p. 426, for an example of the dimensions of the arch.

The clear headroom in the centre is usually, ...	16·00 feet.
To this add, for the thickness of a stone arch, ...	2·00
" " " puddle coating, ...	0·50
" " " roadway, ...	1·00
Total, ...	19·50
For an iron over bridge with flat girders the clear headroom may be reduced to about	14·50
Girders and roadway, say	2·50
Total, ...	17·00

The platform of an over bridge with iron girders may consist either of a series of transverse brick arches spanning across between the girders, which should be about 5 feet apart, and be held together by transverse ties sufficient to resist the thrust of the arches, or of cast iron plates with stiffening ribs above, covered with a layer of asphaltic concrete, or of buckled wrought iron plates, covered with a layer of asphaltic concrete. (See Article 375, p. 545.)

In crossing roadways in and near populous towns, where the ordinary dimensions of bridges would be too small, the width of the roadway, and, in the case of under bridges, the headroom, are usually fixed by agreement with the local authorities.

The *thickness of the abutments* of ordinary road bridges on lines of railway is usually from 1-5th to 1-6th of the span, and the counterforts are altogether of about one-third of the volume of the abutments. The wing walls are retaining walls, as to which, see Articles 265 to 268, pp. 401 to 407. In ordinary cases their thickness at the base is from 1-4th to 3-10ths of their height, and about one-half of that amount at the top, diminishing by steps or scarcements at the back of the wall; the face has a batter, of which 1 in 12 is an usual value.

Bridges over deep and wide cuttings may have three or five arches.

IV. *In selecting the line and levels,* the engineer should have regard to the crossings of existing lines of conveyance which may be required, bearing in mind that the earthwork of the approaches to those crossings, owing to its inconvenient situation, is more expensive than that of the railway itself. He should study to have as few bridges above the minimum size as possible; and with that view he should endeavour, as far as possible, to gain the necessary headroom partly by means of the elevation or depression of the railway above or below the existing road, and partly by means of an alteration of the level of that road.

The level occupied by existing lines of conveyance, if they have been well laid out, is usually the most favourable to economy of works; and for that reason there is generally an advantage in crossing such lines on the level, independently of the saving of the cost of bridges; but the choice between level crossings and bridges must be regulated mainly by considering whether the traffic is such as to make level crossings consistent with the public safety. In comparing the cost of a level crossing with that of a bridge, regard should be had to the necessity of having a gate-keeper at the level crossing.

When the level of an existing road is to be lowered, special care must be taken that the cutting for that purpose can be properly drained.

The line of conveyance which causes the greatest impediment to the passage of a railway is a canal; for it usually occupies precisely the most favourable level for economy of works; its level cannot in general be altered, nor can it be crossed on the level; and it can only be crossed *near* its own level by means of a swing bridge, which is inconvenient if the traffic is great.

In making a bridge over a canal, the span should be sufficient to contain the canal and its towing path without contracting their width; and care must be taken in founding the abutments not to disturb the canal. To prevent the escape of water it may be necessary to use coffer dams. (Article 409, p. 612.) The clear headroom is usually fixed by agreement; in ordinary cases it is 10 feet above the towing path, to be sufficient for a man on horseback.*

A passage under a canal may be made either by tunnelling at a sufficient depth, or by making a temporary or permanent diversion of the canal, and building an aqueduct bridge; which, if the diversion is to be temporary, will be on the original course of the canal, and if permanent, on that of the diversion. Canal and river bridges will be further considered in a later chapter.

437. **Ballast** is that portion of the PERMANENT WAY of a railway which forms a firm and dry foundation for the rails or for the sleepers by which they are supported. It is sometimes distinguished into *ballast* proper, or *under ballast*, which lies wholly below the sleepers or other supports of the rails, and *boxing*, or *upper ballast*, which is packed round the sleepers, chairs, and rails, up to within two or three inches of the upper surface of the rails.

Examples of the breadth of the upper surface of the ballast have already been given in Article 435, pp. 656, 657. Its depth varies in different lines and according to the practice of different engineers; the following may be taken as its ordinary limits:—

	Feet.	Inches.		Feet.	Inches.
Lower ballast,...............from	0	9	to	1	6
Upper ballast, or boxing, „	0	6	to	0	9
Total depth, ... „	1	3	to	2	3

In exploring the course of a projected railway, the engineer should give special attention to the sources from which good ballast can be obtained.

The best material is stone, broken as for road metal, into pieces not exceeding 6 ounces in weight (see Article 420, p. 627); but the stone does not need to be so hard as for roads; it is sufficient if it be of such hardness as would make it suitable for ordinary building purposes. Stone that decays readily by the action of air and moisture ought to be carefully avoided. In the absence or scarcity of suitable stone, the slag of iron works, broken to a proper size, may be used; or alum-work refuse, which is shale burnt to the consistency of brick; or engine ashes. Next to broken stone and slag, as a material for ballast, is clean gravel; the lumps exceeding 6 ounces in weight being broken with the hammer. Clean sharp sand may be used, but it has the disadvantages, that in heavy falls of rain it may be washed away, and that in very dry weather it is blown into the air, and damages the rolling stock by lodging about the bearings of the axles and mechanism. In the absence of all other materials, pieces of clay suitable for bricks may be burnt until they are hard, and then broken down and used as ballast. As to the qualities of such clay, see Article 219, p. 363.

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* To admit of a horse rearing without danger to the rider, 12 feet of headroom should be allowed.

The labour of breaking and spreading a given quantity of ballast is about twice, or 2½ times, that of excavating the same quantity of gravel.

438. **Sleepers** are pieces of material which rest on the ballast, as already stated (being firmly bedded on it by means of a beetle), and support the rails. At an early period in the history of railways, *stone blocks* were used for that purpose; they were placed at 3 feet apart from centre to centre, and measured, on horse-worked railways, about 18 inches × 12 inches × 9 inches, and on steam-worked railways, 2 feet × 2 feet × 1 foot; but they were found to form too hard and unyielding a base for traffic at high speeds, even with the aid of pieces of felt under the chairs, and their use was abandoned in favour of that of *timber sleepers.*

The best materials for timber sleepers are woods which withstand alternate wetness and dryness; and of those, the most generally employed in Europe is Larch. (Article 302, p. 443.) Various substances have been used for the preservation of timber sleepers: the most efficient is "creosote." (Article 450, p. 311.)

Timber sleepers are either transverse or longitudinal. The former afford a ready and efficient means of preserving the gauge; the latter give the most equable and continuous support to the rails.

Transverse or *cross sleepers* are usually about 9 feet long, from 9 to 10 inches broad, and from 4½ to 5 inches deep. Their most common forms of cross-section are the semicircular and the triangular. Semicircular sleepers are made by sawing a round log in two along the middle; they are laid on the ballast with the flat side down; on the upper or convex side are cut with great accuracy two flat surfaces or "seats," made to fit the bases of the chairs, if chairs are used, or of the rails, if flat-bottomed rails are used without chairs; and in those seats are bored holes for the pins or other fastenings by which the chairs or rails are fixed to them. Triangular sleepers are made by sawing a squared log in two by a diagonal cut; they are laid with the right angle downwards and the broadest side upwards. The distance apart at which cross sleepers are laid ranges from about 2½ feet to 4 feet from centre to centre; but it seldom now exceeds 3 feet, as wider spans have been found to cause overstraining of the rails.

Longitudinal sleepers or *bearers* are usually from 12 to 14 inches broad, and from 6 to 7 inches deep, being made by sawing a square balk of timber in two. The rails may either have a continuous bearing on them, or may be supported by chairs at intervals of 30 inches or 3 feet. When the bearing is continuous it is usual to bolt or screw a plank of 7 or 8 inches wide and 1¼ inch thick, or thereabouts, on the top of the sleeper, and upon this plank the base of the rail rests. In order to preserve the gauge, the pair of longitudinal bearers of a track of rails must be connected by means of cross-ties at intervals of about 5 or 6 yards. Special care should be taken that the ballast under the bearers is not impervious to water, lest they should confine water in the middle of the track.

Longitudinal and cross sleepers are sometimes combined, the cross sleepers being laid undermost. In this case the scantlings of the cross sleepers are made less than when cross sleepers are used alone, being usually about 7 feet × 7 inches × 3½ inches.

Cast iron sleepers are used of various forms. In Mr. Greaves's form the chair and sleeper are cast in one piece, the base being like an inverted bowl, near the summit of which are two holes, so that ballast can be put into the hollow and rammed from above. The gauge is preserved by transverse rods. In Mr. Samuel's form the rail is wedged with pieces of wood into a sort of cast iron trough with flat spreading wings. Another form is simply a flat oblong plate, with chairs cast on its upper side; and a fourth is an oblong trough wedged full of pieces of wood, on which the chairs rest. (See Clark *On Railway Machinery*.)

439. **Rails and Chairs.**—The iron from which rails are rolled is either No. 1 bar iron, that is, puddled bars; or No. 2 or 3, that is, bar iron once or twice piled, re-heated, and rolled, as the case may be; so that the rails themselves consist of No. 2, No. 3, or No. 4 bar iron. Piling, re-heating, and rolling increase the compactness and toughness of iron up to the fifth time, as already stated. (See Table, p. 511.) A common practice is, to use No. 1 iron for the interior of the pile from which a rail is to be rolled, and No. 2 or No. 3 for the outside, especially the top or head, and the base; sometimes a flat bar of charcoal iron is used for the head, in order that the surface on which the wheels roll may have greater strength and durability than the rest.

To use inferior sorts of iron in rails, in order to cheapen them, is false economy, as experience has shown; the rails so produced being liable to spread, split, and scale under swift and heavy traffic.

Fig. 280 is an example of an ordinary cross-section for a pile of bars to be rolled into a rail; the top and bottom being each formed of a single bar, and the other bars built so as to break joint.

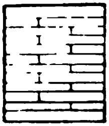

Fig. 280.

The dimensions of the pile range from 6 inches broad by 7 deep, to 9 inches broad by 10 deep; and its length is such as to make its weight *once and a-quarter* that of the rail to be rolled from it, in order to allow for waste.

The rail should be rolled out at one heat, immediately after which it is to be cut to the proper length, by sawing off its ends with a pair of circular saws, already mentioned in Article 366, p. 522. The ends should be exactly perpendicular to the length of the rail.

The ordinary lengths of rails range from 15 to 21 feet.

The sectional area of a rail, in square inches, is almost exactly one-tenth of the weight of one yard of its length in lbs.

On horse-worked railways, the weight of the rails per yard ranges from 28 lbs. to 35 lbs., the former weight being barely sufficient for durability. On the earlier of the high speed locomotive lines, a weight of about 60 lbs. to the yard was adopted; but the continual increase of the weight and speed of engines has rendered necessary a continual increase in the weight of rails; so that it now ranges from 70 to 100 lbs. per yard, or thereabouts.

As a general rule, it may be stated that the *weight of a yard of rail, if supported at intervals, should be* 15 *lbs. for each ton of the greatest load on one driving wheel.* When the bearing is continuous, about five-sixths of that weight is sufficient.

The *head* or top of a rail is usually about 2½ inches broad, and has a very slight convexity in the middle, the radius of which is from 5 to 7 inches. In laying the rails they are carefully adjusted to the true gauge by means of a gauge-rod with shoulders on it at the proper distance apart. On straight lines the heads of the two rails should be exactly at the same level, and in curves they should be set to the proper "cant," as already explained in Article 434, p. 649. If cylindrical wheels are used for the rolling stock, the highest part of the head of each rail should be a tangent to a level line, or a line inclined at the proper cant, as the case may be; but if the wheels are tapered, the rails must be inclined inwards towards each other so as to be tangents at their highest points to the conical treads of the wheels. This inward inclination is given, where chairs are used, by casting the chairs so that their jaws may hold the rail in the proper position.

The figures adopted for the cross-sections of rails vary with the modes in which they are supported. A rail may be either

(1.) Supported on the base and stayed at the sides, or
(2.) Supported on a broad base alone, or
(3.) Hung by the shoulders;

and the bearings may be either

(A) at intervals (generally about 3 feet); or (B) continuous.

In figs. 281, 282, 283, and 284, the cross-section of the rail is of the I-shaped, double T-shaped, or "double-headed" figure, at present more generally used than any other. When this figure was first introduced, it was intended that on the top becoming too much worn for further use, the rail should be turned upside down; but in general, by the time the top is worn out, the rail is unfit for further use. The usual weight of rails of this form is 75 lbs. per yard, and their depth 5 inches. In fig. 281 the rail A is shown as supported by a common cast iron chair B B, which is pinned by two compressed oak treenails, D, D, to a cross sleeper E. The rail rests by its base or foot on the bottom of the chair, and is kept steady by being firmly held between the inner jaw of the chair and a compressed oak wedge or key, C, which is driven between the rail and the outer jaw.

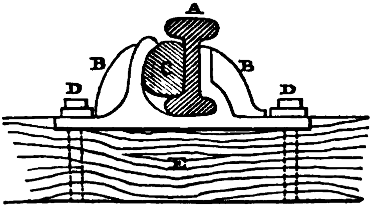

Fig. 281.

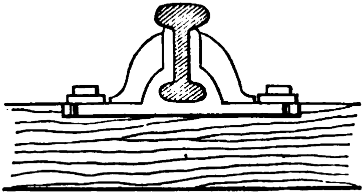
Fig. 282.

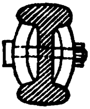
Fig. 283.

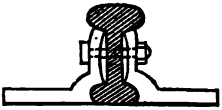
Fig. 284.

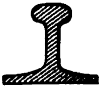
Fig. 285.

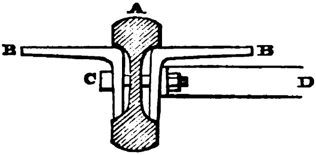

Fig. 286.

The inner faces of the jaws or cheeks of the chair are chilled. (See Article 352, p. 499.) An ordinary chair weighs about as much as *one foot* of the rail which it is intended to support; a *joint-*

chair, for supporting the ends of two adjoining rails, is from one-third to one-half heavier, and its outer end is usually fastened down by two pins instead of one. Fig. 282 represents a sort of joint-chair, introduced by Mr. William Johnstone, which has been found to answer well; it is slid on to the rails, which its jaws fit exactly.

It has now, however, become very generally the practice to connect adjoining rails by the *fish-joint;* the ends of the rails being supported by a pair of ordinary chairs, which they overhang by 12 or 15 inches, and being united to each other by a pair of *fish-pieces*, about 18 or 20 inches long, bolted together through the rails by four bolts. Fig. 283 shows in cross-section the figure of which the fish-pieces are made in order that they may abut at their upper and lower edges against the head and foot of each rail, and thus be wedged into their places. The bolt-holes in the rails are made slightly oblong horizontally, to allow for changes of length by heat and cold, which may amount, in ordinary cases, to about 1-2000th or 1-2500th of the length of each rail (p. 527).

Fig. 284 is the *bracket fish-joint,* in which the fish-pieces are of angle iron, and answer the purpose of a joint-chair; their horizontal bases being bolted down to a pair of cross sleepers.

Among rails which are supported *on a broad base without chairs* may be mentioned the *foot rail*, fig. 285, which is fastened down by means of fang-bolts to longitudinal or cross sleepers; the *bridge rail* (fig. 229, p. 518), and the *Barlow rail* (fig. 230, p. 518).

Bridge rails are made from 3 to 5 inches deep, and from 7 to 6 inches in breadth of base, and are bolted to the sleepers, either through slightly oblong holes, or by fang-bolts holding down the edges of the base. When supported on a continuous bearing, they weigh about 65 lbs. per yard; when supported at intervals on cross sleepers, 82 lbs. per yard. In the latter case each joint is secured by holding the bases of the two rails between a pair of cast iron jaws, drawn together by transverse bolts which pass under the rails. (This system was introduced by Sir John Macneill, and is used on Irish lines).

The Barlow rail is now made with a portion of each wing flat and horizontal, so that it approaches somewhat nearer to the bridge shape than fig. 230 shows. It is about a foot broad, 5 or 6 inches deep over all, and weighs from 90 to 100 lbs. per yard. It rests directly on the ballast, without sleepers or chairs, and the gauge is preserved by means of cross-ties of angle iron. The joint is in fact a sort of fish-joint, the ends of the rails being connected together by being bolted through oblong holes to a *saddle-piece*, 3 feet long, which is a bar somewhat resembling the rails in cross-section, and made exactly to fit the hollow at the under side of the rail.

The system of supporting rails *by the shoulders* of the enlarged head, which is the most favourable of all to steadiness, was practised more than twenty years ago, with the shallow T-shaped rails then used for horse-worked railways, by Mr. David Rankine; but it was not suited to the deep rails employed on locomotive lines without the aid of contrivances which have only of late been invented by Mr. W. B. Adams.

Fig. 286 represents Mr. Adams's *suspended girder* rail, in which the vertical web, not having to sustain compression, is made thinner and deeper than in ordinary rails, so that, for example, a rail of 75 lbs. to the yard is 7 inches deep. The rail A has a continuous bearing at the shoulders upon a pair of angle iron brackets, B, B, whose lower edges press against the foot of the rail, so that they are wedged into the hollow sides of the rail by bolts, C, about 3 feet apart, passing through oblong holes. D is a cross tie-bar, to preserve the gauge. The total breadth across the wings of the brackets ranges from 9 to 14 inches, according to the weight of the traffic; and those wings rest directly on the ballast. The rails and the brackets are laid so as to break joint. Another mode of constructing this sort of permanent way is to substitute pieces of creosoted timber, about 5 inches square, for the angle-iron brackets.

440. **Rails for Level Crossings of Roads.**—When the covering of a road is of broken stone, the ordinary permanent way of the railway may be laid across it, with the heads of the rails rising about $\frac{3}{4}$ of an inch above the road metal. Switches, points, and crossings of rails should not occur on a level crossing of a road, nor should any rails on such a crossing approach nearer to each other than about 6 inches, lest horses should be injured or disabled by their feet being wedged between rails, or the caulkers of their shoes, used in frosty weather, getting jammed in the openings of points, switches, and rail crossings.

Crossings of paved roads also may be made with the ordinary rails, grooves for the flanges of the wheels being cut in the paving-stones alongside of the rails.

Rails of a special form are sometimes used for level crossings of roads. They are usually H-formed, the upper side presenting a groove about $2\frac{1}{2}$ inches broad and $1\frac{1}{2}$ inch deep, between two flanges, the outer of which, being the head of the rail on which the wheel runs, must be $2\frac{1}{2}$ inches broad; while the inner flange may be of the same breadth or of a less breadth, according as the rail is to be reversible or not.

441. **Junctions and Connections of Lines of Rails—Traversers—Turntables.**—I. The *junction* of two lines of rails is effected either by means of a pair of those tapering moveable rails called *switches*,

connected together and worked by the same handle, or by means of a switch at the side *from* which a carriage leaving the main line turns, and a fixed *point* at the other side. The former arrangement is considered the safer where the speed of the traffic is great.

On a narrow gauge line, the ordinary distance between the tip of the switch and the *crossing*, where the two tracks finally become clear of each other, is about 80 feet, so that if one of the tracks is straight the other has a curvature of about 640 feet radius.

Switches are made self-acting by a weight which pulls them into that position which suits the main stream of traffic, so that they require to be held by force in the contrary position. The handles by which switches are worked are so shaped and painted that their position can be distinctly seen from a considerable distance.

It was formerly the practice to notch the sides of the fixed rails so as to receive the tips of the switches; but this has been rendered unnecessary by the introduction of an improved form of switch.*

As far as possible, switches on the main tracks of a line of railway should point in the ordinary direction of the traffic; *facing-points*, as those which point in the contrary direction are called, should only be used in cases of necessity, and then with precautions sufficient to obviate the risk of a train being accidentally turned on to a wrong line.

II. A *connection* between two parallel lines of rails is usually made, where there is room enough, by means of an oblique line of rails with switches at each end. On the narrow gauge, the length of such a connection is about 180 feet.

III. A *traverser* affords the most convenient mode of shifting carriages between parallel lines of rails at a terminal station, where there is not room enough for an ordinary connection. It is a platform supporting a line of rails, long enough for a carriage to stand upon, and supported on wheels which roll on a transverse line of rails at a lower level.

IV. *Turntables* serve to connect lines of rails which cross each other at right angles, or which radiate from a central point. A turntable consists essentially of the following parts:—A foundation of masonry or concrete, a circular cast iron base, having a pivot in the centre, and a race or track for rollers round the circumference; a set of conical rollers, carried in a frame which turns about the pivot; a deck or platform, supported on the pivot at its centre and on the rollers at its circumference, carrying one or more lines of rails, and provided with one or more catches to fix it in different positions. The greater the proportion of the weight borne by the pivot, and the less that borne by the rollers, the less is the friction.

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* First introduced by Messrs. Ransome and May.

A turntable which floats in a cylindrical water-tank was invented by Mr. Adams.

Carriage turntables are usually 12 or 14 feet in diameter, and carry two lines of rails at right angles to each other.

A turntable for an engine and tender is 40 feet in diameter, or thereabouts; it usually carries but one line of rails, and is turned round by the aid of wheelwork. Such turntables are required at stations, to reverse the engines, independently of the connection of lines of rails. Turntables for engine sheds are occasionally made with two parallel lines of rails.

For details as to the construction of these and other railway fittings, see Mr. D. K. Clark's work *On Railway Machinery*.

442. **Stations.**—The best positions for stations, in a purely engineering point of view, and the manner in which they affect questions of curves and gradients, have already been discussed in Article 435, p. 658. It may here be added that the engineer should specially attend to the means of draining the stations, of supplying them with good water, and of getting access to them by roadways, for the arrival and departure of passengers and goods. Passenger platforms are from 2 to 3 feet above the level of the rails, and are best made of strong flags resting on longitudinal walls; at their ends, they should descend gradually to the level of the rails by *ramps* or slopes of about 1 in 10, rather than by flights of steps. They should be, according to Mr. Clark, at least 20 feet broad when used at one side, 30 or 40 when used at both sides; but they are often made of much smaller breadths. The roof should, if possible, be in one span over the whole station; if intermediate pillars are used, they should be placed in the middle of broad platforms, and not near lines of rails if it can be avoided; they should never, on any account, be nearer a line of rails than 4 feet. Care should be taken that sheds have proper means of ventilation. The extent and arrangement of the station and its fittings, as affected by questions of the amount of traffic and the best means of accommodating it, are foreign to the subject of the present work. For examples of stations, see Mr. Clark's article "On Railways" in the *Encyc. Brit.*

442 A. **Pipe-Culverts—Mile-Posts—Gradient-Posts—Telegraph.**—The construction of culverts large enough to be accessible for purposes of repair, in order to carry gas-pipes and water-pipes under a railway, is enjoined by law in Britain; as is also the erection of numbered mile-posts at every quarter of a mile. Gradient-posts, at changes of gradient, having boards to show the direction and rate of inclination, are useful to the engine drivers. Where trains run at very short intervals, a line of electric telegraph may be considered as almost essential to the safe working of the railway.

CHAPTER II.

OF THE COLLECTION, CONVEYANCE, AND DISTRIBUTION OF WATER.

SECTION I.—*Theory of the Flow of Water, or of Hydraulics.*

443. **Pressure of Water—Head.**—The laws of the pressure of a mass of water, when at rest, against any surface which it touches, have already been explained in Article 107, p. 164.

In all questions of hydraulics, it is convenient to express the intensity of the pressure of water in *feet of water;* that is, in terms of the intensity of the pressure of a column of water one foot high upon its base, as an unit. A pressure so expressed is sometimes called a *head of pressure.* In p. 161 two values of that unit, for pure water at the temperatures of 39°·1 and 62° respectively, have been compared with other units; in the following table a comparison of the same sort is given in greater detail, the heaviness assigned to pure water being 62·4 lbs. per cubic foot, which is almost perfectly exact at a temperature of about 52°·3 Fahrenheit, and near enough to the truth for practical purposes at other temperatures, and is also a convenient value for calculation:—

COMPARISON OF HEADS OF WATER IN FEET, WITH PRESSURES IN VARIOUS UNITS.

One foot of water at 52°·3 Fahr. =	62·4	lbs. on the square foot.
” ”	0·4333	lb. on the square inch.
” ”	0·0295	atmosphere.
” ”	0·8823	inch of mercury at 32°.
” ”	773·	feet of air at 32°, and one atmosphere.
One lb. on the square foot,.........	0·016026	foot of water.
One lb. on the square inch, ...	2·308	feet of water.
One atmosphere of 29·922 inches of mercury, ...	33·9	” ”
One inch of mercury at 32°,.......	1·1334	” ”
One foot of air at 32°, and one atmosphere,........................	0·001294	” ”
One foot of average sea water,....	1·026	foot of pure water.

The TOTAL HEAD of a given particle of water is found by adding together the following quantities:—

The *head of pressure*, or intensity of the pressure exerted by the particle, expressed in feet of water.
The *head of elevation*, or actual height of the particle above some fixed or "*datum*" level.

In stating the pressure or head of a particle of water, it is usual *not* to include the *atmospheric pressure*, so that the *absolute* or true pressure exceeds the pressure as stated in the customary way by one atmosphere. When the absolute pressure is exactly one atmosphere, the pressure as stated in the customary way is *nothing;* when the absolute falls short of the atmospheric pressure by so many lbs. on the square inch, or so many feet of water, the customary mode of stating that fact is to say that there are so many lbs. on the square inch, or so many feet, of *vacuum*.

The atmospheric pressure, at the level of the sea, varies from about 32 to 35 feet of water, and diminishes at the rate nearly of 1-100th part of itself for each 262 feet of elevation above that level.

444. **Volume and Mean Velocity of Flow.**—The *volume of flow* or *discharge* of a stream of water is expressed in units of volume per unit of time.

The most convenient unit of volume is the *cubic foot;* but in calculations relating to the water supply of towns it is customary to use the *gallon*.

The following is the relation between those units:—

One gallon = 0·1604 cubic foot (being 10 lbs. of water); and
One cubic foot = 6·2355 gallons;

but in ordinary calculations respecting water-works it is sufficiently accurate to make one gallon = 0·16 cubic foot, and one cubic foot = $6\frac{1}{4}$ gallons.

Of different *units of time*, the *second* is the most convenient in mechanical calculations; the *minute* is the customary unit in stating the discharge of streams; the *hour*, the *day*, and larger periods are used in calculations as to drainage and water supply.

The variety of *units of discharge* is thus very great. The *cubic foot per second* is the most convenient in mechanical calculations.

The *mean velocity* of a stream at a given cross-section is found by dividing the discharge, or volume of flow, by the area of the cross-section, and is most conveniently expressed in feet per second.

445. **Greatest and Least Velocities.**—Inasmuch as every stream of fluid that flows in a channel is retarded by friction against the

material of the channel, the velocity of the fluid particles is different at different points of the same cross-section, being greatest in the centre and least at the border. In open channels, like those of rivers, the ratio of the mean velocity to the greatest or central velocity is given approximately by the following formula of Prony:—

$$\frac{\text{mean velocity}}{\text{greatest velocity}} = \frac{\text{greatest velocity} + 7\cdot71 \text{ feet per second}}{\text{greatest velocity} + 10\cdot28 \text{ feet per second}}. \quad (1.)$$

The least velocity, or that of the particles in contact with the bed, is about as much less than the mean velocity as the greatest velocity is greater than the mean. In ordinary cases, the least, mean, and greatest velocities may be taken as bearing to each other nearly the proportions of 3, 4, and 5. In very slow currents they are nearly as 2, 3, and 4.

446. **General Principles of Steady Flow.**—The *steady* motion of a mass of fluid, as distinguished from unsteady motion, means that kind of motion in which the velocity and direction of motion of a particle depend on its *position* alone, and not jointly on position and time; so that each particle of the series of particles which successively come to a given point, assumes a certain velocity and direction of motion proper to that point. It is, in short, the motion of a *permanent current*, as distinguished from that of a varying current, or that of a wave.

In order to acquire velocity from a state of rest, or an increase of velocity, a fluid particle must pass from a place of *greater total head* to a place of *less total head*. This it may do either by actual descent from a higher to a lower level, or by passing from a place of more intense pressure to a place of less intense pressure, or by both those changes combined. The *loss of head* thus incurred is connected with the velocity produced by the following laws:—

I. In a liquid without friction the loss of head in producing a given increase of velocity is equal to the height of vertical fall which would produce the same increase of velocity in a body falling freely: in other words, the loss of head is equal to the *height due to the acceleration;* and if the particle starts from a state of rest, that height is called the *height due to the velocity*, and is given by the following formula, where v is the velocity in feet per second:—

$$\text{height in feet} = v^2 \div 64\cdot4 \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

II. If the motion of the liquid is impeded by friction, there is an additional loss of head, bearing to the height due to the velocity of flow a certain proportion, depending on the figure and dimensions of the channel and openings traversed by the stream, and other circumstances.

The combination of those two principles may be thus expressed: Let h denote the *loss of head*, in feet; then

$$h = (1 + \mathrm{F}) \frac{v^2}{64 \cdot 4}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

in which F is a factor, determined by experiment, expressing the proportion which the loss of head by friction bears to the height due to the velocity.

The inverse formula, for finding the velocity from the loss of head, is as follows:—

$$v = 8 \cdot 025 \sqrt{\frac{h}{1 + \mathrm{F}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

The velocity computed from a given height, on the supposition that there is no friction, by the formula $v = 8 \cdot 025 \sqrt{h}$, is sometimes called the "theoretical velocity."

In an open channel the loss of head h consists wholly in diminution of the "head of elevation," and is the *actual fall* of the upper surface of the stream. In a close pipe it may consist wholly or partly of diminution of the "head of pressure," and is then called *virtual fall*. To express this in symbols,

Let z_1 denote the elevation above a fixed datum, and
p_1, the head of pressure at a point in the reservoir from which a pipe is supplied, the velocity at that point being insensible, so that
$z_1 + p_1$ is the *total head in still water;* also let
z denote the elevation above the datum, and
p, the head of pressure at a given point in the pipe, at which the loss of head, as computed by equation 2, is h; then the total head at this point is,

$$z + p = z_1 + p_1 - h; \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

and the pressure, in feet of water, is

$$p = z_1 + p_1 - z - h. \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

The pressure of flowing water, as thus diminished by loss of head, is called HYDRAULIC PRESSURE, to distinguish it from the pressure of still water, called *hydrostatic pressure*.

In an open channel, equation 5 is simplified by the fact that for the upper surface of the stream, and all surfaces parallel to it, h is simply $= z_1 - z$; so that $p = p_1$, if the two points are at equal depths below the surface.

If the water has a sensible velocity of flow *at the starting point*, the loss of head required is diminished to the extent of the height due to that *velocity of approach*, as it is called. Thus, let v_0 be the velocity of approach; then, instead of equation 2, we must use the following:—

$$h = (1 + \mathrm{F}) \frac{v^2}{64 \cdot 4} - \frac{v_0^2}{64 \cdot 4}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

and if v_0 bears *a known ratio* to v, let that ratio be $v_0 \div v = r$; then the above equation becomes,

$$h = (1 + \mathrm{F} - r^2) \frac{v^2}{64 \cdot 4}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.)$$

which gives, for the inverse formula,

$$v = 8 \cdot 025 \sqrt{\frac{h}{1 + \mathrm{F} - r^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8.)$$

When a stream flows with an *uniform speed* down an *uniform channel*, and two cross-sections of that channel are compared together, the velocities v_0 and v are equal, and $r = 1$; in this case, the *whole* loss of head between the two cross-sections is expended in overcoming friction; and equations 7 and 8 are reduced to the following:—

$$h = \mathrm{F}\, v^2 \div 64 \cdot 4; \quad \ldots\ldots\ldots\ldots\ldots\ldots (9.)$$

$$v = 8 \cdot 025 \sqrt{h \div \mathrm{F}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (10.)$$

The following table gives examples of heights in feet due to velocities in feet per second, as computed by equation 1. It is exact for latitude 54°¾, and near enough to exactness for practical purposes in all latitudes. The most convenient table, however, for calculating either heights from velocities or velocities from heights is an ordinary table of squares and square roots:—

v	Height.	v	Height.	v	Height.	v	Height.	v	Height.
1	·015528	17	4·4875	32·2	16·100	48	35·776	76	89·688
2	·062111	18	5·0310	33	16·910	49	37·282	78	94·471
3	·13975	19	5·6055	34	17·950	50	38·819	80	99·377
4	·24844	20	6·2111	35	19·021	52	41·987	82	104·41
5	·38819	21	6·8477	36	20·124	54	45·279	84	109·56
6	·55900	22	7·5153	37	21·257	56	48·695	86	114·84
7	·76086	23	8·2141	38	22·422	58	52·235	88	120·25
8	·99377	24	8·9439	39	23·618	60	55·900	90	125·77
9	1·2577	25	9·7048	40	24·844	62	59·688	92	131·43
10	1·5528	26	10·497	41	26·102	64	63·601	94	137·20
11	1·8789	27	11·320	42	27·391	64·4	64·400	96	143·10
12	2·2360	28	12·174	43	28·711	66	67·639	98	149·13
13	2·6241	29	13·059	44	30·062	68	71·800	100	155·28
14	3·0434	30	13·975	45	31·444	70	76·086		
15	3·4937	31	14·922	46	32·857	72	80·496		
16	3·9751	32	15·900	47	34·301	74	85·029		

447. **Friction of Water.**—The following are the values of the factor of friction F in the formulæ of Article 446, as ascertained by experiment, for the cases of most common occurrence in practice.

I. *Friction of an orifice in a thin plate*—

$$F = 0{\cdot}054. \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

II. *Friction of mouthpieces or entrances from reservoirs into pipes.*—Straight cylindrical mouthpiece, perpendicular to side of reservoir—

$$F = 0{\cdot}505. \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

The same mouthpiece making the angle θ with a perpendicular to the side of the reservoir—

$$F = 0{\cdot}505 + 0{\cdot}303 \sin \theta + 0{\cdot}226 \sin^2 \theta. \ldots\ldots\ldots(3.)$$

For a mouthpiece of the form of the "contracted vein," that is, one somewhat bell-shaped, and so proportioned that if d be its diameter on leaving the reservoir, then at a distance $d \div 2$ from the side of the reservoir it contracts to the diameter ${\cdot}7854\, d$,—the resistance is insensible, and F nearly $= 0$.

III. *Friction at sudden enlargements.*—Let A_1 be the sectional area of a channel, discharging Q cubic feet of water per second, in which a sluice, or slide valve, or some such object, produces a sudden contraction to the smaller area a, followed by a sudden enlargement to the area A_2. Let $v = Q \div A_2$ be the velocity in the second enlarged part of the channel. The *effective* area of the orifice a will be $c\,a$, c being a *co-efficient of contraction* of the stream flowing through it, whose value may be taken at ${\cdot}618 \div \sqrt{1 - {\cdot}618 \frac{a^2}{A_1^2}}$. Let the ratio in which the effective area of the channel is suddenly enlarged be denoted by

$$r = A_2 \div c\,a = \frac{A_2}{a}\sqrt{\left(2{\cdot}618 - 1{\cdot}618 \frac{a^2}{A_1^2}\right)}. \;\ldots\ldots(4.)$$

Then $r\,v$ is the velocity in the most contracted part. It appears that all the energy due to the *difference* of the velocities, $r\,v$ and v, is expended in fluid friction, and consequently that there is a loss of head given by the formula—

$$(r - 1)^2 \cdot \frac{v^2}{2\,g}; \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.)$$

so that in this case

$$F = (r - 1)^2. \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.)$$

IV. *Friction in pipes and conduits.*—Let A be the sectional area of a channel; b its *border*, that is, the length of that part of its girth which is in contact with the water; l the length of the channel, so that $l\,b$ is the frictional surface; and for brevity's sake let $A \div b = m$; then, for the friction between the water and the sides of the channel—

$$F = f \cdot \frac{l\,b}{A} = \frac{f\,l}{m}; \text{ ....(7.)}$$

in which the co-efficient f has the following values:—

$$\text{For iron pipes,.....} f = 0{\cdot}0036 + \frac{0{\cdot}0043}{\sqrt{v}}. \text{ ....(8.)}$$

$$\text{For open conduits,} f = 0{\cdot}00741 + \frac{0{\cdot}000227}{v}. \text{ ......(9.)}$$

The quantity $m = A \div b$ is called the "*hydraulic mean depth*" of channel, and for cylindrical and square pipes running full is obviously *one-fourth* of the diameter; and the same is its value for a semicylindrical open conduit, and for an open conduit whose sides are tangents to a semicircle of a diameter equal to twice the greatest depth of the conduit.

In an open conduit, the loss of head,

$$h = \frac{f\,l}{m} \cdot \frac{v^2}{2\,g}, \text{ ....(10.)}$$

takes place as an actual fall in the surface of the water, producing a declivity at the rate

$$i = \frac{h}{l} = \frac{f}{m} \cdot \frac{v^2}{2\,g}; \text{ ....(11.)}$$

and by the last two formulæ are to be determined the fall and the rate of declivity of open channels which are to convey a given flow. In close pipes, the loss of head takes place in the total head; and the ratio $i = h \div l$ is called the *virtual declivity*.

V. For *bends in circular pipes*, let d be the diameter of the pipe, ρ the radius of curvature of its centre line at the bend, θ the angle through which it is bent, π two right angles; then, according to Professor Weisbach,

$$F = \frac{\theta}{\pi} \left\{ 0{\cdot}131 + 1{\cdot}847 \left(\frac{d}{2\,\rho}\right)^{\frac{7}{2}} \right\} \text{............(12.)}$$

VI. For *bends in rectangular pipes*,

$$F = \frac{\theta}{\pi} \left\{ 0{\cdot}124 + 3{\cdot}104 \left(\frac{d}{2\,\rho}\right)^{\frac{7}{2}} \right\} \text{............(13.)}$$

VII. For *knees*, or sharp turns in pipes, let θ be the angle made by the two portions of the pipe at the knee; then

$$F = 0{\cdot}946 \sin^2 \frac{\theta}{2} + 2{\cdot}05 \sin^4 \frac{\theta}{2}. \quad \ldots\ldots\ldots\ldots (14.)$$

VIII. *Summary of losses of head.*—When several successive causes of resistance occur in the course of one stream, the losses of head arising from them are to be added together; and this process may be extended to cases in which the velocity varies in different parts of the channel, in the following manner:—

Let the final velocity at the cross section, where the loss of head is required, be denoted by v;

Let the ratios borne to that velocity by the velocities in other parts of the channel be known; $r_0 v$ being the "velocity of approach" (Article 446, p. 676), $r_1 v$ the velocity in the first division of the channel, $r_2 v$ in the second, and so on; and let F_1 be the sum of all the factors of resistance for the first division, F_2 for the second, and so on; then the loss of head will be—

$$h = \frac{v^2}{64{\cdot}4}(1 - r_0^2 + F_1 r_1^2 + F_2 r_2^2 + \&c.);$$

an expression which may be abbreviated into the following: (15.)

$$h = \frac{v^2}{64{\cdot}4}(1 - r_0^2 + \Sigma \cdot F r^2).$$

448. **Contraction of Stream from Orifice—Co-efficients of Discharge.**—The fact of the contraction of a jet or stream that flows from an orifice has already been referred to. It is caused by the convergence of the particles towards the orifice before they pass through it, which convergence continues for a time after the particles pass the orifice. The result is, that the *effective* area of the orifice, or area of the "*contracted vein,*" which is to be used in computing the discharge, is less than the total area in a proportion which is called the *co-efficient of contraction.*

Sometimes it is impossible to distinguish between the effect of friction in diminishing the velocity (expressed by $1 \div \sqrt{1 + F}$), and that of contraction in diminishing the area of the stream. In such cases the ratio in which the actual discharge is less than the product of the "theoretical velocity" (Article 446, p. 675) and the total area of the orifice, is called the *co-efficient of efflux* or *of discharge.*

The quantities given in the following statements and tables are some of them real co-efficients of contraction, and some co-efficients of discharge. In hydraulic formulæ, such co-efficients are usually denoted by the symbol c.

In sharp-edged orifices the friction is almost inappreciable (see Article 447, Case I.); in those with flat or rounded borders its effects become sensible, and in tubes or other channels of such length as to guide all the particles along their sides there is no contraction, and friction operates alone in diminishing the discharge.

In all the *sharp-edged orifices* here mentioned the edge is supposed to be formed at the *inner* or up-stream side of the plate by chamfering or bevelling the outer side. Were the inner side of the plate chamfered, it would guide the stream, and alter the contraction to an uncertain amount.

I. *Sharp-edged circular orifices in flat plates*; $c = \cdot 618$....(1.)

II. *Sharp-edged rectangular orifices in vertical flat plates.*—In this case the co-efficient depends partly on the proportions of the dimensions of the orifice to each other, and partly on the proportion borne by the breadth of the orifice to the *charge* or head. The co-efficient is intended to be used in the following formula for the discharge in cubic feet per second, A being the area of the orifice in square feet; and h the head, measured from the *centre* of the orifice to the *level of still water*.

$$Q = 8{\cdot}025\, c\, A\, \sqrt{h} \text{ ....(2.)}$$

The co-efficients are given on the authority of experiments of Poncelet and Lesbros on orifices about 8 inches wide. They have not been reduced to a general formula.

CO-EFFICIENTS OF DISCHARGE FOR RECTANGULAR ORIFICES.

Head ÷ Breadth.	Height of Orifice ÷ Breadth.					
	1	0·5	0·25	0·15	0·1	0·05
0·05	...	...	...	...	...	·709
0·10	...	...	...	...	·660	·698
0·15	...	...	...	·638	·660	·691
0·20	...	...	·612	·640	·659	·685
0·25	...	...	·617	·640	·659	·682
0·30	...	·590	·622	·640	·658	·678
0·40	...	·600	·626	·639	·657	·671
0·50	...	·605	·628	·638	·655	·667
0·60	·572	·609	·630	·637	·654	·664
0·75	·585	·611	·631	·635	·653	·660
1·00	·592	·613	·634	·634	·650	·655
1·50	·598	·616	·632	·632	·645	·650
2·00	·600	·617	·631	·631	·642	·647
2·50	·602	·617	·631	·630	·640	·643
3·50	·604	·616	·629	·629	·637	·638
4·00	·605	·615	·627	·627	·632	·627
6·00	·604	·613	·623	·623	·625	·621
8·00	·602	·611	·619	·619	·618	·616
10·00	·601	·607	·613	·613	·613	·613
15·00	·601	·603	·606	·607	·608	·609

The co-efficients in the preceding table include a correction for the error occasioned by measuring the head from the *centre* of the orifice instead of from the point where the mean velocity occurs, which is somewhat above the centre. That correction is inappreciable when the head exceeds 3 times the height of the orifice.

III. *Sharp-edged rectangular notches* (or orifices extending up to the surface) *in flat vertical weir boards.*—The area of the orifice is measured up to the *level of still water* in the pond behind the weir.

Let b = breadth of the notch;
B = total breadth of the weir; then

$$c = \cdot 57 + \frac{b}{10\,B}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

provided b is not less than $B \div 4$.

IV. *Sharp-edged triangular or V-shaped notches in flat vertical weir boards* (from experiments by Professor James Thomson.)—Area measured up to the level of still water.

$$\left.\begin{array}{l}\text{Breadth of notch} = \text{depth} \times 2;\ c = \cdot 595;\\ \text{Breadth of notch} = \text{depth} \times 4;\ c = \cdot 620.\end{array}\right\}\ldots\ldots(4.)$$

V. *Partially-contracted sharp-edged orifice.* (That is to say, an orifice towards part of the edge of which the water is guided in a direct course, owing to the border of the channel of approach partly coinciding with the edge of the orifice).

Let c be the ordinary co-efficient;
n, the fraction of the edge of the orifice which coincides with the border of the channel;
c', the modified co-efficient; then

$$c' = c + \cdot 09\,n. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.)$$

VI. *Flat or round-topped weir,* area measured up to the level of still water—

$$c = \cdot 5 \text{ nearly}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.)$$

VII. *Sluice in a rectangular channel*—

$$\left.\begin{array}{r}\text{vertical; } c = 0{\cdot}7;\\ \text{Inclined backwards to the horizon at } 60^\circ;\ c = 0{\cdot}74;\\ \text{,,} \qquad \text{,,} \qquad \text{,,} \qquad \text{at } 45^\circ;\ c = 0{\cdot}8.\end{array}\right\}(7.)$$

VIII. *Incomplete contraction;* see Article 477, Division III., p. 677.

449. **Discharge from Vertical Orifices, Notches, and Sluices.**—When the height of an orifice in the vertical side of a reservoir

does not exceed about one-half or one-third of its depth below the surface, the head measured from the centre of the orifice to the level of still water may be used, without sensible error, to compute the mean velocity of a flow, and the discharge; so that the formula for the discharge is

$$Q = 8{\cdot}025\, c\, A \sqrt{h}; \ \ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

A being the total area of the orifice, and c the proper co-efficient of contraction.

When the height of the orifice exceeds about one-half of the head of water, and especially when the orifice is a *notch* extending to the surface, it is not sufficiently accurate to measure the head simply from the level of still water to the centre of the orifice; but the area of the orifice is to be conceived as divided into a number of horizontal bands, the area of each such band multiplied by the velocity due to its depth below the surface of still water, the products summed or integrated, and the sum or integral multiplied by a suitable co-efficient of contraction.

To express this in symbols, let b be the breadth, $d\,h$ the height of one of the horizontal bands, so that $b\,d\,h$ is its area; h, the depth of its centre below the level of the surface of still water in the reservoir; h_0, the depth of the upper edge of the orifice, and h_1 that of its lower edge, below the same level; c, the co-efficient of contraction; Q, the discharge in cubic feet per second; then

$$Q = 8{\cdot}025\, c \int_{h_0}^{h_1} b \sqrt{h} \cdot d\,h \ \ldots\ldots\ldots\ldots\ldots(2.)$$

For co-efficients of contraction, see Article 448.

The following are the most important cases:—

I. *Rectangular orifice*; $b =$ constant.

$$Q = 8{\cdot}025\, c \times \frac{2}{3}\, b \left(h_1^{\frac{3}{2}} - h_0^{\frac{3}{2}}\right) = 5{\cdot}35\, c\, b \left(h_1^{\frac{3}{2}} - h_0^{\frac{3}{2}}\right). \ (3.)$$

It is seldom necessary to use this formula in practice; for the co-efficients in the table by Poncelet and Lesbros (see p. 680) comprehend, as has been stated, the correction for the error arising from using the head at the centre of the orifice simply, as in equation 1.

II. *Rectangular notch, with a still pond*; $b =$ constant, $h_0 = 0$; h_1 measured from the lower edge of the notch to the level of still water.

$$\left.\begin{aligned} Q &= 8{\cdot}025\, c \times \frac{2}{3}\, b\, h_1^{\frac{3}{2}} = 5{\cdot}35\, c\, b\, h_1^{\frac{3}{2}} \\ &= \left(3{\cdot}05 + {\cdot}535\, \frac{b}{B}\right) b\, h_1^{\frac{3}{2}} \end{aligned}\right\} \ldots\ldots\ldots\ldots(4.)$$

The last expression is founded on the formula for the co-efficient c, given in Article 448, Division III., p. 681, B being the whole breadth of the weir.

TABLE OF VALUES OF c AND 5·35 c.

$\frac{b}{B}$,……	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·25
c,……	·67	·66	·65	·64	·63	·62	·61	·60	·595
5·35 c,	3·58	3·53	3·48	3·42	3·37	3·32	3·26	3·21	3·18

The *cube of the square root of the head*, $h_1^{\frac{3}{2}}$, is easily computed as follows, by the aid of an ordinary table of squares and cubes: look in the column of squares for the nearest square to h_1; then opposite, in the column of cubes, will be an approximate value of $h_1^{\frac{3}{2}}$.

III. *Rectangular notch, with current approaching it.*—When still water cannot be found, to measure the head h_1 up to, let v_0 denote the velocity of the current at the point up to which the head is measured, or *velocity of approach:* compute the height due to that velocity as follows:—

$$h_0 = v_0^2 \div 64 \cdot 4;$$

then the flow is the difference between that from a still pond due to the height $h_1 + h_0$, and that due to the height h_0; so that it is given by the formula

$$Q = 5 \cdot 35\, c\, b \left\{(h_1 + h_0)^{\frac{3}{2}} - h_0^{\frac{3}{2}}\right\} \ldots\ldots\ldots\ldots (5.)$$

When v_0 cannot be directly measured, it can be computed approximately by taking an approximate value of Q from equation 4, and dividing by the sectional area of the channel at the place up to which the head is measured from the lower edge of the notch.

IV. *Triangular or V-shaped notch, with a still pond;* h_1 measured from the apex of the triangle to the level of still water.

Let a denote the ratio of the *half-breadth* of the notch at any given level to the height above the apex, so that, for example, at the level of still water, the whole breadth of the notch is $2\, a\, h_1$.

$$Q = 8 \cdot 025\, c \times \frac{8}{15}\, a\, h_1^{\frac{5}{2}} = 4 \cdot 28\, c\, a\, h_1^{\frac{5}{2}}; \ldots\ldots\ldots (6.)$$

and adopting the values of c already given in Article 448, p. 681, we have,

$$\text{for } a = 1;\ Q = 2 \cdot 54\, h_1^{\frac{5}{2}}; \ldots\ldots\ldots\ldots\ldots (6\ \text{A.})$$

$$\text{for } a = 2;\ Q = 5 \cdot 3\, h_1^{\frac{5}{2}}. \ldots\ldots\ldots\ldots\ldots\ldots (6\ \text{B.})$$

In the absence of sufficiently extensive tables of squares and fifth powers, the best method of computing the fifth power of the square root of the head is by the aid of logarithms.

V. *Drowned orifices* are those which are below the level of the water in the space into which the water flows as well as in that from which it flows. In such cases the difference of the levels of still water in those two spaces is the head to be used in computing the flow.

VI. *Drowned rectangular notch.*—Let h_1 and h_2 be the heights of the still water above the lower edge of the notch at the up-stream and down-stream sides of the notch-board respectively; the following formula gives the flow in cubic feet per second:—

$$Q = 5{\cdot}35\ c\ b \left(h_1 + \frac{h_2}{2}\right) \sqrt{(h_1 - h_2)} \text{..........(7.)}$$

VII. For *weirs with broad flat crests*, drowned or undrowned, the formulæ are the same as for rectangular notches, except that the co-efficient c is about ·5, as has been stated.

VIII. *Computation of the dimensions of orifices.*—The whole of the preceding formulæ (with the exception of equations 5 and 7) can easily be used in an inverse form, in order to find the dimensions of orifices that are required to discharge given volumes of water per second.

For example, if equation 1 is applicable, we have for the area of the orifice,

$$A = Q \div 8{\cdot}025\ c \sqrt{h} \text{....................(8.)}$$

If equation 3 is applicable, the breadth of the orifice is given as follows:—

$$b = Q \div 5{\cdot}35\ c\ (h_1^{\frac{3}{2}} - h_0^{\frac{3}{2}}). \text{...............(9.)}$$

If equation 4 is applicable, the depth of the bottom of the notch below still water is given by the equation,

$$h_1 = \left\{Q \div 5{\cdot}35\ c\ b\right\}^{\frac{2}{3}}; \text{..................(10.)}$$

if equation 6 is applicable,

$$h_1 = \left\{Q \div 4{\cdot}28\ c\ a\right\}^{\frac{2}{5}}. \text{..................(11.)}$$

IX. *Sluices.*—The opening of a sluice generally acts as a rectangular orifice, drowned or undrowned as the case may be; the value of c being as given in Article 448, p. 681.

450. **Computation of the Discharge and Diameters of Pipes.**—The loss of head by a stream of the velocity v in traversing the length

l of a pipe of the uniform diameter d is given by the following formula, deduced from equations 8 and 10 of Article 447, by putting $d \div 4$ for the hydraulic mean depth m:—

$$h = \frac{4fl}{d} \cdot \frac{v^2}{64 \cdot 4} = \left(\cdot 0144 + \frac{\cdot 0172}{\sqrt{v}} \right) \frac{l}{d} \cdot \frac{v^2}{64 \cdot 4}. \ldots\ldots (1.)$$

From this equation are deduced the solutions of the following problems:—

I. *To compute the discharge of a given pipe;* the data being h, l, and d, all in feet.

Assume an approximate value for $4f$. The value commonly assumed in the first instance is,

$$\cdot 0258.$$

This gives, as a *first approximation* to the velocity,

$$v' = 8 \cdot 025 \sqrt{\frac{h\,d}{\cdot 0258\, l}} = 50 \sqrt{\frac{h\,d}{l}}; \ldots\ldots\ldots\ldots (2.)$$

or, *a mean proportional between the diameter and the loss of head in* 2,500 *feet of length.* With this velocity calculate a new value of $4f$, which is to be used in computing a *second approximation* to the velocity, by means of the formula,

$$v = 8 \cdot 025 \sqrt{\frac{h\,d}{4fl}}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

and this is in almost every case near enough to the truth for practical purposes. Then the discharge is given by the formula,

$$Q = \cdot 7854\, v\, d^2. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

[A *first approximation* to the discharge is given by the following formula,

$$Q' = \cdot 7854\, v'\, d^2 = 39 \cdot 27 \sqrt{\frac{h}{l}} \cdot d^{\frac{5}{2}}. \ldots\ldots (4\text{ A.})]$$

II. *To find (in feet) the diameter* d *of a pipe, so that it shall deliver* Q *cubic feet of water per second, with a loss of head at the rate of* h *feet in each length of* l *feet.*

Supposing the value of $4f$ known,

$$d = \left(\frac{4fl\,Q^2}{39 \cdot 73\, h} \right)^{\frac{1}{5}}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

But $4f$ depends on the velocity, which itself depends on the diameter sought. Assume, as a first approximation, $4\,f = \cdot 0258$. Then compute a *first approximation to the diameter* by the following formula:—

$$d' = 0\cdot 23 \left(\frac{l\,Q^2}{h}\right)^{\frac{1}{5}}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

also a *first approximation to the velocity* by the formula,

$$v' = Q \div \cdot 7854\, d'^2. \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.)$$

From these data, by means of equation 1 of this Article, compute an *approximate loss of head*, h'. If this agrees exactly or very nearly with the given loss of head, h, the first approximation to the diameter is sufficient; if not, a *corrected diameter* is to be found by the following formula:—

$$d = d' \cdot \left(\frac{h'}{h}\right)^{\frac{1}{5}} = d' \cdot \left(\frac{4}{5} + \frac{h'}{5\,h}\right). \ldots\ldots\ldots\ldots\ldots (8.)$$

In the preceding formulæ the pipe is supposed to be free from all curves and bends so sharp as to produce appreciable resistance. Should such obstructions occur in its course, they may be allowed for in the following manner:—Having first computed the diameter of the pipe as for a straight course, calculate the additional loss of head due to curves by the proper formula (Article 447, p. 678); let h'' denote that additional loss of head; then make a further correction of the diameter of the pipe, by increasing it in the ratio of

$$1 + \frac{h''}{5\,h} : 1. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.)$$

By a similar process an allowance may be made for the loss of head on first entering the pipe from the reservoir, viz.:—

$(1 + F)\, v^2 \div 64\cdot 4$; F being the factor of friction of the mouthpiece.

To the diameter of a pipe, as computed by the formulæ, an addition is commonly made in practice, in order to allow for accidental obstructions, for the incrustation of the interior of the pipe, &c. According to some authorities about one-sixth is to be added to the diameter of the pipe for this purpose; but experience seems to show that in general the incrustation, if any, is of equal thickness in pipes of all diameters exposed for equal times to the action of the same water; and therefore that, in a given system of water-pipes, an equal absolute allowance should be made for possible incrustation in pipes of all diameters. In ordinary cases it appears that about *one inch* is sufficient for that purpose.

451. **Discharge and Dimensions of Channels.**—The rate of declivity required for the surface of the current in an uniform

conduit or river-channel is found by dividing the loss of head h (which is all actual fall) by the length l of the channel, and is expressed by the following equation, deduced from equation 11 of Article 447, p. 678:—

$$i = \frac{h}{l} = \frac{f}{m} \cdot \frac{v^2}{64{\cdot}4} = \left({\cdot}00741 + \frac{{\cdot}000227}{v} \right) \cdot \frac{v^2}{64{\cdot}4\, m}; \quad (1.)$$

m being the "hydraulic mean depth." This equation enables the following problems to be solved:—

I. *To compute the discharge of a given stream*, the data being i, m, and the sectional area A. The first step is to find the velocity, which might be done by means of a quadratic equation; but it is less laborious to find it by successive approximations. For that purpose assume an *approximate value* for the co-efficient of friction, such as

$$f' = {\cdot}007565;$$

then the *first approximation* to the velocity is

$$v' = 8{\cdot}025 \sqrt{\frac{i\, m}{{\cdot}007565}} = \sqrt{8512\, i\, m} = 92{\cdot}26 \sqrt{i\, m}; \quad (2.)$$

or, *a mean proportional between the hydraulic mean depth and the fall in* 8,512 *feet.* A *first approximation to the discharge* is

$$Q' = v' A. \quad \ldots\ldots\ldots\ldots (3.)$$

These first approximations are in many cases sufficiently accurate. To obtain second approximations, compute a corrected value of f according to the expression in brackets in equation 1; should it agree nearly or exactly with f', the first assumed value, it is unnecessary to proceed further; should it not so agree, correct the values of the velocity and discharge by multiplying each of them by the factor,

$$\frac{3}{2} - \frac{f}{{\cdot}01513}. \quad \ldots\ldots\ldots\ldots (4.)$$

II. *To determine the dimensions of an uniform channel, which shall discharge* Q *cubic feet of water per second with the declivity* i. —To solve this problem, it is necessary, in the first place, to assume a figure for the intended channel, so that the *proportions* of all its dimensions to each other, and to the hydraulic mean depth m, may be fixed. This will fix also the proportion $A \div m^2$ of the sectional area to the square of the hydraulic mean depth, which will be known although those areas are still unknown; let it be denoted by n.

[The following are examples of the values of n for different figures of cross-section:—

for a semicircle, $n = 6·2832$;
for a half-square, $n = 8$;
for a half-hexagon, $n = 4\sqrt{3} = 6·928$;
for a section (proposed by Mr. Neville) bounded below and at the sides by three straight lines, all tangents to one semicircle which has its centre at the water level, the bottom being horizontal, and the sides sloping at any angle θ (see fig. 288);

$$n = 4\left(\operatorname{cosec}\theta + \tan\frac{\theta}{2}\right).$$

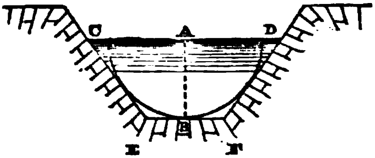

Fig. 288.

In each of the four figures mentioned above, m is one-half of the greatest depth.]

Compute a *first approximation* to the required hydraulic mean depth as follows:—

$$m' = \left(\frac{Q^2}{8,512\, n^2 i}\right)^{\frac{1}{5}}; \text{.....................} (5.)$$

also a first approximation to the velocity,

$$v' = \frac{Q}{n\, m'^2}; \text{.........................} (6.)$$

from these data, by means of equation 1 of this article, compute an *approximate declivity* i'. If this agrees exactly or very nearly with the given declivity, i, the first approximation to the hydraulic mean depth is sufficient; if not, a *corrected hydraulic mean depth* is to be found by the following formula:—

$$m = m'\left(\frac{4}{5} + \frac{i'}{5\,i}\right). \text{....................} (7.)$$

From the hydraulic mean depth, all the dimensions of the channel are to be deduced, according to the figure assumed for it.

452. **Elevation Produced by a Weir.**—When a weir or dam is erected across a river, the following formulæ serve to calculate the height h_1, in feet, at which the water in the pond, close behind the weir, will stand above its crest; Q being the discharge in cubic feet per second, and b the breadth of the weir in feet:—

I. *Weir not drowned*, with a flat or slightly rounded crest—

$$h_1 = \left(\frac{Q^2}{7\,b^2}\right)^{\frac{1}{3}}, \text{ nearly. } \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

II. *Weir drowned.*—Let h_2 be the height of the water in front of the weir above its crest.

$$\textit{First approximation; } h'_1 = h_2 + \left(\frac{Q^2}{7\,b^2}\right)^{\frac{1}{3}}. \ldots\ldots\ldots (2.)$$

$$\textit{Second approximation; } h''_1 = h'_1 - h_2\left(1 - \frac{5}{4}\cdot\frac{h_2}{h'_1 - h_2}\right). \ (3.)$$

Closer approximations may be obtained by repeating the last calculation.

453. **Backwater** is the effect produced by the elevation of the water-level in the pond close behind the weir, upon the surface of the stream at places still farther up its channel.

For a channel of uniform breadth and declivity, the following is an approximate method of determining the figure which a given elevation of the water close behind a weir will cause the surface of the stream farther up to assume.

Let i denote the rate of inclination of the *bottom* of the stream, which is also the rate of inclination of its surface before being altered by the weir.

Let δ_0 be the natural depth of the stream, before the erection of the weir.

Let δ_1 be the depth as altered, close behind the weir.

Let δ_2 be any other depth in the altered part of the stream.

It is required to find x, the distance from the weir in a direction up the stream at which the altered depth δ_2 will be found.

Denote the ratio in which the depth is altered at any point by

$$\delta \div \delta_0 = r;$$

and let ϕ denote the following function of that ratio:—

$$\left.\begin{aligned} \phi = \int\frac{d\,r}{r^3 - 1} = \frac{1}{6}\text{ hyp. log. }\left\{1 + \frac{3\,r}{(r-1)^2}\right\} \\ + \frac{1}{\sqrt{3}}\text{ arc. tan. }\frac{2\,r+1}{\sqrt{3}} \end{aligned}\right\}\ldots\ldots(1.)$$

A convenient approximate formula for computing ϕ is as follows:—

$$\phi \text{ nearly} = \frac{1}{2\,r^2} + \frac{1}{5\,r^5} + \frac{1}{8\,r^8} \text{..............(1 A.)}$$

Compute the values, ϕ_1 and ϕ_2, of this function, corresponding to the ratios

$$r_1 = \delta_1 \div \delta_0 \text{ and } r_2 = \delta_2 \div \delta_0.$$

Then

$$x = \frac{\delta_1 - \delta_2}{i} + \left(\frac{1}{i} - 264\right)\cdot(\phi_1 - \phi_2)\,\delta_0 \text{.........(2.)}$$

The following table gives some values of ϕ:—

r	ϕ	r	ϕ
1·0	∞	1·8	·166
1·1	·680	1·9	·147
1·2	·480	2·0	·132
1·3	·376	2·2	·107
1·4	·304	2·4	·089
1·5	·255	2·6	·076
1·6	·218	2·8	·065
1·7	·189	3·0	·056

The first term in the right-hand side of the formula 2 is the distance back from the weir at which the depth δ_2 would be found if the surface of the water were level. The second term is the additional distance arising from the declivity of that surface towards the weir. The constant 264 is an approximation to $2 \div f$, f being the co-efficient of friction. For a natural declivity of 1 in 264 the second term vanishes. For a steeper declivity it becomes negative, indicating that the surface of the water rises towards the weir; but although that rise really takes place in such cases, the agreement of its true amount with that given by the formula is somewhat uncertain, inasmuch as the formula involves assumptions which are less exact for steep than for moderate natural declivities. It is best, therefore, in cases of natural declivities steeper than 1 in 264, to compute the extent of backwater simply from the first term of the formula.

454. **Stream of Unequal Sections.**—The preceding rule for determining the figure and extent of backwater is the solution of a particular case of the following general problem:—*Given the form of the bed of a stream, the discharge* Q, *and the water-level at one cross-section; to find the form assumed by the surface of the water in an* up-stream *direction from that cross-section.*

In this case the loss of head between any two cross-sections is the sum of that expended in overcoming friction, and of that due to change of velocity, when the velocity increases, or the difference of those two quantities when the velocity diminishes, which difference may be positive or negative, and may represent either a loss or a gain of head. In parts of the stream where the difference is negative, the surface slopes the reverse way. In fig. 289, let O Z be the vertical plane of the cross-section at which the water-level is given; let horizontal abscissæ, such as O X = x, be measured *against* the direction of flow, and vertical ordinates to the surface of the stream, such as X B = z, upwards from a horizontal datum plane. Consider any indefinitely short portion of the stream whose length is $d\,x$, hydraulic mean depth m, and area of section A. The fall in that portion of the stream is $d\,z$, and the acceleration $-\,d\,v$, because of v being opposite to x. Then,

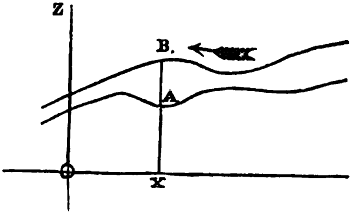

Fig. 289.

$$d\,z = -\frac{v\,d\,v}{32\cdot2} + \frac{f\,d\,x}{m}\cdot\frac{v^2}{64\cdot4}. \ \ldots\ldots\ldots\ldots\ldots(1.)$$

In applying this differential equation to the solution of any particular problem, for v is to be put Q ÷ A, and for A and m are to be put their values in terms of x and z. Thus is obtained a differential equation between x and z, and the constant quantity, Q, which equation, being integrated, gives the relation between x and z, the co-ordinates of the surface of the stream.

455. The **Time of Emptying a Reservoir** is determined by conceiving it to be divided into thin horizontal layers at different heights above the outlet, finding the velocity of discharge for each layer, and thence the time of discharge, and summing or integrating the results.

Let s be the area of any given layer, $d\,h$ its depth, A the effective area of the outlet, h the height of the layer above the outlet; then the velocity of outflow for that layer is $C\sqrt{h}$, C being a multiplier taken from the proper formula in Articles 449, 450, or 451. The time of discharge of the layer is

$$d\,t = \frac{s\,d\,h}{A\,C\sqrt{h}}; \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

and if h_1 be the height of the top water, the whole time is,

$$t = \frac{1}{\mathrm{A\,C}} \int_0^{h_1} \frac{s\,d\,h}{\sqrt{h}}. \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

One of the most convenient ways of expressing this result is to state the ratio which the time of emptying bears to the time of discharging a quantity of water equal to the contents of the reservoir $\left(\text{that is, } \int_0^{h_1} s\,d\,h\right)$, supposing it kept always full. Let that time be called T; its value is $\mathrm{T} = \int_0^{h_1} s\,d\,h \div \mathrm{A\,C}\sqrt{h_1}$, and that of the required ratio is

$$\frac{t}{\mathrm{T}} = \sqrt{h_1} \cdot \int_0^{h_1} \frac{s\,d\,h}{\sqrt{h}} \div \int_0^{h_1} s\,d\,h. \;\ldots\ldots\ldots\ldots(3.)$$

The following are examples:—

Reservoir with vertical sides (s = constant); $t \div \mathrm{T} = 2$.
Wedge-shaped reservoir (s = constant × h); $t \div \mathrm{T} = 1\frac{1}{3}$.
Pyramidal reservoir, the base of the pyramid being the surface, the apex at the outlet (s = constant × h^2);.......................... $t \div \mathrm{T} = 1\frac{1}{5}$.

The division of the reservoir into layers may be facilitated by a plan with contour-lines at a series of different levels.

The time required to empty part of a reservoir is found by computing the time required to empty the whole, and subtracting from it the time which would be required to empty the remaining part.

The time required to *equalize the level of the water* in two adjoining basins with vertical sides (such as lock-chambers on canals), when a communication is opened between them under water, is the same with that required to empty a vertical-sided reservoir of a volume equal to the volume of water *transferred* between the chambers, and of a depth equal to their greatest difference of level.

SECTION II.—*Of the Measurement and Estimation of Water.*

456. **Sources of Water in General—Rain-fall, Total and Available.**—The original source of all supplies of water is the rain-fall. The rain-water which escapes evaporation and absorption by vegetables either runs directly from the surface of the ground or from the pores of the surface-soil into *streams*, or it sinks deeper into the ground, flows through the crevices of porous strata, and escapes at their out-crop in *springs*, or collects in such porous strata, from which it is drawn by means of *wells*.

In what manner soever the water is collected, and whether it is to be used for irrigation, for driving machinery, for feeding a canal, or for the supply of a town, or to be got rid of as in works of mere drainage, the measurement of the rain-fall of the district whence it comes is of primary importance. To complete that measurement two kinds of data are required,—the area of the district, called the *drainage area*, or *catchment-basin*, or *gathering-ground*; and the depth of rain-fall in a given time.

I. *A Drainage Area*, or *Catchment-basin*, is, in almost every case, a district of country enclosed by a *ridge* or *water-shed line* (see Article 58, p. 93), continuous except at the place where the waters of the basin find an outlet. It may be, and generally is, divided by branch ridge-lines into a number of smaller basins, each drained by its own stream into the main stream. In order to measure the area of a catchment-basin a plan of the country is required, which either shows the ridge-lines or gives data for finding their positions by means of detached levels, or of contour-lines. (Article 59, p. 95.)

When a catchment-basin is very extensive it is advisable to measure the several smaller basins of which it consists, as the depths of rain-fall in them may be different; and sometimes, also, for the same reason, to divide those basins into portions at different distances from the mountain-chains, where rain-clouds are chiefly formed.

The exceptional cases, in which the boundary of a drainage area is not a ridge-line on the surface of the country, are those in which the rain-water sinks into a porous stratum until its descent is stopped by an impervious stratum, and in which, consequently, one boundary at least of the drainage area depends on the figure of the impervious stratum, being, in fact, a ridge-line on the upper surface of that stratum, instead of on the ground, and very often marking the upper edge of the outcrop of that stratum. If the porous stratum is partly covered by a second impervious stratum, the nearest ridge-line on the latter stratum to the point where the porous stratum crops out, will be another boundary of the drainage area. In order to determine a drainage area under these circumstances it is necessary to have a geological map and sections of the district.

II. The *Depth of Rain-fall* in a given time varies to a great extent at different seasons, in different years, and in different places. The extreme limits of annual depth of rain-fall in different parts of the world may be held to be respectively nothing and 150 inches. The average annual depth of rain-fall in different parts of Britain ranges from 22 inches to 140 inches, and the least annual depth recorded in Britain is about 15 inches.

The rain-fall in different parts of a given country is, in general, greatest in those districts which lie towards the quarter from which the prevailing winds blow; in Britain, for example, the western districts are the most rainy. Upon a given mountain-ridge, however, the reverse is the case, the greatest rain-fall taking place on that side which lies to leeward, as regards the prevailing winds: thus, in Britain, more rain falls in general on the eastern than on the western slope of a range of hills. The cause of this is probably the fact that the condensation of watery vapour in the atmosphere into rain-clouds arises in general from the ascent of moist and warm air up the slopes of mountains into a cold region; the clouds thus formed are drifted by the wind to the leeward side of the mountains, and there fall in rain. To the same cause may be ascribed the fact that the rain-fall is greater in mountainous than in flat districts, and greater at points near high mountain-summits than at points further from them.

The elevation of the locality where the rain-fall is measured does not appear materially to affect the depth, except in so far as elevation is an usual accompaniment of nearness to a mountain-chain.

A vast amount of detailed information has been collected as to the depth of rain-fall in different places at different times; but there does not yet exist any theory from which a probable estimate of the rain-fall in a given district can be deduced independently of direct observation.

The most important data respecting the depth of rain-fall in a given district, for practical purposes, are the following:—

(1.) The least annual rain-fall.
(2.) The mean annual rain-fall.
(3.) The greatest annual rain-fall.
(4.) The distribution of the rain-fall at different seasons, and, especially, the longest continuous drought.
(5.) The greatest flood rain-fall, or continuous fall of rain in a short period.

The order of importance of these data depends on the purpose of the proposed work. If it is one of water-supply, the least annual rain-fall and the longest drought are the most important data; if it is a work of drainage, the greatest annual rain-fall and the greatest flood are the most important.

Experience shows that to obtain those data completely and exactly for a given district requires at least 20 years of daily rain-gauge observations, if not more. But it very seldom happens that so long a series of observations has been made in the precise spots to which the inquiries of the engineer are directed, and in the absence of such records he may proceed as follows:—

(1.) Obtain a copy of the records of the observations made at the nearest station where the rain-fall has been observed for a long series of years, and from them ascertain the longest drought, and compute the mean annual rain-fall at that station, the greatest and least annual rain-fall, the greatest flood rain-fall, &c. The station in question may be called the "standard station."

(2.) Establish rain-gauges in the district to be examined, at places which may be called the "catchment stations," and have them observed daily by trustworthy persons, taking care to obtain a copy of the records of the observations made at the same time at the standard station; and let that series of simultaneous observations be carried on as long as possible.

(3.) Compute from those simultaneous observations the proportions borne to the rain-fall at the standard station by the rain-fall in the same time at the several catchment stations; multiply the greatest, least, and mean annual depths of rain-fall, the greatest flood, &c., at the standard station by those proportions, and the results will give probable values of the corresponding quantities at the catchment stations.

The positions of the catchment rain-gauge stations must, to a considerable extent, be regulated by the practicability of having them observed once a-day; but they should, as far as practicable, be distributed uniformly over the gathering-ground. If it consists of a number of branch basins, there should, if possible, be one or more rain-gauges in each of them. If it is bounded or traversed by high hills, some gauges should be placed on or near their summits, and others at different distances from them.

Each rain-gauge should be placed in an open situation, that it may not be screened by rocks, walls, trees, hedges, or other objects. Its mouth should be as near the level of the ground as is consistent with security. It may be surrounded with an open timber or wire fence to protect it from cattle and sheep.

A rain-gauge for use in the field consists, in general, of a conical funnel, with a vertical cylindrical rim, very accurately formed to a prescribed diameter, such as 10 or 12 inches, and a collecting vessel for the water, usually cylindrical, and smaller in area than the mouth of the funnel. If this vessel is to be used as a measuring vessel also, the ratio of its area to that of the mouth of the funnel is accurately ascertained, and the depth at which the water stands in it is shown by means of a float with a graduated brass stem rising above the mouth of the gauge. Sometimes the rain collected is measured by being poured into a graduated glass measure, which the observer carries in a case. The most accurate method of graduating the measure is by putting known weights of water into it, and marking the height at which they stand (as recommended by Mr. Haskoll in his *Engineering Field-Work*). In performing this process, the weight of a cubic inch of pure water, at 62° Fahr., may be taken as

252·6 grains.*

The glass measure may either be graduated to cubic inches, which, being divided by the area of the funnel in square inches, will give the depth of rain-fall in inches; or it may be graduated to show at once inches of rain-fall in a funnel of the area employed.

Observations of rain-fall in the field are usually recorded to two decimal places of an inch.

It may be stated as a result of experience, that the proportions of the least, mean, and greatest annual rain-fall at a given spot usually lie between those of the numbers 2, 3, and 4, and those of the numbers 4, 5, and 6.

III. The *Available Rain-fall* of a district is that part of the total rain-fall which remains to be stored in reservoirs, or carried away by streams, after deducting the loss through evaporation, through permanent absorption by plants and by the ground, &c.

The proportion borne by the available to the total rain-fall varies very much, being affected by the rapidity of the rain-fall and the compactness or porosity of the soil, the steepness or flatness of the ground, the nature and quantity of the vegetation upon it, the temperature and moisture of the air, the existence of artificial drains, and other circumstances. The following are examples:—

Ground.		Available Rain-fall. ÷ Total Rain-fall.
Steep surfaces of granite, gneiss, and slate,	nearly	1
Moorland and hilly pasture, ...	from	·8 to ·6
Flat cultivated country, ...	from	·5 to ·4
Chalk, ...		0

Deep-seated springs and wells give from ·3 to ·4 of the total rain-fall.

Such data as the above may be used in roughly estimating the probable available rain-fall of a district; but a much more accurate and satisfactory method is to measure the actual discharge of the streams at the same time that the rain-gauge observations are made, and so to find the actual proportion of available to total rain-fall.

457. **Measurement and Estimation of the Flow of Streams.**—There

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* This is deduced from the value already given in p. 161 for the weight of a cubic foot of pure water at 62° Fahr., viz., 62·355 lbs. avoirdupois, or 436,495 grains. That value is based on data given in Professor Miller's paper on the "Standard Pound" (*Philosophical Transactions*, 1856); it differs slightly from that formerly fixed by statute but since abolished.

are three methods of measuring the discharge of a stream—by weir-gauges, by current meters, and by calculation from the dimensions and declivity.

I. The use of *Weir-gauges* is the most accurate method, but it is applicable to small streams only. The weir is constructed across the stream so as to dam up a nearly still pond of water behind it, from which the whole flow of the stream escapes through a notch or other suitable sharp-edged orifice in a vertical plate or board, the elevation of still or nearly still water being observed on a vertical scale in the pond, whose zero-point is on a level with the bottom of the notch, or with the centre of a round or rectangular orifice. For the laws of the discharge of water through vertical orifices, see Article 449, p. 681.

For streams of very variable flow, it appears from the experiments of Mr. James Thomson, that the right-angled triangular notch is the best form of orifice (see *Reports of the British Association for* 1861), as it measures large and small quantities with equal precision, and has a sensibly constant co-efficient of contraction. Where one such notch is insufficient, he recommends the use of a row of them. The pond may have a flat floor of planks, on a level with the bottom of the triangular notch.

When orifices wholly immersed are used, round or square holes are the best, because their co-efficients of contraction vary less than those of oblong holes (see p. 680). If one round or square hole is insufficient, a horizontal row of them may be used.

A weir-gauge should be placed on a straight part of the channel, because if it is placed on a curved part the rush of water from the outlet may undermine the concave bank of the stream. To prevent the weir itself from being undermined in front, the bottom of the channel below the outlet should be protected by an apron of boards, or a stone pitching, or by carrying the water some distance forward in a wooden shoot or spout, placed so low as not to drown any part of the outlet.

Stream-gauges ought to be observed once a-day at least, and oftener when the flow of the stream is in a state of rapid variation, as it is during the rise and fall of floods.

II. *By Current Meters.*—In large streams the flow can in general be measured only by finding the area of cross-section of the stream, measuring by suitable instruments the velocities of the current at various points in that cross-section, taking the mean of those velocities, and multiplying it by the sectional area. The most convenient instrument for such measurements of velocity is a small light revolving fan, on whose axis there is a screw, which drives a train of wheel-work, carrying indexes that record the number of revolutions made in a given time. The whole apparatus is fixed at the end of a pole, so that it can be immersed to different depths in different parts of the channel. The relation between the number of revolutions of the fan per minute, and the corresponding velocity of the current, should be determined experimentally by moving the instrument with different known velocities through a piece of still water, and noting the revolutions of the fan in a given time.

A straight and uniform part of the channel should always be chosen for experiments on the velocity of a stream.

When from the want of the proper instrument, or any other cause, the velocity of the current cannot be measured at various points, the velocity of its swiftest part, which is at the middle of the surface of the stream, may be measured by observing the motions of any convenient body floating down, and from that greatest velocity the mean velocity may be computed by the formula given in Article 445, p. 674.

III. *By Calculation from the Declivity.*—For this purpose a portion of the stream must be carefully levelled, cross-sections being taken at intervals; and the discharge is to be calculated by the rules of Division I. of Article 451, p. 687. In order that the result may be accurate, the part of the stream chosen should have, as nearly as possible, an uniform cross-section and declivity, and should be free from obstruction to the current, and, above all, from weeds, which have been known to increase the friction nearly tenfold.

IV. *Estimation of Flow in Different Years.*—The discharge of a stream during a certain period of observation having been ascertained, may be used to compute probable values of its least, mean, and greatest discharge in a series of years, by multiplying it by the proportions borne by the rain-fall in those years as observed at the "standard station" (see Article 456, p. 695) to the rain-fall at the same station during the period when the stream was gauged.

458. **Ordinary Flow and Floods.**—Questions often arise between the promoters of a water-work and the owners and occupiers of land on the banks of a stream as to the distinction between the "ordinary" or "average summer discharge" of a stream and the "flood discharge." The distinction is in general not difficult to draw by an engineer who personally inspects the stream at each time that its flow is gauged; but to provide for the case of such inspection being impracticable, Mr. Leslie has proposed an arbitrary rule for drawing that distinction, which many engineers have adopted. It is as follows:—

Range the discharges as observed daily in their order of magnitude.

Divide the list thus arranged into an upper quarter, a middle half, and a lower quarter.

The discharges in the upper quarter of the list are to be considered as *floods.*

For each of the flood discharges thus distinguished substitute the *average of the middle half of the list,* and take the mean of the whole list, as thus modified, for the *ordinary* or *average discharge, exclusive of flood-waters.*

It appears that the ordinary discharge, as computed by this method in a number of examples of actual streams in hilly districts, ranges from *one-third* to *one-fourth* of the *mean discharge, including floods;* being a result in accordance with those arrived at by engineers who have distinguished floods from ordinary discharges to the best of their judgment, without following rules.

459. **Measurement of Flow in Pipes.**—The *Water Meters,* or instruments for measuring the flow in pipes, now commonly used, may be divided into two classes—piston meters and wheel meters.

A piston meter is a small double-acting water-pressure engine, driven by the flow of water to be measured. That of Messrs. Chadwick and Frost records the *number of strokes* made by the piston, each stroke corresponding to a certain volume of water. That of Mr. Kennedy is so constructed that, by means of a rack on the piston-rod driving pinions, the *distance* traversed by the piston is recorded by a train of wheel-work, with dial-plates and indexes.

An example of a wheel meter is that of Mr. Siemens, being a small *reaction turbine* or "Barker's mill," driven by the flow. The revolutions are recorded by a train of wheel-work, with dial-plates and indexes.

Another example of a wheel meter is that of Mr. Gorman, being a small *fan turbine* or *vortex wheel* driven by the flow, and driving the indexes of dial-plates.

The ordinary errors of a good water meter are from ½ to 1 per cent.; in extreme cases of variation of pressure and speed errors may occur of 2½ per cent.

The value of the revolutions of a wheel meter should be ascertained experimentally, by finding the number of revolutions made during the filling of a tank of known capacity.

For descriptions of several kinds of water meters, see the *Transactions of the Institution of Mechanical Engineers for* 1856.

SECTION III.—*Of Store Reservoirs.*

460. **Purposes and Capacity of Store Reservoirs.**—A store reservoir is a place for storing water, by retaining the excess of rain-fall in times of flood, and letting it off by degrees in times of drought. It effects one or more of the following purposes:—

To prevent damage by floods to the country below the reservoir.

To prevent the evil consequences of droughts.

To increase the ordinary or available flow of a stream by adding to it the whole or part of the flood-waters.

To enable water to be diverted from a stream without diminishing the "ordinary" or "average summer flow," as defined in Article 458, p. 698.

To allow mechanical impurities to settle.

The *available capacity* or *storage-room* of a reservoir is the volume contained between the highest and lowest working water-levels, and is less than the *total capacity* by the volume of the space below the lowest working water-level, which is left as a place for the collection of sediment, and which is either kept always full, or only emptied when it is absolutely necessary to do so for purposes of cleansing and repair. It is impossible to lay down an universal rule for the volume of the space so left, or "bottom" as it is called; but in some good examples of artificial reservoirs it occupies about one-sixth of the greatest depth of water at the deepest part of the reservoir.

The absolute storage-room required in a reservoir is regulated by two circumstances:—the *demand* for water, and the extent to which the *supply* fluctuates.

The demand is usually a certain uniform quantity per day. Experience has shown that about 120 *days' demand* is the least storage-room that has proved sufficient in the climate of Britain; in some cases it has proved insufficient; and even a storage equal to 140 days' demand has been known to fail in a very dry season; and consequently some engineers advise that every store reservoir should if possible contain *six months' demand*.

From data respecting various existing reservoirs and gathering-grounds, given by Mr. Beardmore (*Hydraulic Tables*), it appears that the storage-room varies *from one-third to one-half of the available annual rain-fall*.

The best rule for estimating the available capacity required in a store reservoir would probably be one founded upon taking into account the supply as well as the demand. For example—

180 *days of the excess of the daily demand, above the least daily supply*, as ascertained by gauging and computation in the manner described in the preceding section.

In order that a reservoir of the capacity prescribed by the preceding rule may be efficient, it is essential that the *least available annual rain-fall* of the gathering-ground should be sufficient to supply *a year's demand for water*.

To enable the gathering-ground to supply a demand for water corresponding to the *average available annual rain-fall*, the *greatest* *total deficiency* of available rain-fall below such average, whether confined to one year or extending over a series of years, must be ascertained, and an addition equal to such deficiency made to the reservoir room; but it is in general safer, as well as less expensive, to extend the gathering-ground so that the least annual supply may be sufficient for the demand.

The foregoing principles as to capacity have reference to those cases in which the water is to be used to supply a demand for water. When the sole object of the reservoir is to prevent floods in the lower parts of the stream, it ought to be able to contain the ascertained greatest total excess of the available rain-fall during a season of flood above the greatest discharging capacity of the stream consistent with freedom from damage to the country.

461. **Reservoir Sites.**—In choosing the site of a reservoir, the engineer has three things chiefly to consider: the elevation, the configuration of the ground, and the materials, especially those which will form the foundations of the embankment or embankments by which the water is to be retained.

I. The *Elevation* of the site must at once be so high that from the lowest water-level there shall be sufficient fall for the pipes, conduits, or other channels by which the water is to be discharged, and at the same time so low that there shall be a sufficient gathering-ground above the highest water-level.

II. The *Configuration of the Ground* best suited for a reservoir site is that in which a large basin can be enclosed by embanking across a narrow gorge. To enable the engineer to compare such sites with each other, and to calculate their capacities, plans with frequent contour-lines are very useful (Article 59, p. 95), or in the absence of contour-lines, numerous cross-sections of the valleys. The water's edge of the reservoir is itself a contour-line. After the site of a reservoir has been fixed, a plan of it should be prepared with contour-lines numerous and close enough to enable the engineer to compute the capacity of every foot in depth from the lowest to the highest water-level, so that when the reservoir is constructed and in use, the inspection of a vertical scale fixed in it may show how much water there is in store.

Care should be taken to observe whether the basin of a projected reservoir site has, besides its lowest outlet, higher outlets through which the water may escape when the lowest outlet is closed, unless they also are closed by embankments.

The figure of the ground at the site of a proposed reservoir embankment must be determined with care and accuracy, by making not only a longitudinal section along the centre line of the embankment (which section will be a cross-section as regards the valley), but several cross-sections of the site of the embankment, which should be at right angles to the longitudinal section, unless there is some special reason for placing them otherwise. One of these cross-sections of the embankment site should run along the course of the existing outlet of the reservoir site (usually a stream), and another along the course of the intended outlet (usually a culvert containing one or more pipes).

III. *Material.*—The materials of the site of the intended embankment should be either impervious to water or capable of being easily removed so far as they are pervious, in order to leave a water-tight foundation; and their nature is to be ascertained by borings and trial pits, as to which, see Article 187, p. 331, and Article 391, p. 598; and, if necessary, by mines. (Article 392, p. 594.) In many cases it is not sufficient to confine this examination to the site of the embankment; but the bottom and sides of the reservoir-basin must be examined also, in order to ascertain whether they do not contain the outcrop of porous strata, which may conduct away the impounded water. The best material for the foundation of a reservoir embankment is clay, and the next, compact rock free from fissures. Springs rising under the base of the embankment are to be carefully avoided.

The engineer should ascertain where earth is to be found suitable for making the embankment, and especially clay fit for puddle.

462. **Land Awash** means land which lies near the margin of a reservoir, at a height not exceeding three feet above the top water-level, and whose drainage is consequently injured. The promoters of the reservoir are sometimes obliged to purchase such land. Its boundary is of course a contour-line.

463. **Construction of Reservoir Embankments.**—I. *General Figure and Dimensions.*—A reservoir embankment rises at least 3 feet above the top water-level, and in some cases 4, 6, or even 10 feet. It has a level top, whose breadth may be in ordinary cases about *one-third* of the greatest height of the embankment; the outer slope, or that furthest from the water, may have an inclination regulated by the stability of the material, such as 1½ to 1, or 2 to 1; the inner slope, or that next the water, is always made flatter, its most common inclination being 3 to 1.

II. The *Setting-out* of the boundaries of the embankment on the ground (see Article 67, p. 113) is to be performed with great accuracy, by the aid of the cross-sections already mentioned in a preceding article. The following method also has been found convenient in suitable situations. On the side of the valley, at one end of the proposed embankment, erect upon props a wooden rail, with its upper edge exactly horizontal, and exactly in the plane of the slope to be set out. At a convenient distance back from the rail as regards the slope, set up a prop supporting a sight having a small eye-hole, also exactly in the plane of the slope to be set out. A row of pegs ranged from the sight so as to mark points on the ground in a line with the upper edge of the rail will give the foot of the slope.

The same rail (with two different sights) may be used to set out both slopes, if its upper edge coincides with their line of intersection. Let the inner slope be s to 1, the outer s' to 1, the breadth of the top of the embankment b; then the height of that line of intersection above the top of the embankment is,

$$b \div (s + s'); \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

and its horizontal distance outwards from the centre line of the embankment is,

$$b\ (s - s') \div 2\ (s + s'). \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

An instrument consisting of a bar with two sights capable of turning about an axis adjusted so as to be perpendicular to the slope to be ranged has been used for the same purpose.

III. *Preparing the Foundation.*—The foundation is to be prepared by stripping off the soil, and excavating and removing all porous materials, such as sand, gravel, and fissured rock, until a compact and water-tight bed is reached.*

IV. The *Culvert* for the outlet-pipes is next to be built' in cement or strong hydraulic mortar, resting on a base of hydraulic concrete. Its internal dimensions must be sufficient to admit of the access of workmen beside the pipe or pipes which it is to contain. The principles which should regulate its figure and thickness are those which have been explained in Article 297 A, p. 433. The outer or down-stream end of the culvert is usually open, and often has wing-walls sustaining the thrust of part of the outer slope of the embankment; the inner or up-stream end is usually closed with water-tight masonry, through which the lowest or scouring outlet-pipe passes. In some reservoirs there is a water-tight partition of masonry at an intermediate point in the culvert. The culvert is to be well coated with clay puddle. (Article 206, p. 344.) In the best constructed reservoirs a *tower* stands on the inner end of the culvert, to contain outlet-pipes for drawing water from different levels, with valves, and mechanism for opening and shutting them.

Sometimes a cast iron pipe is laid without any culvert.

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* The following method was used by Jardine to clear unsound pieces away from the rock foundation of the embankment of Glencorse reservoir, near Edinburgh. A layer of clay puddle was spread and well rammed over the surface of the rock, and was then torn off, when all the fissured fragments came away adhering to the sheet of puddle, leaving a surface of sound rock for the foundation of the embankment.

V. *Making the Embankment.*—The embankment is to be made of clay in thin horizontal layers, as described in Article 199, Division III., p. 341. The central part of the embankment should be a "*puddle wall*," of a thickness at the base equal to about one-third of its height; it may diminish to about two-thirds or one-half of that thickness at the top. Great care must be taken that the puddle wall makes a perfectly water-tight joint with the ground throughout the whole of its course, and also with the puddle coating of the culvert.*

During the construction of a reservoir embankment care should be taken to provide a temporary outlet for the water of its gathering-ground, sufficient to carry away the greatest flood-discharge. This may be done either by having a pipe sufficient for the purpose traversing the culvert, or by completing a sufficient bye-wash before the embankment is commenced.

VI. *Protection of Slopes and Top.*—The outer slope is usually protected from the weather by being covered with sods of grass. The inner slope is usually *pitched* or faced with dry stone set on edge by hand, about a foot thick, up to about three feet above the top water-level, and as much higher as waves and spray are found to rise. The top of the embankment may be covered with sods like the outer slope; but it is often convenient to make a roadway upon it; in either case it should be dressed so as to have a slight convexity in the middle, like that given to ordinary roads, in order that water may run off it readily.

No trees or shrubs should be allowed to grow on a reservoir embankment, as their roots pierce it and make openings for the penetration of water. For the same reason no stakes should be driven into it.

464. **Appendages of Store Reservoirs.**—I. The *Waste-weir* is an appendage essential to the safety of every reservoir. It is a weir at such a level, and of such a length, as to be capable of discharging from the reservoir the greatest flood-discharge of the streams which supply it, without causing the water-level to rise to a dangerous height. (As to the discharge over a weir, see Article 449, Divisions II., III., VI., and VII., pp. 682 to 684.) The water discharged over the weir is to be received into a channel, open or covered, as the situation may require, and conducted into the natural water-course below the reservoir embankment. The weir is to be built of ashlar or squared hammer-dressed masonry; the bottom of the waste-

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* The late Mr. Smith of Deanston rammed and puddled each successive layer of a reservoir embankment by erecting a rail-fence along each side of it, and driving a flock of sheep several times backwards and forwards along it.

Clay puddle may be protected against the burrowing of rats by a mixture of engine ashes, care being taken not to add so much as to make it pervious to water.

channel, directly in front of it, is best protected by a series of rough stone steps, which break the fall of the water. Instead of a waste-weir, a *waste-pit* has in some cases been used; that is to say, a tower rising through or near the embankment to the top water-level; the waste water falls into this tower and is carried away by a culvert from its bottom; but the efficiency and safety of this contrivance are very questionable, for it seldom can have a sufficient extent of overfall at the top.

II. *Waste-sluices* may be opened to assist the waste-weir in discharging an excessive supply of water. They may either be under the control of a man in charge of the reservoir, or they may be self-acting. The simplest and best self-acting waste-sluice is that of M. Chaubart, as to which, see *A Manual of the Steam Engine and other Prime Movers*, Article 139, p. 153.

III. *Culvert, Valve-Tower, Bridge, Outlet-Pipes and Valves.*—The culvert and its tower have been mentioned in the preceding article. When the tower is imbedded in the embankment, as it sometimes is, it is called the *valve-pit;* but the best position for it is in the reservoir, just clear of the embankment; and then a light *foot-bridge* is required to give access to it from the top of the embankment.

When the object of a store reservoir is simply to equalize the flow of a stream, in order to protect the lower country from floods, and to obtain an increased ordinary flow available for irrigation and water-power, one outlet-pipe may be sufficient, discharging into the natural water-course below the embankment; but if the water is to be used for the supply of a town, or for any other purpose to which cleanness is essential, there must be at least two outlet-pipes,—the ordinary *discharge-pipe*, which takes the water from a point or points not below the lowest water-level of the reservoir, in order to conduct it to the town or place to be supplied; and the *cleansing-pipe,* which takes the water at or near the lowest point in the reservoir, and discharges it into the natural water-course below the embankment, and is only opened occasionally in order to scour away sediment. The water-course, where such scouring discharge falls into it, must have its bottom protected by a stone pitching. As to the discharge of pipes, see Article 450, p. 684.

The mouthpieces of such pipes should be guarded against the entrance of stones, pieces of wood, or other bodies which might obstruct them or injure the valves, by means of convex gratings. The valves best suited for them are slide valves, as to which, see *A Manual of the Steam Engine and other Prime Movers*, Article 120, p. 124.

IV. The *Bye-wash* is a channel sometimes used to divert past the reservoir the waters of the streams which supply it, when these are turbid or otherwise impure. Its dimensions are fixed according to the principles of Article 451, p. 685. Its course usually lies near one margin of the reservoir, and is then conveniently situated for receiving the water discharged by the waste-weir.

In some cases, when a reservoir has been made under a stipulation that only the surplus above a certain quantity was to be allowed to flow into it from the streams, the whole of the streams have been conducted past the reservoir in a bye-wash, having weirs or overfalls along its margin, at certain points in its course above the top water-level of the reservoir. The levels of those weirs were so adjusted that when no more than the prescribed quantity flowed down the bye-wash none escaped over the weirs; but when there was any surplus flow in the bye-wash, the water in it rose above the crests of the weirs, and the surplus escaped over them into the reservoir.

V. *Diversion-cuts* are permanent bye-washes for streams that are so impure as to be rejected altogether.

VI. *Feeders* are small channels for diverting either streams or surface drainage *into* the reservoir, and so increasing its gathering-ground. When used to catch surface drainage, they have been found to conduct to the reservoir from *one-quarter* to *one-half* of the rain-fall.

In connection with feeders for diverting streams into the reservoir may be mentioned what may be called a *separating-weir*, the invention of an assistant of Mr. Bateman, and first used in the Manchester water-works. A weir built across the channel of a stream has in front, and parallel to its crest, a small conduit running along its front slope at such a level that when the stream is in flood, and therefore turbid, the cascade from the top of the weir overleaps the conduit, and runs down the front slope into the natural channel, which conveys it to a reservoir for the supply of mills; but when the flow is moderate, the cascade falls into the small conduit, which leads it into a feeder of the store reservoir for the supply of the city.

The horizontal distance x to which a cascade from the crest of a weir will leap in the course of a given fall z below that crest may be thus calculated. The mean velocity with which the cascade shoots from the weir-crest is nearly

$$v = \frac{2}{3} \times 8{\cdot}025 \sqrt{h_1} = 5{\cdot}35 \sqrt{h_1}; \quad \ldots\ldots\ldots\ldots\ldots(3.)$$

h_1 being the height from the weir-crest to still water in the pond. Then

$$x = \frac{2 v \sqrt{z}}{8{\cdot}025} = \frac{4}{3} \sqrt{z h_1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

465. **Reservoir Walls.**—Retaining walls are often used at the foot of the slopes of a reservoir embankment; they are of course to be built in strong and durable hydraulic mortar, especially at the foot of the inner slope. As to their stability and construction, see Articles 265 to 271, pp. 401 to 410.

Where the gorge to be closed is of small dimensions either horizontally or vertically, a wall may be used instead of an embankment; the formulæ applicable to its stability being deduced from the general formulæ for retaining walls by making the angle of repose = 0, and putting 62·4 for the weight of a cubic foot of the pressing material.

It is advisable, unless the foundation is unusually firm, to limit the deviation of the centre of resistance at the base of the wall, from the centre of the base, to *one-fourth* of the thickness.

The result is, that either for a rectangular wall, or for a trapezoidal wall with the inner face vertical, and the thickness at the top one-fourth of the thickness at the base, the thickness at the base should be about *six-tenths* of the height.

If the wall stands on a soft foundation, it may be advisable to make the centre of resistance coincide with the centre of figure of the base, and for that purpose the wall may be made of an isosceles triangular form of section, with the base = the height × $\sqrt{2}$, or 1·414 nearly.

466. **Lake Reservoirs.**—To convert a natural lake into a reservoir it must be provided with a waste-weir, and with one or more outlets at the intended lower water-level, controlled by valves. As to the waste-weir, two methods may be followed,—one is to leave the natural outlet to act as a waste-weir, but that outlet is seldom broad enough to do so without causing undue variation in the top water-level; the other is to make a dam or embankment across the outlet, with an artificial waste-weir. The waste-weir may be made to answer as a dam. The outlet or outlets may be made either by building a culvert with pipes in an excavation of sufficient depth, or by tunnelling through one of the ridges that enclose the lake.

SECTION IV.—*Of Natural and Artificial Water-Channels.*

467. **Surveying and Levelling of Water-Channels.**—The principles which connect the dimensions, figure, declivity, velocity of current, and discharge of a water-channel have already been fully set forth in Articles 444 and 445, pp. 673 to 674, and Articles 451 to 454, pp. 686 to 691. In the present section are to be explained the principles according to which such-channels, whether natural or artificial, are constructed, preserved, and improved.

The plans of an existing or intended water-channel require no special remark beyond what has already been stated as to plans in general in the first part of this work, except that in the case of existing streams liable to overflow their banks, they should show the boundaries of lands liable to be flooded, and also of those liable to be laid *awash* (see Article 462, p. 702), and that their utility will be greatly increased by contour-lines. The longitudinal section should be made along the centre line of a proposed channel, and along the line of the most rapid current in an existing channel; and it should show the levels of both banks as well as those of the bottom of the channel, and of the surface of the current in its lowest, ordinary, and flooded conditions. It should be accompanied by numerous cross-sections, especially in the case of existing streams of variable sections; and of those cross-sections a sufficient number should extend completely across the lands flooded and awash, to show the figure of their surface. They should include accurate drawings of the archways, roadways, and approaches of existing bridges, also of existing weirs and other obstructions. The nature of the strata should be ascertained, as for any piece of earthwork, by sinking pits and borings, and, in the case of an existing channel, by probing its bottom also, and the results should be shown on the section and plan.

468. **Regime or Stability of a Water-Channel.**—A water-channel is said to be in a state of *régime* or *stability* when the materials of its bed are able to resist the tendency of the current to sweep them forward. The following table shows, on the authority of Du Buat, the greatest velocities *of the current close to the bed*, consistent with the stability of various materials:—

Soft clay,....................................	0·25	foot per second.
Fine sand,....................................	0·50	„ „
Coarse sand, and gravel as large as peas,	0·70	„ „
Gravel as large as French beans,.........	1·00	„ „
Gravel 1 inch in diameter,..................	2·25	feet per second.
Pebbles 1½ inch diameter,..................	3·33	„ „
Heavy shingle,................................	4·00	„ „
Soft rock, brick, earthenware,.............	4·50	„ „
Rock, various kinds,........................	6·00 and upwards.	„ „

As to the relation between the surface velocity, the mean velocity, and the velocity close to the bed, see Article 445, p. 674.

The condition of the channels of streams which have a rocky bed is generally that of stability. When the bed is stony or gravelly the condition is most frequently that of stability in the ordinary state of the river, and instability in the flooded state.

When the bed is earthy its usual condition is either *just stable and no more*, or *permanently unstable*. The former of these conditions arises from the fact of the stream carrying earthy matter in suspension, so that the bed consists of particles which are just heavy enough to be deposited, and which any slight increase of velocity would sweep away.

The bottom of a river in a permanently unstable condition presents, as Du Buat pointed out, a series of transverse ridges, each with a gentle slope at the up-stream side and a steep slope at the down-stream side. The particles of the bed are rolled by the current up the gentle slope till they come to the crest of the ridge, whence they eventually drop down the steep slope to the bottom of a furrow, where they become covered up, and remain at rest till the gradual removal of the whole ridge leaves them again exposed.

When the banks, as well as the bottom, are unstable, the river-channel undergoes a continual alteration of form and position. If the banks are straight, they soon become curved, for a very slight accidental obstacle is sufficient to divert the main current so that it acts more strongly on one bank than on the other: the former bank is scooped away, and becomes concave, and the earthy matter suspended in the stream is deposited in the less rapid part, so as to make the opposite bank convex. A curved part of a river-channel tends to become continually more and more curved; for the centrifugal force (or rather the tendency of the particles of water to proceed in a straight line) causes the particles of water to accumulate towards the concave bank; the current is consequently more rapid there than towards the convex bank, and it scoops away both the bank and the bottom (unless they are able to resist it), and deposits the material in some slower part of the stream: thus the *line of the strongest current* is always *more* circuitous than the centre line of the channel; and the action of the current tends to make the concave banks more concave, the convex banks more convex, and the whole course of the river more serpentine. This goes on until the current meets some material which it cannot sweep away, or until, by the lengthening of the course of the stream and the consequent flattening of its declivity, its velocity is so much reduced that it can no longer scoop away its banks, and stability is established. In some cases stability is never established; but the river presents a serpentine channel which continually changes its form and position.

One of the chief objects of engineering, in connection with the channels of streams, is to protect their banks against the wearing action of the current, so as in some cases to give them that stability which they want in their natural condition, and in other cases to give them the additional stability that is required in order to resist an increased velocity of current, produced by improvements in the course and form of the channel.

469. **Protection of River-Banks.**—The most efficient protection to the banks of a stream is a thick growth of water-plants; but as these form a serious impediment to the current, artificial protection must be substituted for them, at least below the average water-level. Above that level a plantation of small willows forms a good defence against the destructive action of floods; but it is not applicable where there is a towing-path. The means of artificially protecting river-banks may be thus classed:—I. Fascines. II. Timber sheeting. III. Iron sheeting. IV. Crib-work. V. Stone pitching. VI. Retaining walls. VII. Groins.

I. *Fascines*, already referred to in Article 417, p. 625, are bundles of willow twigs from 9 to 12 inches in diameter: the largest are about 20 feet long, but 12 feet is a more common length: they are tied at every 4 feet, or thereabouts. For the protection of a river-bank *below the low water-level* an "apron" or "beard" is laid, consisting of fascines lying with their length up and down the slope of the bank; the upper ends are fastened down to the bank with stakes about 4 four feet long; the lower ends are sunk, and held down under water by loading them with stones. To protect the bank *above the low water-level* fascines are laid horizontally in layers, with their butt ends towards the stream, so as to form a series of steps rising at the same rate with the slope of the lower part of the bank, or nearly so (say from 1 to 1 to 3 to 1); each layer is fastened down with three rows of stakes 4 feet long; the heads of the stakes rise 8 inches or thereabouts above the fascines, and are laced or wattled with wicker-work, so as to form a crib for the retention of a layer of gravel.

Fascines usually last 6 years above the low water-level and 10 years below.

II. *Timber Sheeting* may consist either of sheet-piles (already described in Article 404, p. 605) or of guide-piles and horizontal planks, described in Article 409, p. 613. The wales of the sheet-piling or the guide-piles of the planking must be tied back to anchoring-plates made of planks buried in a firm stratum of earth at a sufficient distance back from the bank. The holding power of such anchoring-plates depends on the same principles as that of iron anchoring-plates, as to which, see Article 272, p. 410.

III. *Iron Sheeting* has already been described in Article 404, p. 606. It is sometimes used for the faces of quays in navigable rivers, being tied back to anchoring-plates. (Article 272, p. 410.)

IV. As to *Crib-work*, see Article 409, p. 614. When used for a quay or river-bank its interstices are rammed full of clay and gravel.

V. *Dry Stone Pitching* is used to protect earthen banks, of slopes ranging from that of 1 to 1 to that of 2 to 1, or flatter. It consists of stones roughly squared, and laid by hand in courses. Its thickness is usually from 8 to 12 inches at the top, and increases in going down at the rate of 2 or 3 inches per yard. The foot of the pitching must abut against a foundation sufficient to prevent it from slipping. Such a foundation may be made by sinking a row of oblong baskets, each containing about 2 cubic yards of gravel, or by driving a row of piles with horizontal wales at the inner side of their heads; the strength of the wales is a matter of calculation; they have to resist a maximum pressure = weight of pitching × rise of slope ÷ length of slope, the friction of the pitching on the earth being neglected for the sake of security.

VI. *Retaining Walls* are used chiefly where quays are required, and will be again mentioned further on.

VII. *Groins* are small dykes projecting at right angles to the bank to be protected, and are made either of loose stones, of piles and planks, or of wattled stakes. Each groin protects a portion of the bank of about *five* times its own length, and usually causes the current that sweeps round its point to scoop out an excavation in the bottom of the channel of a breadth equal to about one-quarter of the length of the groin, the material scooped out being deposited in the space between the groins. Groins, besides being an obstruction to the current, are injurious to the regularity of figure and stability of the bottom of the channel, and should only be used as a temporary expedient to protect the banks, until works of a better description can be completed.

470. **Improvement of River-Channels.**—The defects in a river-channel which are to be removed by improvements are usually of the following kinds:—The channel may be too shallow, either generally or in particular places; it may be too narrow, either generally or in particular places; it may even in particular places be too wide, if the breadth is so great as to cause the formation of shoals by enfeebling the current; its declivity may be too flat, either from the existence of obstacles, such as shoals, islands, weirs, ill-constructed bridges, or the like, or from its course being too circuitous; occasionally, but rarely, the declivity may be too steep at particular places, giving rise to a current so rapid as to make it impossible to preserve the stability of the bed; but this defect generally arises from the declivity being too flat elsewhere; it may contain sharp turns, injurious to the stability of the banks; it may be divided into branches, so as to enfeeble the current.

Setting aside for the present *diversions of the course* of a river, which will be considered in the next article, the works for the improvement of the channel consist mainly of:—I. Excavations to remove islands and shoals, and widen narrow places. II. Regulating dykes, to contract wide shallows. III. Works for stopping useless branches.

Before commencing alterations of any kind in a river-channel careful calculations should be made, according to the principles explained in Section I. of this chapter, of the probable effect of such alterations on the level, declivity, and velocity of the current in different states of the river. The object kept in view should be to obtain a channel either of nearly uniform section, or of a section gradually enlarging from above downwards, with a current that shall be sufficient to discharge flood-waters without overflowing the banks more than can be avoided, and at the same time not so rapid as to make it difficult or impossible to preserve the stability of the channel.

All improvements of river-channels should be begun at the lowest point to be altered, and continued upwards; because every improvement takes effect on the parts of the stream above it.

I. *Excavation* under water, by hand dredging, machine dredging, and blasting, has been described in Article 410, p. 614. When the current is at a low level, it may occasionally be advantageous to excavate parts of the bed by enclosing them with temporary dams as if for foundations (Article 409, p. 611), and laying them dry. Excavation of a muddy, sandy, or gravelly bottom, by the aid of the current, is performed by mooring at the place to be deepened a boat, furnished with a transverse projecting frame covered with boards or canvas; this frame descends to within 3 or 4 inches of the bottom of the channel, and the current, forced through that narrow opening, scoops out the material and sweeps it away. From 30 to 70 cubic yards per day have been excavated in this manner with a single boat.

II. *Regulating Dykes* should be adopted with great caution, and only where the excessive width of the channel is an undoubted cause of shallowness. They should not in any case rise much above the low water-level, lest they contract too much the space for flood-waters. They may be built either of dry stone, with a slope of about 1 to 1, or of wattled piles and gravel. The ordinary rules for the construction of dykes of the latter kind are as follows:—The piles in a double row to be driven into the ground to a depth equal to twice the depth of water; their diameter not less than 1-20th of their length; their distance apart longitudinally to be equal to the depth of water; the distance transversely between the rows of piles to be once and a-half the depth of water. They are to be tied together transversely, and wattled with willow twigs, and the space between the two rows filled with gravel.

III. The *Stopping of Branches* should be performed at their upper ends. In a gentle current it may be effected by means of an embankment of stones and gravel, advancing simultaneously from the two banks until it is closed in the centre; in a more rapid stream a dyke of wattled piles and gravel, made as already described, may be used; should the current be too strong for either of these plans, a raft, boat, or caisson (Article 409, Division III., p. 613), or a crib-work dam (Article 409, Division IV., p. 614), loaded with stones, is to be moored across the stream and sunk. The branch channel having had its current stopped will silt up of itself.

471. **Diversions of River-Channels** are usually adopted for the purpose of rendering the course less circuitous. In designing them regard should be had to the principles already explained in Section I. of this chapter, and in the preceding articles of this section; and care should be taken not to make the course *too direct*, lest the current be rendered too rapid for the stability of the bed. A slightly curved channel is always better than a straight channel; because in the former the main current takes a definite course, being always nearest the concave bank; whereas in a straight channel its course is liable to keep continually changing.

The form of cross-section with a horizontal base and sloping sides which gives the least friction with a given area has already been described in Article 451, p. 688, and it may be adopted if the stream is to act solely as a conduit for the conveyance of water; but should it be navigable, a figure must be adopted suited to the convenience of the navigation. This will be further considered in Chapter III. of this part.

472. A **Weir** is an embankment or dam, usually of stone, sometimes of timber, constructed across the channel of a stream. As to its effect on the water-level, see Articles 452 and 453, pp. 689 and 670.

When erected for purposes of water-power or water-supply, the object of a weir is partly to make a small store reservoir, but principally to prolong a high top water-level from its natural situation at a place some distance up the stream, to a place where water is to be diverted from the stream to drive machinery, or for some other purpose. When erected for purposes of navigation, the object of a weir is to produce a long reach or pond of deep and comparatively still water, in a place where the river is naturally shallow and rapid.

In planning a weir three things are to be considered: its line and position, its form of cross-section, and its construction.

I. *Line and Position of a Weir.*—It is best to avoid sharply curved parts of a river-channel in choosing the site of a weir, lest the rapid current which rushes down its face in times of flood should undermine the concave bank. For the protection of the banks in any case, it is advisable so to form the weir that the cascade from the lateral parts of the crest shall be directed from the banks, and towards the centre of the channel. This may be effected either by making the weir slightly curved in plan, with the concavity at the down-stream side, or by making it like a V in plan, with the angle pointing up stream. Another mode of protecting the banks is to make the crest of the weir slightly higher at the ends than in the middle, so that the lateral parts of the cascade may be too feeble to do damage.

In order to diminish the height and extent of backwater during floods, the crest of the weir is often made considerably longer than the breadth of the channel; this is effected either by making it cross the channel obliquely, or by using the V-shape already described, the latter method being the best for the stability of the banks. The practical advantage of such increased length is doubtful.

II. *Form of Cross-section.*—The back or up-stream side of a weir is usually steep, ranging from vertical to a slope of about 1 to 1; the top is either level or slightly convex, and not less than about 2 or 3 feet broad. In designing the front or down-stream slope of a weir, the principal object is to prevent the cascade that rushes over it from undermining its base. The commonest method is to use a long flat slope of 3 to 1, 4 to 1, or 5 to 1, in order that the speed of the current may be diminished by friction, and that it may strike the bottom of the channel very obliquely. A further protection is given to the river-bed by continuing the front slope a short distance below the bottom of the channel, and then curving it slightly upwards. Another method is to make the front of the weir present a steep or nearly vertical face, over which the water falls on a nearly level apron or pitching of timber or stone. Probably the best method would be to form the front of the weir into a series of steps, presenting steep faces and flat platforms alternately, the general inclination being about 3 to 1; thus a great fall might be broken up into a series of small falls, each incapable of damaging the platform which receives it.

III. *Construction.*—In order that the water of the pond may not force its way under the base of a weir, or round its "roots" (as the ends which join the banks of the stream are called), its foundation should be examined, chosen, and formed with precautions similar to those used in the case of a reservoir embankment, as to which, see Articles 461 and 463, pp. 701 to 704.

To make a weir of *timber*, or of timber, stones, and clay combined, any of the methods may be employed which have been described under the head of "Dams," in Article 409, Divisions II., III., and IV., with the addition that the back, crest, and front of the dam are to be covered with planking laid parallel to the current, to form an overfall for the water; and that the bottom of the channel at the foot of the weir is to be protected either by a platform of planks resting on a timber grating or on piles, or by a stone pitching.

A weir of *fascines* may be built of horizontal layers of fascines, staked down with mixed clay and gravel packed between them, in the manner described under the head of the protection of river-banks, Article 469, p. 710, the crest, front, and foot of the dam being protected with an apron of fascines, like that described in the same article.

A *dry stone* weir is formed like the stone embankments mentioned in Article 412, p. 617, with a steep slope at the bank and a long gentle slope in front, pitched or faced with roughly squared stones set in courses, as in the pitching of a river-bank, Article 469, p. 711. Sometimes a skeleton crib of timber, consisting of piles and longitudinal and transverse horizontal wales is constructed in order to keep the stones of the pitching in their places. As to the pressure against the longitudinal wales, see the article just quoted.

A weir of *solid masonry* may be founded, like other structures under water, on the natural ground, on a bed of concrete, on a timber platform, or on piles, according to circumstances. (See Part II., Chapter VI., Section II., p. 601.) When it has a timber foundation, a row of sheet-piles at the base of the up-stream side will in general be necessary to prevent the passage of water under it; and in the grating of the platform, pieces of timber running continuously through the weir in the direction of the stream should be avoided, lest they should conduct water along their sides. The masonry should be built in cement, or in quickly-setting hydraulic mortar; the heart of the weir may be of coursed rubble, or of concrete laid in layers; but the facing should be of good block-in-course, or of hammer-dressed ashlar, and the crest should form a coping of large stones, all headers, dowelled to each other.

One of the most effectual ways of preventing filtration round the "roots" of a weir is to carry them a considerable distance into the bank; but in the case of a weir of masonry the ends often abut upon a pair of side-walls, running along the banks of the stream, and having counterforts behind them to interrupt filtration.

IV. *Appendages of a Weir—Sluices and Floodgates—Salmon-*

stair.—When a weir is built across a navigable river, it requires a lock for the passage of vessels, which will be again mentioned further on. It may have one or more outlets with valves, like those of a reservoir embankment (Article 464, p. 705), according to the purpose for which it is intended.

It is almost always necessary to provide a weir with waste-sluices or floodgates, to be opened when the river is high, in order to prevent too great a rise of backwater. A sluice is a sliding valve of timber or iron, moving in guides, which are in general vertical, set in a rectangular passage of timber or masonry, and opened and shut by means of a screw, or of a rack and pinion. It is advisable not to make any sluice wider than about 4 or 5 feet. Should a greater width of opening be required, the passage through the weir is to be divided by walls or piers into a sufficient number of parallel passages, each furnished with a sluice. As to the discharge through a sluice, see Articles 448, 449, p. 681.

Another mode of opening and closing floodgates in a weir is by means of *needles*, as they are called. A rectangular channel through the weir is crossed at the bottom by a fixed timber sill, and near the top by a moveable timber sill, resting in two notches. The strength of the sills is a matter of calculation: they have to withstand the pressure of the water on a flat surface closing the passage. That surface is made up of the "needles," which are a set of square bars of wood strong enough to withstand the pressure, which are ranged close together side by side in a vertical position at the up-stream side of the sills. Each needle has a cylindrical handle at its upper end, to hold it by in removing and replacing it.

As to *self-acting waste-sluices*, see *A Manual of the Steam Engine and other Prime Movers*, Article 139, p. 153.

A weir across a river frequented by salmon requires a passage or channel to enable those fish to ascend its front slope. Mr. Smith of Deanston introduced the practice of making that channel of a zig-zag form, so as to reduce its rate of declivity and bring the speed of the current in it within moderate limits.

A *moveable weir* consists in general of a water-tight planked timber gate, placed in a rectangular passage of masonry or timber, and capable of turning upon a horizontal hinge at the floor of the passage, so as to be either laid flat when the channel is to be left clear, or set at any required angle of elevation, sloping against the declivity of the stream, with oblique struts to prop it at the downstream side. In one ingenious modification of this weir the duty of the struts is performed by a second and smaller gate, also turning on a horizontal hinge at the floor of the passage, but so as to slope *with* the stream. When the passage is clear, both gates lie flat in a horizontal recess in the floor of the passage, the smaller gate undermost and the upper surface of the larger gate flush with the floor. When the weir is to be raised, water is admitted through a valve and culvert from the up-stream side of the weir passage into the recess below the gates; its pressure lifts them both until they form a weir of a triangular section, the larger gate making the up-stream slope and the overfall, and the smaller making the down-stream slope, and acting at the same time as a strut to prop the larger gate. When the weir is to be lowered, the mass of water contained below the gates is allowed to escape by opening a valve in a culvert which leads to the down-stream side of the weir; and both gates then fall flat into the recess of the floor.*

473. **River Bridges.**—The construction of the foundations on land and in water, and of the superstructures, of bridges of various materials having been explained in Part II. of this work, and their adaptation to roads and railways in the preceding chapter, it is now only necessary to state those principles which are specially applicable to bridges over rivers.

In choosing the site of a bridge which is to have piers in the river, sharply curved parts of the channel should be avoided, lest the increased rapidity of the current caused by the narrowing of the water-way should undermine the concave bank.

The current should be crossed at right angles, or as nearly so as practicable. The abutments should not contract the water-way.

The piers, if any, should stand with their length exactly in the direction of the current; they should have pointed or cylindrical cutwaters at both ends, to diminish the obstruction to the current which they produce; and they should be no thicker than is necessary for the safety of the bridge. (As to stone piers in particular, see Article 293, p. 428.)

The springing of the arches should be above the highest ordinary water-level, and as much higher as the convenience of the navigation may require; and care should be taken that sufficient water-way is provided for the greatest floods. The crown of the lowest arches should be at least three feet above the flood-level, that they may allow floating bodies to pass through.

It may here be observed that the figure of arch which gives the greatest water-way for a given rise and span is the "hydrostatic arch." (See Article 283, p. 419.)

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* In order to do away as far as possible with the obstruction occasioned by weirs, it has been proposed by Hugh Mackenzie, Esq. of Ardross, that in those cases in which the fall of the stream is sufficiently rapid, and the country in other respects suitable, the diversion of water from a stream for the purpose of obtaining power should be effected by making a tunnel with suitably formed grated apertures in its roof, under the bed of the stream, at a point where its water-level has sufficient elevation, and so conducting the water into a mill-lead of sufficiently large size and moderate declivity.

Should it appear, upon an examination of the land subject to inundation at and near the site of the intended bridge, that such land acts not merely as a reservoir for flood-waters, but as a wide temporary channel for their discharge, that land should be crossed by a viaduct, and not by embanked approaches.

In designing a bridge for carrying an ordinary road over a river, it is usual, in order to obtain the greatest headroom possible consistent with economy in forming the approaches, to give the roadway an ascent from the ends of the approaches to the middle of the bridge, at a rate not exceeding the ruling gradient of the road; and to suit the arches, when there are more than one, to the form of the roadway, the centre arch is made the largest, and the others gradually diminish in size towards the ends of the bridge. They should, at the same time, be so proportioned as to exert as nearly as possible equal horizontal thrust.

Swing bridges for navigable rivers will be again mentioned further on.

Ice-breakers are required for the protection of the piers of bridges across rivers which bring down large masses of ice.

A *stone ice-breaker* usually forms part of the up-stream cutwater of the pier to which it belongs, presenting to the current a ridge sloping at about 45°, up which the flat sheets of ice slide, and break asunder by their own weight. Examples of such ice-breakers are shown in the view of the Victoria Bridge, fig. 249, p. 533.

A *timber ice-breaker* stands usually separate from the pier which it protects, at a short distance up-stream. The sloping ridge is formed by a beam of 12 or 14 inches square, covered with sheet iron. Its base consists of piles, ranged in the form of a long sharp triangle with the point up-stream, connected with the ridge by a strong framework of uprights and diagonals, which are protected against the ice by projecting horizontal wales.

(On the subject of river bridges, see Telford's and Smeaton's *Reports*, and the work *On Bridges* by Mr. Hosking and others).

474. **Artificial Water-Channels—Conduits.**—In laying out and designing artificial water-channels it is advisable, if possible, so to fix the declivity with reference to the length, that the velocity shall not be less than about one foot per second (lest the conduit silt up), nor greater than about four feet per second (lest the current should sweep stones along, and injure the bed).

As to the larger-sized artificial water-channels, and as to those of all sizes which are merely to be used as open drains, when they are wholly in cutting, it is unnecessary to add anything to what has already been stated respecting river-channels, and especially respecting their diversions, Article 471, p. 713. Artificial earthen channels in embankment will be considered under the head of canals.

When a channel is to convey water for the supply of a town, it is usual, with a view to the clearness and purity of the water, as well as to the preservation of the channel, to line it throughout with brick or stone built in cement; and in most cases it is necessary to cover it also, especially if it traverses districts where the air is smoky and otherwise impure. When brick or porous stone is used, the water-way may be lined throughout with a coating of cement, calcareous or asphaltic.

The water-way of a *stone or brick conduit* should be made of one of those forms which give the greatest hydraulic mean depth for a figure of given class and a given area; that is to say, the semicircle, the half-square, or the half-hexagon, already referred to in Article 451, p. 688. To preserve a constant definite flow it may have a series of waste-weirs along its sides, placed in positions where there are convenient channels at hand for discharging the waste water. Should it be necessary to carry it along an embankment, that embankment should be formed in thin layers, each well rammed, and should if possible contain a large mixture of stones with the earth; the breadth at the top should be from 4 to 6 feet at each side of the conduit, so that the total breadth at the brink of the conduit will be = breadth of water-way + from 8 to 12 feet, and the masonry of the conduit should be imbedded in puddle or in hydraulic concrete.

The best form for a *covered conduit* to convey a constant flow, as for the supply of a town, is cylindrical. To guard it against frost it should be completely covered with earth to the depth, in Britain, of about 3 feet, the bank being faced with sods. When it forms a tunnel, or is placed in deep cutting and covered with earth, its strength is regulated by the principles of Article 297 A, p. 433.

One of the largest cylindrical conduits yet executed is that of the Loch Katrine Water-Works, 8 feet in diameter.

A covered conduit should be provided, like a tunnel, with grated ventilating shafts, which will also serve to admit men for the purpose of repairing it.

When the flow varies very much, as in sewers, an egg-shaped section with the small end down is preferred.

A recent invention in conduits is that of Mr. Richardson, in which a cylinder of sheet iron is lined with brickwork in cement. It is suitable for making large conduits possessing great strength and stability with a moderate quantity of materials.

475. **Junctions of Water-Channels.**—In all cases in which a pair of water-channels join together into one, their centre lines, if

possible, should be a pair of curves, or a curve and a straight line touching each other at the junction; or should an angle at the junction be unavoidable, that angle ought to be as acute as possible. This principle applies also to the *divergence* of a branch from a main channel, and to pipes as well as to free channels.

476. **Aqueduct Bridges** differ from viaducts only in supporting a water-conduit instead of a road or a railway, and the mechanical principles of their construction involve nothing that has not been already explained in the Second Part of this treatise.

The water conduit or trough is usually of the same material with the rest of the structure. For example, in a stone aqueduct the conduit is of masonry, imbedded in a mass either of puddle or of concrete, resting on the arch and contained between the external spandril walls.

In some recent examples of wrought iron aqueducts introduced by Mr. Simpson, the water-channel has been made self-supporting by constructing it as a plate iron tubular girder of oval section. In this case the interior of the tube should be smooth, that it may offer no impediment to the current. All T-iron stiffening-ribs, &c., should project outside only.

Pipe-aqueducts will be mentioned further on.

477 **Water-Pipes.**—The diameters of water-pipes are fixed with reference to the vertical declivity and the intended greatest discharge, according to the rules explained in Article 450, p. 684. The materials principally used in making pipes for the conveyance of large quantities of water are earthenware and iron.

I. *Earthenware Pipes* are of various qualities as to texture, from a porous material like that of red bricks, to a hard and compact material, which is glazed to make it water-tight. They are made of various diameters, from 2 inches to nearly 3 feet, and in lengths of from 1 foot to 3 feet. The harder kinds have considerable tenacity, and are capable of bearing the dead pressure of a high column of water; but they are so easily broken by sharp blows and sudden shocks that it is not advisable to expose them to high pressures in situations where their bursting might cause damage or inconvenience. Hence their chief use is as *small covered conduits* for purposes of drainage. Their joints are most commonly of the spigot and faucet form, being made tight, if necessary, with cement, or with a bituminous mastic. (Article 234, p. 376.) Another form, very useful to facilitate laying and lifting is the *thimble-joint*. The lengths of pipe are plain hollow cylinders, and the thimble is a ring embracing and loosely fitting the adjoining ends of a pair of lengths. Sometimes the thimble is in two semicircular halves; and sometimes each pipe has on one end a half-faucet, which is laid downwards; the end of the adjoining pipe rests in the half-faucet, and the joint is completed by a half-thimble above. Curved and acute-angled junction-pieces are made: so also are right-angled junction-pieces; but these last should never be used.

II. *Cast Iron Pipes* should be made of a soft and tough quality of cast iron. (See Article 353, p. 499.) Great attention should be paid to moulding them correctly, so that the thickness may be exactly uniform all round. Each pipe should be tested for air-bubbles and flaws by ringing it with a hammer, and for strength by exposing it to double the intended greatest working pressure.

Cast iron water-pipes are made of various diameters or bores, from 2 inches to 4 feet.

They are usually moulded and cast horizontally, the sand core being supported by a strong horizontal bar with projecting teeth; but advantages in point of accuracy and soundness are possessed by the process of casting them vertically, the faucet being turned downwards, and the plain end upwards.* The pipe is cast with an additional length at the upper end, which acts as a *head* (Article 354, p. 503), compressing the mass below, and receiving the air-bubbles; this *head* is afterwards cut off.

The rule for computing the thickness of a pipe to resist a given working pressure (the factor of safety being *six*) has already been given in Article 150, equation 2, p. 228, the pressure and the tenacity of the iron being expressed in lbs. per square inch; but as it is more convenient to express those quantities in *feet of water*, the following rule is given:—

$$\frac{\text{thickness}}{\text{diameter}} = \frac{\text{greatest working pressure in feet of water}}{12{,}000}. \quad (1.)$$

There are limitations, however, arising from difficulties in casting, and from the fact that the most severe strain on a pipe is often produced by shocks from without, which cause the thickness of cast iron pipes to be often made considerably greater than that given by the above rule. The following empirical rule expresses very accurately the *limit to the thinness of cast iron pipes,* in ordinary practice:—

The thickness of a cast iron pipe is never to be less than a mean proportional between its internal diameter and one-forty-eighth of an inch.

It is very seldom, indeed, that a less thickness than 3-8ths of an inch is used for any pipe, how small soever.

Cast iron pipes are made of various lengths; but the most common length is 9 feet, exclusive of the faucet or socket on one end of each length, for receiving the plain end of the next length. The faucet adds from one-twentieth to one-tenth to the weight of the pipe. The joints are sometimes run up with melted lead, sometimes turned so that the plain end and the faucet fit exactly, and made water-tight with red lead paint. The latter is the easier and quicker process; but the former admits of a certain amount of yielding to expansion and contraction, and to the unequal settlement of the ground, which is an advantage in point of safety.

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* Introduced by Mr. D. Y. Stewart.

III. The best *preservative* for cast iron pipes against corrosion is a coating of pitch, applied both inside and out, by a process which makes it penetrate the pores of the iron to a certain extent, and adhere very firmly. This coating appears to diminish sensibly the friction of the water.

IV. In estimating the greatest working pressure which a water-pipe should be capable of resisting, the *hydrostatic pressure* due to the whole depth below top-water of the reservoir whence the supply enters the pipe, and not the mere *hydraulic pressure* when the water is in motion (Article 446, p. 675), should be taken into account, in order to provide for the contingency of the flow of the water being checked by an obstruction in the pipe.

V. The *loss of head* during the most rapid discharge should be computed for a series of points in the course of an intended pipe by the principles explained in the First Section of this chapter, so as to determine the *line of virtual declivity*, which will commence at a point vertically above the mouthpiece of the pipe, and at a depth below the top-water of the reservoir equal to the loss of head due to the velocity of flow in the pipe and the friction of the mouthpiece. The object of determining that line is to insure that in laying out the levels of the pipe *no part of it shall be made to rise above the line of virtual declivity.* The reason for this rule is, that at all points in a pipe which are above that line, the pressure, when the water is flowing, becomes less than that of the atmosphere (a fact commonly described by saying that there is a "partial vacuum;" see Article 443, p. 673); in consequence of which the air, which all water contains in a diffused state, escapes from the water in bubbles, and eventually accumulates in the highest part of the pipe so as to obstruct the flow of the water.

A pipe thus rising above the line of virtual declivity is called a *siphon*, and is incapable of continuously conveying water unless the air be from time to time exhausted from the summit of the pipe.

Air collects to a certain extent at the *summits* of an undulating pipe even when they are below the line of virtual declivity; but as it exerts a pressure greater than that of the atmosphere, it is easily expelled. A small cylindrical receiver, called an *air-lock*, is placed above the pipe at each such summit, to collect the air, which is from time to time discharged through a valve. That valve may either be opened by hand occasionally, or it may be loaded with a weight equivalent to the hydraulic pressure, and made self-acting.

VI. At the *lowest points* in an undulating line of water-pipe sediment collects, and is to be discharged from time to time through a *cleansing* or *scouring* cock or valve.

VII. As to *slide-valves*, *double-beat-valves*, and other valves and cocks used in connection with water-pipes, see *A Manual of Prime Movers*, Article 116, p. 120, and Articles 119 to 123, pp. 123 to 126.

VIII. *Sheet Iron Water-Pipes* lined with pitch have lately been used in France.

478. **Pipe-Track—Pipe-Aqueducts.**—Care should be taken to bed water-pipes on a firm foundation, and to cover them to a sufficient depth to prevent the action of frost; that is, in Britain, about 2 or 3 feet.

When a water-pipe crosses a valley, or a river-channel, or a line of communication, it may sometimes be advisable to carry it above ground by means of an aqueduct. This may be a bridge of any convenient construction, or it may consist simply of the pipe itself lying on a series of piers, and cased or lined with wood, or other non-conducting material, for protection against heat and cold. For a pipe-aqueduct of wide span, the pipe itself may be made to form a catenarian arch.*

The total thrust at the springing of the arch under an uniform load is to be computed in the usual way, being,

load per foot of span × radius of curvature at crown in feet × secant of inclination at springing;

from which has to be deducted the thrust borne by the water, viz.,

pressure of water × sectional area of pipe;

and the *remainder* only of the thrust has to be borne by the iron of the pipe. In fact, the *arch of water* bears a part of the load.

If the arched pipes be made to carry a roadway, the whole of the stress produced by a partial or travelling load will fall on them; and their strength is to be computed by the formulæ of Article 180, Problems IV. and V., pp. 303 to 308, as explained in treating of cast iron arched ribs, Article 374, Case I., p. 539.

The *wooden lining* referred to as a protection against frost consists of oaken staves about 3 inches thick, packed in a cylindrical form round the interior of each pipe. It is likely to prove more lasting than an outside casing, because it is constantly wet, instead of being alternately wet and dry.

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* Of this there is an example on the Washington Water-Works, designed by Colonel Meigs of the United States' Engineers. The arch is of 200 feet span, and consists of two parallel cast iron pipes of 4 feet diameter, lined with wooden staves.

Section V.—*Of Systems of Drainage.*

479. **General Principles as to Land Drainage.**—The engineer who examines a district with a view to the improvement of its drainage requires the information respecting the features, extent, and levels of the district, its rain-fall, and the course, dimensions, levels, and discharge of its streams, which have already been specified in Articles 456, 457, and 458, pp. 692 to 699, and in Article 467, p. 707. In some cases it is necessary to attend to the question, whether the water to be carried off by the system of drainage comes merely from the apparent gathering-ground bounded by the ridges that surround the district, or whether some of it is brought to the district through porous strata, which have their gathering-ground wholly or partly beyond such ridges.

In order that a district may be in a perfect state as to drainage, the water-level in the branch drains, which directly receive the discharge of the field drains, should be at least about 3 feet below the level of the ground at all times. When it rises above that level the ground becomes *awash* or *flooded*, according as the water-level is below or above its surface.

Each water-channel must have sufficient area and declivity, when at its fullest flow, to discharge all the water that it receives as fast as such water flows in, without its water-level rising so high as to obstruct the flow of the branches it receives, or to lay land awash.

Should it be impossible absolutely to fulfil these conditions, means are to be taken to make the deviation from them as small in extent and as short in duration as possible.

480. **Questions as to Improvement of Drainage.**—Should the drainage of a district be found defective, the engineer will in general have to consider questions of the following kind, as to the causes of such defective condition, and the means of improving it:—

I. Whether, and to what extent, it is practicable to diminish or prevent floods by the construction of store reservoirs.

II. Whether the channels of the streams contain *removable obstructions* such as shelves of rock or other shallows, narrow places, islands, ill-designed weirs and bridges, &c., and how such obstructions are to be removed. This may involve questions as to rebuilding weirs and bridges according to improved designs.

III. Whether the channels are defective and liable to be obstructed through the instability of their beds, and how such instability is to be prevented.

IV. In the case of a smaller stream having too little declivity, which falls into a larger stream, whether that declivity can be increased by diverting the course of the smaller stream so as to remove its outfall to a lower part of the larger stream.

V. Whether the course of a stream, being too circuitous, can be improved by a diversion; and whether, in the event of improvements being required in the channel of a stream, it is best to execute them in the existing channel, or to make a new channel, independently of the question of circuitousness.

All the preceding questions relate to matters which have already been treated of in Sections III. and IV. of this chapter, but the following involve subjects which will be treated of in the ensuing articles:—

VI. Whether the branch drains are of sufficient discharging capacity.

VII. To what extent the water-channels are capable of acting as temporary reservoirs for moderating the rapidity with which flood-waters descend from them into lower and larger channels.

VIII. To what extent the lands adjoining a river which are liable to inundation act in the capacity of a reservoir, and what will be the effect upon the part of the river below them of preventing or diminishing such action.

IX. Whether the drainage can be sufficiently improved by improvements on the water-channels alone, or whether, on the other hand, it is advisable to use embankments for the confinement of floods within certain limits.

481. **Discharging Capacity of Branch Drains.**—If the rain-fall found its way at once from the surface of the ground to the drains, each of these would require to have dimensions and declivity sufficient to discharge the most rapid fall of rain known to take place for any time how short soever. The following data as to the most rapid rain-fall in Britain are given on the authority of Mr. Phillips; they illustrate how the greatest *rate* of rain-fall diminishes according as the period for which it is reckoned is increased:—

Period.	Total depth of Rain-fall. Inches.	Rate of Rain-fall. Inches per Hour.
One hour, ...	1	1·0
Four hours, ...	2	0·5
Twenty-four hours, ...	5	0·2 nearly.

The soil, however, acts as a sort of reservoir to an extent depending on its texture; it keeps from the drains altogether a portion of the rain-fall, which passes off by evaporation, or is absorbed by plants, as stated in Article 456, p. 692; and it discharges the remainder into the drains more or less gradually. The branch drains in country drainage should be made capable of discharging at an uniform rate the greatest *available* rain-fall known to take place in a period whose length is greater according as the soil is more retentive. It is probable that in most cases of cultivated land *twenty-four hours* will be found a sufficiently short period: that is, each drain which directly receives water from the fields should be capable of discharging, in twenty-four hours, the greatest available rain-fall of twenty-four hours; for steep and rocky ground the period must be shortened, in some cases, it is probable, to four hours; but the best method in each case is to ascertain the period by an experimental comparison of the rain-fall with the discharge of drains.

482. **Action of Channels and Flooded Lands as Reservoirs.**—The volume of the space contained between the ordinary water surface of a given portion of a stream and the flood-water surface, whether such space be wholly contained between the banks of that portion of the stream, or partly between such banks and partly over adjoining lands liable to inundation, constitutes a reservoir for retaining *the excess of the total supply of water during a period of flood rain-fall from the district drained by that portion of the stream, above the greatest quantity that the stream is capable of discharging in the same period*, until the flood rain-fall is over, when that excess flows away by degrees. The existence of that reservoir-room thus renders sufficient a water-channel of less discharging capacity than would otherwise be necessary; and if such reservoir-room is diminished, either by improving the channel so as to lower the flood-water surface, or by contracting the space by means of embankments, care should be taken that the discharging capacity of the channel *below* the district in question is increased to a corresponding extent, otherwise the effect of diminishing the extent of floods in that district may be to increase it in some district further down the river. This is one of the reasons for the rule already stated in Article 470, p. 712, that works of river improvement should proceed from below upwards.

483. **River Embankments.**—When the land adjoining a stream cannot be sufficiently guarded from inundation by improvements in the channel, embankments may be erected. In determining the course and site of such embankments regard must be had to the principle stated in the last article—of leaving sufficient reservoir room between them for flood-water. In some cases there may be sufficient room even when the embankments are erected close to the natural banks of the channel; but in general it is advisable to

leave a wider space; and when the river follows a serpentine course sufficient reservoir-room may in many cases be provided by carrying the embankments along the general course of the valley, so as to enclose the windings of the stream without following them, and thus to form not only a reservoir, but a wide and direct channel for the discharge of floods.

The tributary streams which flow into the main streams will in general require branch embankments. Where a main embankment extends for a long distance uninterrupted by a tributary stream, the land protected by it is often divided into portions by means of branch embankments, called "*land arms*," diverging from the main embankment, the object of which is, that, in the event of a breach being made in the main embankment, the inundation may be confined to a limited extent of ground. These "land arms" generally run along the boundaries of separate holdings.

Behind and parallel to each main embankment there runs a "*back drain*," the material dug from which, if suitable, may be used in making the embankment. The use of this back drain is to act not only as a channel for the drainage of the land protected by the embankment, but as a reservoir to collect that drainage when the river is in a state of flood, and its dimensions are to be regulated accordingly. The waters of the back drain are discharged into the river (when its surface is low enough) through a series of pipes traversing the embankment, and having flap-valves opening outwards to prevent the return of water from the river. These valves are made sometimes of iron, sometimes of wood; one of the most efficient consists of an iron grating or perforated plate, covered with a flap of vulcanized indian-rubber. As to the computation of the time required to discharge a given accumulation of water from the back drain through a given outlet, see Article 455, p. 691.

The embankments are to be made of clay rammed in layers one foot deep, or thereabouts. When of moderate height, and not exposed to great pressure, they may have slopes of 1½ to 1 or 2 to 1. When they are liable to be acted upon by a strong current they should be pitched with stone, or otherwise defended like river-banks (Article 469, p. 710): elsewhere they should be covered with sods, and no trees, shrubs, or hedges should be suffered to grow upon them.

484. **Tidal Drainage** is the drainage of lands which are above the low-water-mark of ordinary tides, and either below high-water-mark, or so near that level that their drainage waters can only be discharged in certain states of the tide. Such lands are defended against inundation by the sea by means of embankments, which will be treated of further on.

The best mode of draining a district of this sort is by means of a canal extending completely through it, which acts alternately as a reservoir and as a channel. The *top-water-level* of the canal is to be fixed so as to give sufficient declivity to the branch drains. Its *low-water-level* will be above that of low-water of neap tides to the extent of 1-15th part of the rise of such tides. The space contained in the canal between those levels is the *reservoir-room;* and inasmuch as the length and depth of that space are fixed, the breadth midway between those levels is to be made sufficient to give reservoir-room for the greatest quantity of drainage water that ever collects during one tide. The depth of the canal must be made at least sufficient to enable the whole of that quantity of water to be discharged in the interval between 1 hour before and 1 hour after low-water, the *mean velocity of outflow* being assumed to be about equal to that due to a declivity of the height between high and low-water-levels in the whole length of the canal, and to its hydraulic mean depth when full up to its middle water-level. The outer end of the canal is to have large floodgates capable of throwing its whole width and depth open at once.

For an example of drainage on these principles, see the *Account of the Great Bedford Level.*

485. **Drainage by Pumping** is extensively employed in lands below high-water-mark, especially in Holland. In former times windmills were chiefly used for this purpose, but now they are to a great extent replaced by steam engines. The most economical mode of conducting drainage in this manner is to provide reservoir-room for the greatest floods, and pump constantly at an uniform rate. To provide for the repair of engines, and for accidental stoppages, engines are to be kept in reserve, of power equal to from one-half to the whole of the power of those that are kept at work.

486. **Town Drainage.**—Plans for systems of town drainage require to be on a larger scale, and to have closer contour-lines, than those of any other description of work. (See Article 59, p. 96.) The discharge to be provided for is the natural drainage of the basin which the town occupies, added to the water supply artificially brought into the town.

Inasmuch as the rain-fall in towns finds its way into the sewers almost instantly, their dimensions and declivity must be suited to the heaviest rain-fall in a short period. Authorities differ whether that rain-fall is to be estimated at *one inch* or at *half-an-inch* in depth per hour.

The treatment and disposal of the drainage of towns, after it has been collected by means of a system of sewers, involves chemical and physiological questions into which it is impossible to enter in this treatise.

487. **Sewers**, or main drains of towns, are underground arched brick conduits, designed, laid out, and constructed according to the principles already explained or referred to in Articles 474, 475, pp. 718 to 720. As to their strength, see Article 297 A, p. 433. The cross-section preferred for them in Britain is an oval, with the small end downwards. In order that men may be able to enter them for purposes of cleansing and repair, no sewer should have a less breadth than 2 feet.

The velocity of the current should be not less than 1 foot per second, or more than about 4½ feet per second.

As to the drainage of streets into the sewers, see Article 417, p. 626. Owing to the quantity of mud that is swept into sewers, they are peculiarly liable to be obstructed by collections of sediment: these are swept away by an operation called *flushing* or *flashing*, which consists in placing a temporary dam of timber above the spot where the deposit is, so as to collect a quantity of water, which is allowed suddenly to escape with great speed in order to scour away the deposit.

As the pipes leading into the sewers from the channels of the streets, and also those from the houses, either are or ought to be "trapped" by means of valves or inverted siphons, so as to prevent the escape of foul gas from the sewers, such gas must have openings provided for its escape, either by building chimneys for the purpose, or by connecting the sewer with existing chimneys. Passages for the admission of fresh air to the sewers are also required, and subterranean entrances with trap-doors to give men access to them. As to the use of "side-trenches" and "subways," see Article 421, pp. 629, 630.

488. **Pipe-Drains.**—The earthenware pipes used for drainage have already been described in Article 477, p.720. In town drainage they are chiefly used for the branch drains leading from houses and from the adjoining ground into the main sewers; and they usually range from 4 inches to 18 inches in diameter, according to the quantity which they are to discharge. It is not advisable in any case to use drain-pipes of less than 4 inches in diameter. They should all be laid, as far as possible, at such declivities as to insure a velocity of flow of 4½ feet per second, in order that the formation of deposit may be impossible; and when their proper levels and declivities have been determined by calculation, great care should be bestowed on seeing that they are accurately laid at those levels and declivities: the smaller the diameter of the pipe, the worse is the effect of any inaccuracy in this respect. Obstructions are most likely to occur at the junctions. The importance of making these either curved or acute-angled has already been mentioned; but even at curved or acute-angled junctions deposits may some-

times take place, and a good safeguard against this, when the levels are such as to render it practicable, is to make the junction in a vertical or transversely inclined, instead of a nearly horizontal plane.

The *inverted siphon air-trap*, for preventing the entrance of foul gas from a sewer into a building through a drain-pipe, is an U-shaped tube, in the lower part of the bend of which water lodges, so as to prevent the passage of gas. To insure the efficiency of this trap, it is essential that the sewer should have chimneys for the escape of gas; otherwise the pressure may become sufficient to enable the gas to force its way past the water in the tube.

SECTION VI.—*Of Systems of Water Supply.*

489. **Irrigation.**—It appears that the supply of water required for the irrigation of a district ranges from ·013 to ·008 of a cubic foot of water per second for each acre irrigated; and this is the *demand* to be provided for by reservoirs, or by the use of weirs to divert water from rivers. (Article 460, p. 699; Article 472, p. 713.) The channels by which the water is distributed are to be carried at the highest levels compatible with the minimum velocity of 1 foot per second, in order that as great an area of land as possible may be commanded by them. Their dimensions and declivity are to be determined by the principles of Article 451, p. 686, and they are to be constructed according to the principles of Section IV. of this chapter, especially Article 474, p. 718. When they run between earthen embankments, as is often the case, each embankment should have a vertical puddle wall in its centre, from 2 to 3 feet thick, and the tops of the embankments should not be less than 4 feet wide.

The method of delivering specified supplies of water from an irrigation canal to holders of land is the following:—A small tank at one side of the canal is supplied through a sluice, and the water in it is kept at a constant level by regulating the opening of that sluice. The water is delivered out of the tank through a square or round orifice of constant size under a constant head. Different quantities of water are delivered by varying the *number* of the orifices, and not their dimensions nor the head which causes their discharge.

490. **Water Supply of Towns—Estimation of Demand as to Quantity.**—The supply of water to towns ranges in extreme cases from about 2 gallons to 600 gallons per inhabitant per day. (Gordon *On Civil Engineering.*) In town water-works executed with a due regard to sufficiency of supply on the one hand and economy of

cost on the other, and with a moderate amount of waste, the following may be regarded as fair estimates of the real daily demand for water per inhabitant amongst inhabitants of different habits as to the quantity of water they consume:—

	Gallons per Day. Least.	Average.	Greatest.
Used for domestic purposes, ...	7	10	15
Washing streets, extinguishing fires, supplying fountains, &c., ...	3	3	3
Allowance for trade purposes and for waste,	7	7	7
Total demand in non-manufacturing towns,	17	20	25
Additional demand in manufacturing towns, say	10	10	10
Total demand in manufacturing towns,.....	27	30	35

A liberal supply of water has a tendency to increase its use, and at the same time to bring the daily consumption per head amongst different classes of persons more nearly to an equality; so that, with a view to such improvement in the habits of the population, it is advisable in projecting new water-works to take the highest of the preceding estimates of the demand; that is to say, from 25 to 35 gallons per head per day, according as the town requires less or more water for use in manufactures.

The quantity of water run to waste, however, frequently exceeds enormously that allowed for in the preceding estimate, through ill-constructed fittings and carelessness. It has been proved, for example, that in some parts of London, where only 7 gallons of water per head per day were actually used, 18 gallons ran to waste. The most effectual means of preventing such waste are, the establishment of a regulation or enactment, that domestic water-fittings shall be executed to the satisfaction of the engineer or manager of the water-works; the carrying out, as far as practicable, of the system of selling water by measure to those who require it for other than ordinary domestic purposes (as to water meters, see Article 459, p. 699); and the prevention of excessive pressure in the service-pipes from which houses are directly supplied.

The preceding statements have reference to the daily demand. Regard must also be had to the hourly demand, which fluctuates very much at different times of the day, chiefly because the inhabitants draw nearly the whole of their supply for domestic purposes during a limited number of hours. It is estimated that the most rapid draught for domestic purposes is at such a rate that, if kept up continuously, it would exhaust the whole daily supply for these purposes in 8 hours; that is to say, the maximum hourly demand for domestic purposes is *three times* the average hourly demand.

The effect of this on the greatest hourly demand *for all purposes* is to make it in different cases range from 1·8 to 2·2 times the average hourly demand.

491. **Estimation of Demand as to Head.**—It is considered that the head of pressure in each of the street mains ought, when the flow is most rapid, to be equivalent to an elevation of about 20 feet above the tops of the adjoining houses, in order that their uppermost stories may be directly supplied, and that it may be possible to throw a jet to the top of the highest building without the aid of a fire-engine.

The required virtual head in various districts of the town being fixed, the virtual declivity from the source to each of those districts is to be made as nearly uniform as circumstances will permit, if pipes are used throughout. Should a conduit be used for part of the distance, and pipes for the remainder, the pipes should have the steeper virtual declivity, and consequently the greater share of the total virtual fall in proportion to their length, in order that they may be smaller than the conduit; because their cost is greater in proportion to their size than that of the conduit. No precise rule can be laid down for this distribution of fall between pipes and conduit; but in some good examples the virtual declivity of the pipes has been made *eight times* as steep as the actual declivity of the conduit. As to the discharging capacity and construction of conduits and pipes, see Articles 450, 451, pp. 684 to 688, and Articles 474 to 478, pp. 718 to 724.

In a town of irregular levels, or of great extent, the same virtual declivity which is required in order to give sufficient head of pressure in the higher parts of the town, or in those more distant from the source, may give excessive pressure in the lower or nearer parts. In such cases the excessive pressure in the branch mains and distributing pipes of the latter districts may be moderated by any convenient means of causing loss of head at their inlets, such as passing the water through small orifices, or loaded valves; the latter being the more accurate method in its working.

492. **Compensation Water** is the supply of water which is secured to the owners and occupiers of land and mills, and other parties interested in the sources from which water is diverted to supply a town, in order that they may not suffer damage by such diversion. It must be at least equal to the supply which was beneficially available for their use before the execution of the water-works, or else they must receive compensation in money for the deficiency.

The only means of enabling a source of water to supply a town, besides providing the landholders with compensation water, according to the preceding principle, is to store in reservoirs and discharge by degrees the flood-waters which previously ran to waste. (See Section III. of this chapter, p. 699.)

In providing the daily supply of compensation water to which the landholders on the course of a stream are entitled, different principles have been followed in different cases. The following are three of them:—

I. To secure them the *average summer discharge, exclusive of floods*, as ascertained by gauging. (As to the distinction between flood discharges and ordinary discharges, see Article 458, p. 698).

II. To give them a proportion fixed by agreement (usually *one-third*, or thereabouts) of the whole water impounded.

In some cases a special arrangement has been come to, by which the landholders, on condition of a certain supply being delivered down the stream during the day, have agreed to a less supply being delivered during the night.

III. To make a special compensation reservoir, receiving the discharge from a certain proportion of the gathering-ground, and to hand it over to the landholders, to be managed under their own control.

The usual method adopted in delivering a fixed daily quantity of water into the natural channel of a stream is to construct a tank in which the water is kept at a fixed level by means of the sluice or sluices through which it is supplied, and let the water flow out of that tank through an outlet or outlets of a fixed area and figure, under a fixed head.

493. **Storage-Works** consist of reservoirs with their appurtenances, as described in Section III. of this chapter. In estimating the extent of gathering-ground and capacity of the reservoirs required, regard must be had to the demand of water for compensation (Article 492), as well as for the supply of the town.

In most cases in which a town is supplied from works of this class, the best economy consists in choosing the sites of the store reservoirs, and designing the conduits and principal main pipes, so as to supply every part of the town by means of the gravitation of the water alone. But exceptional cases sometimes occur, in which a great saving may be effected in capital outlay, and especially in the cost of conduits and pipes, by incurring a comparatively small additional annual expenditure in order to supply some limited district that is highly elevated above the rest of the town by means of a pumping steam engine, instead of giving the conduits and principal main pipes the dimensions required in order to supply that limited district by gravitation.

494. **Springs** in many cases are so variable in their discharge that they can only be classed amongst the sources whose waters require to be stored in a reservoir. But occasionally springs are met with which are the outlets of extensive porous strata, forming underground natural reservoirs that maintain a nearly uniform discharge independently of artificial storage. (See Article 456, p. 696.) When the waters of such springs are diverted from the streams into which they naturally flow in order to supply a town, the ordinary summer flow of those streams must be maintained at its original volume by the aid of the flood-waters of a gathering-ground, stored in a reservoir.

495. **River-Works—Pumping.**—A large river may be used for the supply of a town, independently of storage-works, provided the volume of water brought down by it is at all times so great, that the temporary abstraction of a volume sufficient to supply the town will cause no injury to its navigation, or the interests of the inhabitants of its banks.

The works required in order to supply a town from such a river usually comprise a *weir*, for maintaining part of the river at a nearly constant level (Article 472, p. 713); two or more *settling-ponds*, into which the water is conducted, or if necessary, pumped, or otherwise raised by machinery; filtering apparatus; and a sufficient establishment of pumping engines.

It would be foreign to the plan of the present work to enter into details as to the construction and working of pumping steam engines. The following principles, however, must be stated as specially applicable to their use for the supply of a town.

I. The *effective power* required to be in operation may be computed in *foot-pounds per hour*, by multiplying the *weight* of water to be delivered per hour by the *total head* at the engines in feet; such head being measured from the level of the water in the tank whence the engines draw it, to the virtual elevation required in order to give sufficient head in the town and sufficient virtual declivity in the principal main pipes. To find the *effective horse-power*, divide the effective power in foot-pounds per hour by 1,980,000. The *indicated horse-power* is about *once and a-quarter* the effective horse-power.

II. *Reserve power* should be provided to an amount equal to at least one-half of the working power; for example, of three engines of equal power, two are to be kept at work and the third in reserve.

III. *Air-vessels* and *stand-pipes* are contrivances to prevent the shocks to which the pipes would be exposed by the intermittent action of the pumps, and to maintain an uniform head of pressure and velocity of flow in the pipes.

An air-vessel is an air-tight receiver, usually of cast iron, and of the figure of a cylinder standing vertically, with a hemispherical top and bottom. At its lower end are two openings, an inlet through which water enters from a pump, and an outlet from which the water is discharged along a pipe. Its upper portion contains compressed air, which tends continually to diminish in quantity, partly by leakage and partly by absorption in the water, so that a small supply of air should be forced in from time to time by means of a compressing pump. The effect of the air-vessel in moderating fluctuations of pressure is expressed by the following proportion:—

mean volume of air in the vessel : volume of the pump
: : mean head of pressure : greatest fluctuation of the head of pressure.

In some good practical examples, the capacity of the air-vessel is about *fifty times* that of the pump.

A *single stand-pipe* is a vertical cast iron pipe, rising a little higher than the elevation due to the head of pressure, and open at the top. It has at its base an inlet through which it receives water from the pumps, and an outlet or outlets through which it discharges water into the horizontal supply-pipes. Its sectional area varies from once to twice that of its outlets, or thereabouts. It equalizes the pressure and flow even more effectually than an air-vessel, for the rapid entrance of the quantity of water due to one stroke of a pump produces but a slight elevation of the surface of the water in the stand-pipe as compared with its total height.

A *double stand-pipe* has two branches, in one of which the water ascends from the pump, while in the other it descends to the mains: the two branches unite at the top into a vertical stem, which is open above. This construction effects a constant renewal of the water in the stand-pipe.

In estimating the dimensions and speed required for the piston or plunger of a pump that is to deliver a given volume of water in a given time, it is usual to add about *one-fifth* to that volume as an allowance for "*slip*;" that is, water which runs back through the pump-clacks while they are in the act of closing. It appears, however, from experiment, that in the best pumps the slip is not practically appreciable.*

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* The cost of pumping large quantities of water, as ascertained from the accounts of the expenditure of the former Glasgow Water-Works (since superseded by the Loch Katrine Works), during a long series of years, was at the rate of almost exactly 400,000 *gallons raised one foot for a penny*; that is to say, 4,000,000 *foot-pounds of effective work for a penny*.

496. **Wells** may be used as sources for a supply of water, where a water-bearing stratum exists into which they can be sunk. The water in such a stratum has always either an actual or a virtual declivity towards the place where, by the outcrop of the stratum, it makes its escape into a river, or into the sea. Should the water-bearing stratum have its gathering-ground at a high elevation, and should it be covered, in a district far distant from its final outlet, by an impervious stratum, the line of virtual declivity may be above the surface of the ground in that district; so that, on boring or sinking a well through the impervious stratum, the water will spout up in a jet. Such wells are called "Artesian Wells." In other cases the line of virtual or actual declivity is below the surface of the ground, and the water must be raised by pumping (as to which, see the preceding article).

The raising of a large quantity of water from a water-bearing stratum has always the effect of depressing the water-level to an extent which cannot be estimated beforehand.

The quantity of water which a water-bearing stratum is capable of yielding may be estimated in the manner explained in Article 456, p. 696, provided the position and extent of its gathering-ground can be ascertained; but that can seldom be done with precision.

In sinking or boring for well water, it is in general advisable to prevent the surface water from mixing with that of the well. This is done, in the case of a bore, by lining it with iron pipes, and in the case of a shaft, by lining it with brickwork laid in cement.

As to boring and shaft-sinking, see Article 187, p. 331, and Article 391, p. 589.

497. The **Purity of Water** is a subject of which the detailed consideration belongs to chemistry and physiology rather than to engineering. The following general principles, however, may be stated.

For purposes of cleansing, cookery, chemistry, and manufactures, the best water is that which approaches nearest to absolute purity. Such is the water which flows from mountain districts, where granite, gneiss, and slate prevail. Such water usually contains a large quantity of diffused oxygen and carbonic acid. It is the most wholesome for drinking, and the most agreeable to those whose taste does not prefer a certain admixture of earthy salts.

The most common mineral impurities of water are salts of lime and iron, which injure it for all purposes except drinking. Salts of lime, especially the bicarbonate, are the principal causes of the property called "hardness." The bicarbonate of lime can be removed by adding to the water as much lime-water as contains a quantity of lime equal to that already contained in the bicarbonate of lime present. The additional lime thus added combines with one-half of the carbonic acid, thus becoming chalk itself, and reducing the bicarbonate to chalk also; and the chalk, being insoluble, settles, and leaves the water softened. This is Dr. Clarke's process of softening water. The *degrees of hardness* of a specimen of water means the number of grains of chalk which the lime held in solution in a gallon of the water (or 70,000 grains) is capable of forming. Water of less than 5 degrees of hardness may be considered as comparatively soft; that of 12 or 13, as decidedly hard.

The waters collected directly from gathering-grounds are usually the softest, those of rivers harder, those of springs and wells hardest of all.

The drainage waters of cultivated and populous districts, and above all, those of towns and their neighbourhood, are to be avoided, as containing organic matter in the act of decomposition, and being therefore unwholesome, and sometimes highly dangerous.

The taste and smell of a person accustomed to drink pure water and breathe pure air may in general be relied upon for the detection of the presence of impurities in water, though not of their nature or amount; but in persons who have for some time habitually drunk impure water and breathed a foul atmosphere those senses become blunted.

The colouring matter of peat moss, which is a compound of carbon with oxygen and hydrogen, unfits water for many manufacturing purposes. It does not render it unfit for drinking, unless present in considerable quantity, when it produces an unpleasant flatness of taste; but whether that substance is unwholesome or not has not been ascertained. Its appearance is strongly objected to by the inhabitants of most towns. Long exposure to light and air destroys it, probably by oxidating its carbon.

The long-continued action of oxygen decomposes and destroys organic matter in water, and is the principal means of purifying originally impure water. In store reservoirs the presence of a moderate quantity of living plants is favourable to purity of the water, provided there are also animals enough to consume them, so that they may not die and decompose, and that a proper balance is kept up amongst animals of different kinds. The destruction of the fish in a reservoir has been known to lead to an excessive multiplication of the small crustaceous animals upon which the fish had fed, to such an extent that the water acquired a nauseous flavour from the oil which those minute creatures contained. The only remedy was to re-stock the reservoir with fish.*

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* This case was examined into and reported upon, and the remedy discovered, by Dr. H. D. Rogers.

Shallow reservoirs are unfavourable to purity, because the warmth of the water produced by the sun's heat encourages the growth of an excessive quantity of vegetation, most of which dies and decomposes.

On the subject of the purity of water, see Dr. R. Angus Smith's "Report on the Air and Water of Towns," in the *Reports of the British Association* for 1851.

498. **Settling and Filtration.**—A store reservoir generally answers the purpose of a settling-pond also, to clear the water of earthy matter held in suspension. Water pumped from a river generally requires to rest for a time in a settling-pond.

The water both of rivers and of gathering-grounds in most cases requires to be filtered. A filter-bed for that purpose consists of a tank about 5 feet deep, having a paved bottom, covered with open-jointed tubular drains leading into a central culvert; the drains are covered with a layer of gravel about 3 feet deep, and that with a layer of sand 2 or 3 feet deep. The water is delivered upon the upper surface of the sand very slowly and uniformly; it gradually descends, and is collected by the drains into the central culvert. The area of the filter should be such that the water to be filtered may not descend vertically with more than a certain speed; for the whole efficiency of the filtering process depends on its slowness. The speed of vertical descent recommended by the best authorities is *six inches an hour;* in some cases a speed as high as *one foot an hour* has been used.

There should be a sufficient number of filter-beds to enable some to be cleansed whilst others are in use. The cleansing is performed by scraping from the surface of the sand a thin layer, in which all the dirt collects.

It appears that proper filtration not merely removes mechanical impurities from the water, but even organic impurities, by causing their oxidation.

499. **Distributing-Basins or Town Reservoirs.**—It has been explained in Article 490, p. 730, that the *greatest hourly demand* for water is about double of the *average hourly demand;* from which it follows, that the pipe or conduit which *directly* supplies a given town, or part of a town, must have about double the discharging capacity that it would require if the hourly demand were uniform.

The great additional expense which this would cause in the principal conduits and main pipes is saved by the use of *distributing-basins* or *town reservoirs.*

A distributing-basin for a given district is a small reservoir, capable of containing a volume of water *at least* equal to the whole excess of the demand for water during those hours of the day when such demand exceeds the average rate above a supply during the same time at the average rate. The smallest capacity which will enable a distributing-basin to fulfil that condition is about one-half of the daily demand of the district to which it belongs; but to provide for unforeseen contingencies, it may be made to contain a whole day's demand, or even more. It is supplied with water at an uniform rate, by a principal main pipe, which thus only needs to be made capable of supplying the average hourly demand, the distributing-pipes alone requiring to be adapted to the greatest hourly demand. During the night, when the supply exceeds the demand, the water accumulates in the distributing-basin; during the day, when the demand exceeds the supply, that accumulated water is expended.

The area of a distributing-basin should be such, that the variation of its water-level may not cause an inconvenient variation of the head of pressure in the pipes, nor in their virtual declivity.

It may be built and paved with masonry or brickwork lined with cement, in which case the stability of its walls will depend on the principles cited in Article 465, p. 707; or it may be made of rectangular cast iron plates, flanged and bolted together, the opposite sides of the reservoir being tied together by means of wrought iron rods, to enable them to resist the pressure. The figure in plan will in general be regulated by that of the site; but should the engineer be free to choose any figure, the circular figure is obviously the best.

The elevation of the site should be such as to command the district to be supplied from the basin, according to the principles of Article 491, p. 732, and it should be as near that district as possible.

Every distributing-basin should be roofed, that the water may be protected against heat, frost, and the dust and soot which float in the air of populous districts. The most efficient protection against heat and frost is that given by a vaulted roof of masonry or brick, covered with asphaltic concrete to exclude surface water, and with two or three feet of soil, and a layer of turf.

When water is brought to a city from a great distance, it may be useful to construct in the neighbourhood of the city (should the ground afford a suitable site), a large town reservoir or auxiliary store reservoir, capable of holding a store of water for about a month's demand, to be used in the event of an accident happening to the more distant part of the main conduit, until the damage is repaired. From that reservoir to the town the main pipes may form a double line, so that in the event of a failure of one line, a supply, although a diminished one, may be conveyed through the other line until the first line is repaired. The construction of such an auxiliary store reservoir will in general be similar to that of the reservoirs described in Section III. of this chapter.

500. **Distributing-Pipes** must be adapted to the *greatest* hourly demand for water, and to the requisite head in the streets, as already explained in Articles 490 and 491, pp. 730 to 733. In large cities the total length of distributing-pipes required is about a mile for every 2,000 or 3,000 inhabitants. The smaller the town, the smaller in general is the *proportionate* extent of distributing-pipes required.

The distributing-pipes which are laid along the street are classed as *mains* and *service-pipes;* the chief distinction being, that a main either conveys, or is capable of conveying, water along a street to some place beyond it; while a service-pipe is a branch diverging from a main, in order to supply a single or double row of buildings. In wide streets, and in those of great traffic, it is best to have two service-pipes, one for each side, in order that they may be laid so as to be accessible without interrupting the traffic of the street (see Article 421, p. 629), and in order that the house water-pipes may be as short as possible, and may lie as little as possible under the carriage-way.

When a general rate of virtual declivity has been fixed for the distributing-pipes of a town or of a district of a town, and the diameters of the more important mains have been computed by the proper formula, those of all branch mains and service-pipes are easily deduced from them by the rule, that, with equal virtual declivities, the diameters of pipes are to be proportional to the *squares of the fifth roots* of the quantities of water that they are to convey.

When a pipe of uniform diameter has a series of branches diverging from it, so that the flow of water through it becomes less and less at an uniform rate, until the pipe terminates at a "*dead end,*" the virtual declivity goes on diminishing, being proportional to the *square of the distance from the dead end;* the excess of the head at any point above the head at the dead end is proportional to the *cube of the distance from the dead end;* and the total virtual fall, from the commencement of the pipe to the dead end, is *one-third* of what it would have been had the whole quantity of water flowed along the pipe without diverging into branch pipes.

All *dead ends* of pipes should be provided with *scouring-valves,* which should be opened from time to time to prevent the accumulation of deposit there. Pipes should be laid out and connected with each other so as to have as few dead ends as possible; and with that view it is desirable that service-pipes should, if practicable, be connected at both ends with mains.

The use of loaded valves to moderate pressure has already been mentioned in Article 491, p. 732.

The system called that of *constant service*, according to which all distributing-pipes are kept charged with water at all times, is the best, not only for the convenience of the inhabitants, but also for the durability of the pipes, and for the purity of the water; for pipes, when alternately wet and dry, tend to rust; and when emptied of water, they are liable to collect rust, dust, coal-gas, and the effluvia of neighbouring sewers, which are absorbed by the water on its re-admission. In order, however, that the system of constant service may be carried out with efficiency and economy, it is necessary that the diameters of the pipes should be carefully adapted to their discharges, and to the elevation of the district which they are to supply, and that the town should be sufficiently provided with town reservoirs. When these conditions are not fulfilled, it may be indispensable to practise the system of *intermittent service*, especially as regards elevated districts; that is to say, to supply certain districts in succession, during certain hours of the day. The adoption of this system makes it necessary for the inhabitants to have cisterns in their houses for the purpose of holding the daily store of water. In the poorer districts of towns, it is often advisable to have one large tank for a group of small houses, instead of a cistern in each house; the tank may be under the control of the water-work officials, and may be filled once a day, and the householders may be supplied from it through small pipes constantly charged, and may thus have the convenience of constant service although the supply to the tank is intermittent.

500 A. On the subject of the collection, conveyance, and distribution of water generally, special reference may be made to the works of Du Buat, M. D'Aubuisson, Mr. Neville, and Mr. Downing, *On Hydraulics;* Tredgold's *Hydraulic Tracts;* Mr. Beardmore's *Hydraulic Tables;* Professor Becker's "*Wasserbau;*" and Dr. Hagen's "*Handbuch der Wasserbaukunst,*" (Königsberg, 1853 to 1857); and on that of the water supply of towns, to the *Parliamentary Reports* on the supply of water to the metropolis, and to the *Reports* of the Board of Health on the same subject.

CHAPTER III.

OF WORKS OF INLAND NAVIGATION.

SECTION I.—*Of Canals.*

501. **Canals Classed—Selection of Line and Levels.**—Canals may be divided into three classes—

I. *Level Canals*, or *Ditch Canals*, consisting of one *reach* or *pond*, which is at the same level throughout. The most economical course for a canal of this sort is obviously one which nearly follows a contour-line, except where opportunities occur of saving expense by crossing a ridge or a valley so as to avoid a long circuit.

II. *Lateral Canals*, which connect two places in the same valley, and in which, therefore, there is no summit level, the fall taking place in one direction only. A lateral canal is divided into a series of level reaches or ponds, connected by sudden changes of level, at which there are either single locks or flights of locks, or some other means of transferring boats from one level to another. The "lift" of a single lock ranges from 2 feet to 12 feet, and is most commonly 8 or 9 feet. Each level reach is to be laid out on the same principles with a level canal. In fixing the lengths of the reaches and the positions of the locks, the engineer should have regard to the fact that economy of water is promoted by distributing a given fall amongst single locks with reaches between them, rather than concentrating the whole fall at one flight of locks.

III. *Canals with Summits* have to be laid out with a view to economy of works at the passes between one valley and another, and with a view also to the obtaining of sufficient supplies of water at the summit reaches. The subject of the supply of water to canals will be considered further on.

502. **Form and Dimensions of Water-way.**—Although, for the sake of saving expense in aqueducts and bridges, short portions of a canal may be made wide enough for the passage of one boat only, the general width ought to be sufficient to allow two boats to pass each other easily. The depth of water and sectional area of water-way should be such as not to cause any material increase of the resistance to the motion of the boat beyond what it would encounter in open water. The following are the general rules which fulfil these conditions:—

Least Breadth at Bottom = 2 × greatest breadth of a boat.
Least Depth of Water = 1½ foot + greatest draught of a boat.
Least Area of Water-way = 6 × greatest midship section of a boat.

The bottom of the water-way is flat. The sides, when of earth (which is generally the case), should not be steeper than 1½ to 1; when of masonry, they may be vertical; but, in that case, about 2 feet additional width at the bottom must be given to enable boats to clear each other, and if the length traversed between vertical sides is great, as much more additional width as may be necessary in order to give sufficient sectional area.

The customary dimensions of canal-boats have been fixed with a view to horse-haulage. The most economical use of horse-power on a canal is to draw heavy boats at low speeds. The heaviest boat that one horse can draw at a speed of from 2 to 2½ miles an hour weighs, with its cargo, about 105 tons, is about 70 feet long and 12 feet broad, and draws about 4½ feet of water when fully loaded. Smaller boats, which a horse can draw at 3½ or 4 miles an hour, are of about the same length, 6 or 7 feet broad, and draw about 2½ feet of water.

Boats of the greater breadth above-mentioned can easily be adapted to the various methods of propulsion by steam, whether by means of the screw propeller or the warping chain, or fixed engines and endless wire ropes (Mr. Liddell's system).

Ordinary canals are suited to boats such as the above. A larger class of canals are suited to sea-going vessels.

The following are examples of the extreme and ordinary dimensions of canals:—

	Breadth at Bottom.	Breadth at Top-water.	Depth of Water.
Small canal,.......	12 feet, ...	24 feet, ...	4 feet.
Ordinary canal,....	25 „	40 „	5 „
Large canal, ...	50 „	110 „	20 „

503. **Construction of a Canal.**—The least expensive parts of a canal are those in which the upper part of the water-way is contained between two embankments, and the lower part in a cutting, the earth dug from which, together with that dug from the side-drains at the foot of the outer slopes, is just sufficient to form the embankments.

All canal embankments should be formed and rammed in thin layers. (Article 203, p. 341.) The width of the embankment which carries the towing-path is usually about 12 feet at the top; that of the opposite embankment at least 4 feet, and sometimes 6 feet. Each embankment has a vertical puddle wall in its centre from 2 to 3 feet thick.

In cutting, there should be a bench or berm of 12 or 14 feet wide, at one side, for the towing-path, and on the opposite side a bench about 3 or 4 feet wide at the same level. At the feet of the slopes, which terminate at those benches, there are a pair of side-drains, as described in Article 193, p. 335. These side-drains discharge their water at intervals into the canal through tubes.

The surface of the towing-path is usually about 2 feet above the water-level. It is made to slope slightly in a direction away from the canal, in order to give a better foot-hold for the horses, as they draw in an oblique direction.

The slopes are to be pitched with dry stone from 6 to 9 inches thick.

Occasionally it may be necessary to line a canal with concrete, or to face the sides with rows of sheet-piling, in order to retain the water.

Natural water-courses are to be carried below the canal by means of bridges and culverts, and, if necessary, by inverted siphons of masonry or iron. Where such water-courses are above the level of the canal, their waters may be partly used for supplying it; but means should be provided for carrying such waters wholly across the canal when required.

Each reach of a canal should be provided with waste-weirs in suitable positions, to prevent its waters from rising to too high a level; also with sluices, through which it may be wholly emptied of water for purposes of repair; and in a reach longer than two miles, or thereabouts, there may be stop-gates at intervals, so that one division of the reach may be emptied at a time, if necessary. The rectangular channel under a bridge or over an aqueduct is a suitable place for such gates.

Leaks in canals may sometimes be stopped by shaking loose sand, clay, lime, chaff, &c., into the water. The particles are carried into the leaks, which they eventually choke by their accumulation.

504. **Canal Aqueducts and Fixed Bridges.**—A canal aqueduct, like the aqueducts for conduits already mentioned in Article 476, p. 720, is a bridge supporting a water-channel. The trough or channel, for economy's sake, is usually made wide enough for one boat only. Its bottom is flat, or nearly so; its sides vertical or slightly battering. In aqueducts of masonry, the total thickness of material, from the side of the trough to the face of the spandril-wall, is usually 4 feet at least at the side furthest from the towing-path; at the towing-path side it is sufficient for a towing-path of from 6 to 10 feet wide, and a parapet from 15 to 18 inches thick.

In Telford's cast iron aqueduct, known as Pont-y-Cysylte, the channel is a rectangular trough of cast iron, supported on cast iron segmental arched ribs of 45 feet span. The trough is of the whole width of the bridge, about 12 feet, and the towing-path, 5 feet 8 inches wide, covers part of the trough.

The principle of the *suspension bridge* is peculiarly well adapted to aqueducts, because, as each boat displaces its own weight of water, the only disturbance of the uniform distribution of the load is that arising from the passage of men and horses along the towing-path. An aqueduct of this sort, designed by Mr. Roebling, with seven spans of 160 feet, carries a canal 16 feet wide and 8 feet deep, over the Alleghany River at Pittsburg.

Fixed bridges over canals require no special explanation, except to state that, in the older examples of them, the water-way is contracted so as to admit one boat only, and the towing-path is only 6 feet wide, or thereabouts, the headroom over it being about 10 feet. Sometimes the archway admits the water-channel alone, while the towing-path ascends to the approach of the bridge and descends again, the tow-rope being cast loose while the horse passes over. As to bridges for carrying railways over canals, see Article 436, p. 663.

Tunnels for canals usually have the water-way and towing-path contracted as already described; and sometimes the towing-path is dispensed with, the boats being pushed through by means of poles, or by the hands and feet of the boatmen, with the aid of notches in the brickwork, or by means of the various methods of steam propulsion.

505. **Moveable Bridges** cross a canal near its water-level are made of timber or of iron, and are capable of being opened so as to leave the navigation clear, and closed so as to form a passage for a road or railway by one or other of five kinds of movement, viz., I. By turning about a horizontal axis; II. By turning about a vertical axis; III. By rolling horizontally; IV. By lifting vertically; V. By floating in the canal. As regards the adaptation of the strength and stiffness of a moveable bridge to the greatest load which it has to bear when closed, it differs in no respect from a fixed bridge. But, besides having the strength and stiffness required in a fixed bridge, it must fulfil some other conditions, which are as follows:—If it turns about an axis, it must be so balanced that its centre of gravity shall always lie in that axis; if it rolls backwards and forwards it must be so balanced that its centre of gravity shall always lie over the base or platform on which it rolls: in either of those cases it must have strength sufficient to support safely the *overhanging* part of its own structure, when deprived of direct support; if it is lifted vertically, it must be counterpoised; and if it is carried by a pontoon or float, that float must displace a mass of water equal in weight to the bridge, and must have sufficient stability.

I. A bridge which turns about a horizontal axis near an end of its span is called a *draw-bridge.* It is opened by being raised into a vertical position by means of a pinion driving a toothed sector. It is best suited for small spans.

II. A bridge which turns about a vertical axis is called a *swing-bridge.* Its principal parts are as follows :—

A pier of masonry or iron, supporting a circular base-plate of a diameter equal, or nearly equal, to the breadth of the bridge. That base-plate has a pivot in the centre, and a circular race or track for rollers round the circumference, as in a railway turntable:

A roller frame turning about the central pivot, with a set of conical rollers resting on the race:

A circular revolving platform resting on the pivot and rollers:

A toothed arc fixed to the revolving platform, with suitable wheel-work for giving it motion:

A set of parallel girders, resting on and fastened to the revolving platform, of the strength and stiffness required by the principles already stated, and supporting a roadway.

The ends of the superstructure are bounded by arcs of circles, described about the axis of motion, and the ends of the roadway of the approaches must be formed to fit them.*

III. A *rolling bridge* has a strong frame, supported by wheels upon a line of rails, and having an overhanging portion sufficient to span the water-way. When closed, by being rolled forward, the rolling frame leaves a gap between its platform and that of one of the approaches, which gap is filled by rolling in another rolling frame that moves sideways. The latter rolling frame is rolled out of the way before opening the bridge.

IV. A *lifting bridge* is hung by the four corners to four chains, which pass over pullies, and have counterpoises at their other ends.

V. A *floating swing-bridge* rests on a caisson or pontoon: it is opened and closed by means of chains and windlasses, and, when open, lies in a recess in the side of the canal made to receive it. The pontoon, being made of sheet iron, is so designed as to act as a tubular girder when the bridge is closed.

506. **Canal Locks.**—Figs. 290, 291, and 292, show the general arrangement of the parts of a canal lock. Fig. 290 is a longitudinal section, fig. 291, a plan, and fig. 292 a cross-section, looking upwards.

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* For an example of a swing-bridge on a great scale, reference may be made to one planned by Mr. Hemans and constructed by Messrs. Fairbairn, which carries the Midland Great Western Railway of Ireland over the entrance to Lough Atalia. It has two spans of 60 feet each, and is balanced on a central pier of 34 feet diameter. It is described in detail in Mr. Humber's work *On Iron Bridges.*

A is the *lock-chamber; a, a,* its side walls; E, its floor, or invert.

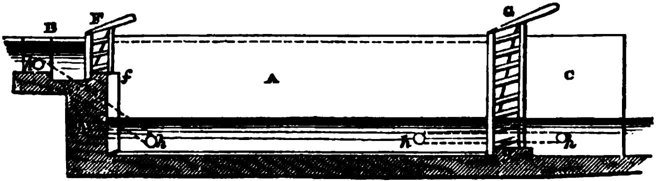

Fig. 290.

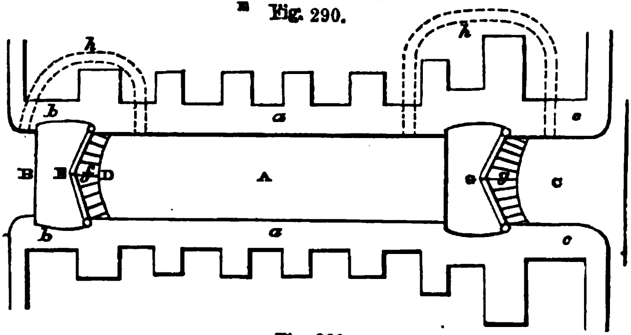

Fig. 291.

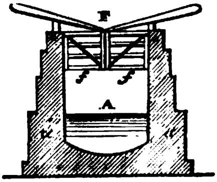

Fig. 292.

Its clear length should be at least equal to that of the longest vessel used on the canal, including the rudder; its clear breadth, one foot more than the greatest breadth of a vessel; its greatest depth of water should be = 1½ foot + greatest draught of a vessel + lift of the lock. Its depth from the cope of the side walls to the bottom may be about 2 feet more.

The side walls and floor are recessed to admit of the opening of the "tail-gates."

The floor is level with the bottom of the lower of the two ponds to be connected.

B is the *head-bay,* with its side walls and floor, which are recessed to admit of the opening of the "head-gates." The side-walls end in curved wings. The floor is level with the bottom of the upper pond.

C, the *tail bay,* with its side walls and floor. The side walls end in curved wings: the floor in a dry stone pitching or *apron.*

D, the *lift-wall,* which is usually built like a horizontal arch.

F, the *head-gates,* whose lower edges, when shut, press against the *head mitre-sill, f.*

G, the *tail-gates*, whose lower edges, when shut, press against the *tail mitre-sill*, g.

The older locks are filled and emptied through sluices in their head and tail-gates; but now the more general practice is to use for that purpose inlet and outlet passages with slide-valves. These passages may either be culverts contained in the thickness of the masonry, or iron pipes in such positions as those marked *h*, *h*, *h*, *h*.

The cylindrical recesses in which the gates are hinged are called the *hollow quoins*.

The following parts of a lock are usually of ashlar:—The quoins, hollow quoins, cope, recesses for the gates (or "gate-chambers"), and mitre-sills.

The mitre-sills are sometimes faced with wood, to enable them the better to withstand the blows which they receive from the gates, and to make a tighter joint.

The floor of the lock is sometimes made of cast iron. (See Article 400, p. 601.)

The gates are made of timber or of iron, and each of them consists of the following principal parts:—

The *heel-post*, about the axis of which the gate turns. This post is cylindrical on the side next the hollow quoins, which it exactly fits when the gate is shut. It is advisable to make it slightly eccentric, so that when the gate is opened, it may cease to rub on the hollow quoins. At its lower end it rests on a pivot, and its upper end turns in a circular collar, which is strongly anchored back to the masonry of the side walls:

The *mitre-post*, forming the outer edge of the frame of the gate, which, when the gate is shut, abuts against and makes a tight joint with the mitre-post of the opposite leaf:

The *cross-pieces*, which extend horizontally between the heel-post and mitre-post:

The *cleading* or covering, which may consist of timber planking or iron plates. When it consists of planks, they run either vertically or diagonally:

The *diagonal bracing*, which, in its simplest form, may consist either of a timber strut extending from the bottom of the heel-post to the top of the mitre-post, or of an iron tie-bar extending from the top of the heel-post to the bottom of the mitre-post.

The gates shown in the sketch are provided with *balance-bars*. A balance-bar is bolted to the top of the mitre-post, slopes slightly upwards, and crosses over the top of the heel-post, which is mortised into it, and has a long and heavy overhanging end, which acts as a counterpoise to bring the centre of gravity of the gate near the heel-post, and as a lever to open and shut it by.

Sometimes the balance-bar is dispensed with, and each gate has one or more *rollers* under its lowest cross-bar, to assist the pivot in supporting its weight. Each of those rollers runs upon a quadrantal iron rail on the floor of the gate-chamber. This mode of construction is almost always adopted in large and heavy gates that require chains and windlasses to open and shut them.

The following are some of the ordinary dimensions and proportions of locks, in addition to those already stated:—

The mitre-sills rise from 6 to 9 inches above the floor:

Versed-sine of mitre-sill, from $\frac{1}{7}$ to $\frac{1}{3}$ of breadth of lock:

Clearance in depth of the recesses for the gates, $\frac{1}{10}$ of thickness of gate; clearance in length, $\frac{1}{7}$ of length of gate:

Least thickness of the side walls at the top, about 4 feet. Greatest thickness at the base, fixed according to the principles of the stability of walls, usually from $\frac{1}{4}$ to $\frac{1}{2}$ of the height:

Length of side walls of head-bay above gate-chamber, about $\frac{1}{3}$ of breadth of lock:

Large counterforts opposite hollow quoins to have stability enough to withstand the calculated *transverse thrust* of the gates.

The *longitudinal thrust* of the head-gates is borne by the side walls of the lock-chamber; that of the tail-gates by the side walls of the tail-bay. To give the latter walls sufficient stability, the rule is to make their length as follows:—

$$\text{Breadth of lock} \times \text{greatest depth of water} \div 15 \text{ feet.}$$

Versed-sine of lift-wall, from 1-12th to 1-7th of breadth of lock.

Floor of head-bay: least thickness, from 10 inches to 14 inches.

Floor of lock-chamber: versed-sine, about 1-15th of breadth; thickness, from 1-15th to 1-3rd of breadth, according to the nature of the foundation.

Foundations of various kinds have been sufficiently explained. It has only to be added that, when a lock is founded on a timber platform, longitudinal pieces of timber extending along the whole length of the foundation are to be avoided, lest they guide streams of water along their sides; that transverse trenches under the foundation, filled with hydraulic concrete, are a good means of preventing leakage; and that, in porous soils, the whole space behind the lift-wall and under the floor of the head-bay may be filled with a mass of concrete.

Length of apron from 15 to 30 feet.

The dimensions of the different parts of the gates are to be computed according to the principles of the strength of materials. It appears that the factor of safety in many actual lock-gates is as low as 3 or 4. This can only be sufficient by reason of the perfect steadiness of the load.

507. **Inclined Planes on Canals.**—To save the time and water

expended in shifting boats from one level to another by means of locks, inclined planes are used on some canals. Their general arrangement is as follows:—The upper and lower reach of the canal, at the places which are to be connected by inclined planes, are deepened sufficiently to admit of the introduction of watertight iron caissons, or moveable tanks, under the boats. Two parallel lines of rails start from the bottom of the lower reach, ascend an inclined plane up to a summit a little above the water-level of the upper reach, and then descend down a short inclined plane to the bottom of the upper reach. There are two caissons, or moveable tanks on wheels, each holding water enough to float a boat. One of these caissons runs on each line of rails; and they are so connected, by means of a chain, or of a wire rope, running on moveable pullies, that when one descends the other ascends. These caissons balance each other at all times when both are on the long incline, because the boats, light or heavy, which they contain, displace exactly their own weight of water. There is a short period when both caissons are in the act of coming out of the water, one at the upper and the other at the lower reach, when the balance is not maintained; and, in order to supply the power required at that time, and to overcome friction, a steam engine drives the main pully, as in the case of fixed-engine planes on railways.*

Boats may be hauled up on wheeled cradles without using caissons; but this requires a greater expenditure of power. Mr. Thomas Grahame has proposed a method of performing this process which would enable a fixed engine to be dispensed with where steamboats are used. It consists in providing each steamer with a windlass, driven by its engine, and the inclined plane simply with a rope, whose upper end is made fast while its lower end is loose. The boat is floated on to the cradle at the bottom of the plane; the loose end of the rope is laid hold of and attached to the windlass, which, being driven by the engine, causes the boat to haul itself up the inclined plane.

On some canals vertical lifts with caissons are used instead of inclined planes.†

508. **Water Supply of Canals.**—Canals are supplied with water from gathering-grounds, springs, rivers, and wells, by the aid of reservoirs and conduits; and their supply involves the same questions of rain-fall, demand, compensation, &c., which have already been treated of in Chapter II. of this Part.

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* For a description of an inclined plane of this sort, used on the Monkland Canal near Glasgow, see the *Transactions of the Royal Scottish Society of Arts* for 1852.

† An improved system of apparatus for such lifts, proposed by Mr. George Simpson, is described in the *Transactions of the Institution of Engineers in Scotland* for 1860-61.

The *demand* for water, in the case of a canal, may be estimated as follows:—

I. *Waste of Water* by leakage of the channel, repairs, and evaporation, *per day* = area of surface of the canal × $\frac{1}{6}$ of a foot, nearly.

II. *Current* from the higher towards the lower reaches, produced by leakage at the lock gates, *per day*, from 10,000 to 20,000 cubic feet, in ordinary cases.

III. *Lockage*, or expenditure of water in passing boats from one level to another.

Let L denote a *lockful* of water; that is, the volume contained in the lock-chamber, between the upper and lower water-levels.

B, the volume displaced by a boat.

Then the quantities of water discharged from the upper pond, at a lock or a flight of locks, under various circumstances, are shown in the following tables. The sign — prefixed to a quantity of water denotes that it is displaced *from the lock into the upper pond.*

SINGLE LOCK.	Lock found,	Water discharged.	Lock left,
One boat descending	empty,	$L - B$	empty.
" "	full,	$- B$	
One boat ascending,	empty or full,	$L + B$	full.
2 *n* boats, descending and ascending alternately,	descending full, ascending empty	$n L$	descending empty. ascending full.
Train of *n* boats descending,	empty,	$n L - n B$	empty.
" " "	full,	$(n - 1) L - n B$	
Train of *n* boats ascending,	empty or full,	$n L + n B$	full.
Two trains, each of *n* boats, the first descending, the second ascending,	full,	$(2 n - 1) L$	full.

FLIGHT OF *m* LOCKS.	Locks found,	Water discharged.	Locks left,
One boat descending,	empty,	$L - B$	empty.
" "	full,	$- B$	
One boat ascending,	empty,	$m L + B$	full.
" "	full,	$L + B$	
2 *n* boats, descending and ascending alternately,	descending full, ascending empty	$m n L$	descending empty. ascending full.
Train of *n* boats descending,	empty,	$n L - n B$	empty.
" " "	full,	$(n - 1) L - n B$	
Train of *n* boats ascending,	empty,	$(m + n - 1) L + n B$	full.
" " "	full,	$n L + n B$	
Two trains, each of *n* boats, the first descending, the second ascending,	full,	$(m + 2 n - 2) L$	full.

From these calculations it appears, as has been already stated, that single locks are more favourable to economy of water than flights of locks; that at a single lock single boats ascending and descending alternately cause less expenditure of water than equal numbers of boats in trains; and that, on the other hand, at a flight of locks, boats in trains cause less expenditure of water than equal numbers of boats ascending and descending alternately.

For this reason, when a long flight of locks is unavoidable, it is usual to make it double; that is, to have two similar flights side by side—using one exclusively for ascending boats and the other exclusively for descending boats.

Water may be saved at flights of locks by the aid of *side ponds* (sometimes called "lateral reservoirs"). The use of a side pond is to keep for future use a certain portion of the water discharged from a lock, when the locks below it in the flight are full, which water would otherwise be wholly discharged into the lower reach. Let a be the horizontal area of a lock-chamber, A that of its side pond; then the volume of water so saved is—

$$\mathrm{L\,A} \div (\mathrm{A} + a).$$

SECTION II.—*Of River Navigation.*

509. An **Open River** is one in which the water is left to take a continuous declivity, being uninterrupted by weirs. On the subject of such streams little has here to be added to what has already been stated in articles 467 to 471, pp. 707 to 713. The towing-path required, if horse haulage is to be employed, is similar to that of a canal.

The effect of the current of the stream on the load which one horse is able to draw against it at a walk may be roughly estimated as follows:—

$$\text{Load drawn against current} = \text{load drawn in still water} \times \left(\frac{3{\cdot}6}{3{\cdot}6 + v}\right)^2;$$

v being the velocity of the current in feet per second.

It would be foreign to the subject of this work to discuss the principles of the propulsion of vessels by steam and sails.

510. A **Canalized River** is one in which a series of ponds or reaches, with a greater depth of water and a slower current than the river in its natural state, have been produced by means of weirs. The construction and effect of weirs have been explained in Article 472, p. 713, and the previous articles there referred to.

Each weir on a navigable river requires to be traversed by a lock for the passage of vessels, the most convenient place for which is usually near one end of the weir, next the bank where the towing-path is. River locks differ from canal locks in having no lift-wall, so that the head-gates and tail-gates are of equal height.

511. **Moveable Bridges over Rivers** are identical in principle with those over canals, and differ from them only in being of greater size. Examples of them have already been cited in Article 505, p. 706.

CHAPTER IV.

OF TIDAL AND COAST WORKS.

SECTION I.—*Of Waves and Tides.*

512. **Motion of Ordinary Waves.**—The following description of wave-motion in water is founded chiefly on the theoretical investigations of Mr. Airy, confirmed by the observations of the Messrs. Weber and of Mr. Scott Russell, with a few additions founded on later researches.

Ordinary waves in water are propagated horizontally; the motion of each particle takes place in a vertical plane, parallel to the direction of propagation; the path or orbit described by each particle is approximately elliptic (see fig. 293), and in water of uniform depth the longer axis of the elliptic orbit is horizontal, and the shorter vertical; the centre of that orbit lies a little above the position that the particle occupies when the water is undisturbed; when at the top of its orbit, the particle moves *forwards* as regards the direction of propagation; when at the bottom, backwards, as shown by the curved arrows in fig. 293, in which the straight feathered arrow denotes the direction of propagation.

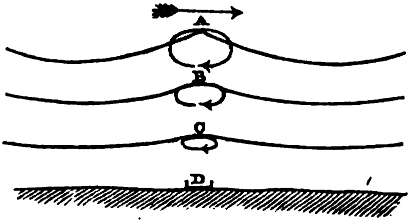

Fig. 293.

The particles at the surface of the water describe the largest orbits; the extent of the motion, both horizontally and vertically, diminishes as the depth below the surface increases; but that of the vertical motion more rapidly than that of the horizontal motion, so that the deeper a particle is situated the more flattened is its orbit, as indicated at A, B, and C; a particle in contact with the bottom moves backwards and forwards in a horizontal straight line, as at D.

In water that is deep, as compared with the length of a wave (or distance between two successive ridges on the surface of the water), the orbits of the particles are nearly circular, and the motion at great depths is insensible.

The *period* of a wave is the time occupied by each particle in making one revolution, and is also the time occupied by a wave in travelling a distance equal to its length. Hence we have the following proportion:—

$$\frac{\text{mean speed of a particle}}{\text{speed of the waves}} = \frac{\text{circumference of particle's orbit}}{\text{length of a wave}}.$$

The *speed of the waves* depends principally on their length and on the depth of water, being greatest for long waves and deep water. When the depth of water is greater than the length of a wave the speed is not sensibly affected by the depth, and is almost exactly equal to the velocity acquired by a body in falling through *half of the radius of a circle whose circumference is the length of a wave.* In water that is very shallow, compared with the length of the waves, the velocity is nearly independent of the length, and is nearly equal to that acquired by a heavy body in falling through *half the depth of the water added to three-fourths of the height of a wave.*

Two or more different series of waves moving in the same, different, and contrary directions, with equal or unequal speeds, may traverse the same mass of water at the same time, and the motion of each particle of water will be the resultant of the respective motions which the several series of waves would have impressed upon it had they acted separately. This is called the *interference* of waves.

When a series of waves advances into water gradually becoming shallower, their *periods* remain unchanged, but their *speed*, and consequently their *length*, diminishes, and their slopes become steeper. The orbits of the particles of water become distorted, as at B, C, D, fig. 294, in such a manner that the front of each wave gradually becomes steeper than the back; the crest, as it were, advancing faster than the trough. At length the front of the wave curls over beyond the vertical, its crest falls forward, and it *breaks* into surf on the beach.

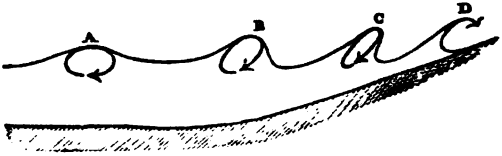

Fig. 294.

As the energy of the motion of a given wave which advances into shallowing water, or up a narrowing inlet, is successively communicated to smaller and smaller masses of water, there is a *tendency* to throw those masses into more and more violent agitation: that tendency may either take effect, or it may be counteracted, or more than counteracted, by the loss of energy which takes place through the production of eddies and surge at sudden changes of depth, and through friction on the bottom.

When waves roll straight against a vertical wall, as in fig. 295, they are reflected, and the particles of water for a certain distance in front of the wall have motions compounded of those due to the direct and to the reflected waves. The results are of the following kind:—The particles in contact with the wall, as at A, move up and down through a height equal to *double* the original height of the waves, and so also do those at half a wave length from the wall, as at C; the particles at a quarter of a wave length from the wall, as at B, move backwards and forwards horizontally, and intermediate particles oscillate in lines inclined at various angles.

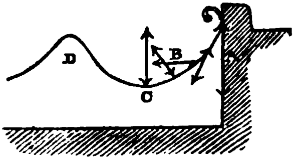

Fig. 295.

In order that a surface may reflect the waves, it is not essential that it should be exactly vertical; according to Mr. Scott Russell, it will do so even with a batter of 45°.

A vertical or steep surface which is wholly covered by the water reflects the wave-motion of those layers of water which lie below its level, and thus a sunken rock or breakwater, even though covered with water to a considerable depth, causes the sea to break over it, and so diminishes the energy of the advancing waves.

The *greatest length* of waves in the ocean is estimated at about 500 feet, which corresponds to a speed of about 50 feet per second, and a period of about ten seconds. Their *greatest height* is given by Scoresby as about 30 feet, and this, with the period just stated, gives 10 feet per second as the velocity of revolution of the particles of water.

In smaller seas the waves are both lower and shorter, and less swift; and, according to Mr. Scott Russell, waves in an expanse of shallow water of nearly uniform depth never exceed in height the undisturbed depth of the water. But the concentration of energy upon small masses of water, which occurs on shelving coasts in the manner already stated, produces waves of heights greatly exceeding those which occur in water of uniform depth, as the following examples show.

Pressures of waves against a vertical surface, at Skerryvore, as observed by Mr. Alan Stevenson:—

	Summer average.	Winter average.	Storm.
In lbs. per square foot, ...	611	2086	6083
In feet of water, ...	9·8	33	97

Greatest height of breakers on the south-west coast of Ireland, as observed by the Earl of Dunraven, 150 feet.

Recent investigations tend towards the conclusion, which is in accordance with observation, that every wave is more or less a "wave of translation," setting down each particle of water, or of matter suspended in water, a little in advance of where it picked that particle up, and thus by degrees producing that heaping up of water which gathers on a lee shore during a storm. This property of waves accounts for the facts, that although they tend to undermine and demolish steep cliffs, they heap up sand, gravel, shingle, or such materials as they are able to sweep along, upon every flat or sloping beach against which they directly roll; that they carry such materials into bays and estuaries; and that when they advance obliquely along the coast they make the materials of the beach travel along the coast in the same direction.

513. **Tides in General.**—The general motion of the tides consists in an alternate vertical rise and fall, and horizontal ebb and flow, occupying an average period of half a lunar day, or about 12·4 hours, and transmitted from place to place in the seas like a series of very long and swift waves, in which the extent of the horizontal motion is very much greater than that of the vertical motion. The extent of motion, both vertical and horizontal, undergoes variations between spring and neap, whose period is half a lunation, and other variations whose periods are a whole lunation and half-a-year. The propagation of the tide-waves is both retarded and deflected in gradually shallowing water, the crests of the waves having a tendency to become parallel to the line of coast which they are approaching.

Tides in narrow seas, and in the neighbourhood of land generally, are modified by the interference of different series of waves arriving by different routes, so as sometimes to present very complex phenomena. (See Mr. Airy's treatise "On Tides and Waves," in the *Encyclopædia Metropolitana.*) In the following examples simple cases only are described.

514. **Tidal Waves in a Clear and Deep Channel** are analogous to ordinary waves, as represented in fig. 293, p. 753; but with the modification that, owing to the enormous length of the waves as compared with the depth of the sea, the extent of horizontal motion is nearly equal at all depths, and the extent of vertical motion in any layer is nearly in the simple proportion of its height above

the bottom. The orbit of each particle is a very long and flat ellipse.

Supposing such a channel as that here considered to have a beach of moderately steep slope at one side, the depth being elsewhere uniform, the particles near that beach move in ellipses situated in planes inclined so as to be nearly parallel to the beach, as represented in plan in figs. 296 and 297. In each of these figures the beach is supposed to be towards the top of the page; in fig. 296 it lies to the right hand of the direction of advance of the tide wave (represented by the feathered arrow); in fig. 297, to the left of that direction. The following are the motions of a particle at different times of the tide:—

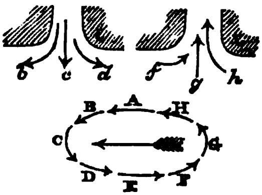

Fig. 296.

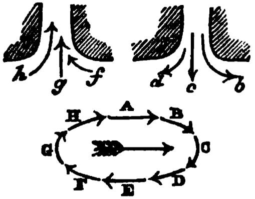

Fig. 297.

Lunar Hours after High-water.	Time commonly called.	Current.	Reference to the Figures.
0	High-water,	Forward,	A
1½	Quarter Ebb,	Forward and Seaward,	B
3	Half Ebb,	Seaward,	C
4½	Three-quarters Ebb,	Backward and Seaward,	D
6	Low-water,	Backward,	E
7½	Quarter Flood,	Backward and Shoreward,	F
9	Half Flood,	Shoreward,	G
10½	Three-quarters Flood,	Forward and Shoreward,	H
12	High-water,	Forward,	A

515. The **Tide in a Short Inlet,** or in any bay, gulf, or estuary of such dimensions and figure that high and low-water occur in all parts of it sensibly at the same instant, is somewhat analogous to a wave rising and falling against a steep wall (fig. 295, p. 755), or to the emptying and filling of a reservoir. Each particle of water moves alternately outwards and inwards during the fall and rise of the tide respectively; and the current is swifter and stronger when the depth of water is greater, that is, *during the second half of flood and the first half of ebb.*

Supposing that the entrance to such an inlet runs at right angles to the line of coast described in the preceding article, the combination of the tidal currents of the inlet with those of the offing, or sea outside, produces the results, as regards the currents at the entrance, indicated by the arrows marked *b*, *c*, *d*, *f*, *g*, *h*, in figs. 296 and 297 (whose lengths denote the strength of the current), and explained in the following table, in which *outward* and *inward* refer to the entrance of the inlet, and *forwards* and *backwards* to the directions of currents as compared with that of the flood-current along the coast:—

	Lunar Hours after High-water.	Time commonly. called	Current.	Reference to the Figures.	
First half of Ebb.	0	High-water,..........	0 (Slack-water),..................	—	
	1½	Quarter Ebb,.........	Outward, turning forward,.....	*b*	Strong.
	3	Half Ebb, ...	Outward,..........................	*c*	
Second half of Ebb.	4½	Three-quarters Ebb,	Outward, turning backward,...	*d*	Weak.
	6	Low-water, ...	0 (Slack-water),..................	—	
First half of Flood.	7½	Quarter Flood,........	Backward, turning inward,....	*f*	Weak.
	9	Half Flood, ...	Inward,............................	*g*	Strong.
Second half of Flood.	10½	Three-quarters Flood	Forward, turning inward,......	*h*	
	12	High-water,..........	0 (Slack-water),..................	—	

The letter J in each figure marks the *up-stream corner* of the entrance as regards the flood-current along the coast.

The *volume of water* which flows alternately in and out at the entrance of a short inlet is nearly equal to the space between the surfaces of high and low-water, as ascertained by levelling and tide-gauges. The *mean velocity* of the current through the entrance is nearly equal to that volume divided by the mean sectional area of the entrance, and by the time of rise or fall; and the *greatest velocity* is nearly equal to 1·57 × mean velocity. It is best to use such calculations only for the purpose of computing the probable effect of alterations. The velocities of actual currents should be found by observation.

516. The **Tides in Long Inlets** are compounded of a simple emptying and filling current like that in a short inlet, and a series of branch tidal waves, propagated up the channel from the waves of the offing. In *river-channels* the alternate currents due to the tides are combined with the downward current due to the flow of fresh water.

The tidal wave which is propagated up a long inlet or river-channel is analogous to those represented as advancing into shallow water in fig. 294, p. 754. It diminishes in length and increases in height until it reaches a limit where its further increase in height is stopped by friction. Its front becomes shorter and steeper, and its back longer and flatter; in other words, the rise of tide occupies a shorter time, and the fall a longer time, as the wave advances up the channel. When a high tidal wave advances into very shallow water, its front sometimes shortens and steepens, until at length it curls over, like the breaker D in fig. 294, and continues to advance rolling and breaking into surf, followed by a very long flat back. The tidal wave is then called a "*bore.*" The back of the wave sometimes breaks up into two or three smaller waves, and then the fall of the tide is interrupted by short intervals of rise.

To estimate by calculation the velocity of the flood and ebb-currents at a given cross-section of a river-channel or other long inlet, two longitudinal sections of the surface of the water must be prepared from two sets of simultaneous tide-gauge observations, made at a series of stations along the channel and above that cross-section, *at the two instants of slack-water at the given cross-section* respectively. The volume contained between the two surfaces thus determined will be the volume of tidal water which runs in and out through the given cross-section; and this, being divided by the duration of flood and ebb respectively, and by the area, will give the probable mean velocities of the currents, which, being multiplied by 1·57, will give, approximately, the probable maximum velocities. The velocity due to the fresh-water stream, if any, is to be subtracted from the flood and added to the ebb. (See the remark at the end of the preceding article.)

The tidal waves in rivers are propagated up the declivity of the stream, which they often affect at points above the level of high water in the sea.

517. **Actions of Tides on Coasts and Channels.**—The flowing tide augments, and the ebbing tide diminishes, the speed and force of storm waves; and hence the observed fact, that the most powerful action of such waves on the coast occurs at half-flood, when the shoreward current is strongest. The tidal currents sweep along with them silt or mud, sand, gravel, and other materials, according to the laws already stated with reference to river currents (Article 468, p. 708); hence the ebbing tide tends to scour and deepen inlets, and the flowing tide to silt them up. From what has been explained in the preceding article, it appears that in shallow water there is a tendency for the flowing tide to become more rapid, and therefore stronger in its action, than the ebbing tide, unless opposed by a sufficiently strong fresh-water current; and hence the prevailing tendency of the tides, like that of the waves, is to choke and fill up estuaries, river-channels, and other inlets, especially such as are already shallow.

A strong fresh-water current may maintain a deep channel against this action of the sea, so far as it is limited in breadth; but where that current escapes into the open sea, and is either enfeebled by spreading laterally, or has its action on the bottom prevented by floating on the salt water, a *bar* is formed by the action of the waves and tides.

One of the chief objects of harbour engineering is so to manage and modify the action of the tidal currents that the ebb shall become stronger than the flood, and shall scour deep channels and remove bars.

SECTION II.—*Of Sea Defences.*

518. **Groins,** running out at right angles to the coast, are constructed in the same manner with groins for river-banks, but more strongly. (Article 469, p. 711.) They not only interrupt the *travelling* of the materials of the beach along the shore under the influence of oblique waves and of the flowing tide, but they also cause a permanent deposit of such materials, and, if gradually extended seaward in shallow water, produce a gain of ground from the sea. After the spaces between the groins have been filled up, the travelling of shingle goes on past their ends as before.

Groins are amongst the most efficient means of protecting dykes, cliffs, and sea-walls, against the undermining action of the sea.

519. An **Earthen Dyke** has usually a long flat slope towards the sea, its inclination ranging from that of 3 to 1 to that of 12 to 1. The top is level, and usually has a roadway upon it: its average usual height above high-water-mark of spring tides, is about 6 feet; it should, if possible, be above the reach of the waves. The back slope has an inclination ranging from that of 1½ to 1 to that of 3 to 1. Behind the dyke is a back drain, or ditch, for the drainage of the land, constructed on the same principles with the back drains mentioned in Article 483, p. 727, and Article 484, p. 728.

In the heart of the dyke is a rectangular wall of fascines, constructed like the fascine-work of a river-bank. (Article 469, p. 710.) The fascines may be made of willow twigs or of reeds. The seaward slope is faced with fascines. If the top is above the reach of the waves the back slope may be turfed; if waves sometimes break over it, the top and back require stone pitching.

520. **Stone Bulwarks** withstand the waves best when either very flat or very steep. They are of two principal kinds—those with a long slope, on which the waves break, as in fig. 294, p. 754, and those with a steep face, which reflect the waves as in fig. 295, p. 755.

I. *Long-sloping Bulwarks* have an inclination which ranges from 3 to 1 to 10 to 1. They are made internally of earth and gravel, or of loose stones, according to the situation, and are faced with blocks, each of which should be able to withstand independently the lifting action of the waves. As to this, see Article 412, p. 618. The foot or "toe" of the slope may be slightly turned up, like that of a weir, to prevent the undermining action of the returning current, or "undertow" from the breakers. (See Article 472, p. 713.)

To prevent breakers or spray from gliding up to the top of the slope, and dashing over the summit of the bulwark, the top of the slope is sometimes curved upwards, so as to present a concave face to the waves; but this is sometimes liable to be knocked down by the shocks which it receives; and in that case it is best to carry up the slope in one plane, with a level *berm* or bench at the top of it, paved with large blocks, and on that berm to erect a strong parapet, set so far back that its cope is below the plane of the slope. A series of level berms, alternating with slopes of 3 to 1 of the same length with the berms, or thereabouts, are very effective in breaking the waves and exhausting their energy; the blocks at the edges of the berms must be larger than the rest.

The largest blocks in the facing of the slope should be at and near half-tide level, because the waves are largest at half-flood.

When a sloping bulwark stands in deep water, the part below low-water-mark may have a steeper slope than that above, as being less violently acted upon by the waves: for example, from 2 to 1 to 3 to 1 below low-water-mark, and 10 to 1 above. The waves will partially break and lose their energy in passing over the place where the inclination changes.

II. A *Steep-faced Bulwark or Sea-Wall* should be proportioned like a reservoir wall. (See Article 465, p. 707.) As to the manner in which it reflects the waves, see Article 512, p. 755. Its cope should either rise above the crests of the highest waves, augmented as they are in height by the reflection, or, should that be impracticable, that cope should be made of stones, each large enough to resist being lifted by the pressure due to the greatest height of a wave above its bed, and dowelled to the adjoining cope-stones. The front edge of the cope may project and form a rounded cornice to throw the crests of the waves back. The remainder of the wall may have a hammer-dressed ashlar or a block-in-course face, backed with coursed rubble or with strong concrete, the whole built in strong hydraulic mortar, and the outer edges of the joints *laid* in cement. (Article 248, p. 389.) The chief danger to the face of such a wall is that the water should penetrate the joints, and, by its pressure and elasticity, cause stones to jump out after receiving the blow of a wave.

The undermining action of the waves on the ground at the foot of a steep wall is very severe, and should be resisted by a *level* stone pitching, (which should have no bond or connection with the wall) and by a series of groins. The undermining action may be somewhat moderated by forming the face of the wall into steps, so as to interrupt the vertical descent of the water. Stones lifted out of a level pitching by the waves fall down into or near their original sites, provided there are groins to prevent them from travelling sideways. Those lifted out of a sloping pitching roll to the foot of the slope.

When there is an earthen embankment behind such a wall, it should have a retaining wall at the landward side also, to prevent the earth from being washed away by water which may collect on the top.

III. *Combined Wall.*—As the expense of erecting a steep or vertical wall in deep water is very great, it is sometimes combined in such situations with a long slope, in the following manner:—From the bottom up to near low-water-mark extends a slope of 2 to 1 or 3 to 1, terminating in a long level or nearly level *berm* or "*foreshore;*" and on that berm, as on a beach in shallow water, is built a steep wall, at a distance back from the edge of the slope equal to twice or thrice the length of the slope.

521. A **Breakwater**, being placed so as to defend a harbour or roadstead from the waves, differs from a bulwark by having sea at both sides of it. The site of a breakwater should be so chosen as to present a barrier to the waves of the prevailing storms, and especially to those which come along with the flood-current. It may be isolated, and in the midst of the entrance of a bay, as at Plymouth and Cherbourg, or it may run out from the shore into deep water. In the latter case, the best position for the junction of a single breakwater with the land is in general at the *up-stream corner* of the entrance to the inlet or harbour (see Article 515, p. 758), for in that position it opposes the strongest flood-current, and does not interfere with the strongest ebb-current. The principles of the construction of the front of a breakwater are the same with those described in the preceding article with reference to bulwarks in deep water. The back of a vertical-fronted breakwater is usually vertical also; that of a sloping or combined breakwater, if intended to be used as a quay, is vertical; in other cases it differs from the front only in having a steeper slope (from 1 to 1 to $1\frac{1}{2}$ to 1) and being faced with smaller blocks. As to embanking and building under water, see Article 412, p. 617. When a stage supported on screw piles is used to tip the stones from, those piles remain imbedded in the breakwater. Their diameter should be about $\frac{1}{20}$th of their height, so that, in very deep water, they may require to be built of several balks of timber hooped together, as at Portland.

Fig. 298 is a section of the Cherbourg breakwater, which combines the long slope and vertical face. The base A F is about 300 feet; the slope A B is $2\frac{1}{2}$ to 1; B C is $5\frac{1}{2}$ to 1; E F, 1 to 1; C D is a nearly level platform, on which stands the wall G, 36 feet thick at its base. Ordinary spring tides rise 19 feet, the depth at low-water being 40 feet.

Fig. 299, a section of the Plymouth breakwater, illustrates the principle of alternate slopes and berms. A B is 5 to 1, B C level, C D 5 to 1, D E level, E F 1½ to 1.

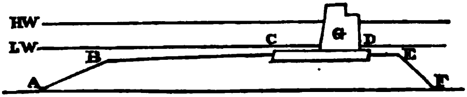

Fig. 298.

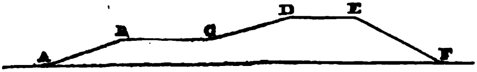

Fig. 299.

(As to breakwaters, and sea defences generally, may be consulted the works of Smeaton and Telford, Sir John Rennie's works *On the Plymouth Breakwater* and *On Harbours*, the *Proceedings of the Institution of Civil Engineers* since the commencement, and Mr. Burnell's *Treatise on Marine Engineering.*) (See also p. 766.)

522. **Reclaiming Land.**—The process of reclaiming or gaining land from the sea is to be undertaken with great caution, especially in river-channels and estuaries, lest it should diminish the tidal scour, and so cause the silting up of channels and harbours; and, in particular, care should be taken that the space for tidal water which is to be lost through the reclaiming of the land, is exactly made up for by deepening or otherwise improving other parts of the estuary or channel. In every instance in which that precaution has been neglected, the damage, and in some cases the ruin, of the harbour has followed. (See *Reports of the Tidal Harbours Commission.*)

The first operation in reclaiming land is usually to raise its level as much as possible by *warping,* or deposition of sediment from the tidal water; with a view to which the land to be reclaimed is intersected by a network of transverse wattled groins, and of longitudinal dykes of the same construction.

The ground having been raised as far as practicable by warping, is enclosed with sea-dykes, and drained in the manner described in Article 484, p. 727.

SECTION III.—*Of Tidal Channels and Harbours.*

523. The **Improvement of Tidal Rivers and Estuaries** depends mainly on the strengthening of the ebbing current, as stated in Article 517, p. 760. With that view, the measures to be adopted

are nearly the same with those already described under the head of Improvements of River-Channels, Article 470, p. 711, with the addition that the space which at each tide is filled and emptied is to be kept as large as possible. For the purpose of concentrating the latter portions of the ebbing current upon the deep-water-channel, *training-dykes* may be required. That these may not diminish the quantity of scouring-water, they should rise but little, if at all, above low-water-mark of ordinary spring-tides, their position being marked by means of rows of beacons.

Should bulwarks or quays be erected, they should either be so placed that the area which they cut off by contracting wide places may be compensated for by widening narrow places, or that the space which they cut off may be compensated for by deepening that part of the space in front of them which is above low-water-mark.

The most important effect of making a deep, direct, and regular channel for a tidal river consists in the increase in the extent of rise and fall of the tide, and the diminution of that steepening action of a shallow channel on the front of the tide-wave which has been described in Article 516, p. 758.

In order to increase the depth over a *bar*, piers or breakwaters must be carried out so as to concentrate the current over it, and it is best, if possible, to make the space between those piers *widen inwards*, in order both to hold scourage-water and to serve as a "wave-trap," or space for storm-waves which roll in at the entrance to spread and expend themselves in. When there is only one pier, it should run from the *up-stream* corner of the entrance, for the reason explained in Article 521, p. 762, observing that in deciding which is the *up-stream* corner, regard must be had to the flood-current *along the shore*, in case, through the action of headlands, its direction should be different from that of the flood-current in the open sea.

The bar may thus be swept into deeper water, although it is in general impossible to remove it altogether.

524. A **Scouring-Basin** is a reservoir by means of which the tidal water is stored up to a certain level, and let out through sluices, in a rapid stream, for a few minutes at low-water, to scour a channel and its bar. The outlets of the basin should face as nearly as possible directly along the channel to be scoured; they should be distributed throughout its whole cross-section, that they may produce an uniform steady current in it like a river, and may not concentrate their action on a few spots. To carry away gravel and large shingle, the scouring stream should flow at 4 or 5 feet per second, and the dimensions of the outlets should be regulated accordingly. One of the best examples of such an arrangement is

at the south entrance of the harbour of Sunderland, described by the engineer, Mr. Murray, in the *Proceedings of the Institution of Civil Engineers* for 1856. The current is let out for 15 minutes at low-water; it runs at about 5 feet per second, and is sensible in the sea 2,000 yards off, although it is confined by piers for 350 yards only.

525. **Quays** of masonry are to be regarded as a class of retaining walls, the stability of which has been treated of in Articles 265 to 269, pp. 401 to 408, and their construction in Articles 271, 272, pp. 409 to 411. Their ordinary thickness at the base is from $\frac{1}{3}$ to $\frac{1}{4}$ of their height. When founded on piles, the timber-work should be always immersed. (See Part II., Chapter VI., Section II., p. 601.) The face of a stone quay is usually protected against being damaged by vessels by means of a network of upright *fender-piles* and horizontal *fender-wales.*

As to timber and iron quays, see Article 469, p. 710, and the other articles there referred to.

The inner side of a breakwater may form a quay, as already mentioned.

526. **Piers** of masonry running out into the sea are to be regarded as upright breakwaters combined with quays, and require here no additional explanation. Those of timber and iron are best formed of a skeleton framework, supported by screw-piles. A timber skeleton-pier is often combined with a loose stone breakwater, in which the lower parts of the posts are imbedded.

527. **Basins and Docks.**—A *deep-water-basin* is a reservoir surrounded by quay-walls, in which the water is retained when the tide falls below a certain level (usually somewhat above half-tide) by a pair of lock-gates opening inwards, of sufficient size and strength. Should the entrance be exposed to waves, a pair of *sea-gates,* or gates opening outwards, are also required, to be closed during storms. A deep-water-basin may also be used as a scouring-basin. (Article 524, p. 764.)

A *dock* differs from a basin in having a *lock* at its entrance, through which ships can pass in all states of the tide. (As to locks, see Article 506, p. 746.) A harbour-lock, like a river-lock, has no lift-wall. In order that vessels may pass easily in and out, the entrances of docks from a river-channel should slant *up-stream as regards the ebb-current.*

One of the best forms of gate for basins and docks is a *caisson-gate,* being a water-tight vessel of plate-iron, which can be floated to or from its seat in the masonry of the entrance, being placed in a recess when open. When closed it is sunk by loading it with water, which is run into a tank on the top of the caisson. In order to open it, it is floated by emptying that tank.

It is often convenient, when practicable, to conduct a supply of fresh water into basins or docks, care being taken that such supply is pure.

528. **Lighthouses.**—The principles which regulate the placing and illuminating of lighthouses form a subject which can be fully considered in a special treatise only, such as that by Mr. Thomas Stevenson. When a lighthouse is exposed to the waves, it may be either a round tower of masonry, built of hewn stones, dove-tailed, tabled, and dowelled to each other, as described in Article 412, p. 618, solid up to the level of high-water of spring tides, and as much higher as ordinary waves rise, and high enough in all to keep the lantern clear of the highest breaking and reflected storm-waves, with an overhanging curved cornice to throw their crests back; or it may consist of a skeleton frame of screw-piles and diagonal bracing, supporting a timber or iron house and platform; and in this case the platform needs only to be high enough to clear the tops of the natural unreflected waves. On the subject of the strength and stability of frames supported on screw-piles, see Article 403, p. 605. In designing the frame of a lighthouse to be supported on them, regard must be had to the pressure of the wind, whose greatest recorded intensity, in Britain, is 55 lbs. per square foot of a flat surface, and about one-half of that intensity per square foot of the plane projection of a cylindrical surface. (See Smeaton's works; Stevenson *On the Bell Rock Lighthouse;* Mr. Alan Stevenson's treatise *On Lighthouses;* and Mr. Mitchell's "Account of Lighthouses on Screw-Piles," in the *Proceedings of the Institution of Civil Engineers* for 1848.)

ADDENDA.

To Article 280, p. 416. **Line of Pressures in an Arch.**—As to the stability of a vertically-loaded arch, reference may be made to a series of papers which have appeared in the *Civil Engineer and Architect's Journal* for 1861, and in which the figure of the line of pressures is shown to be identical with that of a curve whose ordinates represent bending moments.

To Article 357, p. 509. **Strength of Cold-rolled Iron.**—The following results have lately been obtained in some experiments by Mr. Fairbairn on the tenacity of iron. (See *Manchester Transactions*, 10th December, 1861.)

	Tenacity. Lbs. per Square Inch.	Ultimate Extension.
Black bar, ...	58,627	·200
Same bar iron, turned, ...	60,747	·220
Same bar iron, cold-rolled,....................	88,229	·079
Cold-rolled plate, ...	114,912	

To Article 521, p. 763. **Breakwaters.**—As to the construction of a breakwater by means of a wooden cage filled with rough stones, and resting on a rubble base, see a paper by Mr. Michael Scott in the *Proceedings of the Institution of Civil Engineers* for 1858.

APPENDIX.

I.

Table of the Resistance of Materials
Tearing by a Direct Pull, *in pounds av*
inch.

Materials.	Tena or Resist Tear
Stones, Natural and Artificial:	
Brick, Cement,	280 t
Glass,	
Slate,	to 1
Mortar, ordinary,	
Metals:	
Brass, cast,	1
„ wire,	4
Bronze or Gun Metal (Copper 8, Tin 1),	3
Copper, cast,	1
„ sheet,	3
„ bolts,	3
„ wire,	6
Iron, cast, various qualities,	1 to 2
„ average,	1
Iron, wrought, plates,	5
„ joints, double rivetted,	3
„ „ single rivetted,	2
„ bars and bolts,	6 to 7
„ hoop, best-best,	6.
„ wire,	7 to 10
„ wire-ropes,	9
Lead, sheet,	
Steel bars,	10 to 13
Steel plates, average,	8
Tin, cast,	
Zinc,	7,000 to

MATERIALS.	Tenacity, or Resistance to Tearing.	Modulus of Elasticity, or Resistance to Stretching.
TIMBER AND OTHER ORGANIC FIBRE:		
Acacia, false. See "Locust."		
Ash (*Fraxinus excelsior*),..........	17,000	1,600,000
Bamboo (*Bambusa arundinacea*),	6,300	
Beech (*Fagus sylvatica*), ...	11,500	1,350,000
Birch (*Betula alba*),................	15,000	1,645,000
Box (*Buxus sempervirens*),........	20,000	
Cedar of Lebanon (*Cedrus Libani*),	11,400	486,000
Chestnut (*Castanea Vesca*),.......	10,000 to 13,000	1,140,000
Elm (*Ulmus campestris*),..........	14,000	700,000 to 1,340,000
Fir: Red Pine (*Pinus sylvestris*),	12,000 to 14,000	1,460,000 to 1,900,000
„ Spruce (*Abies excelsa*),.......	12,400	1,400,000 to 1,800,000
„ Larch (*Larix Europæa*),....	9,000 to 10,000	900,000 to 1,360,000
Hawthorn (*Cratægus Oxyacantha*),	10,500	
Hazel (*Corylus Avellana*),.........	18,000	
Hempen Cables,....................	5,600	
Holly (*Ilex Aquifolium*),..	16,000	
Hornbeam (*Carpinus Betulus*),...	20,000	
Laburnum (*Cytisus Laburnum*),	10,500	
Lancewood (*Guatteria virgata*),	23,400	
Lignum-Vitæ (*Guaiacum officinale*),............................	11,800	
Locust (*Robinia Pseudo-Acacia*),	16,000	
Mahogany (*Swietenia Mahagoni*),	8,000 to 21,800	1,255,000
Maple (*Acer campestris*), ...	10,600	
Oak, European (*Quercus sessiliflora* and *Quercus pedunculata*),	10,000 to 19,800	1,200,000 to 1,750,000
„ American Red (*Quercus rubra*),............................	10,250	2,150,000
Poplar (*Populus alba*),............	7,200	1,130,000
Sycamore (*Acer Pseudo-Platanus*),	13,000	1,040,000
Teak, Indian (*Tectona grandis*),	15,000	2,400,000
„ African, (?).................	21,000	2,300,000
Whalebone,.........................	7,700	
Yew (*Taxus baccata*),.............	8,000	

II.

TABLE OF THE RESISTANCE OF MATERIALS TO SHEARING AND DISTORTION, *in pounds avoirdupois per square inch.*

MATERIALS.	Resistance to Shearing.	Transverse Elasticity, or Resistance to Distortion.
METALS:		
Brass, wire-drawn,		5,330,000
Copper,		6,200,000
Iron, cast,	27,700	2,850,000
„ wrought,	50,000	8,500,000 to 9,500,000
TIMBER:		
Fir: Red Pine,	500 to 800	
„ Spruce,	600	
„ Larch,	970 to 1,700	
Oak, British,	2,300	
Poplar,	1,800	

III.

TABLE OF THE RESISTANCE OF MATERIALS TO CRUSHING BY A DIRECT THRUST, *in pounds avoirdupois per square inch.*

MATERIALS.	Resistance to Crushing.
STONES, NATURAL AND ARTIFICIAL:	
Brick, weak red,	550 to 800
„ strong red,	1,100
„ fire,	1,700
Chalk,	330
Granite,	5,500 to 11,000
Limestone, marble,	5,500
„ granular,	4,000 to 4,500
Sandstone, strong,	5,500
„ ordinary,	3,300 to 4,400
„ weak,	2,200
Rubble masonry, about four-tenths of cut stone.	
METALS:	
Brass, cast,	10,300
Iron, cast, various qualities,	82,000 to 145,000
„ „ average,	112,000
„ wrought,	about 36,000 to 40,000

MATERIALS.	Resistance to Crushing.
TIMBER,* Dry, crushed along the grain:	
Ash,	9,000
Beech,	9,360
Birch,	6,400
Blue-Gum (*Eucalyptus Globulus*),	8,800
Box,	10,300
Bullet-tree (*Achras Sideroxylon*),	14,000
Cabucalli,	9,900
Cedar of Lebanon,	5,860
Ebony, West Indian (*Brya Ebenus*),	19,000
Elm,	10,300
Fir: Red Pine,	5,375 to 6,200
„ American Yellow Pine (*Pinus variabilis*),	5,400
„ Larch,	5,570
Hornbeam,	7,300
Lignum-Vitæ,	9,900
Mahogany,	8,200
Mora (*Mora excelsa*),	9,900
Oak, British,	10,000
„ Dantzic,	7,700
„ American Red,	6,000
Teak, Indian,	12,000
Water-Gum (*Tristania nerifolia*),	11,000

IV.

TABLE OF THE RESISTANCE OF MATERIALS TO BREAKING ACROSS *in pounds avoirdupois per square inch.*

MATERIALS.	Resistance to Breaking or Modulus of Rupture.†
STONES:	
Sandstone,	1,100 to 2,360
Slate,	5,000

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* The resistances stated are for *dry* timber. Green timber is much weaker, having sometimes only half the strength of dry timber against crushing.

† The modulus of rupture is eighteen times the load which is required to break a bar of one inch square, supported at two points one foot apart, and loaded in the middle between the points of support.

MATERIALS.	Resistance to Breaking, or Modulus of Rupture.
METALS:	
Iron, cast, open-work beams, average,	17,000
„ „ solid rectangular bars, var. qualities,	33,000 to 43,500
„ „ „ „ average,	40,000
„ wrought, plate beams,	42,000
TIMBER:	
Ash,	12,000 to 14,000
Beech,	9,000 to 12,000
Birch,	11,700
Blue-Gum,	16,000 to 20,000
Bullet-tree,	15,900 to 22,000
Cabacalli,	15,000 to 16,000
Cedar of Lebanon,	7,400
Chestnut,	10,660
Cowrie (*Dammara australis*),	11,000
Ebony, West Indian,	27,000
Elm,	6,000 to 9,700
Fir: Red Pine,	7,100 to 9,540
„ Spruce,	9,900 to 12,300
„ Larch,	5,000 to 10,000
Greenheart (*Nectandra Rodiœi*),	16,500 to 27,500
Lancewood,	17,350
Lignum-Vitæ,	12,000
Locust,	11,200
Mahogany, Honduras,	11,500
„ Spanish,	7,600
Mora,	22,000
Oak, British and Russian,	10,000 to 13,600
„ Dantzic,	8,700
„ American Red,	10,600
Poon,	13,300
Poplar, Abele,	10,260
Sycamore,	9,600
Teak, Indian,	14,770
„ African,	14,980
Tonka (*Dipteryx odorata*),	22,000
Water-Gum,	17,460
Willow (*Salix*, various species),	6,600

V.—COMPARATIVE TABLE OF FRENCH AND BRITISH MEASURES.

	No.	Log.	Log.	No.	
Grains in a gramme, ...	15·43235	1·1884320	$\bar{2}$·8115680	0·064799	Gramme in a grain.
Pounds avoird. in a kilogramme,	2·20462	0·3433340	$\bar{1}$·6566660	0·453593	Kilog. in a lb. avoirdupois.
Ton in a tonne, ...	0·984206	$\bar{1}$·9930860	0·0069140	1·01605	Tonnes in a ton.
Feet in a mètre, ...	3·2808992	0·5159929	$\bar{1}$·4840071	0·30479449	Mètres in a foot.
Inch in a millimètre, ...	0·03937079	$\bar{2}$·5951741	1·4048259	25·39954	Millimètres in an inch.
Mile in a kilomètre, ...	0·621383	$\bar{1}$·7933590	0·2066410	1·60932	Kilomètres in a mile.
Square feet in a square mètre,..	10·7643	1·0319858	$\bar{2}$·9680142	0·0928997	Square mètre in a square foot.
Square inch in a square millimètre, ...	0·00155006	$\bar{3}$·1903482	2·8096518	645·137	Square millim. in a square inch.
Cubic feet in a cubic mètre......	35·3166	1·5479787	$\bar{2}$·4520213	0·0283153	Cubic mètre in a cubic foot.
Foot-pounds in a kilogrammètre,	7·23314	0·8593269	$\bar{1}$·1406731	0·138253	Kilogrammètre in a foot-pound.
Pounds-to-the-foot in a kilogramme-to-the-mètre, ...	0·671957	$\bar{1}$·8273411	0·1726589	1·48819	Kilogrammes-to-the-mètre in a pound-to-the-foot.
Pounds-to-the-square-foot in a kilogramme-to-the-square-mètre, ...	0·2048098	$\bar{1}$·3113482	0·6886518	4·88261	Kilogrammes-to-the-square-mètre in a pound-to-the-square-foot.
Pounds-to-the-square-inch in a kilog.-to-the-square-millimètre, ...	1422·28	3·1529858	$\bar{4}$·8470142	0·000703095	Kilog.-to-the-square-millimètre in a pound-to-the-square-inch.
Pounds-to-the-cubic-foot in a kilogramme-to-the-cubic-mètre, ...	0·062425	$\bar{2}$·7953553	1·2046447	16·019	Kilogrammes-to-the-cubic-mètre in a pound-to-the-cubic-foot.
Fahrenheit-degrees in a centigrade-degree, ...	1·8	0·2552725	$\bar{1}$·7447275	$0·5555\dot{5}$	Centigrade-degree in a Fahrenheit degree.
British units of heat in a French unit, ...	3·96832	0·5986065	$\bar{1}$·4013935	0·251996	French units of heat in a British unit.

VI.

Table of Specific Gravities of Materials.

	Weight of a cubic foot in lb. avoirdupois.
Gases, at 32° Fahr., and under the pressure of one atmosphere, of 2116·4 lb. on the square foot:	
Air,	0·080728
Carbonic Acid,	0·12344
Hydrogen,	0·005592
Oxygen,	0·089256
Nitrogen,	0·078596
Steam (ideal),	0·05022
Æther vapour (ideal),	0·2093
Bisulphuret-of-carbon vapour (ideal),	0·2137
Olefiant gas,	0·0795

	Weight of a cubic foot in lb. avoirdupois.	Specific gravity, pure water = 1.
Liquids at 32° Fahr. (except Water, which is taken at 39°·4 Fahr.):		
Water, pure, at 39°·4,	62·425	1·000
„ sea, ordinary,	64·05	1·026
Alcohol, pure,	49·38	0·791
„ proof spirit,	57·18	0·916
Æther,	44·70	0·716
Mercury,	848·75	13·596
Naphtha,	52·94	0·848
Oil, linseed,	58·68	0·940
„ olive,	57·12	0·915
„ whale,	57·62	0·923
„ of turpentine,	54·31	0·870
Petroleum,	54·81	0·878
Solid Mineral Substances, non-metallic:		
Basalt,	187·3	3·00
Brick,	125 to 135	2 to 2·167
Brickwork,	112	1·8
Chalk,	117 to 174	1·87 to 2·78
Clay,	120	1·92
Coal, anthracite,	100	1·602
„ bituminous,	77·4 to 89·9	1·24 to 1·44
Coke,	62·43 to 103·6	1·00 to 1·66
Felspar,	162·3	2·6
Flint,	164·2	2·63

	Weight of a cubic foot in lb. avoirdupois.	Specific gravity, pure water = 1.
SOLID MINERAL SUBSTANCES—*continued.*		
Glass, crown, average,	156	2·5
„ flint, „	187	3·0
„ green, „	169	2·7
„ plate, „	169	2·7
Granite,	164 to 172	2·63 to 2·76
Gypsum,	143·6	2·3
Limestone (including marble),	169 to 175	2·7 to 2·8
„ magnesian,	178	2·86
Marl,	100 to 119	1·6 to 1·9
Masonry,	116 to 144	1·85 to 2·3
Mortar,	109	1·75
Mud,	102	1·63
Quartz,	165	2·65
Sand (damp),	118	1·9
„ (dry),	88·6	1·42
Sandstone, average,	144	2·3
„ various kinds,	130 to 157	2·08 to 2·52
Shale,	162	2·6
Slate,	175 to 181	2·8 to 2·9
Trap,	170	2·72
METALS, solid:		
Brass, cast,	487 to 524·4	7·8 to 8·4
„ wire,	533	8·54
Bronze,	524	8·4
Copper, cast,	537	8·6
„ sheet,	549	8·8
„ hammered,	556	8·9
Gold,	1186 to 1224	19 to 19·6
Iron, cast, various,	434 to 456	6·95 to 7·3
„ average,	444	7·11
Iron, wrought, various,	474 to 487	7·6 to 7·8
„ average,	480	7·69
Lead,	712	11·4
Platinum,	1311 to 1373	21 to 22
Silver,	655	10·5
Steel,	487 to 493	7·8 to 7·9
Tin,	456 to 468	7·3 to 7·5
Zinc,	424 to 449	6·8 to 7·2

TIMBER:*	Weight of a cubic foot in lb. avoirdupois.	Specific gravity, pure water = 1.
Ash,	47	0·753
Bamboo,	25	0·4
Beech,	43	0·69
Birch,	44·4	0·711
Blue-Gum,	52·5	0·843
Box,	60	0·96
Bullet-tree,	65·3	1·046
Cabacalli,	56·2	0·9
Cedar of Lebanon,	30·4	0·486
Chestnut,	33·4	0·535
Cowrie,	36·2	0·579
Ebony, West Indian,	74·5	1·193
Elm,	34	0·544
Fir: Red Pine,	30 to 44	0·48 to 0·7
„ Spruce,	30 to 44	0·48 to 0·7
„ American Yellow Pine,	29	0·46
„ Larch,	31 to 35	0·5 to 0·56
Greenheart,	62·5	1·001
Hawthorn,	57	0·91
Hazel,	54	0·86
Holly,	47	0·76
Hornbeam,	47	0·76
Laburnum,	57	0·92
Lancewood,	42 to 63	0·675 to 1·01
Larch. See "Fir."		
Lignum-Vitæ,	41 to 83	0·65 to 1·33
Locust,	44	0·71
Mahogany, Honduras,	35	0·56
„ Spanish,	53	0·85
Maple,	49	0·79
Mora,	57	0·92
Oak, European,	43 to 62	0·69 to 0·99
„ American Red,	54	0·87
Poon,	36	0·58
Poplar (Abele),	32	0·51
Sycamore,	37	0·59
Teak, Indian,	41	0·66
„ African,	61	0·98
Tonka,	62 to 66	0·99 to 1·06
Water-Gum,	62·5	1·001
Willow,	25	0·4
Yew,	50	0·8

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* The Timber in every case is supposed to be dry.

TABLE OF SQUARES AND FIFTH POWERS.

	Square.	Fifth Power.		Square.	Fifth Power.
10	1 00	1 00000	55	30 25	5032 84375
11	1 21	1 61051	56	31 36	5507 31776
12	1 44	2 48832	57	32 49	6016 92057
13	1 69	3 71293	58	33 64	6563 56768
14	1 96	5 37824	59	34 81	7149 24299
15	2 25	7 59375	60	36 00	7776 00000
16	2 56	10 48576	61	37 21	8445 96301
17	2 89	14 19857	62	38 44	9161 32832
18	3 24	18 89568	63	39 69	9924 36543
19	3 61	24 76099	64	40 96	10737 41824
20	4 00	32 00000	65	42 25	11602 90625
21	4 41	40 84101	66	43 56	12523 32576
22	4 84	51 53632	67	44 89	13501 25107
23	5 29	64 36343	68	46 24	14539 33568
24	5 76	79 62624	69	47 61	15640 31349
25	6 25	97 65625	70	49 00	16807 00000
26	6 76	118 81376	71	50 41	18042 29351
27	7 29	143 48907	72	51 84	19349 17632
28	7 84	172 10368	73	53 29	20730 71593
29	8 41	205 11149	74	54 76	22190 06624
30	9 00	243 00000	75	56 25	23730 46875
31	9 61	286 29151	76	57 76	25355 25376
32	10 24	335 54432	77	59 29	27067 84157
33	10 89	391 35393	78	60 84	28871 74368
34	11 56	454 35424	79	62 41	30770 56399
35	12 25	525 21875	80	64 00	32768 00000
36	12 96	604 66176	81	65 61	34867 84401
37	13 69	693 43957	82	67 24	37073 98439
38	14 44	792 35168	83	68 89	39390 40643
39	15 21	902 24199	84	70 56	41821 19424
40	16 00	1024 00000	85	72 25	44370 53125
41	16 81	1158 56201	86	73 96	47042 70176
42	17 64	1306 91232	87	75 69	49842 09207
43	18 49	1470 08443	88	77 44	52773 19168
44	19 36	1649 16224	89	79 21	55840 59449
45	20 25	1845 28125	90	81 00	59049 00000
46	21 16	2059 62976	91	82 81	62403 21451
47	22 09	2293 45007	92	84 64	65908 15232
48	23 04	2548 03968	93	86 49	69568 83693
49	24 01	2824 75249	94	88 36	73390 40224
50	25 00	3125 00000	95	90 25	77378 09375
51	26 01	3450 25251	96	92 16	81537 26976
52	27 04	3802 04032	97	94 09	85873 40257
53	28 09	4181 95493	98	96 04	90392 07968
54	29 16	4591 65024	99	98 01	95099 00499

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